THE EXAMINATION

XXV INTERNATIONAL PHYSICS OLYMPIAD

BEIJING, PERPLE'S REPUBLIC CHINA

THEORETICAL COMPETITION

July 13, 1994

Time available: 5 hours

READ THIS FIRST!

INSTRUCTIONS:

1. Use only the ball pen provided.
2. Your graphs should be drawn on the answer sheets attached to the problem.
3. Your solutions should be written on the sheets of paper attached to the problems.
4. Write at the top of the first page of each problem:
   - The total number of pages in your solution to the problem
   - Your name and code number

Theoretical Problem 1

RELATIVISTIC PARTICLE

In the theory of special relativity the relation between energy E and momentum P or a free particle with rest mass m_0 is

$$E = \sqrt{p^2c^2 + m_0^2c^4} = mc^2$$

When such a particle is subject to a conservative force, the total energy of the particle, which is the sum of $\sqrt{p^2c^2 + m_0^2c^4}$ and the potential energy, is conserved. If the energy of the particle is very high, the rest energy of the particle can be ignored (such a particle is called an ultra relativistic particle).

1) consider the one dimensional motion of a very high energy particle (in which rest energy can be neglected) subject to an attractive central force of constant magnitude f. Suppose the particle is located at the centre of force with initial momentum p_0 at time t=0. Describe the motion of the particle by separately plotting, for at least one period of the motion: x against time t, and momentum p against space coordinate x. Specify the coordinates of the "turning points" in terms of given parameters p_0 and f. Indicate, with arrows, the direction of the progress of the mothon in the (p, x) diagram. There may be short intervals of time during which the particle is not ultrarelativistic. However, these should be neglected.

   Use Answer Sheet 1.

2) A meson is a particle made up of two quarks. The rest mass M of the meson is equal to the total energy of the two-quark system divided by c^2.

   Consider a one--dimensional model for a meson at rest, in which the two quarks are assumed to move along the x-axis and attract each other with a force of constant magnitude f It is assumed they can pass through each other freely. For analysis of the high energy motion of the quarks the rest mass of the quarks can be neglected. At time t=0 the two quarks are both at x=0. Show separately the motion of the two quarks graphically by a (x, t) diagram and a (p, x) diagram, specify the coordinates of the "turning points" in terms of M and f, indicate the direction of the process in your (p, x) diagram, and determine the maximum distance between the two quarks.

   Use Answer Sheet 2.

3) The reference frame used in part 2 will be referred to as frame S, the Lab frame, referred to as S, moves in the negative x-direction with a constant velocity v=0.6c. the coordinates in the two reference frames are so chosen that the point

x=0 in S coincides with the point $x'=0$ in S'' at time $t=t'=0$. Plot the motion of the two quarks graphically in a (x', t') diagram. Specify the coordinates of the turning points in terms of M, f and c, and determine the maximum distance between the two quarks observed in Lab frame S'.

Use Answer Sheet 3.

The coordinates of particle observed in reference frames S and S'' are related by the Lorentz transformation

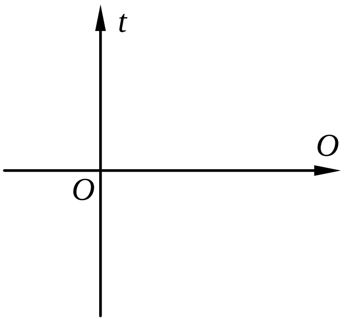
$$\begin{cases} x' = \gamma(x+\beta ct) \\ t' = \gamma(t+\beta \dfrac{x}{c}) \end{cases}$$

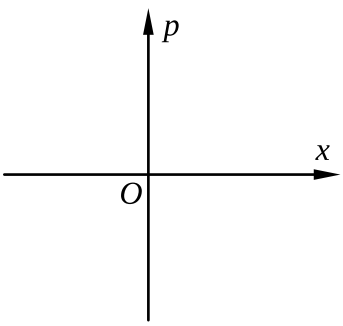
where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$ and v is the velocity of frame S moving relative to the frame S''.

4) For a meson with rest energy Mc^2=140 MeV and velocity 0.60c relative to the Lab frame S'', determine its energy E' in the Lab Frame S''.

ANSWER SHEET 1

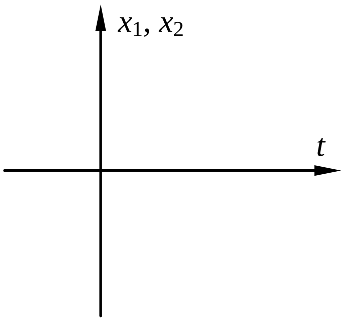
1)

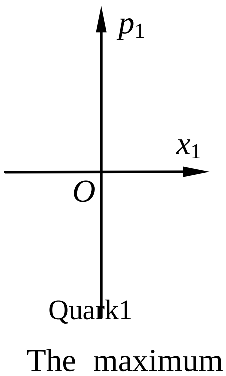


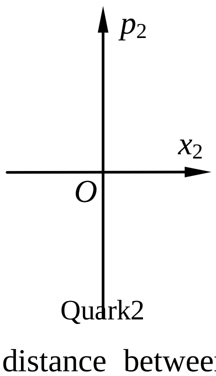


ANSWER SHEET 2

2)



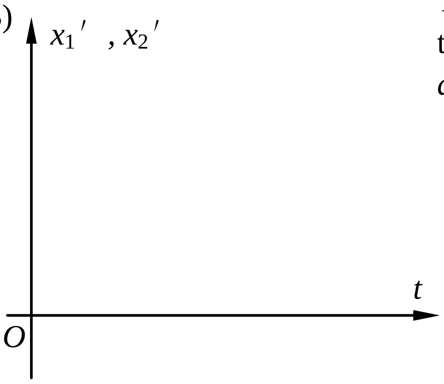




The maximum distance between the two quarks is d=

ANSWER SHEET 3

3)



The maximum distance between the two quarks observed in S′ frame is

d' =

Theoretical Problem 1—Solution

1) **1a. Taking the force center as the origin of the space coordinate** x and the zero potential point, the potential energy of the particle is

$$U(x) = f\,|x| \tag{1}$$

The total energy is

$$W = \sqrt{p^2c^2 + m_0^2c^4} + f\,|x|.$$

1b. Neglecting the rest energy, we get

$$W = |p|c + f\,|x|, \tag{2}$$

Since W is conserved throughout the motion, so we have

$$W = |p|c + f\,|x| = p_0 c, \tag{3}$$

Let the x axis be in the direction of the initial momentum of the particle,

$$\left.\begin{array}{lll} pc + fx = p_0c & \text{when} & x>0,\quad p>0; \\ -pc + fx = p_0c & \text{when} & x>0,\quad p<0; \\ pc - fx = p_0c & \text{when} & x<0,\quad p>0; \\ -pc - fx = p_0c & \text{when} & x<0,\quad p<0. \end{array}\right\} \tag{4}$$

The maximum distance of the particle from the origin, let it be L, corresponds to p=0. It is

$$L = p_0c/f\,.$$

1c. From Eq. 3 and Newton's law

$$\frac{dp}{dt} = F = \begin{cases} -f, & x>0; \\ f, & x<0; \end{cases} \tag{5}$$

we can get the speed of the particle as

$$\left|\frac{dx}{dt}\right| = \frac{c}{f}\left|\frac{dp}{dt}\right| = c, \tag{6}$$

i.e. the particle with very high energy always moves with the speed of light except that it is in the region extremely close to the points $x = \pm L$. The time for the particle to move from origin to the point $x = L$, let it be denoted by τ, is

$$\tau = L/c = p_0/f .$$

So the particle moves to and for between $x = L$ and $x = -L$ with speed c and period $4\tau = 4p_0/f$. The relation between x and t is

$$\left.\begin{array}{ll} x = ct, & 0 \le t \le \tau \\ x = 2L - ct, & \tau \le t \le 2\tau, \\ x = 2L - ct, & 2\tau \le t \le 3\tau, \\ x = ct - 4L, & 3\tau \le t \le 4\tau, \end{array}\right\} \tag{7}$$

The required answer is thus as given in Fig. 1 and Fig. 2.

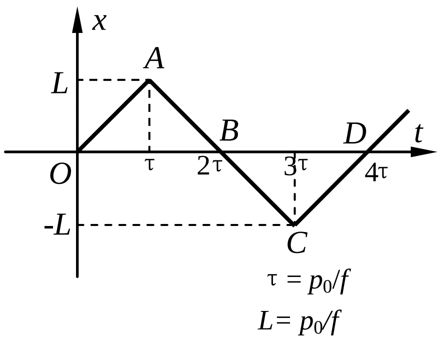


Fig. 1

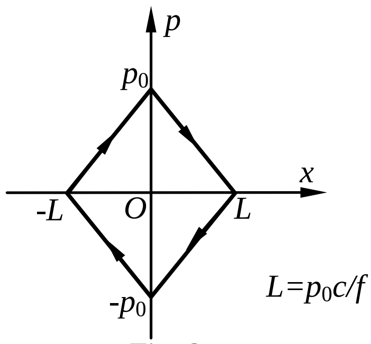


Fig. 2

2) The total energy of the two-quark system can be expressed as

$$Mc^2 = |p_1|c + |p_2|c + f|x_1 - x_2|, \tag{8}$$

where x_1, x_2 are the position coordinates and p_1, p_2 are the momenta of quark 1 and quark 2 respectively. For the rest meson, the total momentum of the two quarks is zero and the two quarks move symmetrically in opposite directions, we have

$$p = p_1 + p_2 = 0, \qquad p_1 = -p_2, \quad x_1 = -x_2 . \tag{9}$$

Let p_0 denote the momentum of the quark 1 when it is at x=0, then we have

$$Mc^2 = 2p_0c \qquad \text{or} \qquad p_0 = Mc/2 \tag{10}$$

From Eq. 8, 9 and 10, the half of the total energy can be expressed in terms of p_1 and x_1 of quark 1:

$$p_0c = |p_1|c + f|x_1|, \tag{11}$$

just as though it is a one particle problem as in part 1 (Eq. 3) with initial momentum

$p_0 = Mc/2$. From the answer in part 1 we get the (x, t) diagram and (p, x) diagram of the motion of quark 1 as shown in Figs. 3 and 4. For quark 2 the situation is similar except that the signs are reversed for both x and p; its (x, t) and (p, x) diagrams are shown in Figs. 3 and 4.

The maximum distance between the two quarks as seen from Fig. 3 is

$$d = 2L = 2p_0c/f = Mc^2/f\ . \tag{12}$$

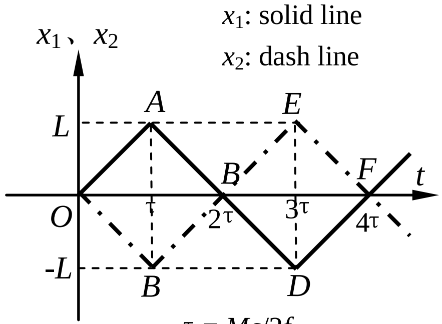


Fig. 3

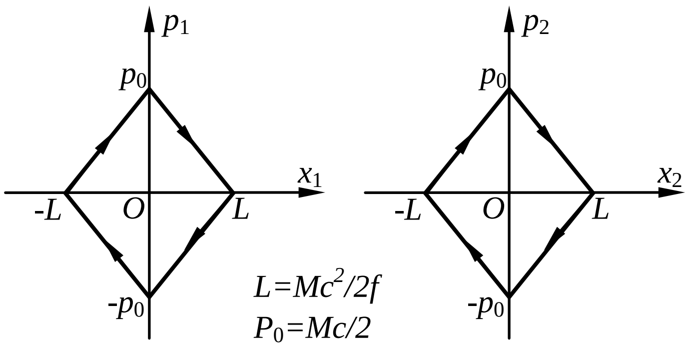


Fig. 4a Quark1

Fig. 4b Quark2

3) The reference frame S moves with a constant velocity V=0.6c relative to the Lab frame S'' in the x' axis direction, and the origins of the two frames are coincident at the beginning ($t = t' = 0$). The Lorentz transformation between these two frames is given by:

$$\begin{aligned} x' &= \gamma(x + \beta ct), \\ t' &= \gamma(t + \beta x/c), \end{aligned} \tag{13}$$

where $\beta = V/c$, and $\gamma = 1/\sqrt{1-\beta^2}$. With $V = 0.6c$, we have $\beta = 3/5$, and $\gamma = 5/4$. Since the Lorenta transformation is linear, a straight line in the (x, t) diagram transforms into a straight line the (x', t') diagram, thus we need only to calculate the coordinates of the turning points in the frame S'.

For quark 1, the coordinates of the turning points in the frames S and S' are as follows:

Frame S		Frame S'	
x_1	t_1	$x_1' = \gamma(x_1 + \beta c t_1)$ $= \frac{5}{4}x_1 + \frac{3}{4}ct_1$	$t_1' = \gamma(t_1 + \beta x_1 / e)$ $= \frac{5}{4}t_1 + \frac{3}{4}x_1 / c$
0	0	0	0
L	τ	$\gamma(1+\beta)L = 2L$	$\gamma(1+\beta)\tau = 2\tau$
0	2τ	$2\gamma\beta L = \frac{3}{2}L$	$2\gamma\tau = \frac{5}{2}\tau$
$-L$	3τ	$\gamma(3\beta - 1)L = L$	$\gamma(3-\beta)\tau = 3\tau$
0	4τ	$4\gamma\beta L = 3L$	$4\gamma\tau = 5\tau$

where $L = p_0 c / f = Mc^2 / 2f$, $\tau = p_0 / f = Mc / 2f$.

For quark 2, we have

Frame S		Frame S'	
x_2	t_2	$x_2' = \gamma(x_2 + \beta c t_2)$ $= \frac{5}{4}x_2 + \frac{3}{4}ct_2$	$t_2' = \gamma(t_2 + \beta x_2 / c)$ $= \frac{5}{4}t_2 + \frac{3}{4}x_2 / c$
0	0	0	0
$-L$	τ	$-\gamma(1-\beta)L = -\frac{1}{2}L$	$\gamma(1-\beta)\tau = \frac{1}{2}\tau$
0	2τ	$2\gamma\beta L = \frac{3}{2}L$	$2\gamma\tau = \frac{5}{2}\tau$
L	3τ	$\gamma(3\beta + 1)L = \frac{7}{2}L$	$\gamma(3+\beta)\tau = \frac{9}{2}\tau$
0	4τ	$4\gamma\beta L = 3L$	$4\gamma\tau = 5\tau$

With the above results, the (x', t') diagrams of the two quarks are shown in Fig. 5.

The equations of the straight lines OA and OB are:

$$x_1'(t') = ct'; \qquad 0 \le t' \le \gamma(1+\beta)\tau = 2\tau; \tag{14a}$$

$$x_2'(t') = -ct'; \qquad 0 \le t' \le \gamma(1-\beta)\tau = \frac{1}{2}\tau \tag{14b}$$

The distance between the two quarks attains its maximum d' when $t' = \frac{1}{2}\tau$, thus we have maximum distance

$$d' = 2c\gamma(1-\beta)\tau = 2\gamma(1-\beta)L = \frac{Mc^2}{2f}. \tag{15}$$

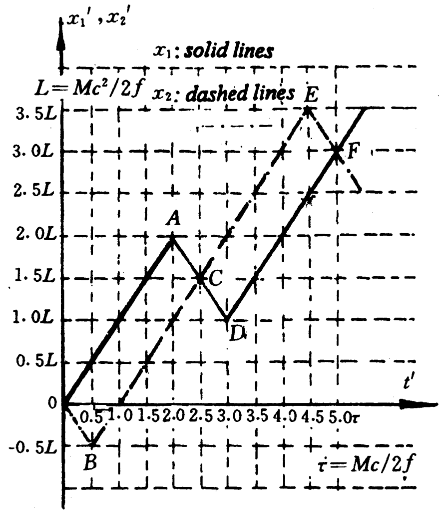


Fig. 5

4) It is given the meson moves with velocity V=0.6 crelative to the Lab frame, its energy measured in the Lab frame is

$$E' = \frac{Mc^2}{\sqrt{1-\beta^2}} = \frac{1}{0.8} \times 140 = 175\,\text{MeV}.$$

Grading Scheme

Part 1 2 points, distributed as follows:

0.4 point for the shape of $x(t)$ in Fig. 1;

0.3 point for 4 equal intervals in Fig. 1;

(0.3 for correct derivation of the formula only)

0.1 each for the coordinates of the turning points A and C, 0.4 point in total;

0.4 point for the shape of $p(x)$ in fig. 2; (0.2 for correct derivation only)

0.1 each for specification of p_0, $L = p_0 c / f$, $-p_0$, $-L$ and arrows, 0.5 point in total.

(0.05 each for correct calculations of coordinate of turning points only).

Part 2 4 points, distributed as follows:

0.6 each for the shape of $x_1(t)$ and $x_2(t)$, 1.2 points in total;

0.1 each for the coordinates of the turning points A, B, D and E in Fig. 3, 0.8 point in total;

0.3 each for the shape of $p_1(x_1)$ and $p_2(x_2)$, 0.6 point in total;

0.1 each for $p_0 = Mc/2$, $L = Mc^2/2f$, $-p_0$, $-L$ and arrows in Fig. 4a and Fig. 4b, 1 point in total;

0.4 point for $d = Mc^2/f$

Part 3 3 point, distributed as follows:

0.8 each for the shape of $x_1'(t')$ and $x_2'(t')$, 1.6 points in total;

0.1 each for the coordinates of the turning points A, B, D and E in Fig. 5, 0.8 point in total; (0.05 each for correct calculations of coordinate of turning points only).

0.6 point for $d' = Mc^2/2f$.

Part 4 1 point (0.5 point for the calculation formula; 0.5 point for the numerical value and units)

Theoretical Problem 2

SUPERCONDUCTING MAGNET

Super conducting magnets are widely used in laboratories. The most common form of super conducting magnets is a solenoid made of super conducting wire. The wonderful thing about a superconducting magnet is that it produces high magnetic fields without any energy dissipation due to Joule heating, since the electrical resistance of the superconducting wire becomes zero when the magnet is immersed in liquid helium at a temperature of 4.2 K. Usually, the magnet is provided with a specially designed superconducting switch, as shown in Fig. 1. The resistance r of the switch can be controlled: either r=0 in the superconducting state, or $r = r_n$ in the normal state. When the persistent mode, with a current circulating through the magnet and superconducting switch indefinitely. The persistent mode allows a steady magnetic field to be maintained for long periods with the external source cut off.

The details of the superconducting switch are not given in Fig. 1. It is usually a small length of superconducting wire wrapped with a heater wire and suitably thermally insulated from the liquid helium bath. On being heated, the temperature of the superconducting wire increases and it reverts to the resistive normal state. The typical value of r_n is a few ohms. Here we assume it to be $5\,\Omega$. The inductance of a superconducting magnet depends on its size; assume it be 10 H for the magnet in Fig. 1. The total current I can be changed by adjusting the resistance R.

This problem will be graded by the plots only!

The arrows denote the positive direction of I, I_1 and I_2.

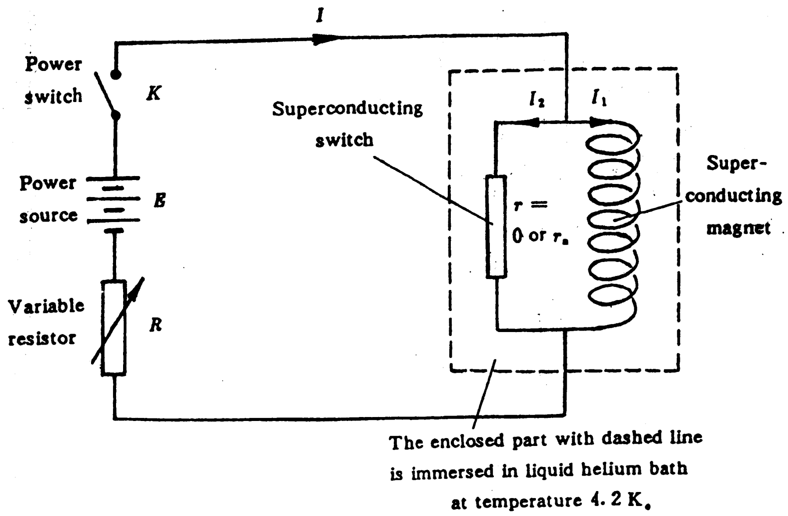


Fig. 1

1) If the total current I and the resistance r of the superconducting switch are controlled

to vary with time in the way shown in Figs, 2a and 2b respectively, and assuming the currents I_1 and I_2 flowing through the magnet and the switch respectively are equal at the beginning (Fig. 2c and Fig. 2d), how do they vary with time from t_1 to t_4? Plot your answer in Fig. 2c and Fig. 2d

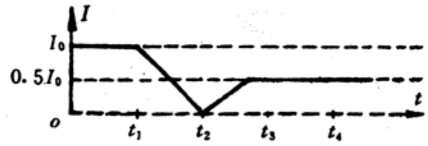
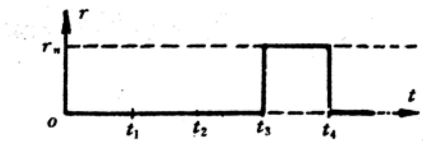
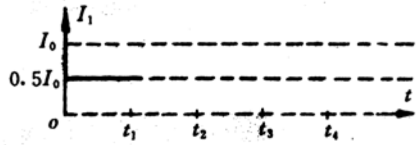


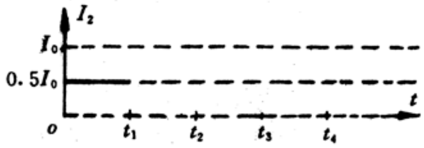
Fig.2a



2b



2c



2d

2) Suppose the power switch K is turned on at time t=0 when r=0, I_1=0 and R=7.5 Ω, and the total current I is 0.5A. With K kept closed, the resistance r of the superconducting switch is varied in he way shown in Fig. 3b. Plot the corresponding time dependences of I, I_1 and I_2 in Figs. 3a, 3c and 3d respectively.

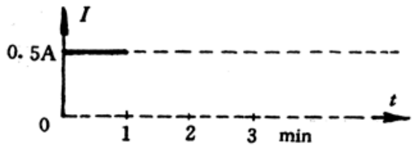
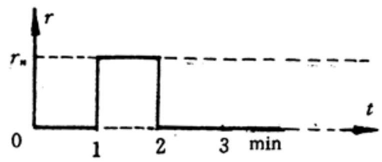
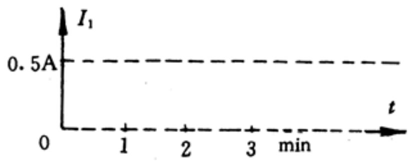


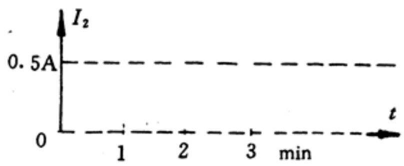
Fig. 3a



3b



3c



3d

3) Only small currents, less than 0.5A, are allowed to flow through the

superconducting switch when it is in the normal state, with larger currents the switch will be burnt out. Suppose the superconducting magnet is operated in a persistent mode, i. e. I=0, and I_1=i_1(e. g. 20A), I_2=-i_1, as shown in Fig. 4, from t=0 to t=3min. If the experiment is to be stopped by reducting the current through the magnet to zero, how would you do it? This has to be done in several operation steps. Plot the corresponding changes of I, r, I_1 and I_2 in Fig. 4

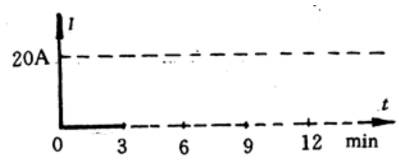
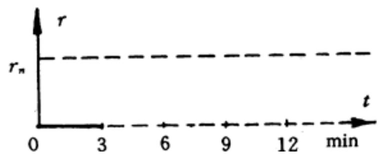
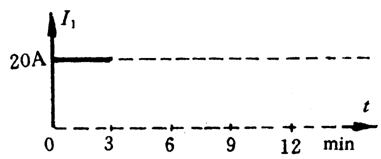


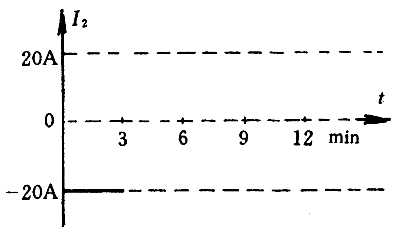
Fig. 4a



4b



4c



4d

4) Suppose the magnet is operated in a persistent mode with a persistent current of 20A (t=0 to t=3min. See Fig. 5). How would you change it to a persistent mode with a current of 30a? plot your answer in Fig. 5.

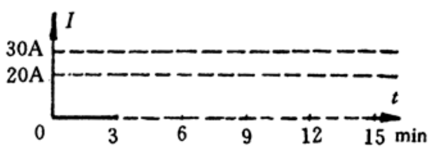
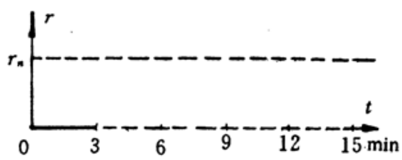
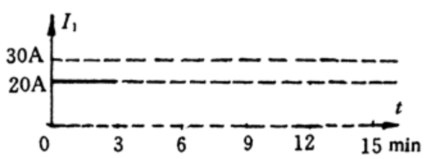


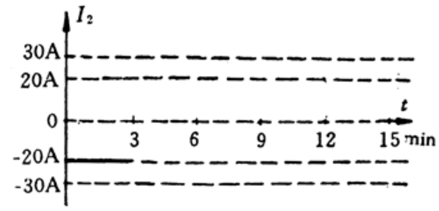
Fig. 5a



5b



5c



5d

Theoretical Problem 2—Solution

1) For t=t_1 to t_3

Since $r=0$, the voltage across the magnet $V_M = LdI_1/dt$ =0, therefore,

$$I_1 = I_1(t_1) = \frac{1}{2}I_0;$$

$$I_2 = I - I_1 = I - \frac{1}{2}I_0.$$

For t=t_3 to t_4

Since I_2=0 at t=t_3, and I is kept at $\frac{1}{2}I_0$ after

$t=t_3$, $V_M = I_2 r_n = 0$, therefore, I_1 and I_2 will not change.

$$I_1 = \frac{1}{2}I_0;$$

$$I_2 = 0$$

These results are shown in Fig. 6.

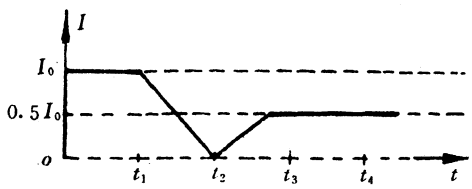
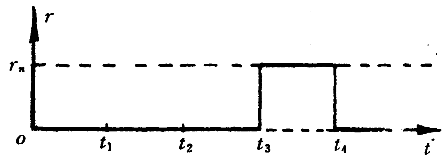
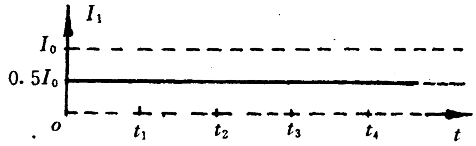


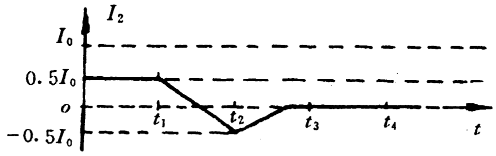
Fig. 6a



6b



6c



6d

2) For $t = 0$ to $t = 1$ min:

Since $r = 0$, $V_M = LdI_1/dt = 0$

$$I_1 = I_1(0) = 0$$

$$I_2 = I - I_1 = 0.5\,\text{A}.$$

At $t = 1$ min, r suddenly jumps from O to r_n, I will drop from E/R to $E/(R + r_n)$ instantaneously, because I_1 can not change abruptly due to the inductance of the magnet coil. For E/R=0.5A, $R = 7.5\Omega$ and $R_n = 5\Omega$. I will drop to 0.3A.

For $t = 1$ min to 2 min:

I, I_1 and I_2 gradually approach their steady values:

$$I = \frac{E}{R} = 0.5\,\text{A},$$

$$I_1 = I = 0.5\,\text{A}$$

$$I_2 = 0.$$

The time constant

$$\tau = \frac{L(R + r_n)}{Rr_n}.$$

When $L = 10$ H, $R = 7.5\Omega$ and $r_n = 5\Omega$, $\tau = 3$ sec.

For $t = 2$ min to 3 min:

Since $r = 0$, I_1 and I_2 will not change, that is

$$I_1 = 0.5\,\text{A and } I_2 = 0$$

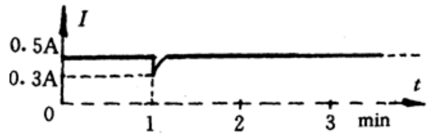
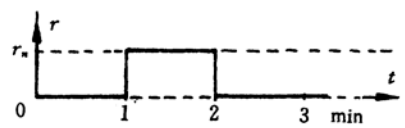
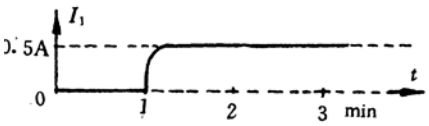


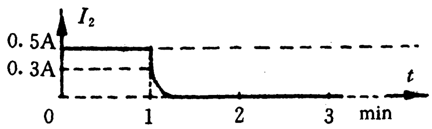
Fig. 7a



7b



7c



7d

3) The operation steps are:

First step

Turn on power switch *K*, and increase the total current *I* to 20 A, i. e. equal to I_1. Since the superconducting switch is in the state $r = 0$, so that $V_M = L \quad dI_1 / dt = 0$, that is, I_1 can not change and I_2 increases by 20A, in other words, I_2 changes from -20 A to zero.

Second step

Switch r from 0 to r_n.

Third step

Gradually reduce I to zero while keeping $I_2 < 0.5$ A: since $I_2 = V_M / r_n$ and $V_m = L\, dI_1 / dt$, when $L = 10$ H, $r_n = 5\Omega$, the requirement $I_2 < 0.5$ A corresponds to $dI_1 / dt < 0.25$ A/sec, that is, a drop of <15A in 1 min. In Fig. 8 dI / dt ～0.1A/sec and dI_1 / dt is around this value too, so the requirement has been fulfilled.

Final step

Switch r to zero when $V_M = 0$ and turn off the power switch *K*. These results are shown in Fig. 8.

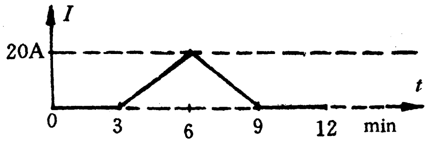
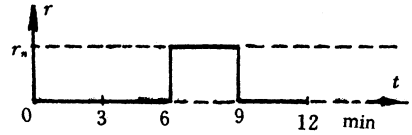
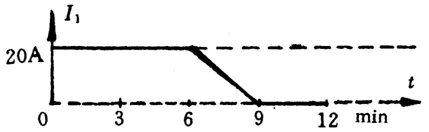


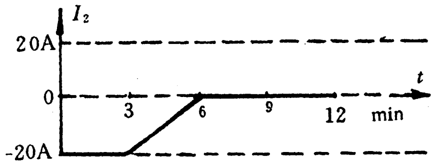
Fig. 8a



8b



8c



8d

4) **First step** and **second step** are the same as that in part 3, resulting in $I_2 = 0$.

Third step Increase I by 10 A to 30 A with a rate subject to the requirement $I_2 < 0.5$ A.

Fourth step Switch r to zero when $V_M = 0$.

Fifth step Reduce I to zero, $I_1 = 30$ A will not change because V_M is zero. $I_2 = I - I_1$ will change to -30 A. The current flowing through the magnet is thus closed by the superconducting switch.

Final step Turn off the power switch *K*. The magnet is operating in the persistent mode.

These results are shown in Fig. 9.

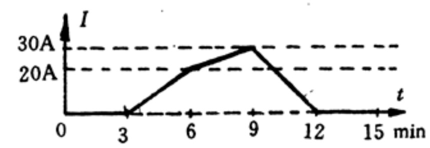
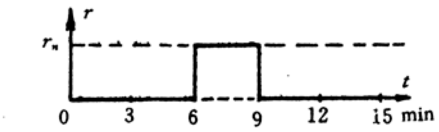
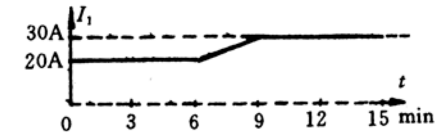


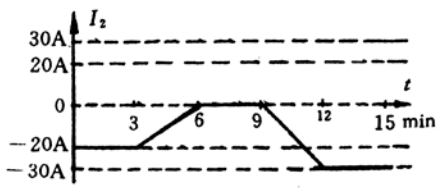
Fig. 9a



9b



9c



9d

Grading Scheme

Part 1, 2 points:

0.5 point for each of I_1, I_2 from $t = t_1$ to t_3 and I_1, I_2 from $t = t_3$ to t_4.

Part 2, 3 points:

0.3 point for each of I_1, I_2 from $t = 0$ to 1 min, I, I_1, I_2 at $t = 1$ min, and I_0, I_1, I_2 from $t=1$ to 2 min;

0.2 point for each of I, I_1, and I_2 from $t=2$ to 3 min.

Part 3, 2 points:

0.25 point for each section in Fig. 8 from $t=3$ to 9 min, 8 sections in total.

Part 4, 3 points:

0.25 point for each section in Fig. 9 from $t=3$ to 12 min, 12 sections in total.

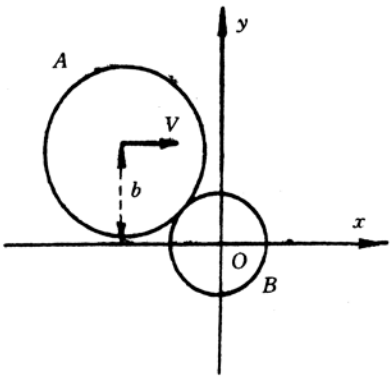
Theoretical Problem 3

COLLISION OF DISCS WITH SURFACE FRICTION

A homogeneous disc A of mass m and radius R_A moves translationally on a smooth horizontal *x*-*y* plane in the *x* direction with a velocity *V* (see the figure on the next page). The center of the disk is at a distance *b* from the x-axis. It collides with a stationary homogeneous disc B whose center is initially located at the origin of the coordinate system. The disc B has the same mass and the same thickness as A, but its radius is R_B. It is assumed that the velocities of the discs at their point of contact, in the direction perpendicular to the line joining their centers, are equal after the collision. It is also assumed that the magnitudes of the relative velocities of the discs along the line joining their centers are the same before and after the collision.

1) For such a collision determine the *X* and *Y* components of the velocities of the two discs after the collision, i. e. V'_{AX}, V'_{AY}, V'_{BX} and V'_{BY} in terms of m, R_A, R_B, V and b.

2) Determine the kinetic energies E'_A for disc A and E'_B for disc B after the collision in terms of m, R_A, R_B, V and b.



Theoretical Problem 3—Solution

1) When disc A collides with disc B, let *n* be the unit vector along the normal to the surfaces at the point of contact and *t* be the tangential unit vector as shown in the figure. Let φ be the angle between *n* and the *x* axis. Then we have

$$b = (R_A + R_B)\sin\varphi$$

The momentum components of *A* and *B* along *n* and *t* before collision are:

$$mV_{An} = mV\cos\varphi,\ mV_{Bn} = 0\,,$$

$$mV_{At} = mV \sin\varphi, \; mV_{Bt} = 0\,.$$

Denote the corresponding momentum components of A and B after collision by mV'_{An}, mV'_{Bn}, mV'_{At}, and mV'_{Bt}. Let ω_A and ω_B be the angular velocities of A and B about the axes through their centers after collision, and I_A and I_B be their corresponding moments of intertia. Then,

$$I_A = \frac{1}{2} mR_A^2, \qquad I_B = \frac{1}{2} mR_B^2$$

The conservation of momentum gives

$$mV \cos\varphi = mV'_{An} + mV'_{Bn}, \tag{1}$$

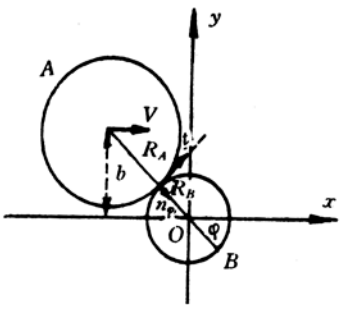
$$mV \sin\varphi = mV'_{At} + mV'_{tn}, \tag{2}$$

The conservation of angular momentum about the axis through O gives

$$mVb = mV'_{At}(R_A + R_B) + I_A\omega_A + I_B\omega_B \tag{3}$$

The impulse of the friction force exerted on B during collision will cause a momentum change of mV'_{At} along t and produces an angular momentum $I_B\omega_B$ simultaneously. They are related by.

$$mV'_{Bt}R_b = I_B\omega_B \tag{4}$$



During the collision at the point of contact A and B acquires the same tangential velocities, so we have

$$V'_{At} - \omega_A R_A = V'_{Bt} - \omega_B R_B \tag{5}$$

It is given that the magnitudes of the relative velocities along the normal direction of the two discs before and after collision are equal, i. e.

$$V \cos\varphi = V'_{Bn} - V'_{An}\,. \tag{6}$$

From Eqs. 1 and 6 we get

$$V'_{An} = 0,$$

$$V'_{Bn} = V\cos\varphi.$$

From Eqs. 2 to 5, we get

$$V'_{At} = \frac{5}{6}V\sin\varphi,$$

$$V'_{Bt} = \frac{1}{6}V\sin\varphi,$$

$$\omega_A = \frac{V\sin\varphi}{3R_A},$$

$$\omega_B = \frac{V\sin\varphi}{3R_B}.$$

The x and y components of the velocities after collision are:

$$V'_{Ax} = V'_{An}\cos\varphi + V'_{At}\sin\varphi = \frac{5Vb^2}{6(R_A+R_B)^2}, \tag{7}$$

$$V'_{Ay} = -V'_{An}\sin\varphi + V'_{At}\cos\varphi = \frac{5Vb\sqrt{(R_A+R_B)^2-b^2}}{6(R_A+R_B)^2}, \tag{8}$$

$$V'_{Bx} = V'_{Bn}\cos\varphi + V'_{Bt}\sin\varphi = \left[1-\frac{5b^2}{6(R_A+R_B)^2}\right], \tag{9}$$

$$V'_{By} = -V'_{Bn}\sin\varphi + V'_{Bt}\cos\varphi = -\frac{5Vb\sqrt{(R_A+R_B)^2-b^2}}{6(R_A+R_B)^2}, \tag{10}$$

2) After the collision, the kinetic energy of disc A is

$$E'_A = \frac{1}{2}m(V'^2_{Ax}+V'^2_{Ay}) + \frac{1}{2}I_A\omega_A^2 = \frac{3mV^2b^2}{8(R_A+R_B)^2} \tag{11}$$

while the kinetic energy of disc B is

$$E'_B = \frac{1}{2}m(V'^2_{Bx}+V'^2_{By}) + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}mV^2\left[1-\frac{11b^2}{12(R_A+R_B)^2}\right] \tag{12}$$

Grading Scheme

1. After the collision, the velocity components of discs A and B are shown in Eq. 7, 8, 9 and 10 of the solution respectively. The total points of this part is 8. 0. If the result in which all four velocity components are correct has not been obtained, the point is marked according to the following rules.

0.8 point for each correct velocity component;

0.8 point for the correct description of that the magnitudes of the relative velocities of the discs along the line joining their centers are the same before and after the collision.

0.8 point for the correct description of the conservation for angular momentum;

0.8 point for the correct description of the equal tangential velocity at the touching point;

0.8 point for the correct description of the relation between the impulse and the moment of the impulse.

2. After the collision, the kinetic energies of disc A and disc B are shown in Eqs. 11 and 12 of the solution respectively.

1.0 point for the correct kinetic energies of disc A;

1.0 point for the correct kinetic energies of disc B;

The total points of this part is 2.0

XXV INTERNATIONAL PHYSICS OLYMPIAD

BEIJING, PEOPLE'S REPUBLIC OF CHINA

PRACTICAL COMPETITION

July 15, 1994

Time available: 2.5 hours

READ THIS FIRST!

INSTRUCTIONS:

1. Use only the ball pen provided.
2. Your graphs should be drawn on the answer sheets attached to the problem.
3. Write your solution on the marked side of the paper only.
4. The draft papers are provided for doing numerical calculations and draft drawings.
5. Write at the top of every page:
   - The number of the problem
   - The number of the page of your report in each problem
   - The total number of pages in your report to the problem
   - Your name and code number

EXPERIMENTAL PROBLEM 1

Determination of light reflectivity of a transparent dielectric surface.

Experimental Apparatus

1. He-Ne Laser(～1.5mW).The light from this laser is not linearly polarized.

2. Two polarizers (**P**$_1$, **P**$_2$) with degree scale disk (Fig. 1), one (**P**$_1$) has been mounted in front of the laser output window as a polarizer, and another one can be fixed in a proper place of the drawing board by push-pins when it is necessary.

3. Two light intensity detectors (D_1, D_2) which consisted of a photocell and a microammeter (Fig. 2).

4. Glass beam splitter(B).

5. Transparent dielectric plate, whose reflectivity and refractive index are to be determined.

6. Sample table mounted on a semicircular degree scale plate with a coaxial swivel arm(Fig. 3).

7. Several push-.pins for fixing the sample table on the drawing board and as its rotation axis.

8. Slit aperture and viewing screen for adjusting the laser beam in the horizontal direction and for alignment of optical elements.

9. Lute for adhere of optical elements in a fixed place.

10. Wooden drawing board.

11. Plotting papers

Experiment Requirement

1. Determine the reflectivity of the p-component as a function of the incident angle (the electric field component, parallel to the plane of incidence is called the *p*-component).
   (a) Specify the transmission axis of the polarizer (A) by the position of the marked line on the degree scale disk in the *p*-componet measurement(the transmission axis is the direction of vibration of the electric field vector of the transmitted light).
   (b) Choose any one of the light intensity detector and set its micro-ammeter at the range of "×5". Verify the linear relation ship between the light intensity and the micro-ammeter reading. Draw the optical schematic diagram. Show your measured data and calculated results(including the calculation formula)in the farm of a table. Plot the linear relationship curve.

(c) Determine the reflectivity of the *p*-component as a function of the incident angle. Draw the optical schematic diagram. Show your measured data and calculated reflectivity(including the calculation formula)in the form of a table. Plot the reflectivity as a function of the incident angle.

2. Determine the refractive index of the sample as accurate as possible.

Explanation and Suggestion

1. Laser radiation avoid direct eye exposure.

2. Since the output power of the laser beam may fluctuate from time to time, the fluctuation of light output has to be monitored during the performance of the experiment and a correction of the experimental results has to be made.

3. The laser should be lighting all the time, even when you finish your experiment and leave the examination hall, the laser should be keeping in work.

4. The reflected light is totally plane polarized at an incident angle θ_B while $\mathrm{tg}\,\theta_B = n$ (refractive index).

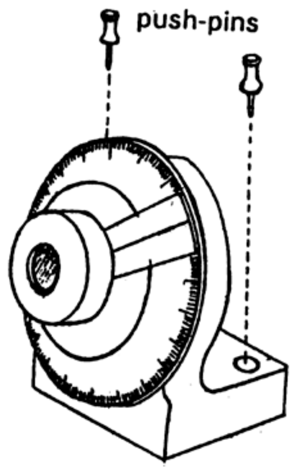


Fig. 1 polarizers with degree scale disk

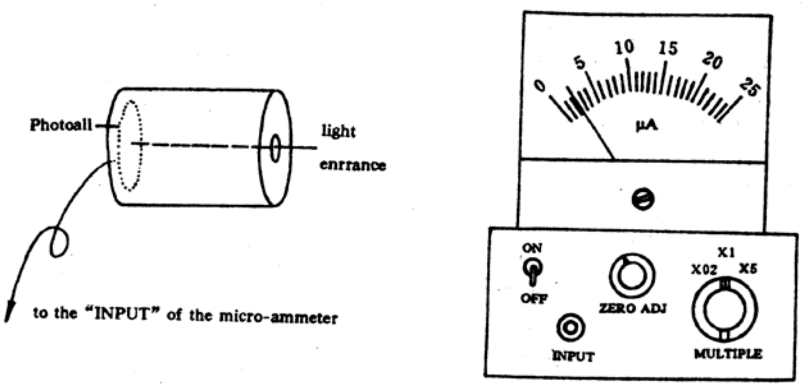


Fig. 2 Light intensity detector

(1) Insert the plug of photocell into the “INPUT” socket of microammeter

(2) Switching on the microammeter.

(3) Blocd off the light entrance hole in front of the photocell and adjust the scale reading of micro ammeter to “0”.

(4) Set the “Multiple” knob to a proper range.

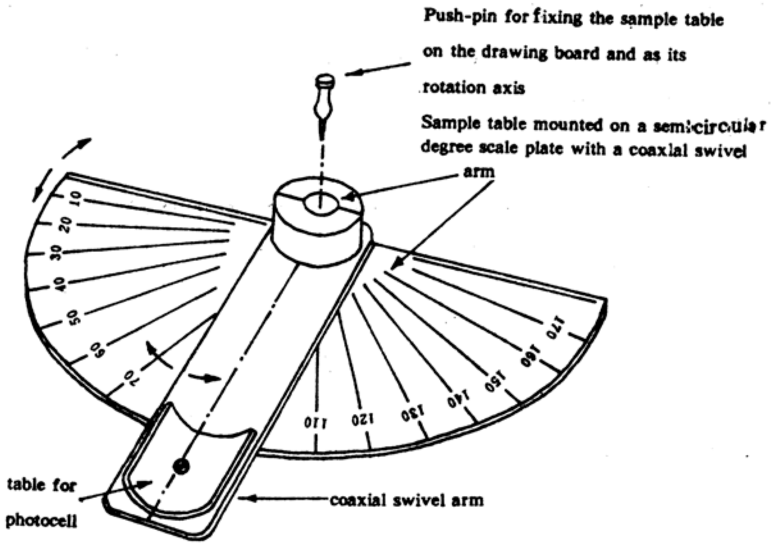
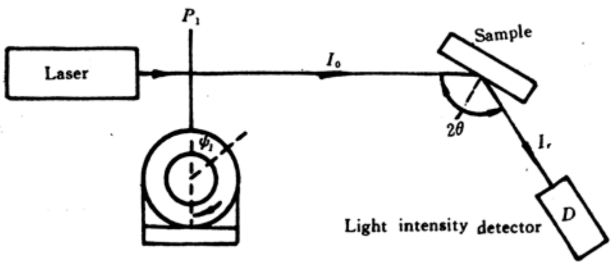


Fig.3 Sample table mounted on a semicircular degree scale plate

Experimental Problem 1——Solution

1. (a) Determine the transmission axis of the polarizer and the Brewster angle θ_B of the sample by using the fact that the rerlectivity of the *p*-component $R_p = 0$ at the Brewster angle.

Change the orientation of the transmission axis of P_1, specified by the position of the marked line on the degree scale disk (ψ) and the incident angle (θ_i) successively until the related intensity $I_r = 0$.



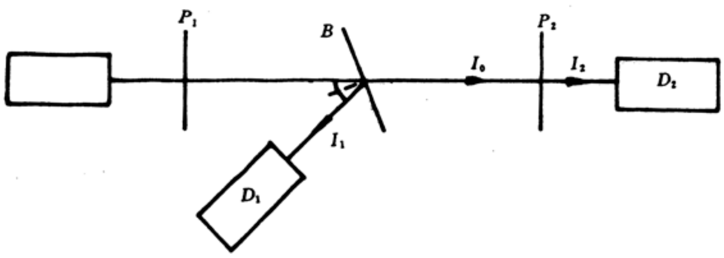
Now the incident light consists of *p*-component only and the incident angle is θ_B, the corresponding values ψ_1 and θ_B are shown below:

ψ_1	140.0°	322.0°	141.0°	322.5°
θ	56.4°	56.4°	56.2°	56.2°

$\psi_1 = 140.5° \pm 0.5°$ or $322.3° \pm 0.1°$

The Brewster angle θ_B is $56.3° \pm 0.1°$

1. (b) Verification of the linear relationship between the light intensity and the microammenter reading.



The intensity the transmitted light passing through two polarized P_1 and P_2 obeys Malus' law

$$I(\theta) = I_0 \cos^2 \theta$$

where I_0 is the intensity of the light polarized by p_1 and incident, I is the intensity of the transmitted light, and θ is the angle between the transmission axes of P_1 and p_2. Thus we can obtain light with various intensities for the verification by using two polarizers.

The experimental arrangement is shown in the figure.

The light intensity detector D_1 serves to monitor the intensity fluctuation of the incident beam (the ratio of I_1 to I_2 remain unchanged), and D_2 measures I_2. Let $i_1(\theta)$ and $i_2(\theta)$ be the readings of D_1 and D_2 respectively, and $\psi_2(\theta)$ be the reading of the marked line position. $i_2 = 0$ when $\theta = 90°$, the corresponding ψ_2 is $\psi_2(90°)$, and the value of θ corresponding to ψ_2 is

$$\theta = |\psi_2 - \psi_2(90°) \pm 90°|$$

Data and results;

$$\psi_2(90°) = 4°$$

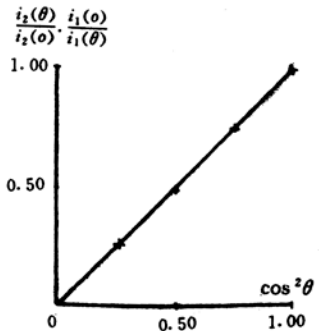
ψ_2	94.0°	64.0°	49.0°	34.0°	4.0°
θ	0.0°	30.0°	45.0°	60.0°	90.0°
$i_1(\theta)\mu A$	6.3×1	5.7×1	5.7×1	5.7×1	5.7×1
$i_2(\theta)\mu A$	18.7×5	12.7×5	8.2×5	4.0×5	0.0×5

From the above data we can obtain the values of $I(\theta)/I_2(\theta)$ from the formula

$$\frac{I(\theta)}{I_0} = \frac{i_2(\theta)}{i_1(\theta)} \cdot \frac{i_1(0)}{i_2(0)}$$

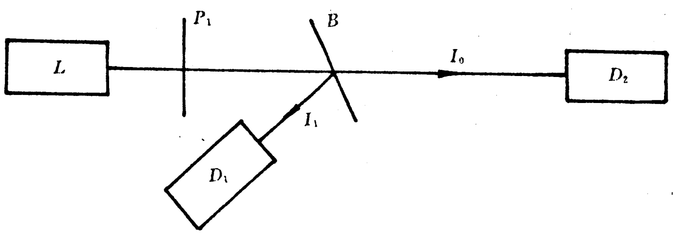
and compare them with $\cos^2\theta$ for examining the linear relationship. The results obtained are:

θ	0.0°	30.0°	45.0°	60.0°	90.0°
$\cos^2\theta$	1.00	0.75	0.50	0.25	0.00
$I(\theta)/I_0$	1.00	0.75	0.49	0.24	0.00

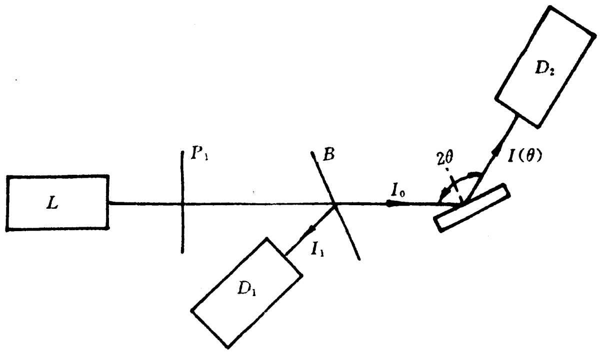


1. (c) Reflectivity measurement

The experimental arrangement shown below is used to determine the ratio of I_0 to I_1 which is proportional to the ratio of the reading (i_{20}) of D_2 to the corresponding reading (i_{10}) of D_1.



Then used the experimental arrangement shown below to measure the relativity R_p of the sample at various incident angle (θ) while the incident light consists of p-component only. Let $i_1(\theta)$ and $i_2(\theta)$ be the readings of D_1 and D_2 respectively.



Then the reflectivity is

$$R_p(\theta) = \frac{I(\theta)}{I_0} = \frac{i_2(\theta)}{i_1(\theta)} \cdot \frac{i_{10}}{i_{20}}$$

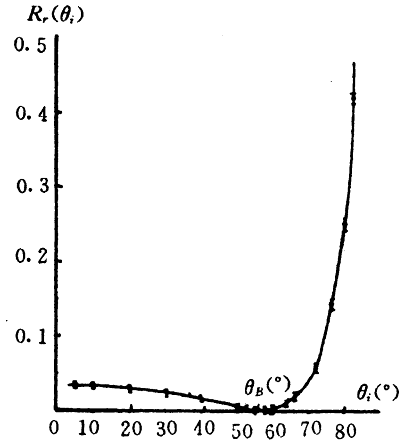
Data and results:

$$\psi_1 = 140.5°$$
$$i_{20} = 19.8 \times 5 \mu A$$
$$i_{10} = 13.3 \mu A$$

θ(°)	$i_2(\theta)$	$i_1(\mu A)$	$R_p(\theta)$
5	15.1×0.2	11.1	0.037
10	14.9×0.2	11.2	0.036
20	13.3×0.2	11.1	0.032
30	11.4×0.2	12.2	0.025
40	7.8×0.2	14.7	0.014
50	2.3×0.2	16.9	0.0037
53	0.7×0.2	11.3	0.0017
55	0.3×0.2	11.3	0.00059
56.3（dark）	～0	11.5	～0
58	0.3×0.2	11.5	0.0007
60	1.1×0.2	13.5	0.0024
64	6.5×0.2	16.7	0.011
66	7.8×0.2	11.8	0.018
68	16.3×0.2	15.0	0.029

72	5.3×0.1	11.7	0.061
76	13.1×1	14.0	0.13
80	4.4×5	11.7	0.25
84	9.1×5	14.5	0.42

The curve of reflectivity of p-component as a function of incident in plexiglass



2. The Brewster angle θ_B can be found from the above date as

$$\theta_B = 56.3° \pm 0.2°$$

The index of refraction can be calculated as

$$n = \tan\theta_B = 1.50 \pm 0.01$$

The sources of errors are:

1. Detector sensitivity is low.
2. The incident light does not consist of *p*-component only.
3. The degree scales are not uniform.

EXPERIMENTAL PROBLEM 1: Grading Scheme(10 points)

Part 1. Reflectivity of the *p*-component. 7 points, distributed as follows.

a. Determination of the transmission axis of the polarizer (A) in *p*-component measurement, 1 point.

(Error less than ±2°, 1.0point;
error less than ±3°, 0.7point;
error less than ±4°, 0.3point;
error less than ±5°, 0.1 point.)

b. Verification of the linearity of the light intensity detector(2 points). Draws the optical schematic diagram correctly, 1.0 point; (Without the correction of the fluctuation of the light intensity, 0.4 point only);

Uses $I/I_0 \sim \cos^2\theta$ figure to show the "linearity", 0.5 point;

Tabulate the measured data(with 5 points at least)correctly, 0.5 point.

c. Determination of the reflectivity of the p-component of the light as a function of incident angle, 4 points, distributed as follows.

Draws the optical schematic diagram correctly and tabulate the measured data perfectly, 2.0 points;

Plot the reflectivity as the function of incident angle with indication of errors, 2 points.

Part 2. Determination of the refractive index of sample, 3 point.

Brewster angle of sample, 1 point;

(Error less than ±1°,	1.0point;
error less than ±2°,	0.5point;
error less than ±3°,	0.2point;
error larger than ±3°,	0 point.)
The refractive index of sample,	0.5 point.

Discussion and determination of errors, 1.5 points.

EXPERIMENTAL PROBLEM 2

Black Box

Given a black box with two similar terminals. There are no more than three passive elements inside the black box. Find the values of elements in the equivalent circuit between the terminals. This box is not allowed to be opened.

Experimental Apparatus

1. Double channel oscilloscope with a panel illustration, showing the name and function of each knob
2. Audio frequency signal generator with a panel illustration, showing the name and function of each knob
3. Resistance box with a fixed value of l00 ohm(< ±0.5%)
4. Several connecting wires
5. For the coaxial cables, the wire in black color at the terminal is grounded.
6. Log-log paper, semi-log paper, and millimeter paper are provided for use if necessary

Note: The knobs, which were not shown on the panel illustration of the "signal generator" and "oscilloscope", have been set to the correct positions. It should not be touched by the student.

Experimental Requirements

1. Draw the circuit diagram in your experiment.

2. Show your measured data and the calculated results in the form of tables. Plot the experimental curves with the obtained results on the coordinate charts provided(indicate the title of the diagram and the titles and scale units of the coordinate axes)
3. Given the equivalent circuit of the black box and the names of the elements with their values in the equivalent circuit(write down the calculation formulas).

Instructions

1. Do your experiment in the frequency range between 100 Hz and 50kHz.
2. The output voltage of the signal generator should be less than 1.0V (peak-to-peak). Set the "Out Attenuation" switch to "20" db position and it should not be changed.
3. On connecting the wires, be careful to manage the wiring so as to minimize the 50Hz interference from the electric mains.

Instruction for Using XD2 Type Frequency Generator

1. Set the "Out Attenuation" to "20" db position and it should not be changed.
2. Set the "Damping Switch" to "Fast" position.
3. The indication of the voltmeter of the signal generator is the relative value, but not the true value of the output.
4. Neglect the error of the frequency readings.

Note: For XD22 Type Audio Frequency generator, there is no "Damping Switch", and the "output" switch should be set to the sine "~" position.

Instruction for Using SS-5702 Type Oscilloscope

1. Keep the "V mode" switch in "Dual" position.
2. The "Volts/div" (black) and the "variable control" (red) vary the gain of the vertical amplifier, and when the "variable control" (red) is ill the fully clockwise position, the black setting are calibrated.
3. The "Times/div" (Black) varies the horizontal sweep rate from 0.5μs/div to 0.2s/div, and they are calibrated when the "variable control" (red) is in the fully clockwise CAL position.
4. The "Trigging Source" (Trigging sweep signal) is used to select the trigging signal channel and the" level" control is used to adjust the amplitude of the trigging signal.
5. Measuring accuracy: ±4%.

Instruction for Using "Resistance Box"

The resistance of the "Resistance Box" has been set to a value of 100ohm, and it should not be changed.

Experimental problem 2...... Solution

1. The circuit diagram is shown in Fig. 1

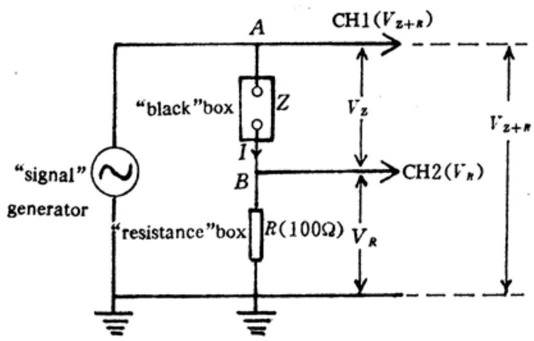


Fig. 1

We have the relation:

$$I = \frac{V_R}{R};$$

$$Z + R = \frac{V_{Z+R}}{I} = \frac{V_{Z+R}}{V_R} R$$

2. Measure the values of V_{Z+R} and V_R at various frequencies (*f*), the measured data and calculated value of *Z*+*R* are shown in table l. "The *Z*+*R*-*f* curve is plotted in Fig. 2

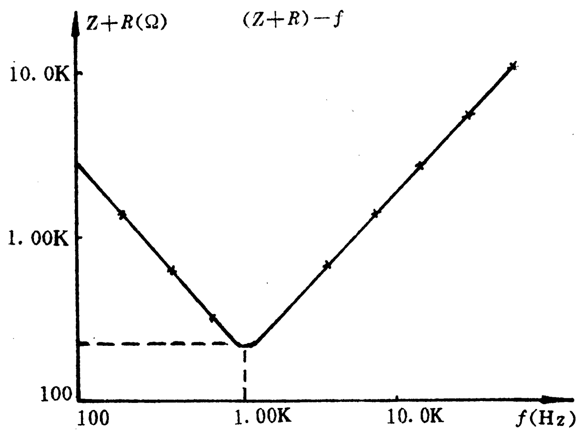


Table l. The magnitude of impedance verus frequency

$f\,(\times 10^3\,\text{Hz})$	$U_{Z+R}(V_{pp})$	U_R mV$_{pp}$	$Z + R(\times 10^3\,\Omega)$
0.100	0.600	22.0	2.73
0.200	0.600	45.0	1.33
0.400	0.600	94.0	0.638
0.700	0.300	92.0	0.326
0.900	0.300	121	0.248

1.00	0.300	136	0.220
1.10	0.300	140	0.214
1.16	0.300	141	0.213
1.25	0.300	140	0.214
1.50	0.300	120	0.250
2.00	0.300	88.0	0.341
4.00	0.300	78.0	0.769
8.00	0.600	38.0	1.58
15.0	0.600	20.0	3.00
30.0	0.600	10.0	6.00
50.0	0.600	6.0	10.0

From table 1 and Fig. 2, we got the conclusions:

(1) Current resonance (minimum of Z) occurs at $f_0 \cong 1.16 \times 10^3$ Hz.

(2) $f \langle\langle f_0$, $Z \propto f$, $\Delta\varphi \approx -\pi/2$. The impedance of the "black box" at low frequency is dominated by a inductance.

(3) $f \rangle\rangle f_0$, $Z \propto f$, $\Delta\varphi \approx \pi/2$. The impedance of the "black box" at high frequency is dominated by a inductance.

(4) Equivalent circuit of the "black box"; r, L and C connected in series shown in Fig. 3.

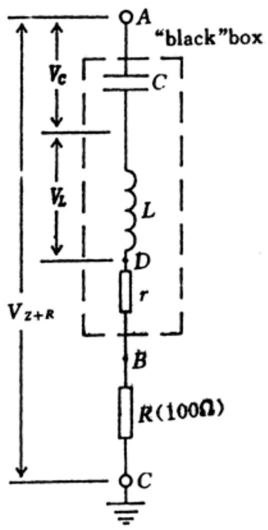


Fig. 3

3. Determination of the values of r, L and C.

(a) r

At resonance frequency f_0

$$V_C = -V_L$$

Then

$$Z + R = \frac{V_{Z+R}}{I} = \frac{V_{Z+R}}{V_R} R = r + R$$

From table 1, $r + R = 213\Omega$, it is given $R = 100\Omega$, so the equivalent resistance r in Fig. 3 is equal $113\,\Omega$.

(b) C

At low frequency, $z_L \approx 0$ in Fig. 3. So the circuit could be considered as a series RC circuit.

From phasor diagram, Fig. 4,

$$\frac{1}{\omega C} = Z_C = \frac{V_C}{I} = \frac{\sqrt{V_{Z+R}^2 - V_{R+r}^2}}{I}$$

Since $V_{R+r}^2 / V_{Z+R}^2 \approx 6 \times 10^{-3}$ at $f = 100\,\text{Hz}$, V_{R+r}^2 can beneglected with respect to V_{Z+R}^2, so

$$\frac{1}{\omega C} \approx \frac{V_{Z+R}}{I} \approx Z + R = 2.73 \times 10^3\,\Omega$$

$$C \approx \frac{1}{\omega(Z+R)} = 0.58\mu f\ .$$

$$C \cong 0.58\mu f\ .$$

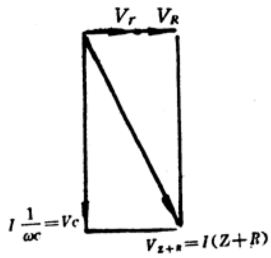


Fig. 4

(c) L

At high frequency, $Z_L \approx 0$ in Fig. 3. So the circuit could be considered as a series RL circuit.

From phasor diagram, Fig. 5,

$$|V_L| = \sqrt{V_{Z+R}^2 - V_{r+R}^2}\ ,$$

Since $V_{r+R}^2 / V_{Z+R}^2 \approx 4.5 \times 10^{-4}$ at $f = 50$ kHz, V_{r+R}^2 can be neglected with respect to V_{Z+R}^2, so

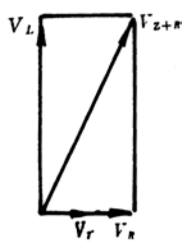


Fig. 5

$$\omega L = Z_L = \frac{V_L}{I} = \frac{|V_{Z+R}|}{I} \approx Z + R = 10^4\,\Omega \qquad (3)$$

$$L = \frac{Z+R}{\omega} = 31.8\,\text{mH}.$$

Error estimation:

It is given, precision of the resistance box reading $\Delta R / R \approx 0.5\%$

precision of the voltmeter reading $\Delta V / V \approx 4\%$

(1) Resistance r: at resonance frequency f_0

$$r + R = \frac{V_{Z+R}}{V_R} R$$

$$\frac{\Delta(r+R)}{r+R} = \frac{\Delta V_{Z+R}}{V_{Z+R}} + \frac{\Delta V_R}{V_R} + \frac{\Delta R}{R} \approx 4\% + 4\% + 0.5\% = 8.5\%$$

$$\Delta r = 16\Omega$$

(2) Capacitance C: (Neglect the error of the frequency reading)

$$\frac{1}{\omega C} \cong Z_C = \frac{V_{Z+R}}{V_R} R$$

$$\frac{\Delta C}{C} = \frac{\Delta V_{Z+R}}{V_{Z+R}} + \frac{\Delta V_R}{V_R} + \frac{\Delta R}{R} \approx 8.8\%$$

The approximation $V_C \approx V_{Z+R}$ will introduce apercentageerror 0.3%

(3) Inductance L: Similar to the results of capacitance C, but the percentage error introduced by the approximation $V_L \approx V_{Z+R}$ is much small (0.003%) and thus negligible.

$$\frac{\Delta L}{L} \approx 8.5\%.$$

Experimental Problem 2: Grading Scheme (10 points maximum)

1. Measuring circuit is correct as shown in Fig.(a)

……2.0point

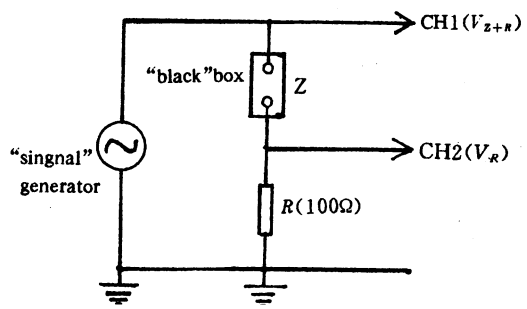


Fig. a

2. Correct data table and figure to show the characteristic of the black box

……2.0 points

3. The equivalent circuit of the black box, and the names of the elements with their values in the equivalent circuit are correct

total 6.0 points

(a) R, L and C are connected in series

……1.5 point

(L and C are connected in series

……1.0 point)

(b) Correct value (error less than 15%) for each element

……0.5 point (×3)

(error between 15% and 30% 0.3)

(error between 30% and 50% 0.1)

(c) Correct calculation formula for each element

……0.5 point (×3)

(d) Error estimate is reasonable for each element

……0.5 points (×3)

□□□□□□□□□