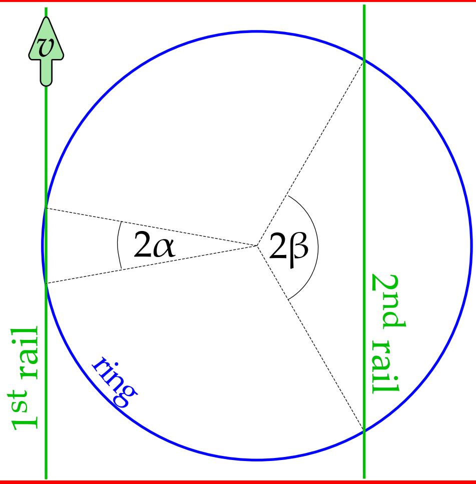
Problem 7

A homogeneous ring lays horizontally on two identical parallel rails. The first rail moves parallel to itself, with a constant speed v; the second rail is at rest. The angular distance between the ring-rail contact points, as seen from the centre of the ring, is 2α for the first rail, and 2β for the second rail, see figure. Assuming that $\alpha \ll 1$ and $\beta = \pi/3$, find the speed of the centre of the ring.



Hints after 1st week: This problem can be solved by using a brute force approach, i.e. writing down two equations for two unknown angles. However, the solution can be significantly simplified once a useful geometrical fact is noticed: then, it is enough to write down only one equation for one unknown quantity.

Hints after 2nd week: Typically in the case of static's problems, it is convenient to start with a torque balance, because the origin for the balance equation can be chosen in such a way that arms of at least two forces become equal to zero; also, you are free to choose, which forces you want to disappear from your torque balance (which are the least desirable). In particular, if there are only three forces applied to a rigid body at an equilibrium, the lines along which these forces are applied intersect always in a single point. Here, in order to derive the "geometrical fact" mentioned in the previous hint, study the torque balance with respect to the intersection point of the lines along which two forces (e.g. the friction forces due to the 1st rail) are applied.

Results of the 7th problem.

Correct solutions have been submitted by

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