Modeling Bottom Shear Stress for Transient Wave Events

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A correct estimate of the bottom friction effect is important when calculating the propagation of long waves in shallow water over large distances. Traditional methods for modeling the bottom shear stress take the form of $C_f\,\rho|u_b|u_b$, where ρ is water density, u_b is the horizontal velocity at the bottom, and C_f is the friction coefficient. If tuned appropriately, this formulation can capture the long term average energy dissipation, which in many cases is adequate to obtain a satisfactory damping rate of the wave amplitude. However, it does not reproduce the correct phase of the bottom shear stress, nor does it take into account the historic development of the velocity field. These factors are of particular interest when modeling groups of transient long-waves, such as wakes from high-speed ferries or tsunami events, which often disrupt otherwise stable sediments at the sea floor.

Recently Liu and Orfila[1] suggested a new model for the bottom friction, based on analysis of the bottom boundary layer. The resulting bottom shear stress is formulated as

$$\tau_b(x,t) = \frac{u_\alpha(x,0)}{\sqrt{\pi t}} + \frac{1}{\sqrt{\pi}}\int_0^t \frac{u_{\alpha,\tau}(x,\tau)}{\sqrt{t-\tau}}\,d\tau + \mathcal{O}(\mu^2)\,,$$

where u_α is the horizontal velocity evaluated at $z = z_\alpha$, and t is time. Since this formulation includes a convolution integral in time, the computational effort in calculating the bottom shear stress by a direct method is usually very large. Torsvik and Liu[2] presented an iterative method for estimating the value of the convolution integral, thereby greatly reducing the cost of the computation.

The method has been tested for several idealized test cases, involving both periodic and transient waves. All test cases show good agreement between results using the direct and iterative method. Compared to simulations without bottom friction, the (wall-clock) execution time for the test cases increased by 970%–1105% using the direct method, whereas using the iterative method increased the execution time by about 20%.

References

[1] P. L.-F. Liu and A. Orfila. Viscous effects on transient long-wave propagation. *J. Fluid Mech.*, 520:83–92, 2004.

[2] T. Torsvik and P. L.-F. Liu. An efficient method for the numerical calculation of viscous effects on transient long waves. *Coastal Engineering*, 54:263–269, 2007.