Estonian-Finnish Olympiad - 2009

1. Boat *(9 points)* Consider a boat of effective mass m, which moves along the (horizontal) x-axes, and for which the friction force of the water is given by $F_f = -\alpha v$, where v is the velocity of the boat. (Effective mass is somewhat larger than the real mass, because any change of the boat's velocity incurs a velocity change of those regions of water, which are close to the boat; hence, the boat's inertia is complemented by the inertia of the surrounding water.)
i) Prove that during this motion,

$$v + kx = \text{Const},$$

and find the factor k.
ii) Now, a boy of mass M moves inside the boat, back and forth, with velocity $u(t)$ (also, along the x-axis). What term must be added to the conservation law $v + kx =$ Const, to describe this situation?
iii) Now, let the boat rest initially close to the coast and ready to depart. The boy jumps from the coast into the boat, turns around and jumps back to the coast. The horizontal component of the boy's velocity during the first jump was u_1, and during the second jump $-u_2$. What is the maximum distance s travelled by the boat? Neglect the effect of a vertical motion on the horizontal friction force of the water.

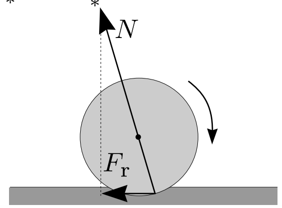
2. Nanoclock *(10 points)*

Nanotechnology allows controllable fabrication of very small structures. Let us consider a tiny homogeneously charged thin ring, having a radius R and carrying a positive net charge Q.
i) Find the electric potential φ at a point P, which is on the axes of the ring, at a distance z from the centre of the ring.
ii) Find the electric field E at the point P.
iii) Show that the force acting on an electron moving along the symmetry axes in the vicinity of the centre of the ring ($|z| \ll R$) is harmonic (i.e. depends linearly on z).
iv) Define the frequency of oscillations of such an electron. Use numerical values $R = 1\,\mu\text{m}$ and $Q = 1.0 \times 10^{-13}$ C.
v) Now, let us assume that the electron can perform also off-axis movements. Is the position at the symmetry centre of the ring (on-axis, $z = 0$) stable or unstable? Motivate your answer.

Hint: You may use the approximate formula

$$(1+x)^\alpha \approx 1 + \alpha x + \tfrac{1}{2}\alpha(\alpha-1)x^2.$$

3. Ball *(8 points)* A homogeneous ball of radius R and mass m is thrown horizontally onto a table at the moment of time $t = 0$. Its initial velocity before contact is purely horizontal and equal to v; it is non-rotating. Coefficient of kinetic friction between the table and the ball is μ.
i) Find the moment of time t, when the ball stops sliding, i.e. starts rolling without sliding.
ii) Calculate the angular velocity of the ball ω_* and its total mechanical energy E_* at the moment when it stops sliding. In the case of a hollow sphere, would the energy E_* be larger or smaller than in the case of a homogeneous ball?
iii) Now, assume that the horizontal surface is treated so that the coefficient of kinetic friction depends on the horizontal coordinate x as $\mu = a + b\cos x$ (with $a > b$). Find the expression for the terminal mechanical energy E'_* in this new case.
iv) Now, let us return to the case of constant kinetic friction coefficient μ. However, let us assume that the surface is not perfectly rigid (e.g. covered by a felt cloth). This gives rise to a second friction force — the rolling friction force $F_r = \mu_r mg$. Unlike the kinetic friction force, it is not tangential to the touching point of the ball and the surface. Instead, it can be interpreted as the horizontal component of the surface reaction force (the entire reaction force is, of course, normal to the surface), see Fig. Find the expression for the terminal mechanical energy E''_* in this case. What is the most important (qualitative) difference between the expressions for E_* and E''_*?



Hint: The moment of inertia of a ball is $I = \frac{2}{5}MR^2$.

4. Black box *(9 points)* *Equipment:* electrical black box with three outlets, battery, voltmeter.

It is known that inside the black box, there are three resistors (connected with wires in an unknown manner), the smallest of which is $R_1 = 100\,\Omega$. Find the value of the largest resistor R_3. What can be said about the middle-valued resistor R_2? Estimate the uncertainties of your results.

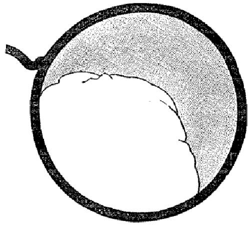
5. Pencil ***(6 points)*** *Equipment:* pencil, paper, ruler.

Determine the coefficient of friction of the pencil's graphite core against the paper. Estimate the uncertainty.

6. Spring ***(7 points)*** *Equipment:* helical spring of known mass $m = 19 \pm 0.5\,\mathrm{g}$, measuring tape, a load of unknown mass.

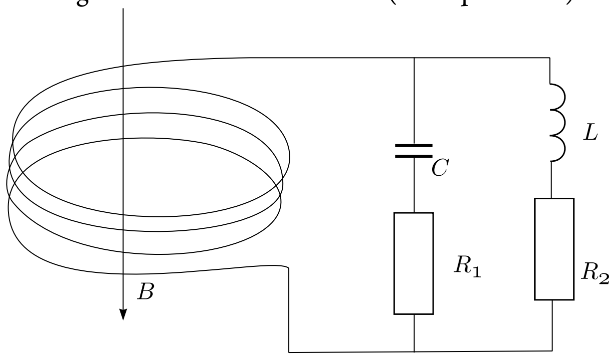
Determine the mass of the load. Estimate the uncertainty.

7. Soap film ***(6 points)*** Lord Rayleigh had in 1891 a lecture about taking photos of physical processes. Among others, he showed a photo of a soap film, which is falling apart (see Fig.). Instead of a flash, he used an electric spark (well, nowadays the flashes are also based on electric sparks). Estimate, how precise must have been the timing, i.e. estimate the time for a soap film to fall apart. Let the thickness of the soap film be $h = 1\ \mu\mathrm{m}$, the ring diameter $D = 10\,\mathrm{cm}$ and the surface tension $\sigma = 0.025\,\mathrm{N/m}$.



Hint: you may use a model, according to which the already broken part of the soap film gathers into a single front and moves all together towards the still preserved part of the film.

8. Magnetic pulse ***(7 points)*** Consider an electric circuit consisting of a coil of negligibly small inductance, consisting of $N = 10$ turns and with the surface area of a single loop $S = 10\,\mathrm{cm}^2$), resistors $R_1 = R_2 = 3\,\Omega$, capacitor $C = 0.2\,\mathrm{F}$, and an inductance $L = 1H$, connected as shown in Fig. At the moment of time $t = 0$, a magnetic field, parallel to the axis of the coil is switched on. The inductance of the magnetic field starts growing linearly, starting from $B = 0$ until the maximal value $B = 1\,\mathrm{T}$ is achieved at $t = 10\,\mathrm{ms}$. Further, the inductance of the magnetic field remains constant (and equal to $1\mathrm{T}$).



i) Find the current through the resistors R_1 and R_2 at the moment of time $t_1 = 5\,\mathrm{ms}$.
ii) Find the current through the resistors R_1 and R_2 at the moment of time $t_2 = 15\,\mathrm{ms}$.
iii) What is the net charge passing through the resistor R_2?

9. Stratostat ***(5 points)***
i) Show that the pressure of an isothermal gas of molar mass μ follows the law $p = p_0 e^{-\alpha z}$, where p_0 is the pressure at the origin, and z is the height. Find the constant α. The temperature is T, the free fall acceleration is g.
ii) Consider a stratostat, the envelope of which (a freely deformable non-elastic sack) is filled at the Earth's surface by helium to the volume fraction of $\beta = 10\%$. At which height h does the helium expand so that it fills the entire volume of the stratostat? The molar masses of the air and helium are $\mu_a = 29\,\mathrm{g/mol}$ and $\mu_{He} = 4\,\mathrm{g/mol}$, respectively. You may neglect the temperature variations of the atmosphere, and use the value $T = 250\,\mathrm{K}$.

10. Wedge ***(5 points)*** A wedge of mass M is kept at rest on an horizontal surface, and a block of mass m is kept on the wedge at the height h from the surface. The angle of the wedge is α, see Fig. There is no friction neither between the block and wedge nor between the surface and the wedge. The system is released into a free motion. Find the time t needed for the block to reach the surface.