Eksam ainest Loogika arvutiteaduses WAI 3720 8.1.2003 kl 10.00.
Lahendused

1.

$$\dfrac{\dfrac{\dfrac{\overset{+2}{p \lor r} \qquad \dfrac{\dfrac{\dfrac{\overset{+1}{(p \supset q) \land (r \supset s)}}{p \supset q}\,\land\mathcal{E} \quad \overset{+3}{p}}{q}\,\supset\mathcal{E}}{q \lor s}\,\lor\mathcal{I} \qquad \dfrac{\dfrac{\dfrac{\overset{+1}{(p \supset q) \land (r \supset s)}}{r \supset s}\,\land\mathcal{E} \quad \overset{+4}{r}}{s}\,\supset\mathcal{E}}{q \lor s}\,\lor\mathcal{I}}{q \lor s}\,\lor\mathcal{E}, -3, -4}{p \lor r \supset q \lor s}\,\supset\mathcal{I}, -2}{(p \supset q) \land (r \supset s) \supset (p \lor r \supset q \lor s)}\,\supset\mathcal{I}, -1$$

$$\dfrac{\dfrac{\dfrac{\overset{+1}{q}}{p \supset q}\,\supset\mathcal{I}, -3}{(p \supset q) \lor (q \supset r)}\,\lor\mathcal{I} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\overset{+2}{\neg q} \quad \overset{+4}{q}}{\bot}\,\neg\mathcal{E}}{r}\,\bot\mathcal{E}}{q \supset r}\,\supset\mathcal{I}, -4}{(p \supset q) \lor (q \supset r)}\,\lor\mathcal{I}}{(p \supset q) \lor (q \supset r)}\,\text{Dilemma}, -1, -2$$

$$\dfrac{\dfrac{\overset{+1}{\exists x.\,\exists y.\,(p(f(x), y) \lor p(y, g(x)))} \qquad \dfrac{\overset{+2}{\exists y.\,(p(f(x'), y) \lor p(y, g(x')))} \qquad \dfrac{\overset{+3}{p(f(x'), y') \lor p(y', g(x'))} \qquad \dfrac{\dfrac{\overset{+4}{p(f(x'), y')}}{\exists y.\,p(f(x'), y)}\,\exists\mathcal{I}}{\exists x.\,\exists y.\,p(x, y)}\,\exists\mathcal{I} \qquad \dfrac{\dfrac{\overset{+5}{p(y', g(x'))}}{\exists y.\,p(y', y)}\,\exists\mathcal{I}}{\exists x.\,\exists y.\,p(x, y)}\,\exists\mathcal{I}}{\exists x.\,\exists y.\,p(x, y)}\,\lor\mathcal{E}, -4, -5}{\exists x.\,\exists y.\,p(x, y)}\,\exists\mathcal{E}, -3, y'}{\exists x.\,\exists y.\,p(x, y)}\,\exists\mathcal{E}, -2, x'}{\exists x.\,\exists y.\,(p(f(x), y) \lor p(y, g(x))) \supset \exists x.\,\exists y.\,p(x, y)}\,\supset, -1$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{\text{Id.}}{r \supset s, p \to p, q, s} \quad \overset{\text{Id.}}{q, r \supset s, p \to q, s}}{p \supset q, r \supset s, p \to q, s}\,\supset\mathcal{L} \qquad \dfrac{\overset{\text{Id.}}{p \supset q, r \to r, q, s} \quad \overset{\text{Id.}}{p \supset q, s, r \to q, s}}{p \supset q, r \supset s, r \to q, s}\,\supset\mathcal{L}}{p \supset q, r \supset s, p \lor r \to q, s}\,\lor\mathcal{L}}{(p \supset q) \land (r \supset s), p \lor r \to q, s}\,\land\mathcal{L}}{(p \supset q) \land (r \supset s), p \lor r \to q \lor s}\,\lor\mathcal{R}}{(p \supset q) \land (r \supset s) \to p \lor r \supset q \lor s}\,\supset\mathcal{R}}{\to (p \supset q) \land (r \supset s) \supset (p \lor r \supset q \lor s)}\,\supset\mathcal{R}$$

$$\dfrac{\dfrac{\dfrac{\overset{\text{Id.}}{p, q \to q, r}}{p \to q, q \supset r}\,\supset\mathcal{R}}{\to p \supset q, q \supset r}\,\supset\mathcal{R}}{\to (p \supset q) \lor (q \supset r)}\,\lor\mathcal{R}$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{Id.}}{p(f(x'),y') \to p(f(x'),y'),\ldots,\ldots}\ \exists\mathcal{R}}{p(f(x'),y') \to \exists y.\ p(f(x'),y),\ldots}\ \exists\mathcal{R}}{p(f(x'),y') \to \exists x.\ \exists y.\ p(x,y)} \qquad \dfrac{\dfrac{\dfrac{\text{Id.}}{p(y',g(x')) \to p(y',g(x')),\ldots,\ldots}\ \exists\mathcal{R}}{p(y',g(x')) \to \exists y.\ p(y',y),\ldots}\ \exists\mathcal{R}}{p(y',g(x')) \to \exists x.\ \exists y.\ p(x,y)}}{p(f(x'),y') \lor p(y',g(x')) \to \exists x.\ \exists y.\ p(x,y)}\ \lor\mathcal{L}}{\exists y.\ (p(f(x'),y) \lor p(y,g(x'))) \to \exists x.\ \exists y.\ p(x,y)}\ \exists\mathcal{L},y'}{\exists x.\ \exists y.\ (p(f(x),y) \lor p(y,g(x))) \to \exists x.\ \exists y.\ p(x,y)}\ \exists\mathcal{L},x'}{\to \exists x.\ \exists y.\ (p(f(x),y) \lor p(y,g(x))) \supset \exists x.\ \exists y.\ p(x,y)}\ \supset\mathcal{R}
$$

2.

$$
\begin{array}{c}
\mathbf{F}(p \supset (q \supset r)) \supset (p \supset (r \supset q)) \\
\mathbf{T}p \supset (q \supset r) \\
\mathbf{F}p \supset (r \supset q) \\
\mathbf{T}p \\
\mathbf{F}r \supset q \\
\mathbf{T}r \\
\mathbf{F}q \\
\hline
\begin{array}{c|c}
\mathbf{F}p & \mathbf{T}q \supset r \\
\times & \begin{array}{c|c} \hline \mathbf{F}q & \mathbf{T}r \\ \circ & \circ \end{array}
\end{array}
\end{array}
$$

Kontramudel on I, kus $I(p) = 1$, $I(q) = 0$, $I(r) = 1$.

$$
\begin{array}{c}
\mathbf{F}\forall x.\ \forall y.\ (p(x,y) \supset \exists z.\ (p(x,z) \land p(z,y))) \\
\mathbf{F}\forall y.\ (p(x',y) \supset \exists z.\ (p(x',z) \land p(z,y))) \\
\mathbf{F}p(x',y') \supset \exists z.\ (p(x',z) \land p(z,y')) \\
\mathbf{T}p(x',y') \\
\mathbf{F}\exists z.\ (p(x',z) \land p(z,y')) \\
\mathbf{F}p(x',x') \land p(x',y') \\
\hline
\begin{array}{c|c}
\begin{array}{c}
\mathbf{F}p(x',x') \\
\mathbf{F}p(x',y') \land p(y',y') \\
\hline
\begin{array}{c|c} \mathbf{F}p(x',y') & \mathbf{F}p(y',y') \\ \times & \circ \end{array}
\end{array}
&
\begin{array}{c} \mathbf{F}p(x',y') \\ \times \end{array}
\end{array}
\end{array}
$$

Kontramudel on $M = (D, I)$, kus $D = \{x', y'\}$ ja $I(p)(x', x') = 0$, $I(p)(x', y') = 1$, $I(p)(y', y') = 0$ ja $I(p)(y', x')$ võib olla suvalise väärtusega.

3. Täielik konjunktiivne normaalkuju: $(\neg p \lor \neg q \lor r) \land (\neg p \lor q \lor \neg r) \land (\neg p \lor q \lor r) \land (p \lor \neg q \lor r) \land (p \lor q \lor r)$. Lühike: $(\neg p \lor q) \land r$.

4. (i) $\forall x.\ (p(x) \supset a(m,x)$, (ii) $\exists x.\ (p(x) \land a(x,m)$, (iii) $a(m,m)$, (iv) $\neg\exists x.\ (s(x) \land \forall y.\ (l(y) \supset b(x,y)))$, (v) $\neg\exists y.\ (l(y) \land \forall x.\ (s(x) \supset b(x,y)))$, (vi) $\neg\exists x.\ (s(x) \land \exists y.\ (l(y) \land b(x,y)))$.

5. Võimalik teisendus:

$$
\begin{array}{l}
\forall x.\ \neg(\exists y.\ p(x,y) \land \forall z.\ (q(z,y) \lor \neg\forall x.p(x,z))) \\
\forall x.\ \neg(\exists y'.\ p(x,y') \land \forall z.\ (q(z,y) \lor \neg\forall x'.p(x',z))) \\
\forall x.\ \forall y'.\ \exists z.\ \forall x'.\ \neg(p(x,y') \land (q(z,y) \lor \neg p(x',z))) \\
\forall x.\ \forall y'.\ \exists z.\ \forall x'.\ (\neg p(x,y') \lor \neg(q(z,y) \lor \neg p(x',z))) \\
\forall x.\ \forall y'.\ \exists z.\ \forall x'.\ (\neg p(x,y') \lor (\neg q(z,y) \land p(x',z))) \\
\forall x.\ \forall y'.\ \exists z.\ \forall x'.\ ((\neg p(x,y') \lor \neg q(z,y)) \land (\neg p(x,y') \lor p(x',z))) \\
(\neg p(x,y') \lor \neg q(f(y,x,y'),y)) \land (\neg p(x,y') \lor p(x',f(y,x,y')))
\end{array}
$$

6. Nii $\Diamond p \vee \Diamond q$ kui ka $\Diamond(p \vee \Diamond q)$ kehtivad maailmades a, b, d.

7. (i) Suvalise struktuuri (W, R, I) suvalises maailmas w kehtib $\Box(p \wedge q)$ parajasti siis, kui igas w-st saavutatavas maailmas w' kehtib $p \wedge q$ ehk siis kehtivad nii p kui ka q. Valem $\Box p \wedge \Box q$ agas kehtib w-s siis, kui seal kehtivad nii $\Box p$ kui ka $\Box q$ ehk kui ühelt poolt igas w-st saavutatavas w'-s kehtib p ja teiselt poolt igas w-st saavutatavas w'-s kehtib q ning see teeb sama välja.

   (ii) Suvalise struktuuri (W, R, I) suvalises maailmas w kethib $\Diamond\top$ parajasti siis, kui leidub mingi w-st saavutatav maailm w'. Kui sellel asjaolul w-s kehtib $\Box p$, siis peab p kehtima igas w-st saavutatavas maailmas, teiste hulgas ka w'-s. Siis aga kehtib w-s $\Diamond p$, sest w' näol on olemas üks w-st saavutatav maailm, kus p kehtib.

8. (i) Tingimus: ühestki tipust ei välju ühtki kaart, ehk ühegi $w \in W$ korral ei leidu $w' \in W$, nii et wRw''. (ii) Tingimus: kui tipust väljub kaar, viib ta samasse tippu tagasi, ehk iga $w, w' \in W$ korral kui wRw', siis $w' = w$. (NB: See ei ole refleksiivsus, mis ütleb, et iga $w \in W$ korral wRw).

9. $\forall w.\ [\forall w'.\ (wRw' \supset (p(w') \supset \exists w''.(w'Rw'' \wedge q(w'')))) \vee \exists w'.\ (wRw' \wedge (\forall w''.(w'Rw'' \supset q(w''))) \supset r(w'))]$

10. (i) Agent 1 teab, et p, või ta teab, et mitte-p; teisisõnu, agent 1 teab, kas p. (ii) Agent 2 ei tea, et agent 1 teab, kas p. (iii) Kui agent 2 teab, et p, siis agent 1 teab seda.