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# LOGICS OF ACTION

- happening vs doing vs action
- <u>action</u> as successful <u>trying</u> (agency, intentionality)
- <u>action</u> as exercised <u>ability</u>
- action $\neq$ act

an <u>act</u> is what ~~takes~~ place when an <u>action</u> is <u>performed</u>

- actions
  - <u>executions</u> (defined by what is executed)
  - <u>accomplishments</u> (defined by what is achieved)

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## Two streams of logics

- logics of <u>agency</u>

  von Wright, Fitch, Pörn, Brown; Chellas' $\Delta$, Belnap and Perloff's STIT

- logics of <u>actions</u>

  dynamic logic, Segerberg's $\delta$

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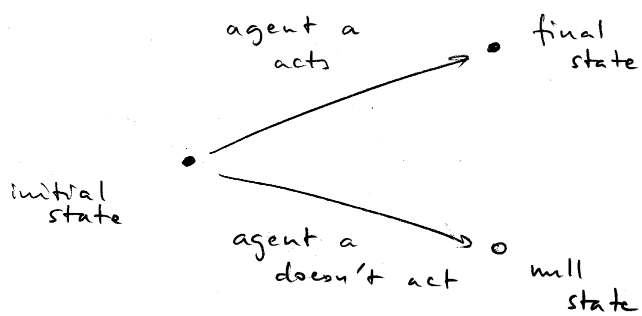
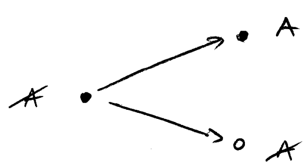
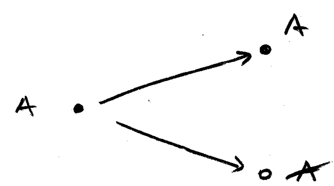
## Von Wright's intuition

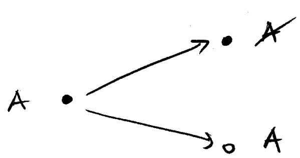


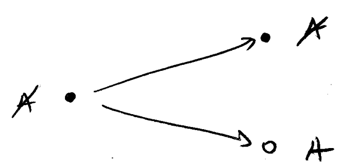

a produces A
(brings it about that)




a sustains A




a destroys A (= produces ¬A)




a suppresses A (= sustains ¬A)

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## Chellas' logic of seeing-to-it-that (Δ-logic)

- seeing-to-it-that is done
  by exercising control over the present (narrowing the set of yet possible histories at the time immediately preceding the current time)
- avoidability is not required

## Syntax of $\Delta$-logic

Assumed are a denumerable set $Fma_0$ of prop. letters and a non-empty set $Agt$ of agent names.

The set $Fma$ of formulae is defined as follows:

- if $p \in Fma_0$, then $p \in Fma$;
- $\top, \bot \in Fma$;
- if $A \in Fma$, then $\neg A \in Fma$;
- ... ;
- if $a \in Agt$, $A \in Fma$, then $\Delta_a A$ ($a$ sees to it that $A$) $\in Fma$;
- if $A \in Fma$, then $\boxdot A$ (it is historically necessary that $A$) $\in Fma$.

## $\Delta$- structures

A $\Delta$-structure is a 6-tuple $\langle T, \leq, S, H, R, V \rangle$ where

- $T$ is a (non-empty) set of something called times or time instants;
- $\leq \in \mathcal{P}(T \times T)$ is a binary relation on times which is reflexive, antisymmetric, transitive and linear, i.e. a linear ordering of $T$; if $t \leq t'$, say that $t$ is not later than $t'$;
- $S$ is a (non-empty) set of something called states-of-affairs (events);
- $H \in \mathcal{P}([T \to S])$ is a set of functions from $T$ to $S$; elements of $H$ are called (possible) histories;



define a function $\equiv \in [T \to \mathcal{P}(H \times H)]$ from times to binary relations on histories by letting

* $h =_t h'$ iff, for any $t' < t$, $h_{t'} = h'_{t'}$;

If $h =_t h'$ say that $h, h'$ have the same past at $t$;

$H$ is required to meet the future branching only condition:

* for any $t \in T$, $h, h' \in H$,
if $h_t = h'_t$, then $h =_t h'$;

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- $R \in [Agt \times T \to \mathcal{P}(H \times H)]$ is a function from agent names and times to binary relations on histories called the <u>instigative alternativeness relations</u>; for any $a \in Agt$, $t \in T$, the relation $R_{a,t}$ is required to be <u>reflexive</u> and $R$ must meet the <u>historical relevance</u> condition:

  * for any $a \in Agt$, $t \in T$, $h, h' \in H$, if $h R_{a,t} h'$, then $h =_t h'$.

- $V \in [Fma_0 \to \mathcal{P}(S)]$ is a function from prop. letters to sets of states-of-affairs called the <u>valuation</u>.

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<u>Intuition</u>

- $h \; R_{a,t} \; h'$

  $\Leftrightarrow$ history $h'$ is an instigative alternative to history $h$ for agent $a$ at time $t$

  $\Leftrightarrow$ agent $a$ at $t$ would not be able to prevent $h'$ from appearing to be the actual history, if $h$ were the actual history

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## Satisfaction in Δ-structures

Given a Δ-structure $M = \langle T, \leq, S, H, R, V \rangle$, define the corresponding interpretation function $[\![ - ]\!]^M \in [\mathrm{Fma} \to \mathcal{P}(T \times H)]$ from formulae to time-history pairs as follows:

* If $p \in \mathrm{Fma}_0$, then: $\models^M_{t,h} p$ iff $h_t \in V(p)$;
* $\models^M_{t,h} \top$ is true;
* $\models^M_{t,h} \bot$ is false;
* $\models^M_{t,h} \neg A$ iff not $\models^M_{t,h} A$;
* ... ;
* $\models^M_{t,h} \Delta_a A$ iff, for any $h' \in H$ st $h R_{a,t} h'$, $\models^M_{t,h'} A$;
* $\models^M_{t,h} \odot A$ iff, for any $h' \in H$ st $h =_t h'$, $\models^M_{t,h'} A$.

A formula $A$ is valid in $M$ (notation $\models^M A$), if $\models^M_{t,h} A$, for any $t \in T$, $h \in H$.

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## Intuition

- $\Delta_a A$ holds at time $t$ in history $h$

  $\iff$ $A$ holds at time $t$ in any history $h'$ that is an instigative alternative to $h$ for agent $a$ at $t$

  $\iff$ $A$ would hold at $t$ in any history $h'$ that agent $a$ at $t$ would not be able to prevent from appearing to be the actual history, if the actual history were $h$

## Some observations

- The following hold in any $\Delta$-structure $M$:

  * $\models^M \Delta_a(A \supset B) \supset (\Delta_a A \supset \Delta_a B)$;
    if $\models^M A$, then $\models^M \Delta_a A$; $\models^M \Delta_a \top$;
  * $\models^M \Delta_a A \supset A$
    (<u>reliability</u>);
  * $\models^M \Delta_a \Delta_b A \supset \Delta_a A$
    (<u>qui facit per alium</u>, facit per se; if a sees to it that b sees to it that something, then a sees to it, too);
  * $\models^M \boxdot A \supset \Delta_a A$;
  * $\models^M \boxdot A \supset A$;
    $\models^M \boxdot A \supset \boxdot\boxdot A$;
    $\models^M \neg\boxdot A \supset \boxdot\neg\boxdot A$.



## Belnap and Perloff's logic of seeing-to-it-that (stit-logic)

- seeing-to-it-that results from a <u>prior</u> <u>choice</u>
  (the choices at any time are given by a <u>partition</u> of the histories yet possible at that moment)
- avoidability is required

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## Syntax of stit-logic

Assumed are again a denumerable set $Fma_0$ of prop. letters and a non-empty set $Agt$ of agent names.

The set $Fma$ of formulae is defined as follows:

- If $p \in Fma_0$, then $p \in Fma$;
- $\top, \bot \in Fma$;
- If $A \in Fma$, then $\neg A \in Fma$;
- ... ;
- If $a \in Agt$, $A \in Fma$, then $stit_a A$ (a sees to it that $A$) $\in Fma$.

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## stit-structures

A stit-structure is a 6-tuple $\langle T, \leq, S, H, E, V \rangle$ where

- $T$ is a (non-empty) set of something called <u>times</u> or <u>time instants</u>;
- $\leq \in \mathcal{P}(T \times T)$ is a <u>linear ordering</u> of $T$, the <u>no-later-than</u> relation;
- $W$ is a (non-empty) set of something called <u>states-of-affairs</u>
- $H \in \mathcal{P}([T \to S])$ is a set of functions from times to worlds; elements of $H$ are called <u>(possible) histories</u>;

define a function $\equiv\, \in [T \to \mathcal{P}(H \times H)]$ from times to binary relations on histories by letting

* $h \equiv_t h'$ iff, for any $t' \leq t$, $h_{t'} = h'_{t'}$;

if $h \equiv_t h'$ say that $h, h'$ have the <u>same past and present</u> at $t$;

$H$ is required to meet the <u>future branching only</u> condition;

* for any $t \in T$, $h, h' \in H$, if $h_t = h'_t$, then $h \equiv_t h'$;



- $E \in [Agt \times T \to \mathcal{P}(H \times H)]$ is a function from agent names and times to binary relations on histories called the <u>same choice</u> relations;

for any $a \in Agt$, $t \in T$, the relation $E_{a,t}$ is required to be <u>reflexive</u>, <u>symmetric</u> and <u>transitive</u> (ie, an <u>equivalence</u>) and $E$ has to meet the following conditions:

* for any $a \in Agt$, $t \in T$, $h, h' \in H$, if $h\, E_{a,t}\, h'$, then $h \equiv_t h'$ (<u>historical relevance</u>);
* for any $a \in Agt$, $t \in T$, $t' > t$, $h, h' \in H$, if $h \equiv_{t'} h'$, then $h\, E_{a,t}\, h'$ (<u>no choice between undivided histories</u>);
* for any $t \in T$, $\{h_a\}_{a \in Agt} \subseteq H$, there exists a $h \in H$, st $h_a\, E_{a,t}\, h'$ for all $a \in Agt$ (<u>something happens</u>).

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- $V \in [Fma_0 \to P(S)]$ is a function from prop. letters to sets of state's-of-affairs called the <u>valuation</u>.

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<u>Intuition</u>

- $h \; E_{a,t} \; h'$

  $\Leftrightarrow$ if $h$ were the actual history and agent $a$ made a choice at $t$, then, by this choice, he would not be able to prevent $h'$ from appearing to be the actual history

# Satisfaction in stit-structures

Given a stit-structure $M = \langle T, \leq, S, H, E, V \rangle$, define the interp. function $[\![ - ]\!]^M \in [Fm \to \wp(T \times H)]$ from formulae to time-history pairs as follows:

* if $p \in Fm_0$, then: $\models^M_{t,h} p$ iff $h_t \in V(p)$;
* $\models^M_{t,h} \top$ is true;
* $\models^M_{t,h} \bot$ is false;
* $\models^M_{t,h} \neg A$ iff not $\models^M_{t,h} A$;
* ... ;
* $\models^M_{t,h}$ stit$_a$ $A$ iff, for some $t' < t$ (a witnessing choice point)
  (1) for any $h' \in H$ st $h \, E_{a,t'} \, h'$, $\models^M_{t,h'} A$ (the positive condition);
  (2) for some $h' \in H$ st $h \equiv_{t'} h'$, not $\models^M_{t,h'} A$ (the negative condition).

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## Intuition

- $\mathrm{stit}_a\, A$ holds at time $t$ in history $h$

$\Leftrightarrow$ there is a prior time $t'$ st,
if $h$ were the actual history and agent $a$ had made a choice at time $t'$, then

(1) $A$ would hold at $t$ in any history $h'$ that $a$, by his choice at $t'$, would not have been able to prevent from becoming actual;

(2) $A$ would not hold at $t$ in some history $h'$ that, at $t'$, would have had the same past and present as $h$.

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## Observations

For any stit-structure $M$, the following are true:

* if $\vDash^M A \supset B$ and $t_1$ witnesses $\vDash^M_{t,h} \mathrm{stit}_a A$, $t_2$ witnesses $\vDash^M_{t,h} \mathrm{stit}_a B$, then $t_1 \geq t_2$; (1)

* if $\vDash^M A \equiv B$ and $t_1$ witnesses $\vDash^M_{t,h} \mathrm{stit}_a A$, $t_2$ witnesses $\vDash^M_{t,h} \mathrm{stit}_a B$, then $t_1 = t_2$;

* if $\vDash^M_{t,h} \mathrm{stit}_a A$, then there is exactly one witness for this.

Verification of (1): (107)

Suppose that, to the contrary, we have $A, B \in Fma$, $t, t_1, t_2 \in T$, $h \in H$ st $\models^M A \supset B$, $t_1$ witnesses $\models^M_{t,h}$ stit$_a$ $A$, $t_2$ witnesses $\models^M_{t,h}$ stit$_a$ $B$ and $t_2 > t_1$. Let us see that this leads to a contradiction.

From the negative condition for $t_2$ witnessing $\models^M_{t,h}$ stit$_a$ $B$, it follows that there is a $h \in H$ st $h \equiv_{t_2} h'$ and not $\models^M_{t,h'} B$. Since $\models^M A \supset B$, it must also be the case that not $\models^M_{t,h'} A$. From $M$ meeting the no choice between undivided histories condition, and $t_2 > t_1$, it follows that $h \, E_{a,t_1} \, h'$. This, together with the positive condition for $t_1$ witnessing $\models^M_{t,h}$ stit$_a$ $A$, yields $\models^M_{t,h'} A$. Contradiction.

Observations (108)

For any stit-structure $M$, the following are true:

* if $\models^M A \equiv B$, then $\models^M$ stit$_a$ $A \equiv$ stit$_a$ $B$;
* $\models^M$ stit$_a$ $A \supset A$;
* $\models^M \neg$ stit$_a$ $\top$;

* $\models^M$ stit$_a$ $A \wedge$ stit$_a$ $B \supset$ stit$_a$ $(A \wedge B)$;
* $\models^M$ stit$_a$ $A \supset$ stit$_a$ stit$_a$ $A$;
* $\models^M$ stit$_a$ stit$_b$ $A \supset$ stit$_a$ $A$;
* $\models^M \neg$ stit$_a$ stit$_b$ $A$ (whenever $a \neq b$).

# Segerberg's logic of bringing it about (109)

(δ-logic)

- essentially a version of <u>dynamic logic</u>

- $\delta A$ - a name for the action of bringing it about that $A$

(cf in Chellas' logic,

$\Delta A$ - a name for the proposition that $A$ is, or has been, brought about)

# Syntax of δ-logic (110)

Assumed is a denumerable set $Fma_0 = \{p_0, p_1, \ldots\}$ of <u>prop. letters</u>.

The set $Fma$ of <u>formulae</u> and the set $Prg$ of <u>programs</u> are defined as follows:

* if $P, Q \in Prg$, then $P;Q$, $P \cup Q \in Prg$;
* if $P \in Prg$, then $P^* \in Prg$;
* if $A \in Fma$, then $\delta A$ (bringing it about that $A$) $\in Prg$
* if $p \in Fma_0$, then $p \in Fma$;
* $\top, \bot \in Fma$;
* if $A \in Fma$, then $\neg A \in Fma$;
* ...
* if $P \in Prg$, $A \in Fma$, then $[P]A$, $\langle P \rangle A \in Fma$.

## $\delta$-structures

A $\delta$-structure is a triple $M = (W, D, V)$ where

- $W$ is a non-empty set of something called <u>states</u>;
- $D \in [\mathcal{P}(W) \to \mathcal{P}(W, W)]$ is a function from sets of states to binary relations on states;
- $D$ is required to meet two conditions:
  - \+ for any $X \subseteq W$, $w \in W$,
    for any $w' \in W$ st $wD(X)w'$, $w' \in X$
    (<u>reliability</u>)
  - \+ for any $X, Y \in W$, $w \in W$,
    if, for any $w' \in W$ st $wD(X)w'$, $w' \in Y$,
    then, for any $w' \in W$ st $wD(X)w'$, $wD(Y)w'$
    (<u>weak maximality</u>)
- $V \in [Fm_{\alpha_0} \to \mathcal{P}(W)]$ is a function from prop. letters to sets of states.

## Satisfaction in $\delta$-structures

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Given a $\delta$-structure $M = (W, D, V)$, the corresponding interpretation functions $\{\!|-|\!\}^M \in [Prg \to \mathcal{P}(W \times W)]$ from programs to binary relations on states and $[\![-]\!]^M \in [Fma \to \mathcal{P}(W)]$ from formulae to sets of states are defined as follows ($\models^M_w A$ is short for $w \in [\![A]\!]^M$):

* $w \{\!|P; Q|\!\}^M w'$ iff, for some $w'' \in W$, $w \{\!|P|\!\}^M w''$ and $w'' \{\!|Q|\!\}^M w'$;
* $w \{\!|P \cup Q|\!\}^M w'$ iff $w \{\!|P|\!\}^M w'$ or $w \{\!|Q|\!\}^M w'$;
* $w \{\!|P^*|\!\}^M w'$ iff, for some $n \geq 0$, $w_0, \ldots, w_n \in W$, $w = w_0$, $w_0 \{\!|P|\!\}^M w_1, \ldots, w_{n-1} \{\!|P|\!\}^M w_n$, $w_n = w'$;
* $w \{\!|\delta A|\!\}^M w'$ iff $w\, D([\![A]\!]^M)\, w'$;
* if $p \in Fma_0$, then: $\models^M_w p$ iff $w \in V(p)$;
* $\models^M_w \top$ is true;
* $\models^M_w A \wedge B$ iff $\models^M_w A$ and $\models^M_w B$;
* $\ldots$;
* $\models^M_w [P] A$ iff, for any $w' \in W$ st $w \{\!|P|\!\}^M w'$, $\models^M_{w'} A$;
* $\models^M_w \langle P \rangle A$ iff, for some $w' \in W$ st $w \{\!|P|\!\}^M w'$, $\models^M_{w'} A$.

## Observations

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For any $\delta$-structure $M$, the following hold:

* $\models^M [P](A \supset B) \supset ([P]A \supset [P]B)$;
* if $\models^M A$, then $\models^M [P] A$;
* $\models^M [\delta A] A$; (1)
* $\models^M [\delta A] B \supset ([\delta B] C \supset [\delta A] C)$; (2)
* if $\models^M A \supset B$, then $\models^M [P] A \supset [P] B$;
* if $\models^M A \supset B$, then $\models^M [\delta B] C \supset [\delta A] C$.

Verification for (1):

Consider any $w \in W$. We have to show that $\models^M_w [\delta A] A$, ie, that, for any $w' \in W$ st $w D([\![A]\!]^M) w'$, $\models^M_{w'} A$. But this is guaranteed by the reliability condition which is met by M.

Verification of (2):

Consider any $w \in W$. Assume that $\models^M_w [\delta A] B$, ie, that, for any $w' \in W$. st $w D([\![A]\!]^M) w'$, $\models^M_{w'} B$. Since M meets the weak maximality condition, this entails that, for any $w' \in W$ st $w D([\![A]\!]^M) w'$, $w D([\![B]\!]^M) w'$. Assume now also that $\models^M_w [\delta B] C$. We have to show that $\models^M_w [\delta A] C$, ie, that, for any $w' \in W$ st $w D([\![A]\!]^M) w'$, $\models^M_{w'} C$. Consider any such $w' \in W$ st $w D([\![A]\!]^M) w'$. We know that $w D([\![B]\!]^M) w'$. Since $\models^M_w [\delta B] C$, from this it follows that $\models^M_{w'} C$, and this was what we wanted to prove.

## Axiomatization

$\delta$-logic is (soundly & completely) axiomatized by $\mathbb{K}_{Prg}$ plus

$$[P;Q]A \equiv [P][Q]A$$

$$[P \cup Q]A \equiv [P]A \wedge [Q]A$$

$$[P^*]A \supset A \wedge [P][P^*]A$$

$$\frac{B \supset A \wedge [P]B}{B \supset [P^*]A}$$

$$[\delta A]A$$

$$[\delta A]B \supset ([\delta B]C \supset [\delta A]C)$$