Dispersive Numerical Schemes for Schrödinger Equations

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ABSTRACT

In this lecture we shall present recent work in collaboration with L.Ignat on the numerical approximation of Schrödinger equations.

We first consider finite-difference space semi-discretizations and show that the standard conservative scheme does not reproduce at the discrete level the properties of the continuous Schrödinger equation. This is due to high frequency numerical spurious solutions. In order to damp out these high-frequencies and to reflect the properties of the continuous problem we add a suitable extra numerical viscosity term at a convenient scale.

We prove that the dispersive properties of this viscous scheme are uniform when the mesh-size tends to zero. We show that similar convergence results may be obtained by a two-grid algorithm based on the idea of resolving on a fine grid slow oscillations of the initial data and nonlinearity. We also prove the convergence of these schemes for a class of nonlinear Schöцdinger equations with nonlinearities that may not be handeled by standard energy methods and that require the so-called Strichartz inequalities. We prove a polynomial convergence rate for smooth initial data. By the contrary we show that the convergence rate for stnard non-dispersive numerial schemes is at most logarithmic.