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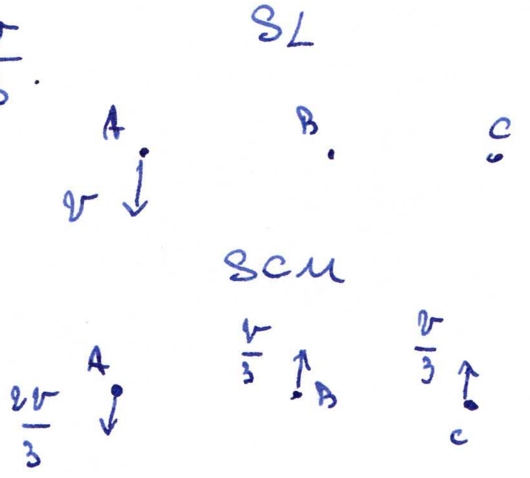
In the initial position $v_{cm} = \dfrac{mv}{3m} = \dfrac{v}{3}$.

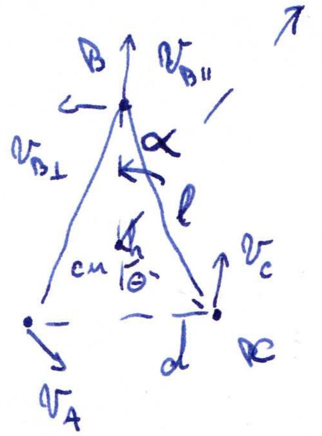
In the center of mass system the velocities are: $v_A = \dfrac{2v}{3}$

$v_B = \dfrac{v}{3}$

$v_C = \dfrac{v}{3}$



Let us consider a random position and calculate the Lagrangian of the system. The total degrees of freedom are reduced from 6 to 2 due to the pinning down of the CM and due to the bars.



Let α and d be the coordinates:

$$\cancel{L = \frac{1}{2}m(v_A^2 + v_B^2 + v_C^2) = \frac{1}{2}m\left(\frac{2}{9}h^2\dot{\alpha}^2 + 2\dot{d}^2 + \frac{4}{9}h^2\dot{\alpha}^2 + \frac{4}{9}\dot{h}^2\right)}$$

$\frac{7}{9}h^2 + d^2$

$$\cancel{h^2 + d^2 = l^2 \Rightarrow \dot{h} = -\frac{d\dot{d}}{\sqrt{l^2-d^2}} \Rightarrow \dot{h}^2 = \frac{d^2\dot{d}^2}{l^2-d^2}}$$

$$\cancel{\text{Since } \alpha \text{ is cyclic: } \frac{\partial L}{\partial \dot{\alpha}} = ct. \Rightarrow}$$

$$\cancel{\Rightarrow \frac{4}{9}h^2\dot{\alpha} + \frac{8}{9}h^2\dot{\alpha} = ct. \Rightarrow h^2\dot{\alpha} = ct. \Rightarrow h^2\dot{\alpha} =}$$

$$\cancel{\Rightarrow \frac{4}{9}h^2\dot{\alpha} + \frac{8}{9}h^2\dot{\alpha} + 4d^2\dot{\alpha} = ct \Rightarrow \left(\frac{1}{3}h^2 + d^2\right)\dot{\alpha} = ct.}$$

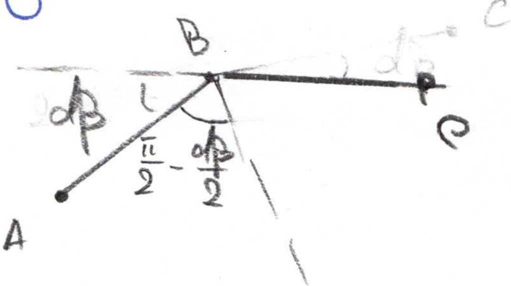
$$\cancel{\Rightarrow \left(\frac{1}{3}h^2 + d^2\right)\dot{\alpha} = l^2\frac{v}{6l} = \frac{1}{6}lv \quad \left(\dot{\alpha}\big|_{t=0} = \frac{v}{6l} \text{ as observed in the picture}\right)}$$

$$L = \frac{1}{2}m\left(v_A^2+v_B^2+v_C^2\right) = \frac{1}{2}m\left(2d^2\dot\alpha^2 + \frac{2}{9}h^2\dot\alpha^2 + 2\dot d^2 + \frac{4}{9}h^2\dot\alpha^2 + \frac{4}{9}\dot h^2\right)$$

α - cyclic $\Rightarrow \frac{\partial L}{\partial \dot\alpha} = ct. \Rightarrow 4d^2\dot\alpha + \frac{4}{9}h^2\dot\alpha + \frac{8}{9}h^2\dot\alpha = ct. \Rightarrow$

$\Rightarrow \left(d^2 + \frac{1}{3}h^2\right)\dot\alpha = ct.$

Figure at $t = dt$



At $t=0$: $\dot\alpha = \frac{v}{2l}$

$\Rightarrow \left(d^2 + \frac{1}{3}h^2\right)\dot\alpha = l^2 \frac{v}{2l} = \frac{vl}{2}$

$d\alpha = d\beta + \frac{\pi}{2} - \frac{d\beta}{2} - \frac{\pi}{2} = \frac{1}{2}d\beta \Rightarrow$

$\Rightarrow \dot\alpha = \frac{1}{2}\dot\beta \overset{t=0}{=} \frac{3}{2}\frac{v}{3l} = \frac{v}{2l}$

Since the energy is conserved:

$\frac{1}{2}m\left(\frac{4v^2}{9} + \frac{v^2}{9} + \frac{v^2}{9}\right) =$

$= \frac{1}{2}m\left(2d^2\dot\alpha^2 + \frac{6}{9}h^2\dot\alpha^2 + 2\dot d^2 + \frac{4}{9}\dot h^2\right)$

When the distance is the smallest we have $\dot d = \dot h = 0$

$\Rightarrow \frac{2}{3}v^2 = 2d^2\dot\alpha^2 + \frac{6}{9}h^2\dot\alpha^2 = \cancel{\left(2d^2 + \frac{6}{9}h^2\right)\frac{v^2}{4l^2}} \Rightarrow$

$\cancel{\Rightarrow \frac{8}{3}l^2 = 2d^2 + \frac{6}{9}\left(l^2 - d^2\right) \Rightarrow \frac{6}{3}l^2 = d^2 \Rightarrow d = \sqrt{2}\,l}$

~~Since the answer suggests d>l the balls can get to d=0 (A and C collide), so there is no minimum distance~~

$\Rightarrow \frac{2}{3}v^2 = \left(2d^2 + \frac{6}{9}h^2\right)\frac{v^2l^2}{4\left(d^2 + \frac{1}{3}h^2\right)^2} = \frac{1}{2}\frac{v^2l^2}{d^2 + \frac{1}{3}h^2} \Rightarrow \frac{4}{3}d^2 + \frac{4}{9}\left(l^2 - d^2\right) = l^2$

$\Rightarrow \frac{8}{9}d^2 = \frac{5}{9}l^2 \Rightarrow d = \sqrt{\frac{5}{8}}\,l$