Solitary Pulse Interaction Theory for the Generalized Kuramoto–Sivashinsky Equation

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ABSTRACT

We study weak interaction of solitary pulses for the generalized Kuramoto–Sivashinsky equation, which is one of the simplest prototypes describing active media with energy supply, dissipation, dispersion, and nonlinearity. A particular example of such a medium is that of a liquid film flowing down an inclined planar substrate. The spatio–temporal evolution of this system is dominated by localized quasi-stationary nonlinear pulses which continuously interact with each other, see [1]. We represent the solution of the generalized Kuramoto–Sivashinsky equation as a superposition of such solitary pulses and an overlap function. Under the assumption that the pulses are sufficiently separated, we derive a linearized equation for the overlap function in the vicinity of each pulse. We project the dynamics of this function onto the discrete part of the spectrum of the linearized interaction operator. This leads to a coupled system of ordinary differential equations describing the evolution of the locations of the pulses, or, alternatively, the evolution of the separation distances. By analyzing the fixed points of the latter system, we obtain bound states of any number of pulses. In particular, we analyze in detail bound states of two and three pulses. For two pulses, we formulate and prove a criterion for the existence of a countable infinite or finite number of bound states, depending on the strength of the dispersive term in the equation. Interestingly, this criterion coincides exactly with Shilnikov's criterion on the existence of subsidiary homoclinic orbits, see [2]. However, our approach not only provides an existence result but also describes the dynamic interaction of the pulses. The interaction theory and resulting bound states are corroborated by computational experiments, see [3].

REFERENCES

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