

PREDIKAATLOOGIKA: HILBERTI SÜSTEEM

- Predikaatloogika Hilberti süsteem (üks mitmetest võimalikest)
 - Aksiomiskeemid:

$$\begin{aligned}
 & A \supset (B \supset A) \\
 & (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C)) \\
 & (A \supset \neg B) \supset ((A \supset B) \supset \neg A) \\
 & \neg\neg A \supset A \\
 & \top \\
 & \perp \supset C \\
 & A \supset (B \supset A \wedge B) \\
 & A \wedge B \supset A \\
 & A \wedge B \supset B
 \end{aligned}$$

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$$\begin{aligned}
 & A \supset A \vee B \\
 & B \supset A \vee B \\
 & (A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C)) \\
 & \forall x. (C \supset A) \supset (C \supset \forall x. A) \quad (x \notin \text{FV}(C)) \\
 & \forall x. A \supset A[t/x] \\
 & A[t/x] \supset \exists x. A \\
 & \forall x. (A \supset C) \supset (\exists x. A \supset C) \quad (x \notin \text{FV}(C))
 \end{aligned}$$

- Reeglid:

$$\frac{A \quad A \supset B}{B} \quad (\text{modus ponens})$$

$$\frac{A}{\forall x. A} \quad \text{üldistamisreegel}$$

Viimast reeglit on lubatud rakendada ainult siis, kui x ei esine vaba muutujana üheski kasutada olevas eelduses (assumption).

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PREDIKAATLOOGIKA: LOOMULIK TULETUS

- Standardesitus: tuletusreeglid [tuletusreeglid kasutavad eeldusi (assumptions)].

$$\begin{array}{c}
 \neg \neg \mathcal{I} \\
 \\
 \frac{A \quad B}{A \wedge B} \wedge \mathcal{I} \\
 \\
 \frac{A}{A \vee B} \vee \mathcal{I}_1 \quad \frac{B}{A \vee B} \vee \mathcal{I}_2 \\
 \\
 \frac{A \quad \dots \quad B}{A \supset B} \supset \mathcal{I}
 \end{array}
 \qquad
 \begin{array}{c}
 - \\
 \frac{}{C} \perp \mathcal{E} \\
 \\
 \frac{A \wedge B}{A} \wedge \mathcal{E}_1 \quad \frac{A \wedge B}{B} \wedge \mathcal{E}_2 \\
 \\
 \frac{A \quad B \quad \dots \quad C \quad C}{A \vee B \quad C} \vee \mathcal{E} \\
 \\
 \frac{A \supset B \quad A}{B} \supset \mathcal{E}
 \end{array}$$

$$\begin{array}{c}
 \frac{A[y/x]}{\forall x. A} \forall \mathcal{I}^* \quad \frac{\forall x. A}{A[t/x]} \forall \mathcal{E} \\
 \\
 \frac{A[t/x]}{\exists x. A} \exists \mathcal{I} \quad \frac{\exists x. A \quad C}{C} \exists \mathcal{E}^\dagger \\
 \\
 \frac{A \quad \dots \quad \perp}{\neg A} \neg \mathcal{I} \quad \frac{\neg A \quad A}{\perp} \neg \mathcal{E} \quad \frac{A \quad \neg A \quad \dots \quad C \quad C}{C} \text{dil.}
 \end{array}$$

* y ei tohi vabalt esineda valemis $\forall x. A$ ja kasutada olevates eeldustes

† y ei tohi vabalt esineda valemities $\exists x. A, C$ ja kasutada olevates eeldustes

- Loomuliku tuletuse sekventsiesitus: aksiomiskeemid ja tuletusreeglid.

$$\begin{array}{c}
 \overline{\Gamma \rightarrow \top} \top\mathcal{I} \\
 \\
 \frac{\Gamma \rightarrow A \quad \Gamma \rightarrow B}{\Gamma \rightarrow A \wedge B} \wedge\mathcal{I} \\
 \\
 \frac{\Gamma \rightarrow A}{\Gamma \rightarrow A \vee B} \vee\mathcal{I}_1 \quad \frac{\Gamma \rightarrow B}{\Gamma \rightarrow A \vee B} \vee\mathcal{I}_2 \\
 \\
 \frac{\Gamma, A \rightarrow B}{\Gamma \rightarrow A \supset B} \supset\mathcal{I} \\
 \\
 \overline{\Gamma, A \rightarrow A} \text{ ass.} \\
 \\
 \frac{\Gamma \rightarrow \perp}{\Gamma \rightarrow C} \perp\mathcal{E} \\
 \\
 \frac{\Gamma \rightarrow A \wedge B}{\Gamma \rightarrow A} \wedge\mathcal{E}_1 \quad \frac{\Gamma \rightarrow A \wedge B}{\Gamma \rightarrow B} \wedge\mathcal{E}_2 \\
 \\
 \frac{\Gamma \rightarrow A \vee B \quad \Gamma, A \rightarrow C \quad \Gamma, B \rightarrow C}{\Gamma \rightarrow C} \vee\mathcal{E} \\
 \\
 \frac{\Gamma \rightarrow A \supset B \quad \Gamma \rightarrow A}{\Gamma \rightarrow B} \supset\mathcal{E}
 \end{array}$$

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$$\begin{array}{c}
 \frac{\Gamma \rightarrow A[y/x]}{\Gamma \rightarrow \forall x. A} \forall\mathcal{I}^* \\
 \\
 \frac{\Gamma \rightarrow A[t/x]}{\Gamma \rightarrow \exists x. A} \exists\mathcal{I} \\
 \\
 \frac{\Gamma \rightarrow \forall x. A}{\Gamma \rightarrow A[t/x]} \forall\mathcal{E} \\
 \\
 \frac{\Gamma \rightarrow \exists x. A \quad \Gamma, A[y/x] \rightarrow C}{\Gamma \rightarrow C} \exists\mathcal{E}^\dagger \\
 \\
 \frac{\Gamma, A \rightarrow \perp}{\Gamma \rightarrow \neg A} \neg\mathcal{I} \\
 \\
 \frac{\Gamma \rightarrow \neg A \quad \Gamma \rightarrow A}{\Gamma \rightarrow \perp} \neg\mathcal{E} \\
 \\
 \frac{\Gamma, A \rightarrow C \quad \Gamma, \neg A \rightarrow C}{\Gamma \rightarrow C} \text{dil.}
 \end{array}$$

* y ei tohi vabalt esineda valemis $\forall x. A$ ja valemihulgas Γ

† y ei tohi vabalt esineda valemites $\exists x. A, C$ ja valemihulgas Γ

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PREDIKAATLOOGIKA: SEKVENTSIARVUTUS

- Aksiomiskeemid ja tuletusreeglid (tagasisuunalisele otsimisele orienteeritud süsteem):

$$\begin{array}{c}
 \overline{\Gamma, A \rightarrow A, \Delta} \text{ id.} \\
 \overline{\Gamma \rightarrow \top, \Delta} \top\mathcal{R} \\
 \frac{-}{\Gamma \rightarrow A, \Delta} \quad \frac{-}{\Gamma \rightarrow B, \Delta} \quad \frac{-}{\Gamma \rightarrow A \wedge B, \Delta} \wedge\mathcal{R} \\
 \frac{\Gamma \rightarrow A, B, \Delta}{\Gamma \rightarrow A \vee B, \Delta} \vee\mathcal{R} \\
 \frac{\Gamma, A \rightarrow B, \Delta}{\Gamma \rightarrow A \supset B, \Delta} \supset\mathcal{R} \\
 \frac{\Gamma, A \rightarrow \Delta}{\Gamma \rightarrow \neg A, \Delta} \neg\mathcal{R} \\
 \frac{-}{\Gamma, \perp \rightarrow \Delta} \perp\mathcal{L} \\
 \frac{\Gamma, A, B \rightarrow \Delta}{\Gamma, A \wedge B \rightarrow \Delta} \wedge\mathcal{L} \\
 \frac{\Gamma, A \rightarrow \Delta \quad \Gamma, B \rightarrow \Delta}{\Gamma, A \vee B \rightarrow \Delta} \vee\mathcal{L} \\
 \frac{\Gamma \rightarrow A, \Delta \quad \Gamma, B \rightarrow \Delta}{\Gamma, A \supset B \rightarrow \Delta} \supset\mathcal{L} \\
 \frac{\Gamma \rightarrow A, \Delta}{\Gamma, \neg A \rightarrow \Delta} \neg\mathcal{L}
 \end{array}$$

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$$\begin{array}{c}
 \frac{\Gamma \rightarrow A[y/x], \Delta}{\Gamma \rightarrow \forall x. A, \Delta} \forall\mathcal{R}^* \\
 \frac{\Gamma \rightarrow A[t/x], \exists x. A, \Delta}{\Gamma \rightarrow \exists x. A, \Delta} \exists\mathcal{R} \\
 \frac{\Gamma, \forall x. A, A[t/x] \rightarrow \Delta}{\Gamma, \forall x. A \rightarrow \Delta} \forall\mathcal{L} \\
 \frac{\Gamma, A[y/x] \rightarrow \Delta}{\Gamma, \exists x. A \rightarrow \Delta} \exists\mathcal{L}^\dagger
 \end{array}$$

* y ei tohi vabalt esineda valemis $\forall x. A$ ja valemihulkades Γ, Δ

† y ei tohi vabalt esineda valemis $\exists x. A$ ja valemihulkades Γ, Δ