

Kontrolltöö 12.11.2012
Lahendused

1.

$$\begin{array}{c}
 \frac{\frac{\frac{\forall x (p(x) \supset q(f(x, a)))}{p(y') \supset q(f(y', a))} \forall \mathcal{E} \quad \frac{+3}{p(y')} \supset \mathcal{E}}{\frac{+2}{\exists y p(y)} \quad \frac{\frac{q(f(y', a))}{\exists z q(z)} \exists \mathcal{I}}{\exists z q(z)} \exists \mathcal{E}, -3, y' \text{ värske}}{\supset \mathcal{I}, -2}} \\
 \frac{\supset \mathcal{I}, -1}{\forall x (p(x) \supset q(f(x, a))) \supset (\exists y p(y) \supset \exists z q(z))}
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{p(y') \rightarrow p(y'), q(f(y', a)) \text{ hyp.} \quad \frac{q(f(y', a)), p(y') \rightarrow p(y') \text{ hyp.}}{p(y') \supset q(f(y', a)), p(y') \rightarrow q(f(y', a))} \forall \mathcal{L}}{\frac{p(y') \supset q(f(y', a)), p(y') \rightarrow \exists z q(z)}{\forall x (p(x) \supset q(f(x, a))), p(y') \rightarrow \exists z q(z)} \exists \mathcal{R}} \exists \mathcal{L}, y' \text{ värske} \\
 \frac{\frac{\frac{\frac{\forall x (p(x) \supset q(f(x, a))), \exists y p(y) \rightarrow \exists z q(z)}{\forall x (p(x) \supset q(f(x, a))), \exists y p(y) \rightarrow \exists z q(z)} \supset \mathcal{R}}{\forall x (p(x) \supset q(f(x, a))) \rightarrow \exists y p(y) \supset \exists z q(z)} \supset \mathcal{R}}{\rightarrow \forall x (p(x) \supset q(f(x, a))) \supset (\exists y p(y) \supset \exists z q(z))} \supset \mathcal{R}
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{\forall x p(x) \wedge \exists y q(y)}{p(f(y')) \wedge q(y')} \wedge \mathcal{E}_L \quad \frac{+2}{q(y')} \wedge \mathcal{I}}{\frac{+1}{\exists y q(y)} \wedge \mathcal{E}_R \quad \frac{p(f(y')) \wedge q(y')}{\exists z (p(f(z)) \wedge q(z))} \exists \mathcal{I}}{\exists z (p(f(z)) \wedge q(z))} \exists \mathcal{E}, -2, y' \text{ värske}}{\supset \mathcal{I}, -1}} \\
 \frac{\supset \mathcal{I}, -1}{\forall x p(x) \wedge \exists y q(y) \supset \exists z (p(f(z)) \wedge q(z))}
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{p(f(y')), q(y') \rightarrow p(f(y')) \text{ hyp.} \quad \frac{p(f(y')), q(y') \rightarrow q(y') \text{ hyp.}}{p(f(y')), q(y') \rightarrow p(f(y')) \wedge q(y')} \wedge \mathcal{R}}{\frac{\forall x p(x), q(y') \rightarrow p(f(y')) \wedge q(y')}{\forall x p(x), q(y') \rightarrow \exists z (p(f(z)) \wedge q(z))} \exists \mathcal{R}} \exists \mathcal{L}, y' \text{ värske} \\
 \frac{\frac{\frac{\forall x p(x), \exists y q(y) \rightarrow \exists z (p(f(z)) \wedge q(z))}{\forall x p(x) \wedge \exists y q(y) \rightarrow \exists z (p(f(z)) \wedge q(z))} \wedge \mathcal{L}}{\rightarrow \forall x p(x) \wedge \exists y q(y) \supset \exists z (p(f(z)) \wedge q(z))} \supset \mathcal{R}
 \end{array}$$

2.

$$\begin{aligned}
 & \forall x (\exists y (p(x, y) \supset \forall z q(y, z)) \supset \exists w r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w (\neg p(x, y) \vee \forall z q(y, z) \supset r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w (\forall z (\neg p(x, y) \vee q(y, z)) \supset r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w \exists z (\neg p(x, y) \vee q(y, z) \supset r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w \exists z (\neg(\neg p(x, y) \vee q(y, z)) \vee r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w \exists z ((p(x, y) \wedge \neg q(y, z)) \vee r(w)) \\
 \Leftrightarrow & \quad \forall x \forall y \exists w \exists z ((p(x, y) \vee r(w)) \wedge (\neg q(y, z) \vee r(w))) \\
 \text{samakeht.} & \quad (\neg p(x, y) \vee r(g(x, y))) \wedge (q(y, f(x, y)) \vee r(g(x, y)))
 \end{aligned}$$

Kvantorite jaoks on ka teisi korrektseid järjekordi, mis annab erinevad tulemused, nt:

$$\begin{aligned}
 & \dots \\
 \Leftrightarrow & \quad \forall x \exists w \forall y \exists z ((p(x, y) \vee r(w)) \wedge (\neg q(y, z) \vee r(w))) \\
 \text{samakeht.} & \quad (\neg p(x, y) \vee r(g(x))) \wedge (q(y, f(x, y)) \vee r(g(x)))
 \end{aligned}$$

- 3.
- Keegi on õnnelik ainult siis, kui ta pole murelik.
 $\forall x (h(x) \supset \neg a(x))$
 - Kui keegi on murelik, siis pole ta õnnelik.
 $\forall x (a(x) \supset \neg h(x))$
 - Kõik on murelikud, kui keegi on murelik.
 $\exists x a(x) \supset \forall x a(x)$
 - Kellel on murelik sõber, see pole murelik.
 $\forall x (\exists y (a(y) \wedge f(x, y)) \supset \neg a(x))$ või $\forall x \forall y (a(y) \wedge f(x, y) \supset \neg a(x))$
 - Tom on õnnelik või Jane on murelik.
 $h(t) \vee a(j)$
 - Kui Tom on õnnelik, on kõik õnnelikud.
 $h(t) \supset \forall x h(x)$