Solitary Shock Waves and Periodic Shock Waves in a Compressible Mooney-Rivlin Elastic Rod

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ABSTRACT

Weakly nonlinear waves in elastic rods have been studied by many authors. Most studies concentrated on smooth solutions only. However, when waves are of moderate length, non-smooth waves, such as peakons (or solitary shock waves), can arise (see Dai and Huo 2000). For strongly nonlinear waves (not assuming the smallness of the wave amplitude), as far as the authors are aware, only incompressible hyperelastic materials were considered. Wright (1985) pointed out the existence of a variety of travelling waves. Coleman and Newman (1990) derived the one-dimensional rod equation for a general incompressible hyperelastic material and gave the explicit solitary and periodic wave solutions for the case of a neo-Hookean material. For a rod composed of an incompressible Mooney-Rivlin material, Dai (2001) showed that under the strong nonlinearity there exist a number of travelling wave solutions, including some interesting non-smooth solutions. Here, we study strongly nonlinear axisymmetric waves in a circular cylindrical rod composed of a compressible Mooney-Rivlin material. To consider the travelling wave solutions for the governing partial differential system, we first reduce it to a nonlinear ordinary differential equation. By using the bifurcation theory of planar dynamical systems, we show that the reduced system has seven periodic annuluses with different boundaries which depend on four parameters. We further consider the bifurcation behavior of the phase portraits for the reduced one-parameter vector fields when other three parameters are fixed. Corresponding to seven different periodic annuluses, we obtain seven types of travelling wave solutions, including solitary waves of radial contraction, solitary waves of radial expansion, solitary shock waves of radial contraction, solitary shock waves of radial expansion, periodic waves and two types of periodic shock waves. These are physically acceptable solutions by the governing partial differential system. The rigorous parameter conditions for the existence of these waves are given.

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