KONTROLLTÖÖ

1. Määra semantiliste tabelitega, kas järgmised valemid on üldkehtivad või vääratavad. Vääratavuse korral anna kontramudel.

$$p \wedge (q \vee r) \supset (p \wedge q) \vee r$$
$$\forall x.\ \exists y.\ p(x, y) \supset \exists x.\ \forall y.\ p(x, y)$$

2. Tõesta loomulikus tuletuses:

$$(p \supset q \vee r) \supset ((q \supset r) \supset (p \vee q \supset r))$$
$$\exists x.\ p(g(x), a) \supset \exists y.\ \exists z.\ p(y, z)$$

3. Tõesta sekventsiarvutuses:

$$\neg(p \wedge \neg r) \wedge p \supset r$$
$$\neg\forall x.\ \neg p(f(x)) \supset \exists x.\ p(x)$$

4. Teisenda klauselkujule:

$$\forall y.\ (\exists x.\ p(x, y) \supset q(y, z)) \wedge \exists y.\ (\forall x. r(x, y) \vee q(x, y))$$

5. Leia kõige üldisem unifikaator atomaarvalemitele $p(f(a, c), g(x), x)$, $p(y, g(h(z)), w)$.

6. Resolveeri klauslid $p(a, y) \vee r(y, f(y))$, $\neg r(g(z), w) \vee s(z) \vee \neg t(w, v)$.

LAHENDUSED

1.

$$\mathbf{F}(p \wedge (q \vee r) \supset (p \wedge q) \vee r)$$
$$\mathbf{T}(p \wedge (q \vee r))$$
$$\mathbf{F}((p \wedge q) \vee r)$$
$$\mathbf{T}p$$
$$\mathbf{T}(q \vee r)$$
$$\mathbf{F}(p \wedge q)$$
$$\mathbf{F}r$$

$\mathbf{T}q$		$\mathbf{T}r$
$\mathbf{F}p$	$\mathbf{F}q$	$\times$
$\times$	$\times$	

Valem on üldkehtiv.

$$\mathbf{F}(\forall x.\ \exists y.\ p(x,y) \supset \exists x.\ \forall y.\ p(x,y))$$
$$\mathbf{T}(\forall x.\ \exists y.\ p(x,y))$$
$$\mathbf{F}(\exists x.\ \forall y.\ p(x,y))$$
$$\mathbf{T}(\exists y.\ p(x_0,y))$$
$$\mathbf{F}(\forall y.\ p(x_0,y))$$
$$\mathbf{T}(p(x_0,x_{01}))$$
$$\mathbf{F}(p(x_0,x_{02}))$$
$$\ldots$$
$$\mathbf{T}(\exists y.\ p(x_s,y))$$
$$\mathbf{F}(\forall y.\ p(x_s,y))$$
$$\mathbf{T}(p(x_s,x_{s1}))$$
$$\mathbf{F}(p(x_s,x_{s2}))$$
$$\ldots$$
$$\bigcirc$$

Valem on vääratav. Tabelist tulenev kontramudel on (D, I), kus $D = \{x_0, x_{01}, x_{02}, x_{011}, x_{012}, x_{021}, x_{022}, \ldots\}$ ja $I(p)(x_s, x_{s1}) = 1$, $I(p)(x_s, x_{s2}) = 0$.

On ka lihtsamaid kontramudeleid, juba kahest elemendist põhihulgas piisab.

2.

$$
\dfrac{
\dfrac{
\overset{+3}{p \lor q}
\quad
\dfrac{
\dfrac{\overset{+1}{p \supset q \lor r} \quad \overset{+4}{p}}{q \lor r}\supset\mathcal{E}
\quad
\dfrac{\overset{+2}{q \supset r} \quad \overset{+6}{q}}{r}\supset\mathcal{E}
\quad
\overset{+7}{r}
}{r}\lor\mathcal{E}, -6, -7
\quad
\dfrac{\overset{+2}{q \supset r} \quad \overset{+5}{q}}{r}\supset\mathcal{E}
}{
\dfrac{r}{p \lor q \supset r}\supset\mathcal{I}, -3
}\lor\mathcal{E}, -4, -5
}{
\dfrac{(q \supset r) \supset (p \lor q \supset r)}{(p \supset q \lor r) \supset ((q \supset r) \supset (p \lor q \supset r))}\supset\mathcal{I}, -1
}\supset\mathcal{I}, -2
$$

$$
\dfrac{
\dfrac{
\overset{+1}{\exists x.\, p(g(x), a)}
\quad
\dfrac{\dfrac{\overset{+2}{p(g(x'), a)}}{\exists z.\, p(g(x'), z)}\exists\mathcal{I}}{\exists y.\, \exists z.\, p(y, z)}\exists\mathcal{I}
}{\exists y.\, \exists z.\, p(y, z)}\exists\mathcal{E}, -2, x' \text{ fresh}
}{\exists x.\, p(g(x), a) \supset \exists y.\, \exists z.\, p(y, z)}\supset\mathcal{I}, -1
$$

3.

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\overset{\text{Id}}{p \to p, r}
\quad
\dfrac{\overset{\text{Id.}}{p, r \to r}}{p \to \neg r, r}\neg\mathcal{R}
}{p \to p \land \neg r, r}\land\mathcal{R}
}{\neg(p \land \neg r), p \to r}\neg\mathcal{L}
}{\neg(p \land \neg r) \land p \to r}\land\mathcal{L}
}{\to \neg(p \land \neg r) \land p \supset r}\supset\mathcal{R}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{Id.}}{p(f(x')) \to p(f(x')), \exists x.\, p(x)}}{p(f(x')) \to \exists x.\, p(x)}\ \exists\mathcal{R}}{\to \neg p(f(x')), \exists x.\, p(x)}\ \neg\mathcal{R}}{\to \forall x.\, \neg p(f(x)), \exists x.\, p(x)}\ \forall\mathcal{R}, x' \text{ fresh}}{\neg\forall x.\, \neg p(f(x)) \to \exists x.\, p(x)}\ \neg\mathcal{L}}{\to \neg\forall x.\, \neg p(f(x)) \supset \exists x.\, p(x)}\ \supset\mathcal{R}$$

4.

$$\forall y.\, (\exists x.\, p(x,y) \supset q(y,z)) \wedge \exists y.\, (\forall x.\, r(x,y) \vee q(x,y))$$
$$\forall y.\, (\exists x'.\, p(x',y) \supset q(y,z)) \wedge \exists y'.\, (\forall x''.\, r(x'',y') \vee q(x,y'))$$
$$\forall y.\, (\neg\exists x'.\, p(x',y) \vee q(y,z)) \wedge \exists y'.\, (\forall x''.\, r(x'',y') \vee q(x,y'))$$
$$\forall y.\, (\forall x'.\, \neg p(x',y) \vee q(y,z)) \wedge \exists y'.\, (\forall x''.\, r(x'',y') \vee q(x,y'))$$
$$\forall y.\, \forall x'.\, (\neg p(x',y) \vee q(y,z)) \wedge \exists y'.\, \forall x''.\, (r(x'',y') \vee q(x,y'))$$
$$(\forall z.\, \forall x.)\forall y.\, \forall x'.\, \exists y'.\, \forall x''.\ ((\neg p(x',y) \vee q(y,z)) \wedge (r(x'',y') \vee q(x,y')))$$
$$((\neg p(x',y) \vee q(y,z)) \wedge (r(x'', f(z,x,y,x',x'')) \vee q(x, f(z,x,y,x',x''))))$$

5. Kahe atomaarvalemi kõige üldisem unifikaator on $[f(a,c)/y, h(z)/x, h(z)/w]$.
6. Konfliktsed literaalid on A ja $\neg A'$, kus $A = r(y, f(y))$ ja $A' = r(g(z), w)$.
   A ja A' kõige üldisem unifikaator on $[g(z)/y, f(g(z))/w]$.
   Klauslite resolvent on $p(a, g(z)) \vee s(z) \vee \neg t(f(g(z)), v)$.