Turbulent Mixing of Passive Scalars: Evolution of Discontinuity Fronts and Material Lines

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ABSTRACT

Mixing of passive scalars can be considered as a wave propagation process, which is induced by the competing effects of chaotic forcing and small-scale dissipation. The evolution of the scalar fields depends qualitatively on the features of the mixing flow. Here we focus on the case of fully developed incompressible turbulent flows, which are known to lead to highly intermittent scalar fields. The main feature of these fields is the presence of a cascade of *fractal discontinuity fronts* of the tracer density. While most of the fronts are characterized by relatively small values of the density drop, there are also some segments of the fronts (the so-called *mature fronts*), where the density drop is of the order of the amplitude of the global density variations [1]. In particular, the presence of these fronts is the cause for the anomalous scaling of the structure function scaling exponents.

It has bee widely believed that almost all the features of the passive scalar turbulence can be explained on the basis of the stretching statistics of material elements. We use this approach to construct a simple, analytically tractable model of passive scalar turbulence. Based on that model, we derive the probability distribution function for the height distribution of the tracer discontinuity fronts. We are also able to obtain an analytic expression for the structure function scaling exponent ζ_p, which is in a good agreement with the experimental and numerical data [2].

Based on our model, we argue that while the stretching of material elements is indeed the driving force of the geometrical complexity of the passive tracer fields, some aspects of that complexity cannot be explained without evoking the dissipation effects, even at the formal limit of infinite Peclet' numbers. We provide a further proof of that conclusion by studying the evolution of material lines in fully developed two-dimensional turbulent flows. These lines are stretched and folded according to the action of the flow; if two segments of a line approach each other (to a Batchelor scale), a reconnection takes place, and a material loop separates from the ”parent” line. We show that the fractal dimension of a single material line (i.e. the ”parent” line without the sprouted loops) differs significantly from the fractal dimension of the tracer isodensity lines. This observation leads us to the conclusion that the seed diffusivity (even if vanishingly small) gives rise to the reconnection of the isodensity lines, which plays a crucial role in determining the statistical topography of the passive scalar fields.

REFERENCES

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