LAUSELOOGIKA: SEKVENTSIARVUTUS

- Sekventsiarvutused on kolmas alternatiiv tõestussüsteeme Hilberti süsteemide ja loomuliku tuletuse süsteemide kõrval.

  Neid ei tohi segamini ajada loomuliku tuletuse sekventsiesitustega.

- Kui loomulikus tuletuses olid igal konnektiivil oma sissetoomise ja väljaviimise reeglid, siis sekventsiarvutustes on igal konnektiivil oma parema ja vasaku poole reeglid.

- Sekventsid on sedapuhku nö mitmejärelduselised: sekvents on figuur kujul $\Gamma \to \Delta$, kus nii Γ kui ka Δ on lõplikud (võibolla tühjad) hulgad valemeid (taas hulgad, mitte listid või multihulgad, st järjestus, kordsus ei loe).

- Valem A loetatakse tõestatavaks, kui on tõestatav sekvents $\to A$.

- Sekventsi $A_1, \ldots, A_n \to B_1, \ldots, B_m$ võib samastada valemiga $A_1 \wedge \ldots \wedge A_n \supset B_1 \vee \ldots \vee B_m$; meenutame, et 0 valemi konjuktsioon on $\top$ ja 0 valemi disjunktsioon on $\bot$.

- Sekventsiarvutuse aksioomiskeemid ja tuletusreeglid (tagasisuunalisele otsimisele häälestatud versiooni jaoks):

$$\frac{}{\Gamma, A \to A, \Delta}\ \text{hyp}$$

$$\frac{}{\Gamma \to \top, \Delta}\ \top\mathcal{R} \qquad -$$

$$- \qquad \frac{}{\Gamma, \bot \to \Delta}\ \bot\mathcal{L}$$

$$\frac{\Gamma \to A, \Delta \quad \Gamma \to B, \Delta}{\Gamma \to A \wedge B, \Delta}\ \wedge\mathcal{R} \qquad \frac{\Gamma, A, B \to \Delta}{\Gamma, A \wedge B \to \Delta}\ \wedge\mathcal{L}$$

$$\frac{\Gamma \to A, B, \Delta}{\Gamma \to A \vee B, \Delta}\ \vee\mathcal{R} \qquad \frac{\Gamma, A \to \Delta \quad \Gamma, B \to \Delta}{\Gamma, A \vee B \to \Delta}\ \vee\mathcal{L}$$

$$\frac{\Gamma, A \to B, \Delta}{\Gamma \to A \supset B, \Delta}\ \supset\mathcal{R} \qquad \frac{\Gamma \to A, \Delta \quad \Gamma, B \to \Delta}{\Gamma, A \supset B \to \Delta}\ \supset\mathcal{L}$$

$$\frac{\Gamma, A \to \Delta}{\Gamma \to \neg A, \Delta}\ \neg\mathcal{R} \qquad \frac{\Gamma \to A, \Delta}{\Gamma, \neg A \to \Delta}\ \neg\mathcal{L}$$

- Mõned näited:

$$
\dfrac{
  \dfrac{}{p \wedge q \supset r, p \to p, r}\ \text{hyp.}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{}{q, p \to p, r}\ \text{hyp.}
      \quad
      \dfrac{}{q, p \to q, r}\ \text{hyp.}
    }{q, p \to p \wedge q, r}\ \wedge\mathcal{R}
    \qquad
    \dfrac{}{q, r, p \to r}\ \text{hyp.}
  }{q, p \wedge q \supset r, p \to r}\ \supset\mathcal{L}
}{
  \dfrac{
    \dfrac{
      \dfrac{p \supset q, p \wedge q \supset r, p \to r}{p \supset q, p \wedge q \supset r \to p \supset r}\ \supset\mathcal{R}
    }{(p \supset q) \wedge (p \wedge q \supset r) \to p \supset r}\ \wedge\mathcal{L}
  }{\to (p \supset q) \wedge (p \wedge q \supset r) \supset (p \supset r)}\ \supset\mathcal{R}
}\ \supset\mathcal{L}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p \to p, q}\ \text{hyp.} \qquad \dfrac{}{q, p \to q}\ \text{hyp.}}{p \supset q, p \to q}\ \supset\mathcal{L}}{p \supset q, \neg q, p \to}\ \neg\mathcal{L}}{p \supset q, \neg q \to \neg p}\ \neg\mathcal{R}}{p \supset q \to \neg q \supset \neg p}\ \supset\mathcal{R}}{\to (p \supset q) \supset (\neg q \supset \neg p)}\ \supset\mathcal{R}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{q \supset r, p \to p, r}\ \text{hyp.} \qquad \dfrac{}{r, q \supset r, p \to r}\ \text{hyp.}}{p \supset r, q \supset r, p \to r}\ \supset\mathcal{L} \qquad \dfrac{\dfrac{}{p \supset r, q \to q, r}\ \text{hyp.} \qquad \dfrac{}{p \supset r, r, q \to r}\ \text{hyp.}}{p \supset r, q \supset r, q \to r}\ \supset\mathcal{L}}{p \supset r, q \supset r, p \vee q \to r}\ \vee\mathcal{L}}{p \supset r, q \supset r \to p \vee q \supset r}\ \supset\mathcal{R}}{p \supset r \to (q \supset r) \supset (p \vee q \supset r)}\ \supset\mathcal{R}}{\to (p \supset r) \supset ((q \supset r) \supset (p \vee q \supset r))}\ \supset\mathcal{R}
$$

PREDIKAATLOOGIKA: SEKVENTSIARVUTUS

- Predikaatarvutuse sekventiarvutuses lisanduvad lauseloogika sekventsiarvutusele järgmised reeglid kvantorite kohta:

$$\frac{\Gamma \to A[y/x], \Delta}{\Gamma \to \forall x\ A, \Delta}\ \forall\mathcal{R}^\star \qquad \frac{\Gamma, \forall x\ A, A[t/x] \to \Delta}{\Gamma, \forall x\ A \to \Delta}\ \forall\mathcal{L}$$

$$\frac{\Gamma \to A[t/x], \exists x\ A, \Delta}{\Gamma \to \exists x\ A, \Delta}\ \exists\mathcal{R} \qquad \frac{\Gamma, A[y/x] \to \Delta}{\Gamma, \exists x\ A \to \Delta}\ \exists\mathcal{L}^\dagger$$

* y ei tohi vabalt esineda valemis $\forall x\ A$ ja valemihulkades Γ, Δ

† y ei tohi vabalt esineda valemis $\exists x\ A$ ja valemihulkades Γ, Δ

- Näide:

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{p(f(x'),y') \to p(f(x'),y')}\ \text{hyp.}}{p(f(x'),y') \to \exists y\, p(f(x'),y)}\ \exists\mathcal{R}}{p(f(x'),y') \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{R} \quad \dfrac{\dfrac{\dfrac{}{p(y',g(x')) \to p(y',g(x'))}\ \text{hyp.}}{p(y',g(x')) \to \exists y\, p(y',y)}\ \exists\mathcal{R}}{p(y',g(x')) \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{R}}{\dfrac{p(f(x'),y') \lor p(y',g(x')) \to \exists x\, \exists y\, p(x,y)}{\dfrac{\exists y\, (p(f(x'),y) \lor p(y,g(x'))) \to \exists x\, \exists y\, p(x,y)}{\dfrac{\exists x\, \exists y\, (p(f(x),y) \lor p(y,g(x))) \to \exists x\, \exists y\, p(x,y)}{\to \exists x\, \exists y\, (p(f(x),y) \lor p(y,g(x))) \supset \exists x\, \exists y\, p(x,y)}\ \supset\mathcal{R}}\ \exists\mathcal{L}, x'}\ \exists\mathcal{L}, y'}\ \lor\mathcal{L}
$$