Eksam ainest Loogika arvutiteaduses WAI 3720 24.1.2003 kl 10.00.
Lahendused

1.

$$
\begin{prooftree}
\AxiomC{$\overset{+1}{p \vee (p \wedge q)}$}
\AxiomC{$\overset{+2}{p}$}
\AxiomC{$\overset{+3}{p \wedge q}$}
\RightLabel{$\wedge\mathcal{E}$}
\UnaryInfC{p}
\RightLabel{$\vee\mathcal{E}, -2, -3$}
\TrinaryInfC{p}
\RightLabel{$\supset \mathcal{I}, -1$}
\UnaryInfC{$p \vee (p \wedge q) \supset p$}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$\overset{+1}{\neg(\neg p \vee q)}$}
\AxiomC{$\overset{+2}{\neg p}$}
\RightLabel{$\vee\mathcal{I}$}
\UnaryInfC{$\neg p \vee q$}
\RightLabel{$\neg\mathcal{E}$}
\BinaryInfC{$\bot$}
\RightLabel{$\bot\mathcal{E}$}
\UnaryInfC{p}
\AxiomC{$\overset{+3}{p}$}
\RightLabel{Dilemma, $-2, -3$}
\BinaryInfC{p}
\RightLabel{$\supset \mathcal{I}, -1$}
\UnaryInfC{$\neg(\neg p \vee q) \supset p$}
\end{prooftree}
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\begin{prooftree}
\AxiomC{$\overset{+1}{\forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x)}$}
\RightLabel{$\wedge\mathcal{E}$}
\UnaryInfC{$\exists x.\, p(x)$}
\AxiomC{$\overset{+1}{\forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x)}$}
\RightLabel{$\wedge\mathcal{E}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x)))$}
\RightLabel{$\forall\mathcal{E}$}
\UnaryInfC{$p(f(x')) \supset p(f(f(x')))$}
\AxiomC{$\overset{+1}{\forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x)}$}
\RightLabel{$\wedge\mathcal{E}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x)))$}
\RightLabel{$\forall\mathcal{E}$}
\UnaryInfC{$p(x') \supset p(f(x'))$}
\AxiomC{$\overset{+2}{p(x')}$}
\RightLabel{$\supset \mathcal{E}$}
\BinaryInfC{$p(f(x'))$}
\RightLabel{$\supset \mathcal{E}$}
\BinaryInfC{$p(f(f(x')))$}
\RightLabel{$\exists\mathcal{I}$}
\UnaryInfC{$\exists x.\, p(f(f(x)))$}
\RightLabel{$\exists\mathcal{E}, -2, x'$}
\BinaryInfC{$\exists x.\, p(f(f(x)))$}
\RightLabel{$\supset \mathcal{I}, -1$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x) \supset \exists x.\, p(f(f(x)))$}
\end{prooftree}
$$

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\begin{prooftree}
\AxiomC{}
\RightLabel{Id.}
\UnaryInfC{$p \to p$}
\AxiomC{$\overset{\text{Id.}}{p, q \to p}$}
\RightLabel{$\wedge\mathcal{L}$}
\UnaryInfC{$p \wedge q \to p$}
\RightLabel{$\vee\mathcal{L}$}
\BinaryInfC{$p \vee (p \wedge q) \to p$}
\RightLabel{$\supset \mathcal{R}$}
\UnaryInfC{$\to p \vee (p \wedge q) \supset p$}
\end{prooftree}
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\begin{prooftree}
\AxiomC{$\overset{\text{Id.}}{p \to p, q}$}
\RightLabel{$\neg\mathcal{R}$}
\UnaryInfC{$\to p, \neg p, q$}
\RightLabel{$\vee\mathcal{R}$}
\UnaryInfC{$\to p, \neg p \vee q$}
\RightLabel{$\neg\mathcal{L}$}
\UnaryInfC{$\neg(\neg p \vee q) \to p$}
\RightLabel{$\supset \mathcal{R}$}
\UnaryInfC{$\to \neg(\neg p \vee q) \supset p$}
\end{prooftree}
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\begin{prooftree}
\AxiomC{$\overset{\text{Id.}}{\ldots, p(x') \to p(x'), p(f(x')), p(f(f(x'))), \ldots}$}
\AxiomC{$\overset{\text{Id.}}{p(f(x')), \ldots, p(x') \to p(f(x')), p(f(f(x'))), \ldots}$}
\RightLabel{$\supset \mathcal{L}$}
\BinaryInfC{$p(x') \supset p(f(x')), \ldots, p(x') \to p(f(x')), p(f(f(x'))), \ldots$}
\RightLabel{$\forall\mathcal{L}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))), p(x') \to p(f(x')), p(f(f(x'))), \ldots$}
\AxiomC{$\overset{\text{Id.}}{p(f(f(x'))), \ldots, p(x') \to p(f(f(x'))), \ldots}$}
\RightLabel{$\supset \mathcal{L}$}
\BinaryInfC{$p(f(x')) \supset p(f(f(x'))), \forall x.\, (p(x) \supset p(f(x))), p(x') \to p(f(f(x'))), \ldots$}
\RightLabel{$\forall\mathcal{L}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))), p(x') \to p(f(f(x'))), \ldots$}
\RightLabel{$\exists\mathcal{R}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))), p(x') \to \exists x.\, p(f(f(x)))$}
\RightLabel{$\exists\mathcal{L}, x'$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))), \exists x.\, p(x) \to \exists x.\, p(f(f(x)))$}
\RightLabel{$\wedge\mathcal{L}$}
\UnaryInfC{$\forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x) \to \exists x.\, p(f(f(x)))$}
\RightLabel{$\supset \mathcal{R}$}
\UnaryInfC{$\to \forall x.\, (p(x) \supset p(f(x))) \wedge \exists x.\, p(x) \supset \exists x.\, p(f(f(x)))$}
\end{prooftree}
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2.

$$
\begin{array}{c}
\mathbf{T}p \wedge (\neg q \vee \neg p) \\
\mathbf{T}p \\
\mathbf{T}\neg q \vee \neg p \\
\hline
\begin{array}{c|c}
\mathbf{T}\neg q & \mathbf{T}\neg p \\
\mathbf{F}q & \mathbf{F}p \\
\circ & \times
\end{array}
\end{array}
$$

Mudel on I, kus $I(p) = 1$, $I(q) = 0$.

$$
\begin{array}{c}
\mathbf{T}\exists x.\ p(x) \wedge \exists x.\ q(x) \wedge \forall x.\ (\neg p(x) \vee \neg q(x)) \\
\mathbf{T}\exists x.\ p(x) \\
\mathbf{T}\exists x.\ q(x) \\
\mathbf{T}\forall x.\ (\neg p(x) \vee \neg q(x)) \\
\mathbf{T}p(x') \\
\mathbf{T}q(x'') \\
\mathbf{T}\neg p(x') \vee \neg q(x') \\
\hline
\begin{array}{c|c}
\begin{array}{c}
\mathbf{T}\neg p(x') \\
\mathbf{F}p(x') \\
\times
\end{array}
&
\begin{array}{c}
\mathbf{T}\neg q(x') \\
\mathbf{F}q(x') \\
\mathbf{T}\neg p(x'') \vee \neg q(x'') \\
\hline
\begin{array}{c|c}
\mathbf{T}\neg p(x'') & \mathbf{T}\neg q(x'') \\
\mathbf{F}p(x'') & \mathbf{F}q(x'') \\
\circ & \times
\end{array}
\end{array}
\end{array}
\end{array}
$$

Mudel on (D, I), kus $D = \{x', x''\}$ ja $I(p)(x') = 1$, $I(p)(x'') = 0$, $I(q)(x') = 0$, $I(q)(x'') = 1$.

3. Täielik DNK: $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$. Lühem: $(p \wedge q \wedge \neg r) \vee (\neg q \wedge r) \vee (\neg p \wedge r)$.

4. (i) $\forall y.\ \exists x.\ m(x, y)$,
(ii) $\forall y.\ (\exists x.\ m(x, y) \wedge \exists x.\ f(x, y))$,
(iii) $\forall y.\ (\exists x.\ m(x, y) \supset \exists x.\ f(x, y))$,
(iv) $\exists y.\ \exists z.\ (f(e, y) \wedge (f(y, z) \vee m(y, z)))$,
(v) $\neg\exists x.\ (\exists y.\ \exists z.\ [m(x, y) \wedge (f(y, z) \vee m(y, z))] \wedge \forall y.\ f(x, y))$,
(vi) $\forall x.\forall y.\ (f(x, y) \supset f(x, y) \vee m(x, y))$.

5.

$$
\begin{array}{l}
\forall x.\ \exists y.\ p(x, y) \vee \neg\exists x.\ \forall y.\ q(x, y) \\
\forall x.\ \exists y.\ p(x, y) \vee \neg\exists x'.\ \forall y'.\ q(x', y') \\
\forall x.\ \exists y.\ \forall x'.\ \exists y'\ (p(x, y) \vee \neg q(x', y')) \\
p(x, f(x)) \vee q(x', g(x, x'))
\end{array}
$$

6. $\Box(p \wedge \neg\Diamond q)$ ei kehti kuskil. (Võimalik põhjendus: $\vee q$ on kõikjal tõene, mis teeb, et $\neg\Diamond q$ on kõikjal väär ja samuti siis $p \wedge \neg\Diamond q$. Kuna igast maailmast on mõni maailm saavutatav, siis on kõikjal väär ka $\Box(p \wedge \neg\Diamond q)$.) $\Diamond p \supset \Box q$ kehtib maailmades a, c, d.

7. (i) Oletame, et maailmas w kehtib $\Box p$, ehk siis et igas w-st saavutatavas maailmas kehtib p. Peame näitama, et w-s kehtib $\Box\Box\Box p$. Selleks peab p kehtima igas maailmas w', mis on w-st saavutatav kolme sammuga. Iga w-st ühe või enama sammuga saavutatav maailm on aga transitiivsuse põhjal saavutatav ka ühe sammuga ning selliste jaoks teame eeldusest, et neis p kehtib.

(i) Oletame, et maailmas w kehtib $\Box(p \supset q)$, siis kehtib $p \supset q$ igas w-st saavutatavas maailmas ning transitiivsuse tõttu seega ka igas w-st kahe sammuga saavutatavas maailmas. Peame näitama, et $\Box(\Box p \supset \Box q)$. Selleks peab $\Box p \supset \Box q$ kehtima igas maailmast w ühe sammuga saavutatavas maailmas w'. Kujutame ette vastupidist, st et mingi w' jaoks nii ei ole. Siis peab $\Box q$ w'-s olema väär, st mingi w'-st saavutatava w'' puhul peab q olema w''-s väär. Samas $\Box p$ peab w'-s olema tõene, mis tingib, et p peab w''-s tõene olema. Niisiis ei kehti w''-s $p \supset q$. Kuna w'' on w-st kahe sammuga saavutatav, on ta w-st saavutatav ka ühe sammuga ja oleme saavutanud vastuolu oma eeldusega.

8. (i) Tingimus: igast tipust väljub ülimalt üks kaar, ehk: iga $w, w', w'' \in W$ korral kui wRw' ja wRw'', siis $w'' = w'$. (ii) Tingimus: igast tipust väljub ülimalt kaks kaart, ehk: iga $w, w', w'', w''' \in W$ korral kui wRw', wRw'' ja wRw''', siis $w''' = w'$ või $w''' = w''$.

9. $\forall w.\ (\exists w.'\ (wRw' \wedge q(w')) \supset \forall w'.\ (wRw' \supset \exists w.''\ (w'Rw'' \wedge r(w'')) \wedge \neg p(w')))$

10. (i) Agent 1 ei tea, et agent 2 teab, et p. (ii) Kui agent 1 teab, et p, siis agent 2 ei tea vastupidist (tema teadmised on agendi 1 omadega p suhtes kooskõlas. (iii) Kui agent 1 või agent 2 teab, et p, siis agent 3 teab ka.