Functional Quantum Programming

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University of Nottingham

based on joint work with Jonathan Grattage

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Background

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no Nature is classical after all!

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- Nielsen and Chuang, p.7, *Coming up with good quantum algorithms is hard.*
- Richard Josza, QPL 2004: *We need to develop quantum thinking!*

QML

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- Analogy with classical computation

$\mathbf{FCC}$	Finite classical computations
$\mathbf{FQC}$	Finite quantum computations

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  FCC Finite classical computations

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- Important issue: **control of decoherence**
- Compiler under construction (Jonathan)

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Matrix

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QML

$$\begin{aligned} &had : Q_2 \multimap Q_2 \\ &had\ x = \mathbf{if}^{\circ}\ x \\ &\qquad\qquad \mathbf{then}\ \{\,\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\,\} \\ &\qquad\qquad \mathbf{else}\ \ \{\,\mathrm{qfalse} \mid \mathrm{qtrue}\,\} \end{aligned}$$

Deutsch algorithm

$$deutsch : 2 \multimap 2 \multimap Q_2$$

$$
\begin{array}{l}
deutsch\ a\ b = \\
\quad \textbf{let}\ (x, y) = \textbf{if}^\circ \{\,\mathrm{qfalse} \mid \mathrm{qtrue}\,\} \\
\qquad\qquad\qquad \textbf{then}\ (\mathrm{qtrue}, \textbf{if}\ a \\
\qquad\qquad\qquad\qquad\qquad \textbf{then}\ \{\,\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\,\} \\
\qquad\qquad\qquad\qquad\qquad \textbf{else}\ \ \{\,(-1)\ \mathrm{qfalse} \mid \mathrm{qtrue}\,\} \\
\qquad\qquad\qquad \textbf{else}\ (\mathrm{qfalse},\ \textbf{if}\ b \\
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\qquad\qquad\qquad\qquad\qquad \textbf{else}\ \ \{\,\mathrm{qfalse} \mid (-1)\ \mathrm{qtrue}\,\} \\
\quad \textbf{in}\ H\ x
\end{array}
$$

Overview

1. Finite classical computation
2. Finite quantum computation
3. QML basics
4. Compiling QML
5. Conclusions and further work

1. Semantics

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- **However:** Newtonian mechanics, Maxwellian electrodynamics are also time-reversible. . .
- . . . hence classical computation **should be** based on reversible operations.

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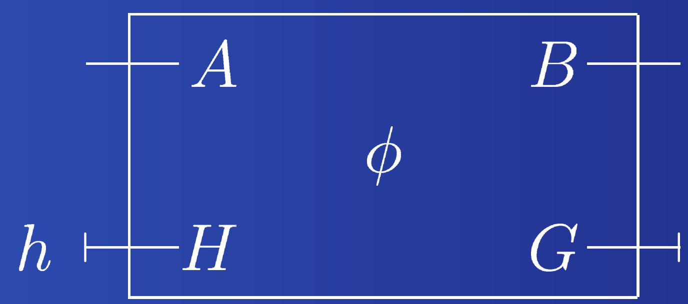

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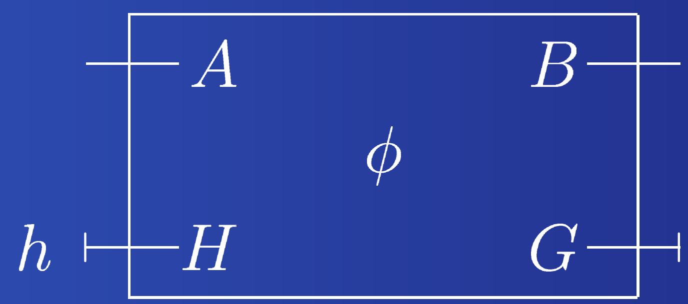

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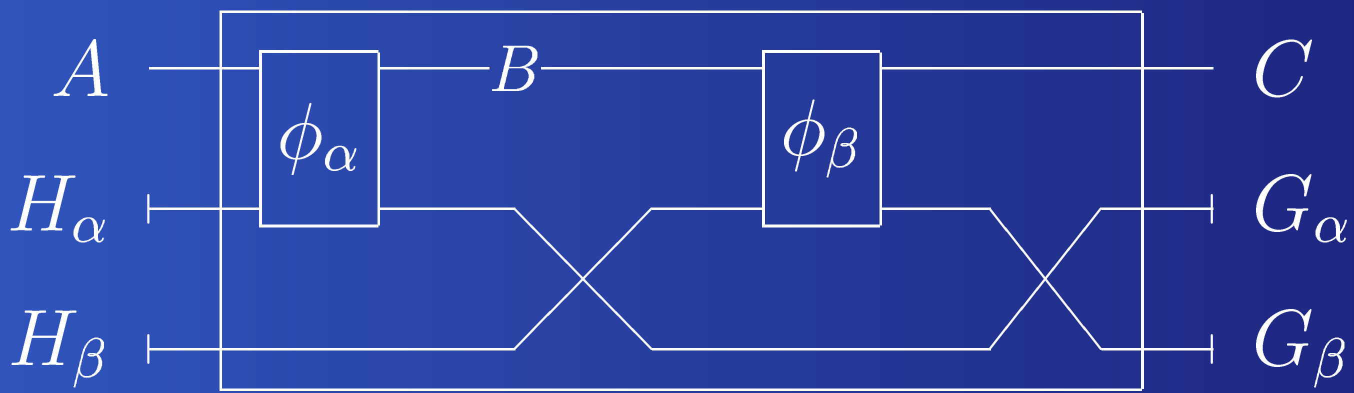
Classical computation (FCC)

Given finite sets A (input) and B (output):

- a finite set of initial heaps H,
- an initial heap $h \in H$,
- a finite set of garbage states G,
- a bijection $\phi \in A \times H \simeq B \times G$,

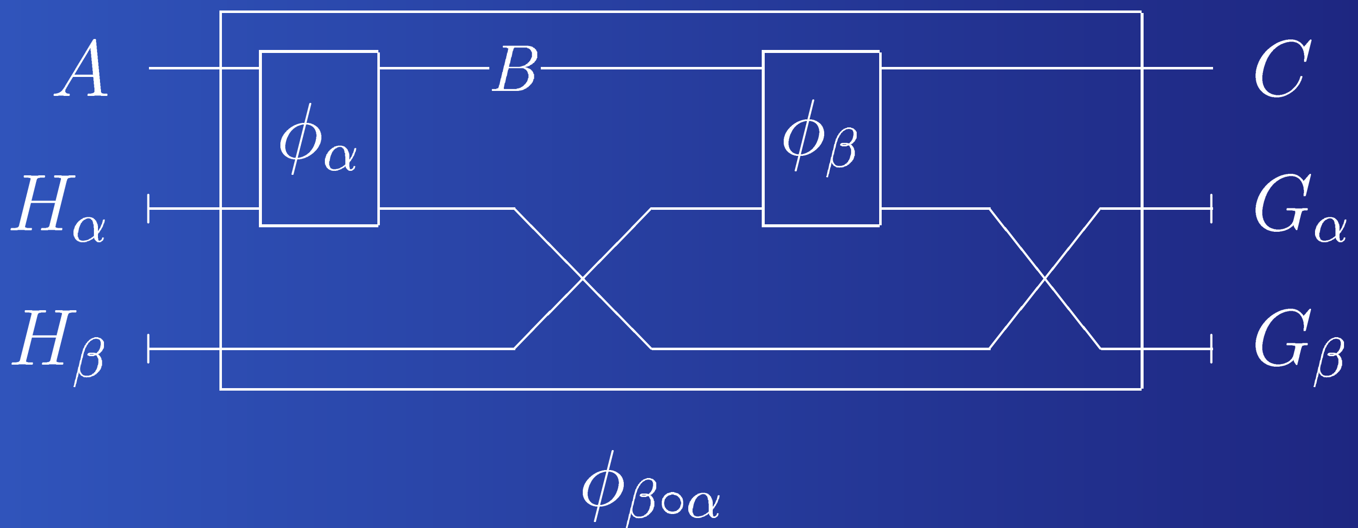
Composing classical computations

Composing classical computations



$\phi_{\beta\circ\alpha}$

Composing classical computations



Theorem:

$$\mathbf{U}\,(\beta \circ \alpha) = (\mathbf{U}\,\beta) \circ (\mathbf{U}\,\alpha)$$

Extensional equality

Extensional equality

- A classical computation $\alpha = (H, h, G, \phi)$ induces a function $\mathsf{U}\alpha \in A \to B$ by

$$\begin{array}{ccc} A \times H & \xrightarrow[\phi]{} & B \times G \\ \uparrow{\scriptstyle (-,h)} & & \downarrow{\scriptstyle \pi_1} \\ A & \xrightarrow[\mathsf{U}\,\alpha]{} & B \end{array}$$

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\end{array}$$

- We say that two computations are **extensionally equivalent**, if they give rise to the same function.

Extensional equality . . .

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- **Theorem:** Any function $f \in A \to B$ on finite sets A, B can be realized by a computation.
- *Translation for Category Theoreticians:* U is full and faithful.

Example π_1 :

function

$$\pi_1 \in (2, 2) \to 2$$
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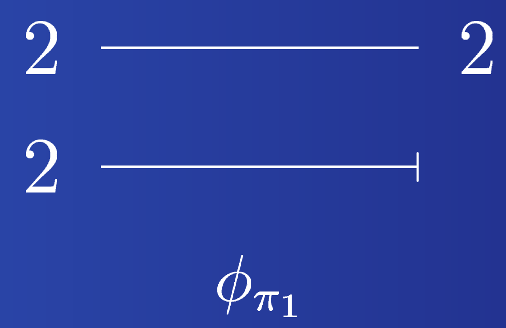

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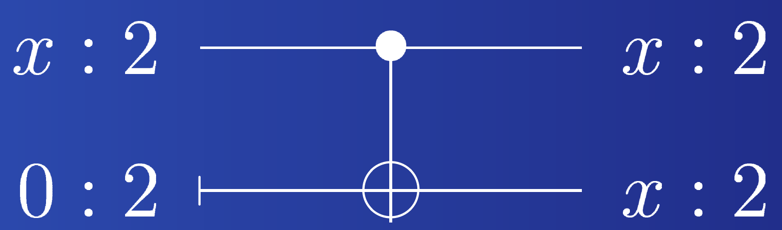

ϕ_δ

$$\phi_\delta \in (2, 2) \to (2, 2)$$

$$\phi_\delta\ (0, x) = (0, x)$$

$$\phi_\delta\ (1, x) = (1, \neg\ x)$$

2. Finite quantum computation

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Norm of a vector:
$\|v\| = \Sigma_{a \in A}(va)^*(va) \in \mathbb{R}^+$,
Unitary operators:

A unitary operator $\phi \in A \multimap_{\text{unitary}} B$ is a linear isomorphism that preserves the norm.

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- A **pure state** over A is a vector $v \in \mathbb{C}^A$ with unit norm $\|v\| = 1$.

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- A **reversible computation** is given by a unitary operator $\phi \in A \multimap_{\text{unitary}} B$.

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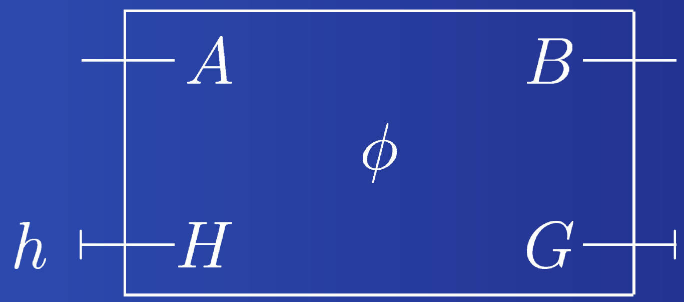

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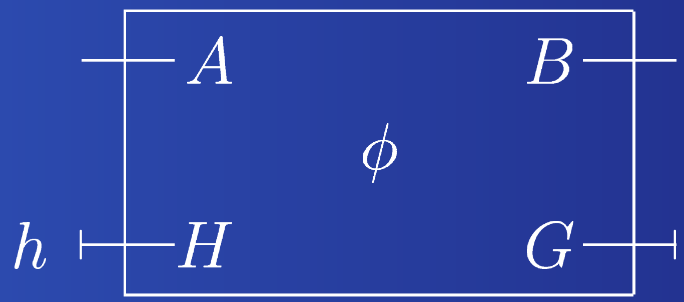

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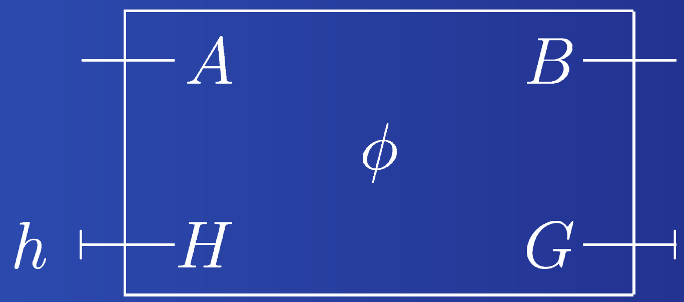

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- a unitary operator $\phi \in A \otimes H \multimap_{\text{unitary}} B \otimes G$.

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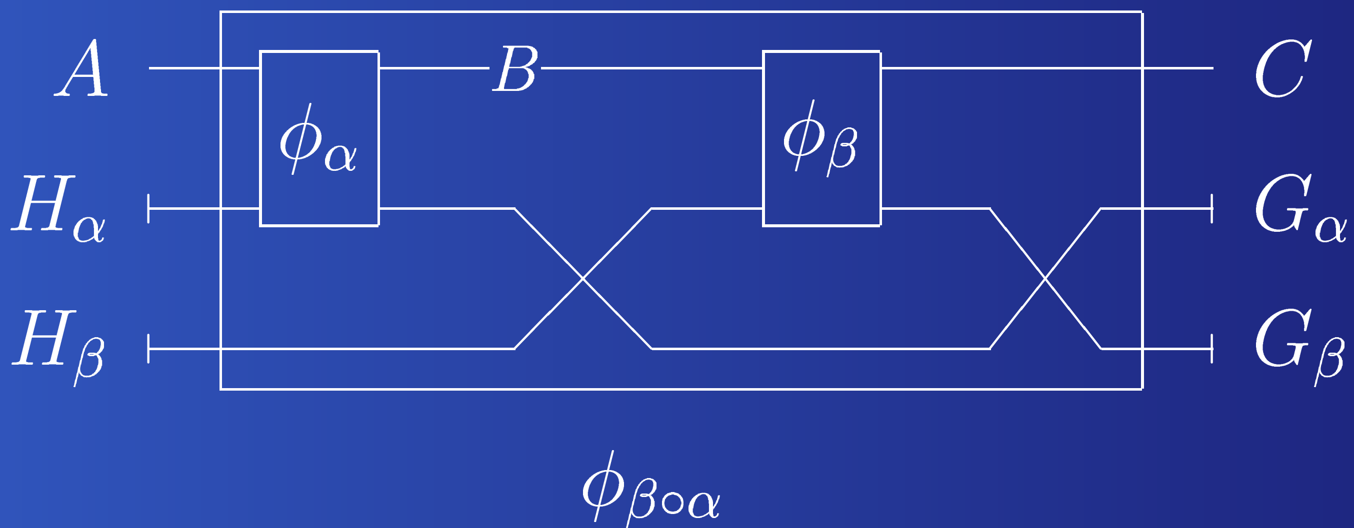

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- . . . is a bit more subtle.
- There is no (sensible) operator on vector spaces, replacing $\pi_1 \in B \times G \to B$.
- **Indeed:** Forgetting part of a **pure state** results in a **mixed state**.

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- Eigenvalues have to be positive and their sum (the trace) is 1.

Example: forgetting a qbit

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- EPR is represented by $\rho \in \mathcal{Q}_2 \otimes \mathcal{Q}_2 \multimap \mathcal{Q}_2 \otimes \mathcal{Q}_2$:

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

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- $\rho\left(\frac{1}{\sqrt{2}}\left|00\right\rangle + \frac{1}{\sqrt{2}}\left|11\right\rangle\right) = \frac{1}{\sqrt{2}}\left|00\right\rangle + \frac{1}{\sqrt{2}}\left|11\right\rangle$

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- After measuring one qbit we obtain $\rho' \in \mathcal{Q}_2 \multimap \mathcal{Q}_2$:

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$$\begin{aligned} \rho' \, |0\rangle &= \frac{1}{2} \, |0\rangle \\ \rho' \, |1\rangle &= \frac{1}{2} \, |1\rangle \end{aligned}$$

Superoperators

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- Every unitary operator ϕ gives rise to a superoperator $\widehat{\phi}$.

Superoperators. . .

- There is an operator

$$\mathrm{tr}_{B,G} \in B \otimes G \multimap_{\text{super}} B$$

called *partial trace*.

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- E.g. $\mathrm{tr}_{\mathcal{Q}_2,\mathcal{Q}_2} \in \mathcal{Q}_2 \otimes \mathcal{Q}_2 \multimap_{\text{super}} \mathcal{Q}_2$ is represented by a 16×4 matrix.

Semantics

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Every quantum computation α gives rise to a superoperator $\mathsf{U}\,\alpha \in A \multimap_{\mathsf{super}} B$

$$
\begin{array}{ccc}
A \otimes H & \xrightarrow[\widehat{\phi}]{} & B \otimes G \\
\uparrow {\scriptstyle -\otimes\widetilde{h}} & & \downarrow {\scriptstyle \mathrm{tr}_G} \\
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Theorem: Every superoperator $F \in A \multimap_{\mathsf{super}} B$ (on finite Hilbert spaces) comes from a quantum computation.

Classical vs quantum

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classical (FCC)	quantum (FQC)

Classical vs quantum

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finite sets	

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projections	partial trace

Decoherence

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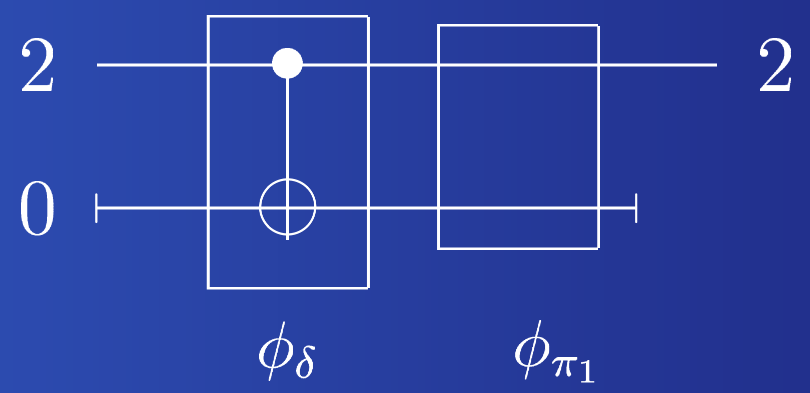

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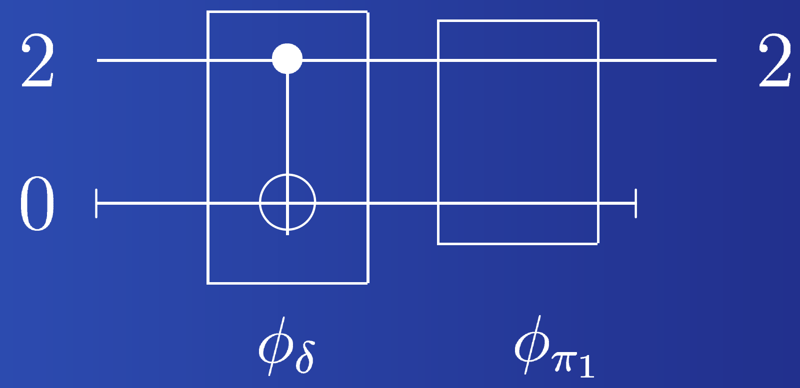
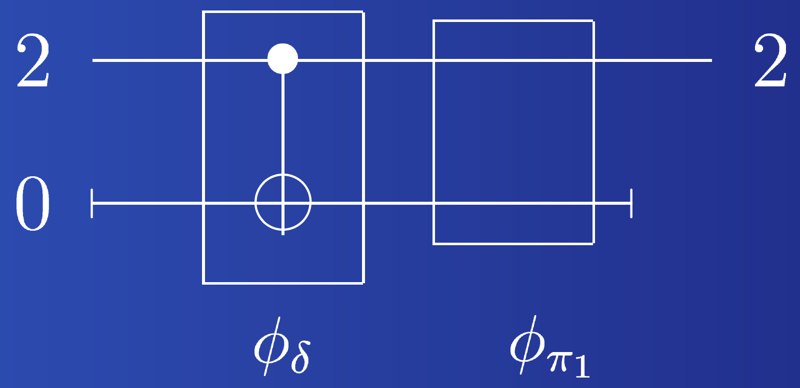


Classically

$$\pi_1 \circ \delta = \mathrm{I}$$

Decoherence



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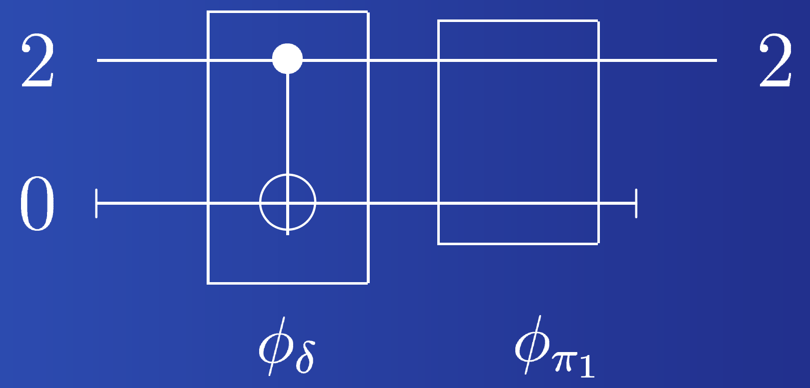

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input: $\{\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|0\rangle\}$

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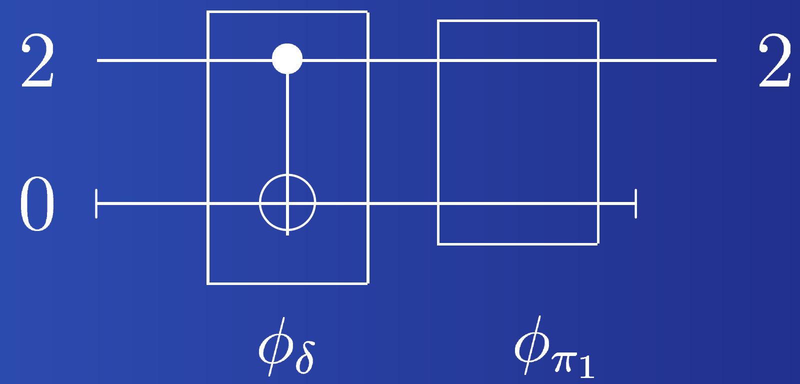

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output: $\frac{1}{2}\{|0\rangle\} + \frac{1}{2}\{|1\rangle\}$

3. QML basics

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- Qbytes
  $\mathcal{Q}_2^8 = \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2$.

QML basics . . .

- A QML program is an expression in a context of typed variables, e.g.

$$
\begin{aligned}
qnot &: Q_2 \multimap Q_2 \\
qnot\ x &= \mathbf{if}^{\circ}\ x \\
&\quad\ \mathbf{then}\ \mathrm{qfalse} \\
&\quad\ \mathbf{else}\ \ \mathrm{qtrue}
\end{aligned}
$$

QML basics . . .

- A QML program is an expression in a context of typed variables, e.g.

$$
\begin{aligned}
&qnot : Q_2 \multimap Q_2 \\
&qnot\ x = \mathbf{if}^\circ\ x \\
&\qquad\qquad \mathbf{then}\ \mathrm{qfalse} \\
&\qquad\qquad \mathbf{else}\ \ \mathrm{qtrue}
\end{aligned}
$$

- We can compile QML programs into quantum computations (i.e. quantum circuits).

QML basics . . .

- Forgetting variables has to be explicit.

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$$

is just the identity, but

$$
\begin{aligned}
&meas : Q_2 \multimap Q_2\\
&meas\ x = \mathbf{if}\ x\\
&\qquad\qquad \mathbf{then}\ \mathrm{qtrue}\\
&\qquad\qquad \mathbf{else}\ \ \mathrm{qfalse}
\end{aligned}
$$

introduces a measurement (end hence decoherence).

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$$
\begin{aligned}
&cswap : Q_2 \otimes Q_2 \multimap Q_2 \multimap Q_2 \otimes Q_2 \\
&cswap\ (x, y)\ c = \mathbf{if}^{\circ}\ c \\
&\quad \mathbf{then}\ (y, x) \\
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QML basics . . .

- We can introduce superpositions, e.g.

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  However, the terms in the superposition have to be orthogonal.

4. Compiling QML

1. Finite classical computation
2. Finite quantum computation
3. QML basics
4. Compiling QML
5. Conclusions and further work

Compilation

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- Correct QML programs are defined by typing rules, e.g.

$$\frac{\Gamma \vdash t : \sigma \otimes \tau \qquad \Delta, x : \sigma, y : \tau \vdash u : C}{\Gamma \otimes \Delta \vdash \texttt{let}\ (x, y) = t\ \texttt{in}\ u : C} \otimes \text{elim}$$

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For each rule we can construct a quantum computation, i.e. a circuit.

$\otimes$-elim

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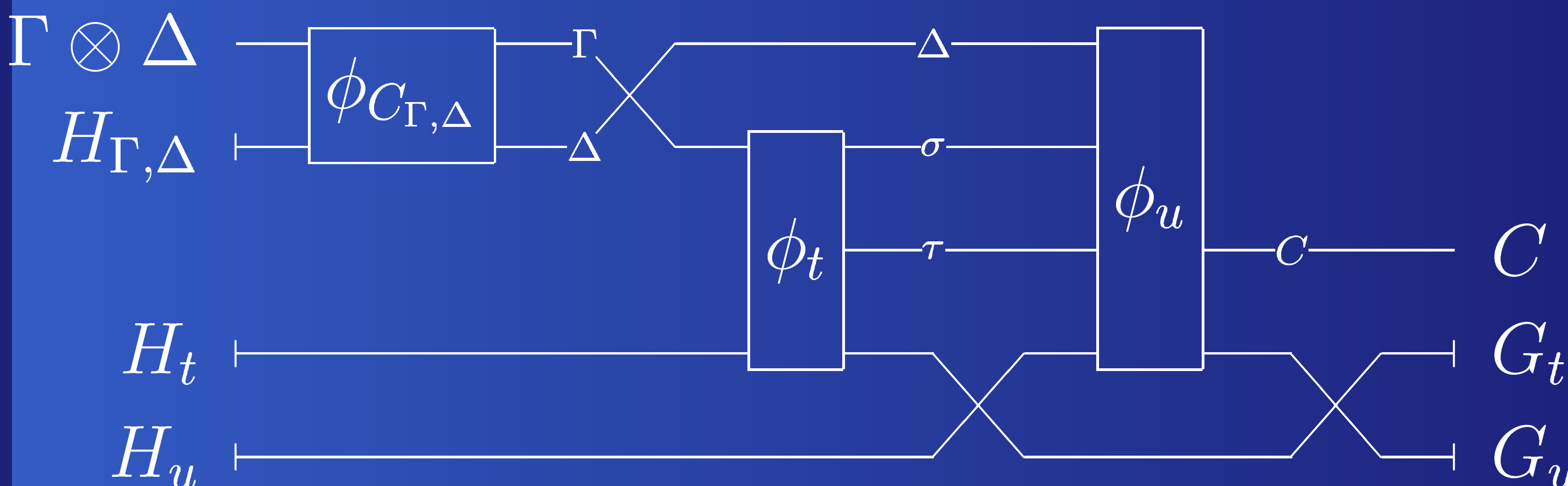

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- Amr Sabry and Juliana Vizotti (Indiana University) embarked on an independent implementation of QML based on our paper.

5. Conclusions

1. Semantics of finite classical and quantum computation
2. QML basics
3. Compiling QML
4. Conclusions and further work

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- Our analysis also highlights the differences between classical and quantum programming.
- We have developed an *algebra of quantum programs* which for pure programs is complete wrt the semantics and a normalisation algorithm.
- Quantum programming introduces the problem of *control of decoherence*, which we address by making forgetting variables explicit and by having different if-then-else constructs.

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- Investigate the similarities/differences between FCC and FQC from a categorical point of view.

The end

Thank you for your attention.

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`//www.cs.nott.ac.uk/˜txa/publ/`

A functional quantum programming language LICS 2005

with J.Grattage

Structuring Quantum Effects: Superoperators as Arrows

MFCS 2006

with J.Vizzotto and A.Sabry

An Algebra of Pure Quantum Programming QPL 2005

with J.Grattage, J.Vizzotto and A.Sabry