Models for Essentially Nonlinear Strain Waves in Complex Materials

Alexey V. Porubov

Institute of Problems in Mechanical Engineering
Bolshoy av. 61, V.O., Saint-Petersburg 199178
RUSSIA
e-mail: porubov.math@mail.ioffe.ru

ABSTRACT

The interest in the dynamic behaviour of micro- and nano- structured materials has grown in the last years. The classical theories of elasticity and magnetism cannot explain some of the experimental data concerning the properties of such materials with complicated microstructure. In particular, different from classic elasticity, nonlinearity in complex materials may be strong. Sometimes, it is caused by the presence of the components with contrasting elastic features including cracks, intergranular contacts, dislocations at the grain boundaries of polycrystals, see, e.g.[1] . An influence of magnetic field revealed essentially nonlinear strains in paramagnetic crystals [2]. In all above mentioned cases there exists dispersion caused by various reasons. It gives rise to a balance between nonlinearity and dispersion thus to a possible existence of localized strain waves.

In this paper it is shown that essentially nonlinear models for solids with complex internal structure may be studied using phenomenological approach based on the formal use of the stress-strain expansion obtained for the weakly nonlinear strains. One way is to substitute this expansion into the equations for macro-strains while another approach is based on an introduction of a micro-field, like it was done in the weakly nonlinear case in [3] using the Mindlin theory. It is found that both approaches give rise to the same nonlinear equation for travelling longitudinal macro-strain waves whose solitary wave solutions differ from those of the model equation in the weakly nonlinear case. In particular, essential nonlinearity provides simultaneous existence of both compression or tensile localized strain waves.

Numerical simulations are performed in order to see how an arbitrary perturbation evolves into a sequence of localized waves and how they are interact each other. Some preliminary results may be found in [4,5].

The work has been supported by a grant of the Russian Science Support Foundation and by the grants of the Russian Foundation for Basic Researches No 07-01-00213a and 09-01-00469a.

REFERENCES

[1] V.Yu. Zaitsev, V.E. Nazarov, V.I. Talanov. Nonclassical' manifestations of microstructure-induced nonlinearities: new prospects for acoustic diagnostics. *Physics-Uspekhi,* **49**, 89-94, 2006.

[2] A.N. Bugai, S.V. Sazonov. Soliton-like propagation modes of picosecond acoustic pulses in a paramagnetic crystal. *Physics of the Solid State*, **49**, 118–125, 2007.

[3] J. Janno, J. Engelbrecht. Solitary waves in nonlinear microstructured materials. *J. Phys. A: Math. Gen.*, **38**, 5159–5172, 2005.

[4] A.V. Porubov, G.A. Maugin. Longitudinal strain solitary waves in presence of cubic non-linearity. *Intern. J. Non-Linear Mech.* , **40**, 1041–1048, 2005.

[5] A.V. Porubov, G.A. Maugin. Improved description of longitudinal strain solitary waves. *J. of Sound and Vibration*, **310**, 694–701, 2008.