Emergence of Solitary Deformation Waves in Mindlin-type Microstructured Solids

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ABSTRACT

In the present paper propagation of 1D deformation waves in microstructured media is modelled making use of Mindlin theory of continua with microstructure and hierarchical approach by Engelbrecht and Pastrone [1]. If to introduce the free energy function in the simpliest nonliner form

$$W = \frac{A}{2}u_x^2 + \frac{B}{2}\varphi^2 + \frac{C}{2}\varphi_x^2 + D\varphi u_x + \frac{N}{6}u_x^3 + \frac{M}{6}\varphi_x^3, \tag{1}$$

one gets from Euler–Lagrange equations system of equations of motion

$$\begin{aligned} \rho u_{tt} &= D\varphi_x + Au_{xx} + Nu_x u_{xx}, \\ I\varphi_{tt} &= C\varphi_{xx} + M\varphi_x\varphi_{xx} - B\varphi - Du_x. \end{aligned} \tag{2}$$

Here φ is microdeformation, u – macrodisplacement and x – material coordinate. Making use of the slaving principle a nonlinear hierarchical wave equation

$$U_{TT} - bU_{XX} - 0.5\mu(U_X^2)_X = \delta\left[\beta U_{TT} - \gamma U_{XX} - 0.5\delta^{1/2}\lambda(U_{XX}^2)\right]_{XX} \tag{3}$$

can be derived from the latter. Here $b, \mu, \beta, \gamma, \delta, \lambda$ are free energy related parameters, U is dimensionless macrodisplacement, X – dimensionless coordinate and T – dimensionless time. The full derivation of eqs. (2) and (3) can be found in papesr [2] and [3] and some preliminary results of numerical simulations in [4].

Equations (2) (in dimensionless form) and (3) are solved numerically under harmonic and localized initial conditions. For numerical integration Fourier transform based pseudospectral method is used. The influence of free energy parameters on the character of the solution is studied. Numerical results of hierarchical approximation (3) and full equation system (2) will be compared. Special attention is paid to solitonic solutions.

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