Numerical Stability of Mass Lumping Schemes for Higher Order Finite Elements

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ABSTRACT

Discretization of a continuous medium by the finite element method introduces dispersion error to numerical solutions of stress waves propagation. The paper opens with summarization of recent results by these authors [1] on the extension of dispersion theory to quadratic finite elements, following the lines of reasoning introduced by Belytschko and Mullen [2] for one-dimensional elements and those of Abboud and Pinsky [3] for the scalar Helmholtz equation. The finite elements investigated in a 2D elasto-dynamic context are the eight-node serendipity elements.

In the second part, the paper narrows to a detailed study of the mass matrix lumping schemes suitable for wave propagation simulations. To this end, a variable parameter, x, is defined whose role is to distribute total mass between the elements corner and midside nodes. Based on that, dispersion analysis is carried out for varying x. For example, it is shown in terms of dispersion curves that the HRZ mass ratio ($x = 16/76 = 0.21$) is far from optimum and, on the contrary, the most accurate travelling wave-train representation is surprisingly obtained for $x = 0.23$ mass ratio, that is, when 92% of total mass is coalesced into four midside nodes, whereas only the 8% share is placed to the corner nodes.

Finally, the issues of numerical stability are addressed. Rigorous stability theory is invoked to assess stability properties of various lumping methods given by the choice of x, directly employing dispersion diagrams. The results obtained are compared against the approximate expression by the Fried theorem which makes use of eigenvalue estimates of individual (disconnected) elements. It turns out that the well known lack of stability of quadratic elements is caused by the presence of optical modes in the spectrum, which reduces the time step about three to five times compared to 4-node elements. It should be kept in mind, however, that the nodal distance is half the one in the bilinear mesh and much better accuracy is gained as a trade-off. This was shown earlier by the same authors in [1].

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