On the dominance of nonlinear transfer for wind-driven seas

Sergei I. Badulin (1), Vladimir E. Zakharov (2,3,4)

(1) P.P.Shirshov Institute of Oceanology Moscow, Russia,
(2) Waves and Solitons, LLC, USA,
(3) The Lebedev Physics Institute, Moscow, Russia,
(4) University of Arizona, USA

Key words:

- Together
- Separately

Before we unite we must be separated

Bolshevik’s newspaper “Iskra”, 1900

Gentlemen, we must all hang together or we shall most assuredly all hang separately

In the Continental Congress just before signing
the Declaration of Independence, 1776.

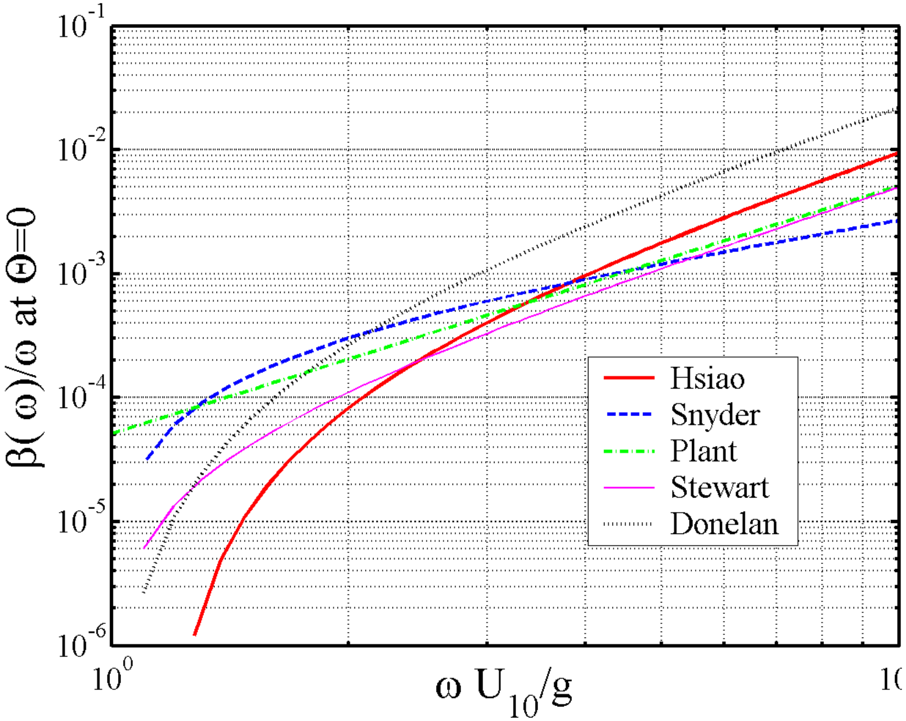
Kinetic equation for deep water waves (the Hasselmann equation, 1962)

$$\frac{dn_k}{dt} = S_{nl} + S_{input} + S_{diss}$$

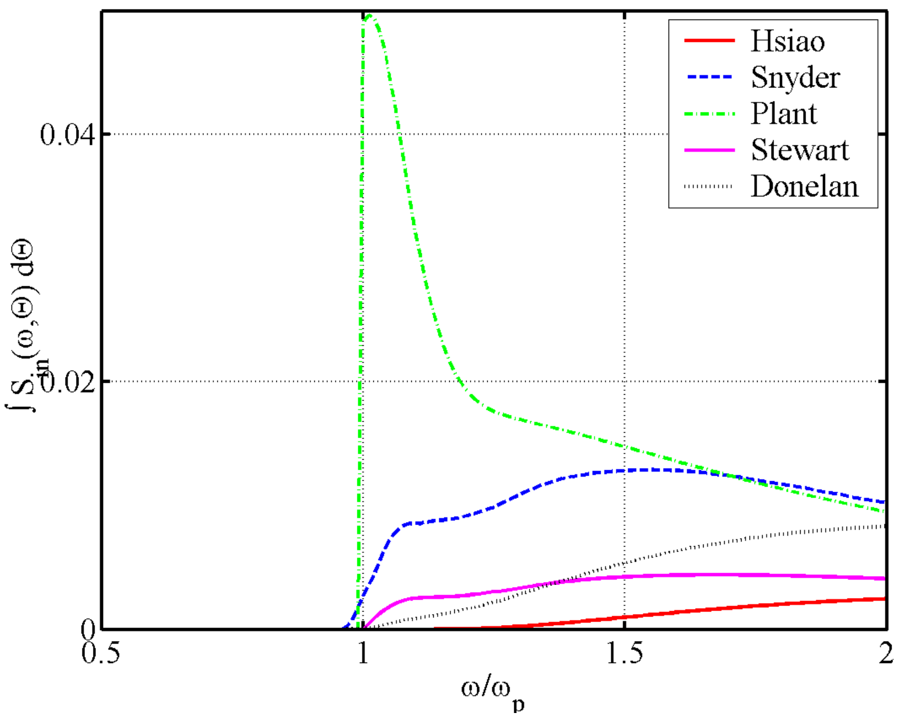
$$S_{nl} = 2\pi \int |T_{0123}|^2 (n_0 n_2 n_3 + n_1 n_2 n_3 - n_0 n_1 n_2 - n_0 n_1 n_3) \times \delta(\omega_0 + \omega_1 - \omega_2 - \omega_3)\delta(\mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

S_{input}, S_{diss} - empirical dependences

Non-dimensional wave input rates



Wave input term S_{in} for $U_{10}\omega_p/g=1$



S_{in} is estimated at specific conditions (e.g. low winds).

Our knowledge of S_{in} (and S_{diss}) is “based on extrapolation” (Snyder et al.1981, fig.26)

- **Hypothesis**

Nonlinear transfer term is a leading term of wind-wave balance $S_{nl} >> (S_{in}+S_{diss})$

A1: Badulin et al. *Self-similarity of wind-driven seas. Nonlinear Proc.Geoph.*, ***12***, *891-945, 2005 –*

A2: Badulin et al. *Weakly turbulent laws of wind-wave growth, J. Fl. Mech.* 591, pp. 339–378.

Message 1

Term-to-term comparison
is misleading for
kinetic equation for deep water waves

Take available parameterizations of wind-wave spectra and wave input and dissipation and

compare

$$S_{nl}\,,\, S_{in}\,,\, S_{diss}$$

JONSWAP spectra

$$E(\omega) = \alpha g^2 \omega_p^{-5} \eta^d \Phi(\xi)$$

Incomplete self-similarity !!!

$\xi = \omega/\omega_p$ – `inner variable', $\eta = \omega_p U_{10}/g$ – `outer variable', wave age

0.55 < d < 1.45

α = 0.006 – Donelan et al., 1985

α = 0.0127 – Babanin & Soloviev, 1998

N.B.

- Parameters can vary essentially
- Self-similar form of the spectra implies the leadership of S_{nl}

S_{in} - quasi-linear empirical parameterizations

$$S_{in} \sim \gamma_{in}(\omega) E(\omega)$$

- Cherenkov-like increments

$$\gamma_{in}(\omega,\theta) / \omega = \begin{cases} 0, & \beta\xi\eta < 1 \\ A(\beta\xi\eta - 1)^n, & \text{otherwise} \end{cases}; \qquad n=1,2$$

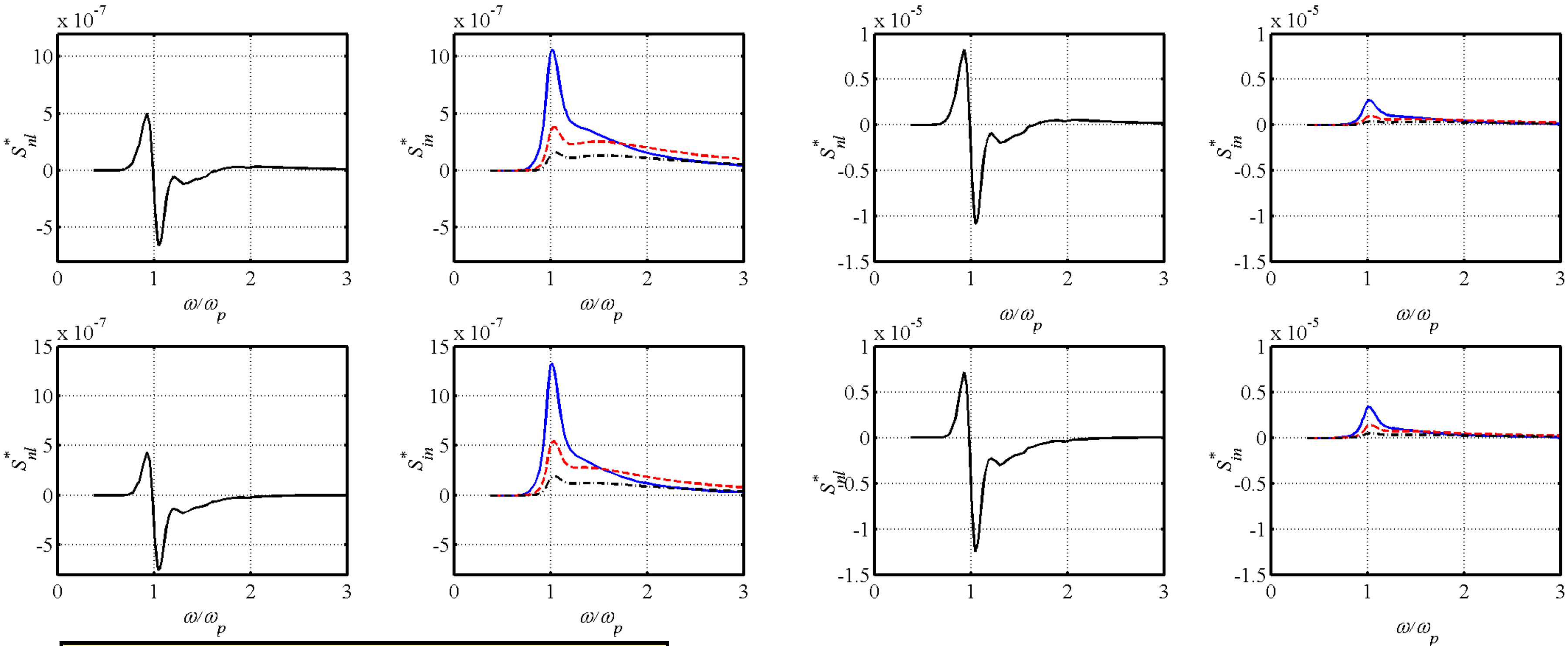
$$\xi = \omega / \omega_p$$ non-dimensional frequency

$$\eta = U_h \omega_p / g$$ inverse wave age

$\beta \sim 1$ for U_h - wind speed

JONSWAP (α_0=0.006, d=0.55) vs JONSWAP (α_0=0.0127; d=1)

$$E^* = \frac{E(\omega)\omega_p^5}{g^2}; \quad S_{nl}^* = \frac{S_{nl}^*(\omega)\omega_p^5}{g^2}; \quad S_{in}^* = \frac{S_{in}^*(\omega)\omega_p^5}{g^2}$$



$\omega_p U/g=1.5$, $\gamma=3.3$

Blue – Snyder et al. 1981;

Red – Donelan et al., 1987;

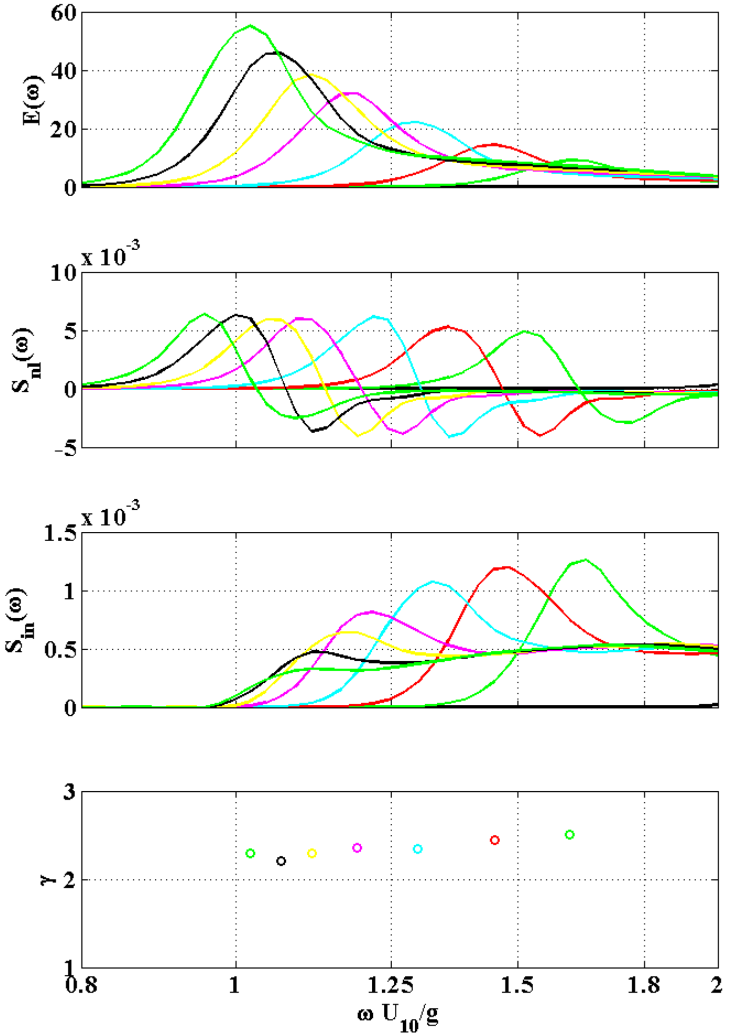
Black – Hsiao & Shemdin, 1983

Message 2
Terms in the KE are not independent due to nonlinearity

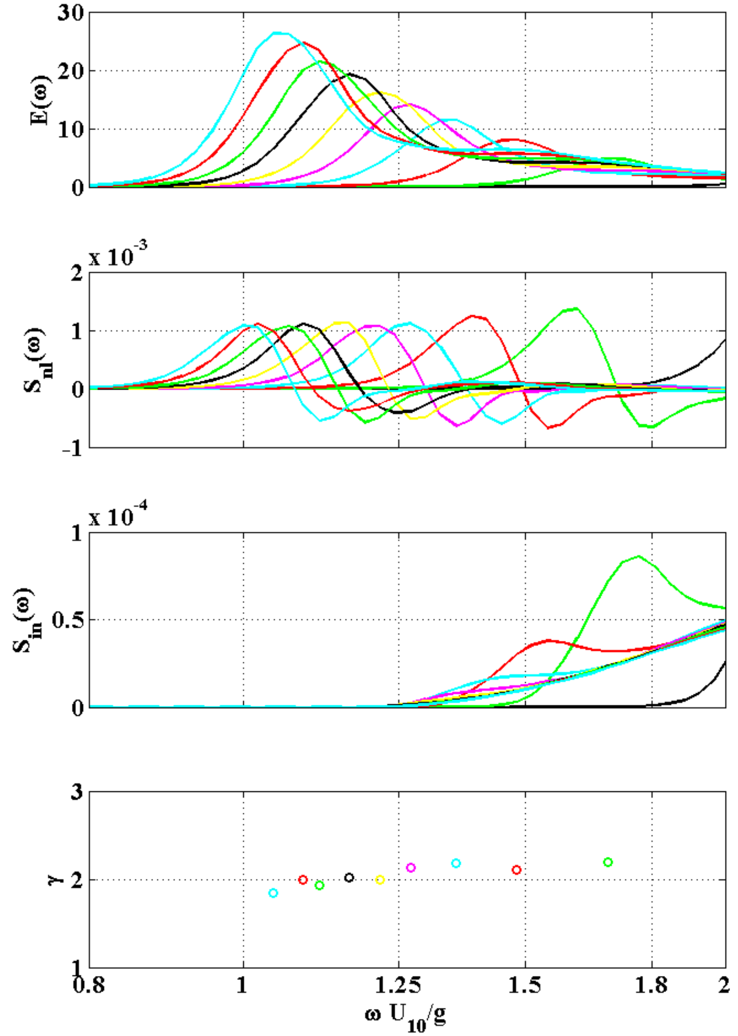
"All the terms hang together" is more adequate approach to the problem

The nonlinear transfer is dominating near the spectral peak only

S_{in} by Snyder et al. 1981, $U_{10}=20$ m/s



S_{in} by Hsiao & Shemdin 1983, $U_{10}=20$ m/s



Message 3

The collision integral itself is a sum of two great terms that should be analysed separately.
These terms diverge at high frequencies.

$$S_{nl} = F_k - \gamma_k n_k$$

Nonlinear relaxation rate

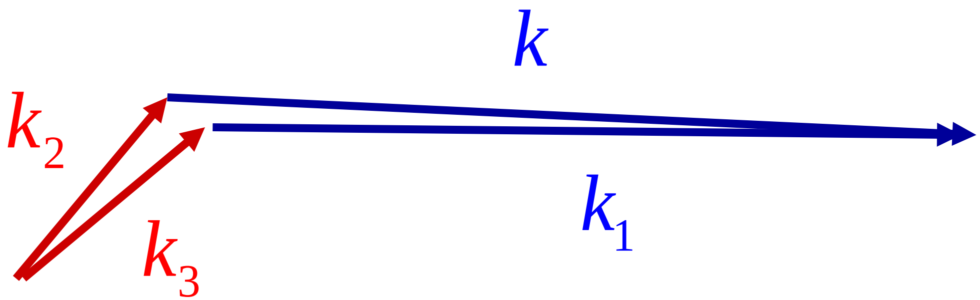
$$\Gamma_k = -2\pi \int |T_{0123}|^2 (n_2 n_3 - n_1 n_2 - n_1 n_3) \delta_{\omega_0 + \omega_1 - \omega_2 - \omega_3} \delta_{\mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3} d\mathbf{k}_{123}$$

Nonlinear forcing

$$F_k = 2\pi \int |T_{0123}|^2 n_1 n_2 n_3 \delta_{\omega_0 + \omega_1 - \omega_2 - \omega_3} \delta_{\mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3} d\mathbf{k}_{123}$$

Hypothesis

Quasi-identical interactions of long and short waves give key contribution into F_k and Γ_k



$$T_{0123} \approx \frac{k_2^2 k}{4\pi^2}\cos\theta$$

$$\Gamma_k = \Lambda\left(\frac{\omega}{\omega_p}\right)^3 \mu^4$$

$\Lambda = 36\pi \approx 113 >> 1$ - narrow spectrum

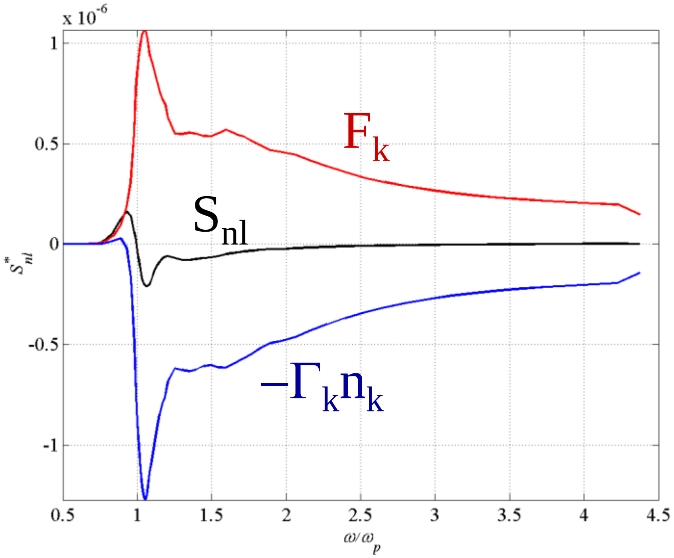
$\Lambda = 9\pi \approx 28.3 >> 1$ - isotropic spectrum

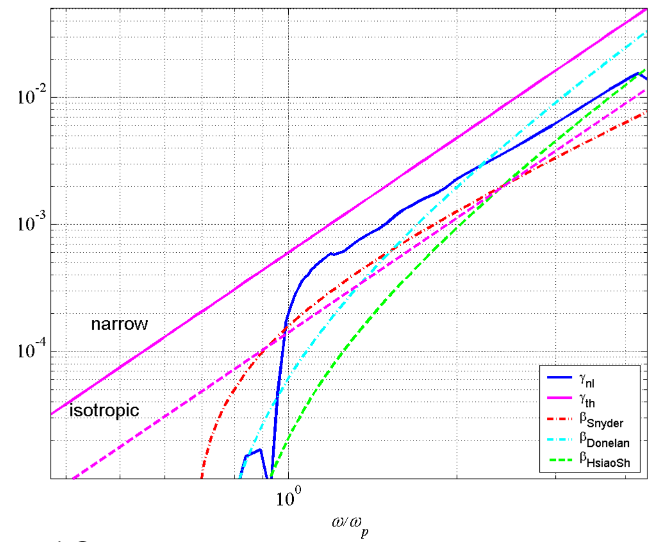
$\mu = \sqrt{\frac{E\omega^4}{g^2}}$ - wave steepness

Collision integral

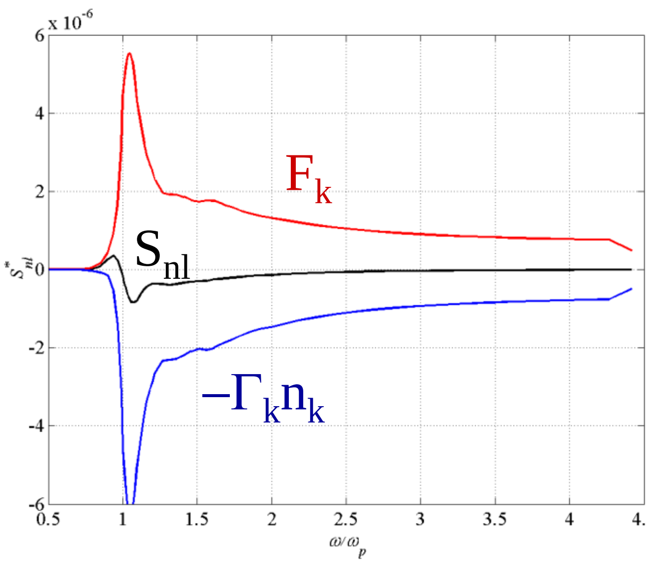
$E(\omega) \sim \cos^2 \Theta$

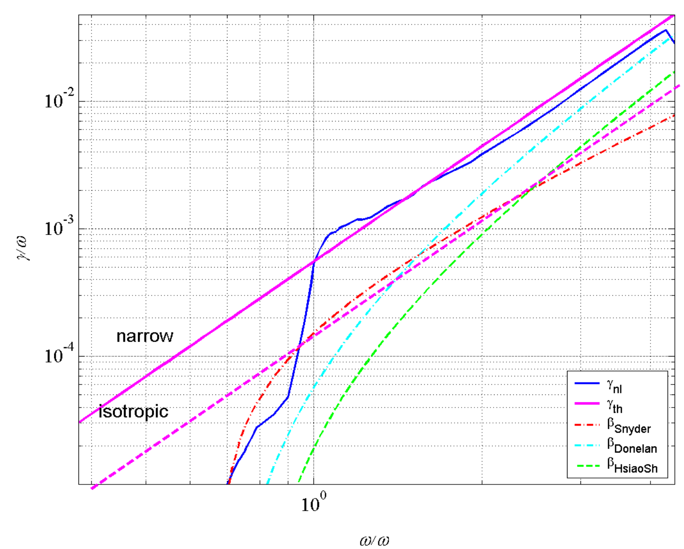
Nonlinear damping increment



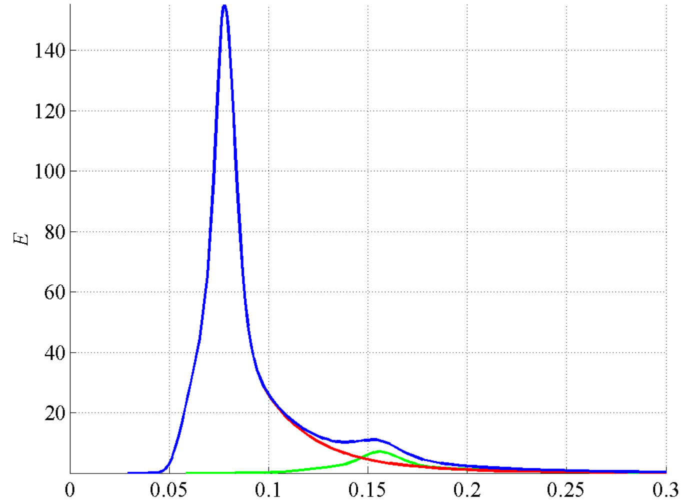


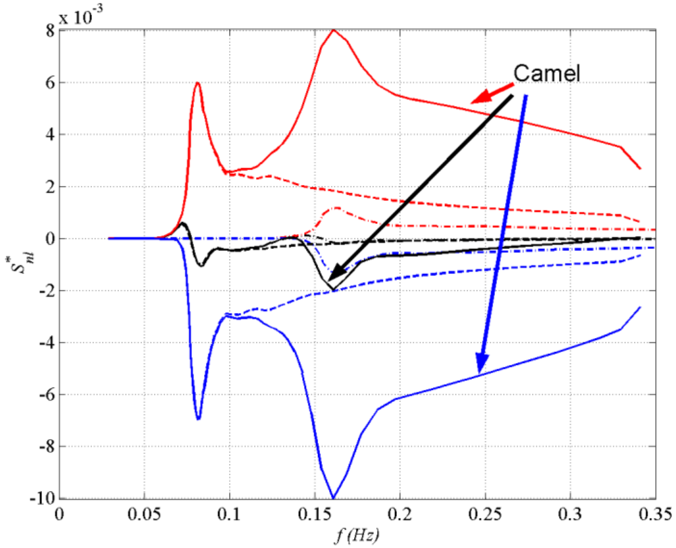
$E(\omega) \sim \cos^{10} \Theta$

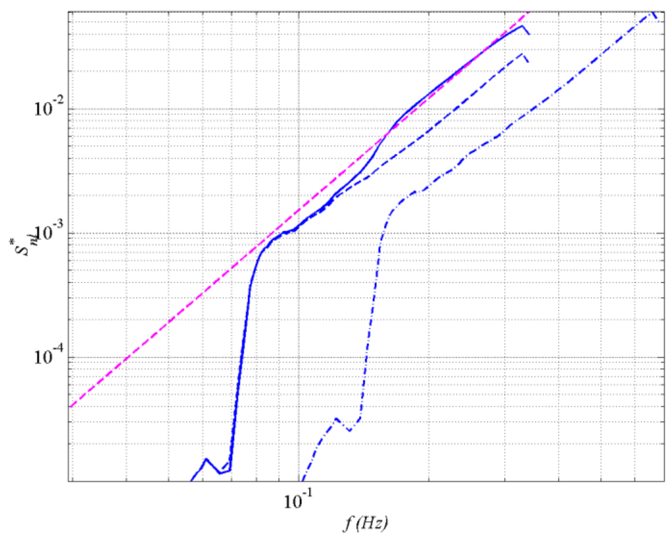




Example 1:
Camel-like spectra







<u>N.B.</u>

- Small perturbation = strong feedback
- Cumulative effect of nonlinearity on high frequency range

Kahma & Pettersson, unpublished in JPO since 2004

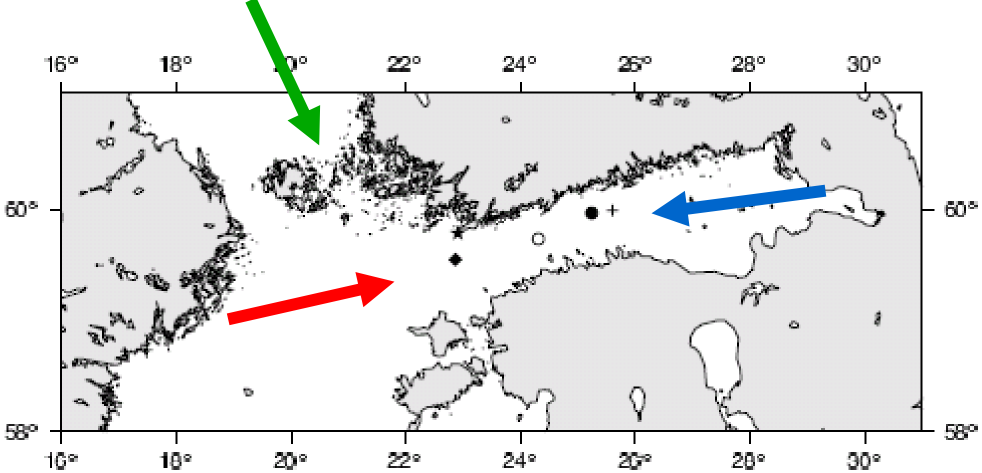
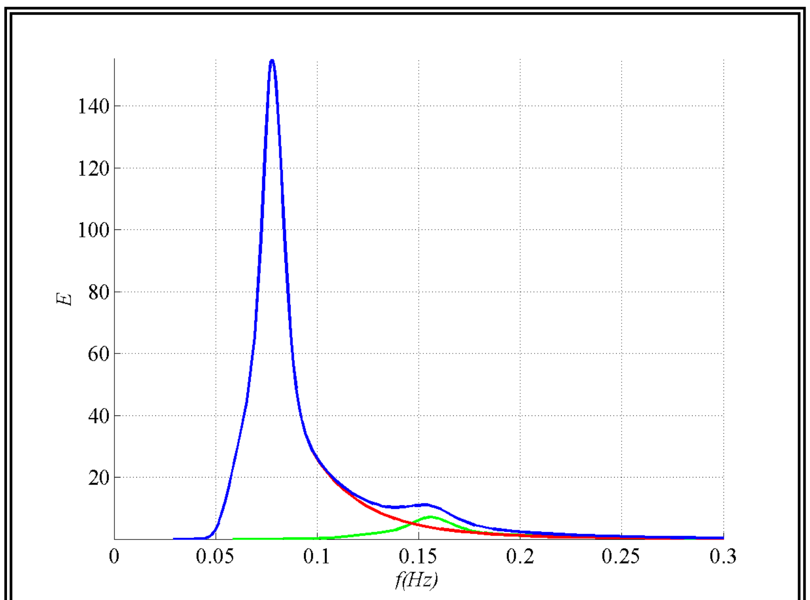


Fig. 1. Wave measuring sites in the Gulf of Finland, Hanko (diamond), Porkkala (open circle) and Helsinki (bullet). The automatic weather stations Tulliniemi and Kalbådagrund are denoted by a star anc



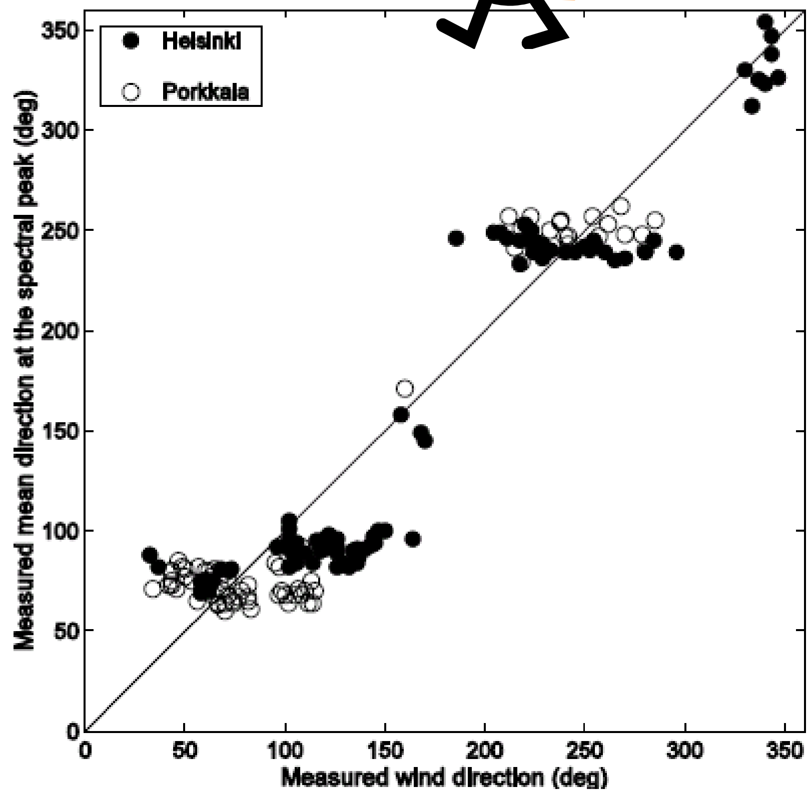
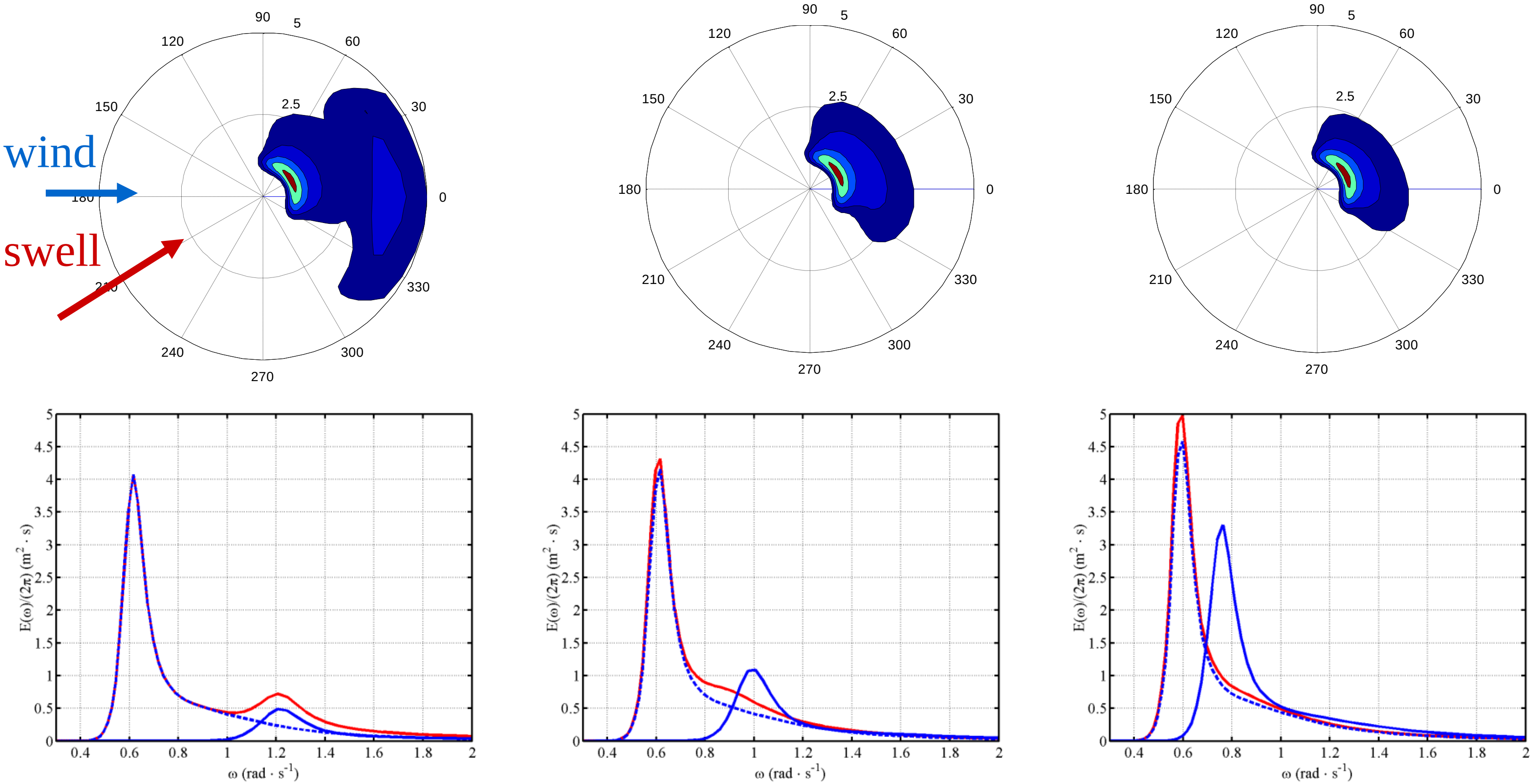


Fig. 5. The mean direction at the spectral peak versus wind direction. The directions indicate the approach directions. Open circle denotes data from station Porkkala and the bullet data from station Helsinki.

t=0 t=522 sec (~ 100 T_{0ww}) t=2000 sec (> 400 T_{0ww})

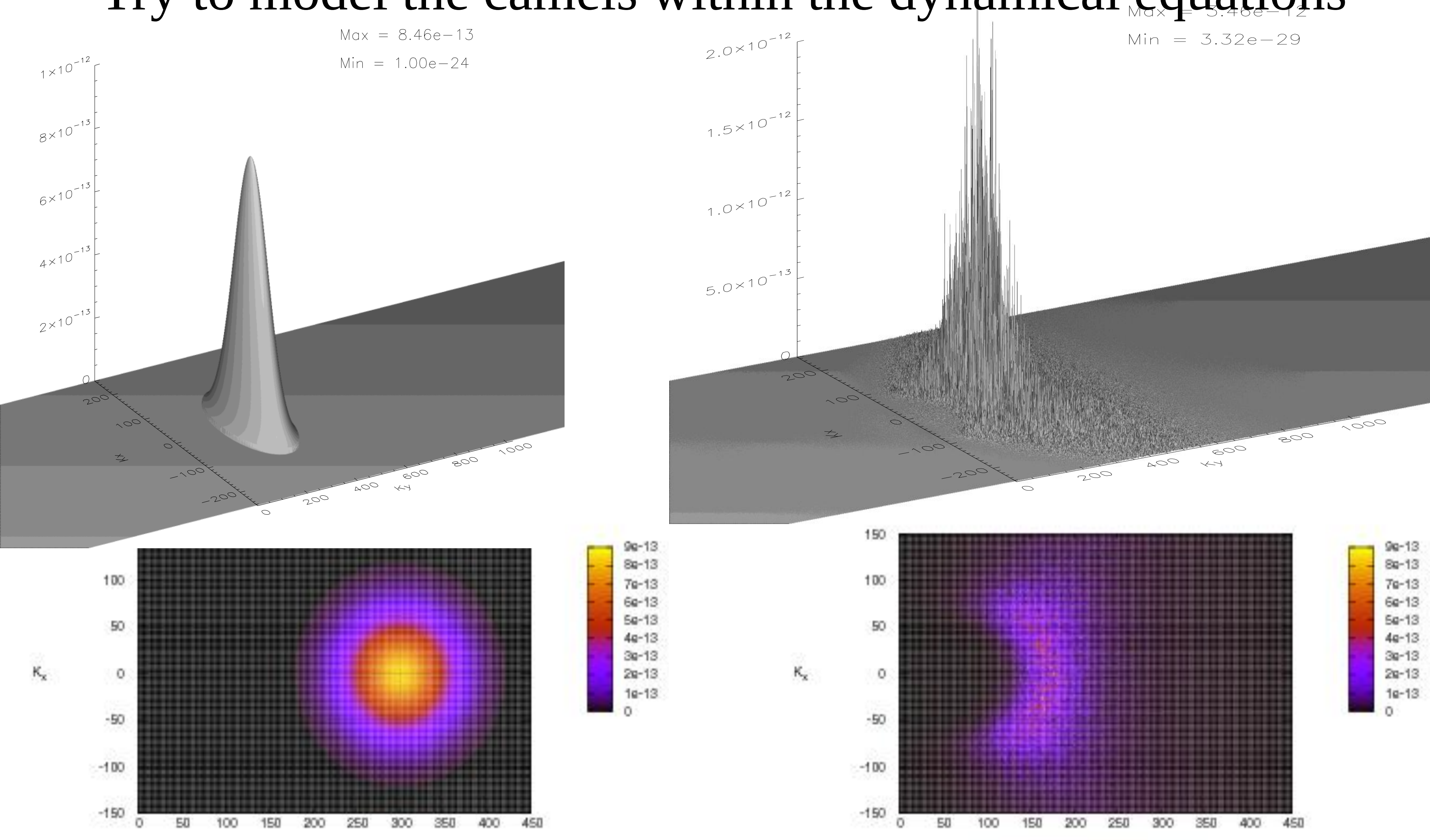


RED = together; BLUE – separately

Steering, absorbing, reduction of direct wind input

A big question – the KE validity

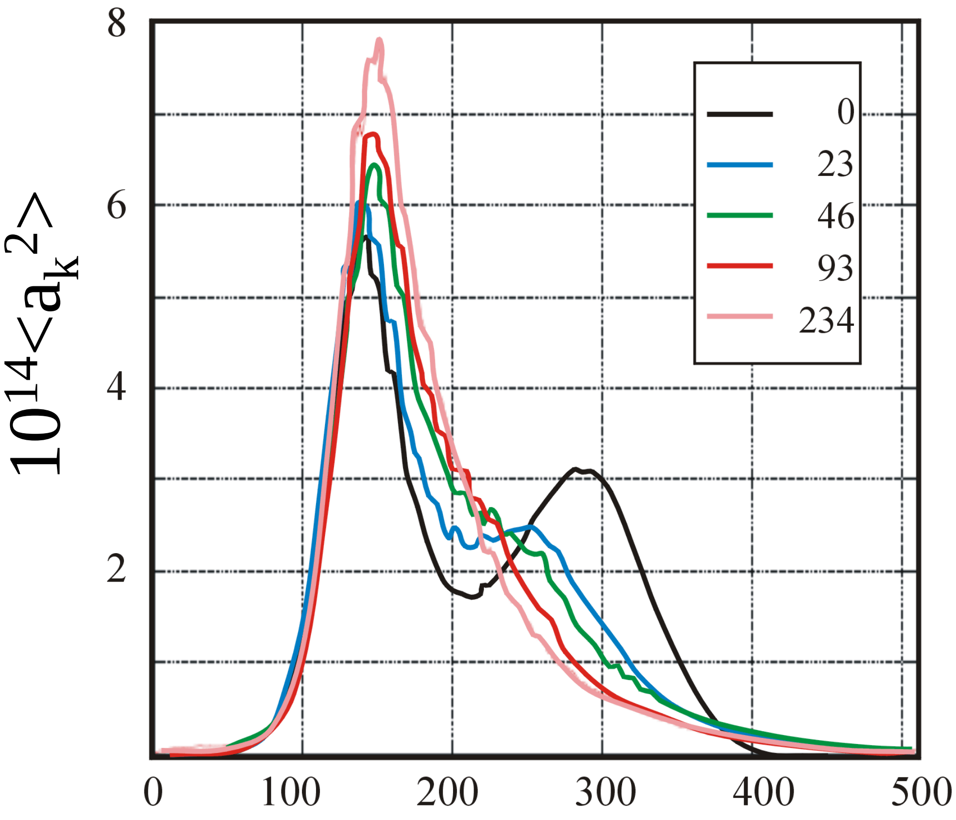
Try to model the camels within the dynamical equations



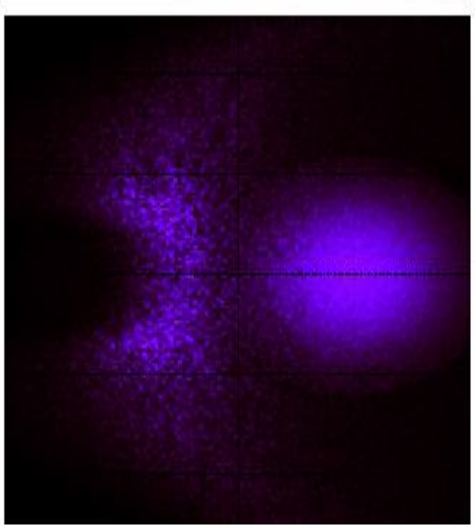
Korotkevich, Pushkarev, Resio, Zakharov, 2007,

Direct numerical simulation of the Hasselmann equation

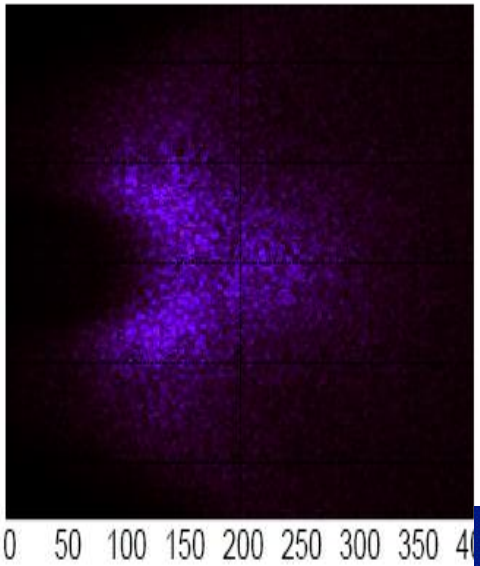
Camel-like spectra in the numerical tank



The same fast evolution within dynamical approach

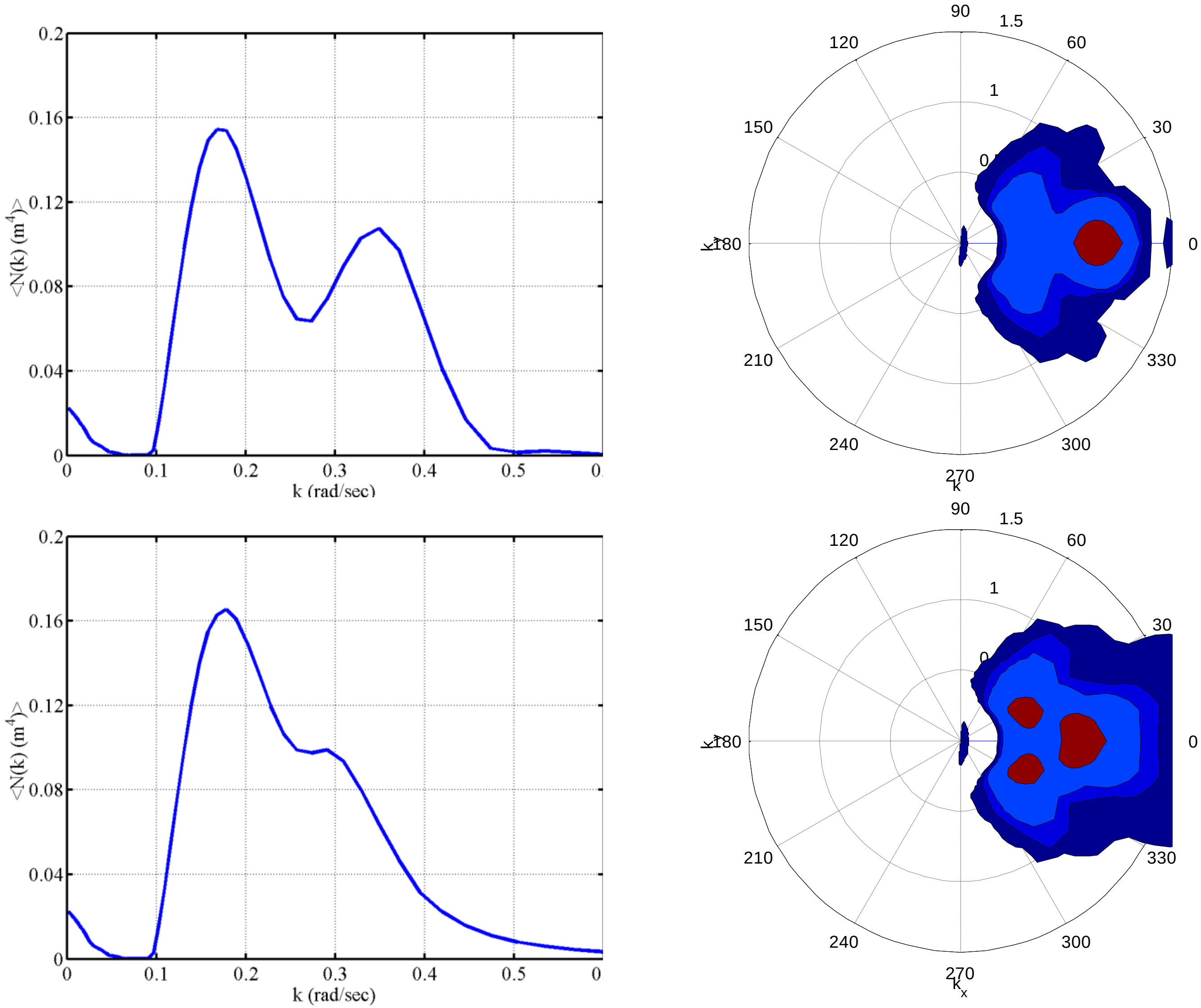


t=0



t=$46T_0$

The same fast evolution within the statistical approach (the Hasselmann equation)



t=0

$=51T_0$

Summary

- Nonlinear transfer is leading term in the KE for wind waves;
- "Fine structure" of the collision integral should be taken into account when considering the wind-wave balance;
- Kinetic equation for wind waves is still working beyond its formal validity

Thank you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JONSWAP (α_0=0.006, d=0.55) vs JONSWAP (α_0=0.0127; d=1)

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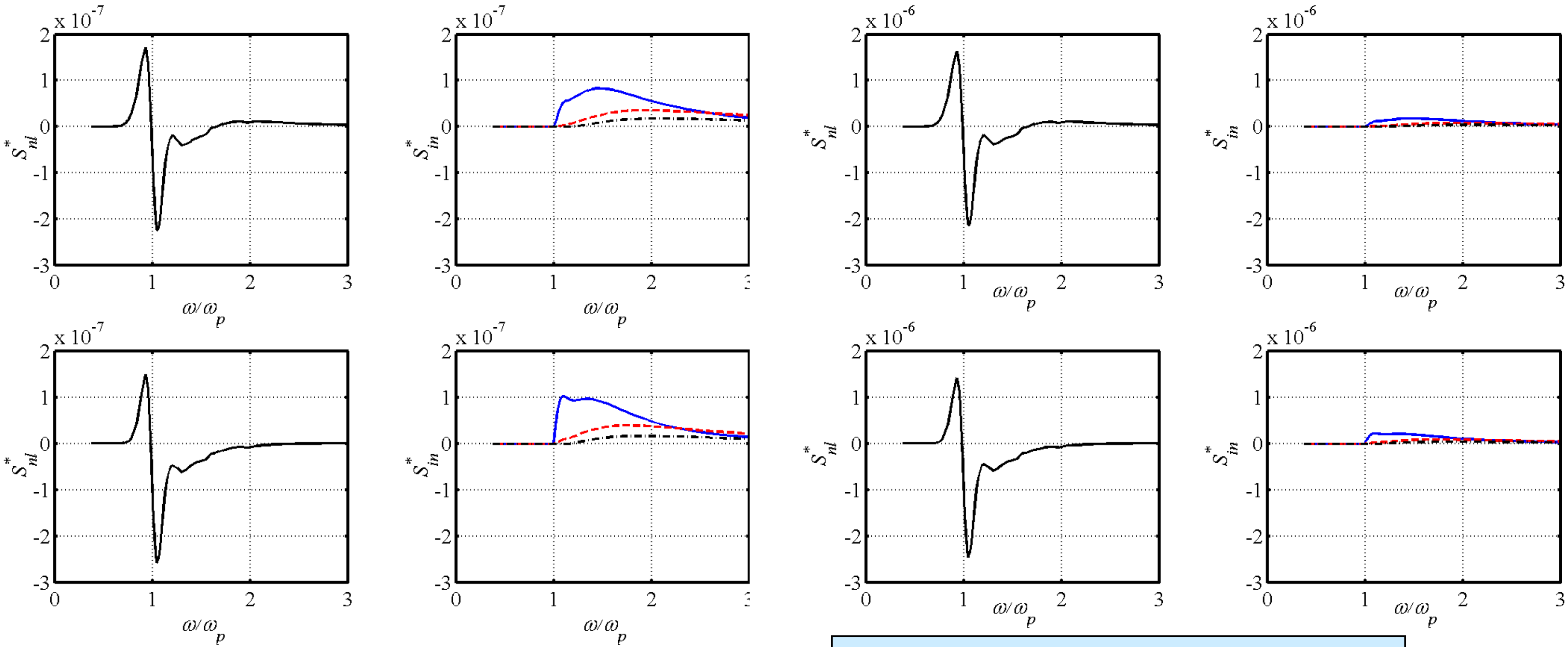

$\omega_p U/g=1., \gamma=3.3$

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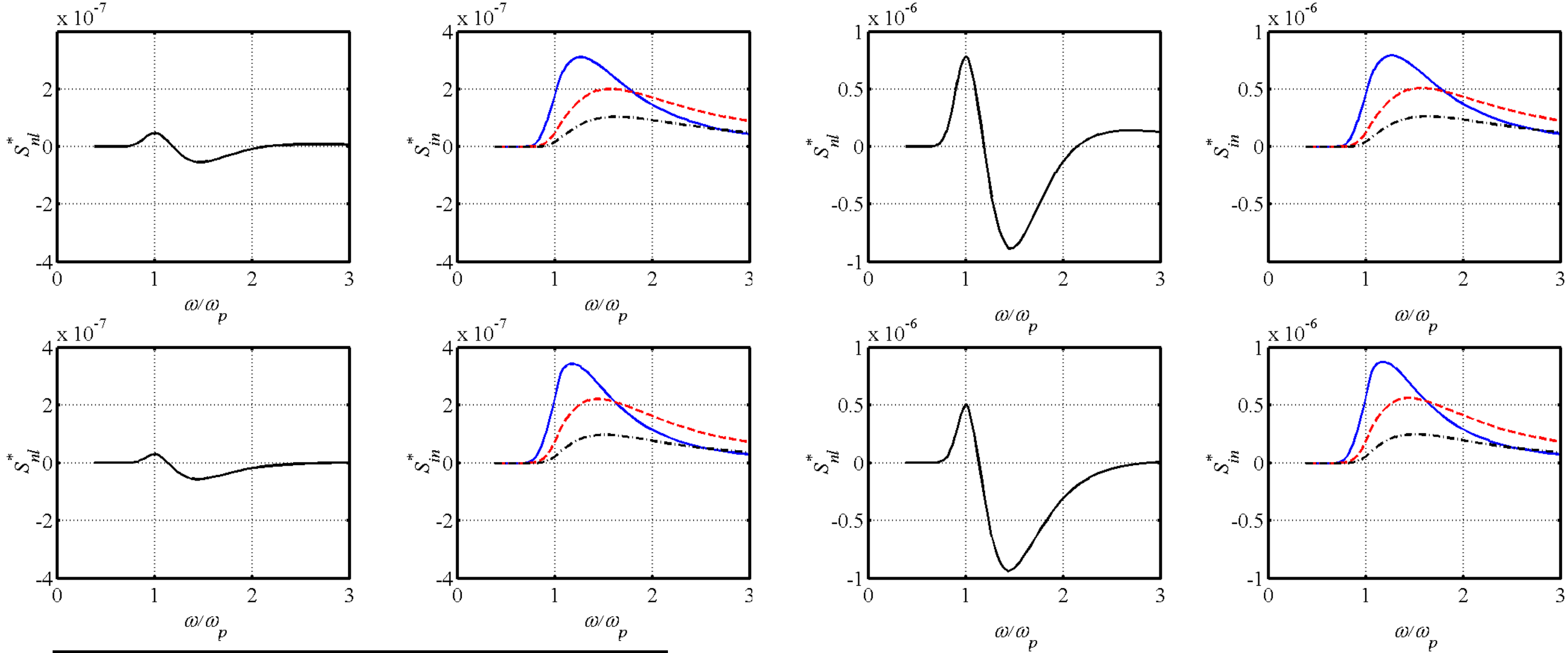

$\omega_p U/g$=1.5, γ=1

Blue – Snyder et al. 1981;

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