Final exam in Advanced Discrete Mathematics, 20 May 2005, 10:00-13:00

Any written materials are allowed. Conversation and SMSing are strictly forbidden.

Please write your full name and student number on every sheet.

Please make sure all solutions are numbered and clearly separated from each other.

1. (3 p) Prove that, for any natural number n, $n^2+7n+12$ is an even number.
2. (5 p) How many five-digit telephone numbers have a digit that occurs more than once?
3. (4 p) Professor McBrain has taught the same course for the last 12 years and tells 3 jokes each year. He has never told the same set of three jokes twice (the order of the jokes is unimportant). How many jokes must he know?
4. (6 p) Prove by induction or otherwise that $4^{2n} - 1$ is a multiple of 15 for all natural numbers n.
5. (3 p) How many points must be chosen inside a square of sidelength 2 to ensure that at least one pair of them are not more than $\sqrt{2}$ apart?

   (Hint: use the pigeonhole principle.)
6. (6 p) How many integers in the range 1..1000 are divisible by 2 or 3 or 5?

   (Hint: use inclusion-exclusion.)
7. (6 p) Find a solution in integers to the equation

   $$325x + 26y = 91$$

   (Hint: use the gcd algorithm.)
8. (6 p) Calculate 3^{50} (mod 13) in any way you please.

   (Hint: make good use of modular arithmetic.)
9. (5 p) Construct a graph with 5 vertices and 6 edges which contains no 3-cycles (i.e. sets of three mutually adjacent vertices).
10. (6 p) On a chessboard of size 7x7 can you arrange for a traversal of all squares starting from the top left corner and ending at the bottom left corner visiting each square exactly once and always proceeding from a square to a square with whom it shares a side?

    (Hint: This is a chessboard so the squares are two-colored.)