PREDIKAATLOOGIKA: LOOMULIK TULETUS

- Standardesitus: lauseloogika loomuliku tuletuse standardesituse reeglitele lisanduvad järgmised:

$$\frac{A[y/x]}{\forall x\ A}\ \forall\mathcal{I}^{\star} \qquad\qquad \frac{\forall x\ A}{A[t/x]}\ \forall\mathcal{E}$$

$$\frac{A[t/x]}{\exists x\ A}\ \exists\mathcal{I} \qquad\qquad \frac{\exists x\ A \quad \begin{array}{c} A[y/x] \\ \vdots \\ C \end{array}}{C}\ \exists\mathcal{E}^{\dagger}$$

* y ei tohi vabalt esineda valemis $\forall x\ A$ ja kasutada olevates hüpoteesides

† y ei tohi vabalt esineda valemites $\exists x\ A$, C ja kasutada olevates hüpoteesides

- Näiteid tõestustest:

$$\dfrac{\overset{+1}{\exists x\, \forall y\, p(x,y)} \quad \dfrac{\dfrac{\overset{+2}{\forall y\, p(x',y)}}{p(x',y')}\,\forall\mathcal{E}}{\exists x\, p(x,y')}\,\exists\mathcal{I}}{\dfrac{\dfrac{\exists x\, p(x,y')}{\forall y\, \exists x\, p(x,y)}\,\forall\mathcal{I}, y' \text{ värske}}{\exists x\, \forall y\, p(x,y) \supset \forall y\, \exists x\, p(x,y)}\,\supset\mathcal{I}, -1}\,\exists\mathcal{E}, -2, x' \text{ värske}$$

$$\dfrac{\dfrac{\dfrac{\overset{+1}{\exists x\, p(x)} \quad \dfrac{\dfrac{\overset{+2}{\forall x\, \neg p(x)}}{\neg p(x')}\,\forall\mathcal{E} \quad \overset{+3}{p(x')}}{\bot}\,\neg\mathcal{E}}{\bot}\,\exists\mathcal{E}, -3, x' \text{ värske}}{\neg \forall x\, \neg p(x)}\,\neg\mathcal{I}, -2}{\exists x\, p(x) \supset \neg\forall x\, \neg p(x)}\,\supset\mathcal{I}, -1$$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \overset{+1}{\neg\forall x\ \neg p(x)}
      \qquad
      \dfrac{
        \dfrac{
          \dfrac{
            \overset{+2}{\neg\exists x\ p(x)}
            \qquad
            \dfrac{\overset{+4}{p(x')}}{\exists x\ p(x)}\ \exists\mathcal{I}
          }{\bot}\ \neg\mathcal{E}
        }{\neg p(x')}\ \neg\mathcal{I}, -4
      }{\forall x\ \neg p(x)}\ \forall\mathcal{I}, x' \text{ värske}
    }{
      \dfrac{\bot}{\exists x\ p(x)}\ \bot\mathcal{E}
    }\ \neg\mathcal{E}
    \qquad
    \overset{+3}{\exists x\ p(x)}
  }{\exists x\ p(x)}\ \text{dilemma}, -2, -3
}{\neg\forall x\ \neg p(x) \supset \exists x\ p(x)}\ \supset\mathcal{I}, -1
$$

- Veel näiteid:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\overset{+1}{\exists x\, p(x) \supset q} \qquad \dfrac{\overset{+1}{p(x')}}{\exists x\, p(x)}\ \exists\mathcal{I}}{q}\ \supset\mathcal{E}}{p(x') \supset q}\ \supset\mathcal{I}, -2}{\forall x\,(p(x) \supset q)}\ \forall\mathcal{I}, x' \text{ värske}}{(\exists x\, p(x) \supset q) \supset \forall x\,(p(x) \supset q)}\ \supset\mathcal{I}, -1
$$

$$
\dfrac{\dfrac{\overset{+1}{\exists x\, p(f(x))} \qquad \dfrac{\overset{+2}{p(f(x'))}}{\exists x\, p(x)}\ \exists\mathcal{I}}{\exists x\, p(x)}\ \exists\mathcal{E}, x' \text{ värske}, -2}{(\exists x\, p(f(x)) \supset \exists x\, p(x)}\ \supset\mathcal{I}, -1
$$

- Sekventsiesitus: lauseloogika loomuliku tuletuse sekventsiesituse reeglitele lisanduvad järgmised:

$$\frac{\Gamma \to A[y/x]}{\Gamma \to \forall\, x\ A}\ \forall\mathcal{I}^\star \qquad \frac{\Gamma \to \forall x\ A}{\Gamma \to A[t/x]}\ \forall\mathcal{E}$$

$$\frac{\Gamma \to A[t/x]}{\Gamma \to \exists\, x\ A}\ \exists\mathcal{I} \qquad \frac{\Gamma \to \exists x\ A \quad \Gamma, A[y/x] \to C}{\Gamma \to C}\ \exists\mathcal{E}^\dagger$$

* y ei tohi vabalt esineda valemis $\forall x\ A$ ja valemihulgas Γ

† y ei tohi vabalt esineda valemites $\exists x\ A$, C ja valemihulgas Γ

PREDIKAATLOOGIKA: SEKVENTSIARVUTUS

- Predikaatarvutuse sekventiarvutuses lisanduvad lauseloogika sekventsiarvutusele järgmised reeglid kvantorite kohta:

$$\frac{\Gamma \to A[y/x], \Delta}{\Gamma \to \forall x\, A, \Delta}\ \forall\mathcal{R}^{\star} \qquad \frac{\Gamma, (\forall x\, A,)\, A[t/x] \to \Delta}{\Gamma, \forall x\, A \to \Delta}\ \forall\mathcal{L}$$

$$\frac{\Gamma \to A[t/x], (\exists x\, A,)\, \Delta}{\Gamma \to \exists x\, A, \Delta}\ \exists\mathcal{R} \qquad \frac{\Gamma, A[y/x] \to \Delta}{\Gamma, \exists x,\ A \to \Delta}\ \exists\mathcal{L}^{\dagger}$$

* y ei tohi vabalt esineda valemis $\forall x\ A$ ja valemihulkades Γ, Δ

† y ei tohi vabalt esineda valemis $\exists x\ A$ ja valemihulkades Γ, Δ

- Näiteid tõestustest:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p(x',y') \to p(x',y')}\ \text{hyp.}}{\forall y\, p(x',y) \to p(x',y')}\ \forall\mathcal{L}}{\forall y\, p(x',y) \to \exists x\, p(x,y')}\ \exists\mathcal{R}}{\exists x\, \forall y\, p(x,y) \to \exists x\, p(x,y')}\ \exists\mathcal{L}, x'\ \text{värske}}{\exists x\, \forall y\, p(x,y) \to \forall y\, \exists x\, p(x,y)}\ \forall\mathcal{R}, y'\ \text{värske}}{\to \exists x\, \forall y\, p(x,y) \supset \forall y\, \exists x\, p(x,y)}\ \supset\mathcal{R}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p(x') \to p(x')}\ \text{hyp.}}{p(x'), \neg p(x') \to}\ \neg\mathcal{L}}{p(x'), \forall x\, \neg p(x) \to}\ \forall\mathcal{L}}{\exists x\, p(x), \forall x\, \neg p(x) \to}\ \exists\mathcal{L}, x'\ \text{värske}}{\exists x\, p(x) \to \neg\forall x\, \neg p(x)}\ \neg\mathcal{R}}{\to \exists x\, p(x) \supset \neg\forall x\, \neg p(x)}\ \supset\mathcal{R}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p(f(x'),y') \to p(f(x'),y')}\ \text{hyp.}}{p(f(x'),y') \to \exists y\, p(f(x'),y)}\ \exists\mathcal{R}}{p(f(x'),y') \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{R} \qquad \dfrac{\dfrac{\dfrac{}{p(y',g(x')) \to p(y',g(x'))}\ \text{hyp.}}{p(y',g(x')) \to \exists y\, p(y',y)}\ \exists\mathcal{R}}{p(y',g(x')) \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{R}}{p(f(x'),y') \vee p(y',g(x')) \to \exists x\, \exists y\, p(x,y)}\ \vee\mathcal{L}}{\exists y\, (p(f(x'),y) \vee p(y,g(x'))) \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{L}, y' \text{ värske}}{\exists x\, \exists y\, (p(f(x),y) \vee p(y,g(x))) \to \exists x\, \exists y\, p(x,y)}\ \exists\mathcal{L}, x' \text{ värske}}{\to \exists x\, \exists y\, (p(f(x),y) \vee p(y,g(x))) \supset \exists x\, \exists y\, p(x,y)}\ \supset\mathcal{R}$$