On Periodic Waves Governed by the Extended Korteweg-deVries Equation

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ABSTRACT

Janno and Engelbrecht [1] have shown that the propagation of nonlinear one-dimensional waves in microstructured solids leads to a generalized Korteweg-deVries equation. After suitable transformation of the variables, the evolution equation of the waves can be written in the form [2]

$$y_t + 3\left(y^2\right)_x + y_{xxx} + 3\varepsilon\left(y_x^2\right)_{xx} = 0. \tag{1}$$

The additional term involving the small parameter ε reflects the nonlinearity in the microscale. Solutions representing asymmetric solitary waves have been analyzed both qualitatively and numerically in [1]. An approximate solution in analytic form has been given in [2].

Solitary waves can be considered as the long-wave limit of periodic solutions which, in the KdV case, have the form of cnoidal waves. The present paper is devoted to periodic solutions $y = q(x - ct)$ of the *extended* KdV equation (1), which emerge from the cnoidal waves for $\varepsilon > 0$. The corresponding phase curves $q'(q)$ are represented by a cubic equation in q' which makes the final integration step unlikely to be accomplished in closed form. Therefore the cubic equation is solved approximately for small values of the parameter ε. In this approximation, the final integration can be performed analytically and yields an implicit representation of the periodic solutions.

Compared with the cnoidal waves the periodic waves for $\varepsilon > 0$ have steeper slope at the leading flank while the trailing flank falls off gentler. The relations between amplitude, period, and propagation speed are not influenced by the additional nonlinear term. Qualitatively the behavior is like expected from the solitary-wave limit.

REFERENCES

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