Problem 9: high-efficiency light emitting diode (idea contributed by Mihkel Heidelberg)

Introduction. As compared to ordinary light bulbs, light emitting diodes (LED) provide very high lighting efficiency. The reason is that the spectral energy distribution of ordinary lamps is close to black body radiation, in which case one can say that the photons are in thermal equilibrium with the black body. Then, the total energy radiated by a black body per unit area, unit time, and unit frequency interval is given by Planck's law

$$I = \frac{2\pi h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1},$$

where ν is the frequency, $h = 6.626\times10^{-34}\,\mathrm{J\cdot s}$ — the Planck constant, $c = 2.997\times10^{8}\,\mathrm{m/s}$ — the speed of light, $k = 1.38\times10^{-23}\,\mathrm{J\cdot K^{-1}}$ — the Boltzmann constant, and T — the temperature; note that

$$\int_0^\infty I d\nu = \sigma T^4,$$

where $\sigma = 5.678\times10^{-8}\,\mathrm{W\cdot m^{-2}\cdot K^{-4}}$ is the Stefan-Boltzmann constant. With a black body radiation, a lot of energy is wasted by radiating non-visible light. Meanwhile, LED-s can be constructed so that they radiate almost only visible light.

In recent experiments [1], it has been reported that such LED-s have been constructed which have efficiency higher that 100%. Here the efficiency is defined as the ratio of the radiated light energy to the consumed electrical energy.

Problem. Based on reasonable approximations, find what is the theoretically highest possible efficiency of a LED assuming that:
(a) the LED has a heat sink which is kept at the room temperature $T_0 = 293\,\mathrm{K}$ (via a fast enough heat exchange with the surrounding medium);
(b) the LED emits light at wavelengths smaller than $\lambda_0 = 700\,\mathrm{nm}$
(b) the surface area of the light-emitting part of the LED is $S = 1\,\mathrm{mm}^2$;
(c) the light emission power of the LED is $P = 1\,\mu\mathrm{W}$.

Hints after the first week. Since the efficiency is larger than 100%, there needs to be another source of energy (apart from the electrical energy), and with the given experimental setup, there is only one possibility: the energy is taken from the surroundings in the form of heat. This is perfectly possible, because heat energy can be easily transferred to the diode via the heat sink. However, there is a limit to how much heat can be transferred. That theoretical limit depends on the working regime of the LED. In particular, a higher light radiation power by a fixed surface area of the LED will result in a lower theoretical limit for the efficiency.

Hints after the second week. Since some heat is taken from the environment, the entropy of the environment is decreased respectively. However, entropy cannot decrease in a closed system, so at least some of the LED light needs to be radiated in the form of heat. Your first sub-task is to figure out, what is the optimal temperature of that heat.

Hints after the third week. So, the LED converts using electrical energy a certain amount of environment's heat (at T_0) into heat in the form of electromagnetic radiation at temperature T_1. This process needs to conserve energy and cannot decrease the overall entropy. These conditions put limit to the heat conversion efficiency. Note also that the density of thermal electromagnetic radiation falls rapidly with temperature, and too low values of T_1 cannot yield the required output power of the LED.

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NB! Problem No 10 will be uploaded on the forthcoming Sunday, 30th June, 14.00 GMT.

[1] P. Santhanam et al, *Thermoelectrically Pumped Light-Emitting Diodes Operating above Unity Efficiency*, Phys. Rev. Lett. **108**, 097403 (2012)