Hierarchy of Nonlinear Waves in Complex Microstructured Solids

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ABSTRACT

As pointed out in [1], in non classical mechanics naturally arises the problem of the propagation of non linear waves in solids with different internal structural scales. Here we make use of the model of one dimensional microstructured solids as developed in [2], and references therein quoted, to describe the behaviour of internal structures with two different scales. Hence we have an elastic material composed by a macrostructure, a first microstructure (say a mesostructure) and a second microstructure at some smaller scale.

The choice of suitable microstrains functions φ and ψ at the two levels respectively, of the microdisplacement *u*, of their time derivatives as strain velocities, allows us to obtain the field equations via a variational principle (see [2], [3]).

By means of the slaving principle, as introduced and used mainly by Engelbrecht [1] and references quoted therein, a single PDE is obtained in terms of a rescaled displacement field. This equation is a 6^{th} order PDE, with characteristic hierarchical structure, where the three levels hierarchy and the various coefficients may reflect the dominance of one structural level over the other ones in wave propagation.

The introduction of the phase variable $z = x \pm Vt$, where x is the one dimensional coordinate, t the time and V the wave speed, followed by twice integration with suitable conditions at infinity, reduces the 6^{th} order ODE to a 4^{th} order ODE, which can be integrated in terms of elliptic functions, following a general method introduced by Samsonov [4], that may be applied in many different problems.

REFERENCES

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