A Study of Dispersive Effects in Hydraulic Jumps

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ABSTRACT

A hydraulic jump is the transition from supercritical to subcritical flow in an open channel flow. Hydraulic jumps are often observed behind dams and they are also used as an energy-dissipation mechanism in spillways. A hydraulic jump is characterized by the upstream Froude number $Fr = \frac{u}{\sqrt{gh_0}}$, where u is the (uniform) upstream horizontal flow velocity, g is the gravitational constant, and h_0 is the flow depth. Depending on the upstream Froude number, different types of jumps are possible , including undular jumps, and fully developed turbulent jumps [2]. All hydraulic jumps experience some form of energy loss. While the energy loss in a turbulent hydraulic jump can be attributed to turbulent dissipation, an undular hydraulic jump loses energy due to an increasing number of waves in the downstream part. The present study is concerned with weak hydraulic jumps, where it is assumed that no wave-breaking is taking place.

The common explanation for the energy loss in an undular hydraulic jump is that energy is radiated from the jump by an increasing number of oscillations traveling downstream. The energy loss can be computed by an analysis of the conservation equations associated with a jump solution of the shallow-water system

$$\eta_t + h_0 u_x + (u\eta)_x = 0, \tag{1}$$

$$u_t + g\eta_x + uu_x = 0, \tag{2}$$

where η denotes the deflection of the free surface. As is well known, conservation of mass and momentum across the jump necessitate a loss of energy, and this energy loss can be computed explicitly [3]. Through a quantitative analysis of the time development of the energy associated to the higher-order dispersive model equation,

$$\eta_t + h_0 w_x + (w\eta)_x - \frac{h_0^2}{6}\eta_{xxt} = 0, \tag{3}$$

$$w_t + g\eta_x + ww_x - \frac{h_0^2}{6}w_{xxt} = 0, \tag{4}$$

it is found that there is no energy loss if dispersive effects are taken into account. This finding confirms that the energy lost at the jump goes into the downstream oscillations, and there is no need to consider other effects, such as bottom boundary layers.

This system (3),(4) was derived in [1], and is one of a general class of dispersive partial differential equations depending on three parameters, including the height at which the horizontal velocity w is evaluated. In the case of (3),(4), this height is $\sqrt{\frac{2}{3}}h_0$. This system is amenable to the study of a hydraulic jump because Dirichlet boundary conditions are easily implemented.

REFERENCES

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