From Phase-Transformation Fronts to the Growth of Long Bones

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ABSTRACT

The mathematical and physical modelling of the propagation of phase transformation fronts is a multiscale problem that can be examined at several scales with different styles of approach including that of discontinuity surfaces (homothermal shock waves) of vanishing thickness, that of a structured transition zone in which dissipation works against dispersion (a model of applied mathematics per se), that of a smooth nondissipative solitary wave in a regular lattice with both nonlinearity and dispersion, and finally, that of a quasi-particle in a more or less inertial motion [1]. Having perused these different viewpoints and using some analogies, we shall address a problem issued from mechano-biology, that of the growth of long bones in mammals. This growth (lengthening) takes place at the *growth plate* in the so-called physeal region between cartilage and forming bone. The modelling aims at explaining the various phases of the evolution until the end of growth (in the early twenties of most adult humans) in nonpathological situations as also with the possibility of some pathological development (instabilities, untimely arrest of growth). This problem represents, at the various degrees of description, a true benchmark for the evolution of such transition regions with resemblance and difference with what occurs in inanimate matter (crystals, alloys; see above). The modelling and problem study are divided in three stages of increasing complexity (a long project that will simply be sketched out). The first relatively simple model sees the growth plate as a practically zero-thickness transition zone moving steadily although slowly (here the time scale is in years) under the action of **a driving force** related to the mechanical environment (mechanical loading of the long bone). This exploits arguments from the theory of Eshelbian-(configurational) material forces as fruitfully developed in recent years by the author and co-wrkers. An interesting question to be answered here is the **stability** of this evolution (i.e., the absence of pathological development; work in co-operation with A.B. Freidin's team in St Petersburg)) The second modelling should see the growth plate as a transition zone with relatively smoothly varying properties through the small albeit finite thickness. The modelling inside this transition zone may rely on the **gradient theory of elasticity** or a refined microstructured theory such as that of so-called **Cosserat continua**. The third and final modelling views the growth plate as an evolving dissipative structure with thickness variation in time until final closing of the plate (end of growth). The last two modellings will bring into the picture the evolution of "**nonlinear structures**" of varying complexity.

REFERENCES

[1] G.A.Maugin. Multiscale approach to a basic problem of materials mechanics (Propagation of phase-transition fronts). In: A.Sandig, W.Schielen, and W.L.Wendland, (eds.) *Multifield Problems: State of the Art*. pp. 11-22, Springer, 2000.