Nonlinear Waves: Examples of Complexity

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ABSTRACT

Contemporary complexity science deals with problems involving many variables which interact with each other (and with the environment) in such a way that new quality appears. Nonlinearity is a cornerstone of complex systems which as a rule are far from equilibrium and exhibit properties of emergence of coherent structures, possibly over many scales. Although nowadays complexity studies stress beside physics and mathematics also biology, chemistry, environment (including weather and climate), social sciences, etc. [1], mechanics is definitely a part of complexity science. Even the earlier studies demonstrate, for example, the complexity of wave motion [2]. Now complexity in mechanical systems has been studied in much more details.

Generalized continuum theories incorporate intrinsic microstructural and nonlinear effects in the mechanical behaviour of solids. The concept of internal variables, multiscale approach and hierarchical governing equations allow analysing waves in microstructured solids with clear physical reasoning. The effects like emergence of solitary waves and solitary wave structures, scale-dependence and wave hierarchy clearly demonstrate that the contemporary mechanics exhibits many complex effects which now are intensively studied in other fields.

In this talk nonlinear wave propagation in microstructured solids is analysed as a basic example of complexity but some insights to fluid dynamics [3] and biophysics [4] are also given. The basic models are derived from the canonical (material) momentum equation where the interaction forces are clearly separated. Novel concepts like wave hierarchy [5] and dual internal variables [6] are described and several examples together with numerical results are presented.

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