On Periodic Waves Governed by the Extended Korteweg-de Vries Equation

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CENTRE FOR NONLINEAR STUDIES

Microstructured Materials

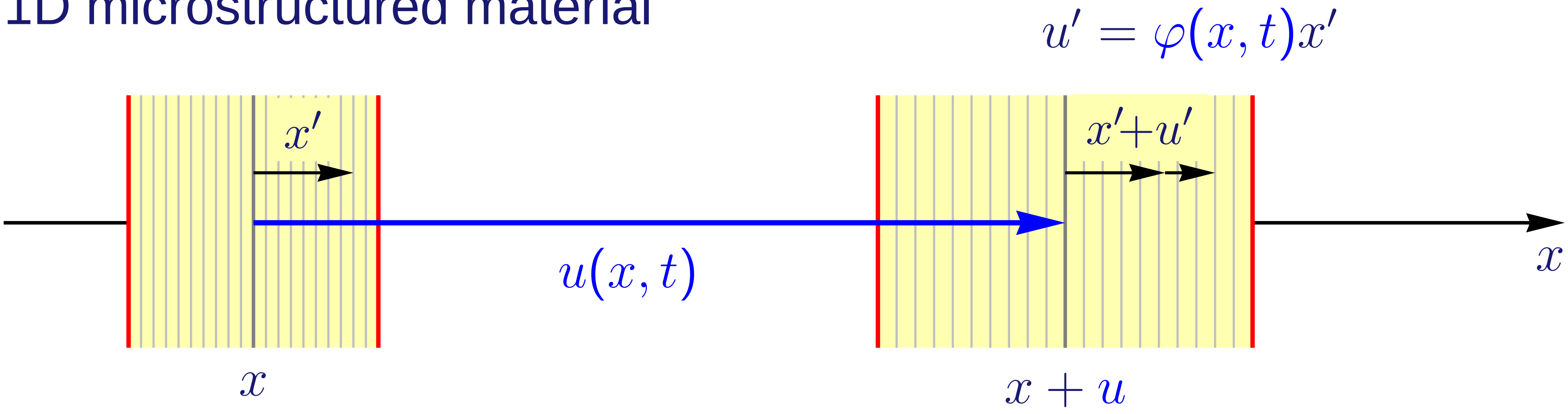
Mindlin 1964: Microstructure in linear elasticity

Engelbrecht & Pastrone 2003: Waves in microstructured solids with strong nonlinearities in microscale
Mindlin's model specialized to 1D, nonlinear terms included

Janno & Engelbrecht 2005: Solitary waves in nonlinear microstructured materials
study of parameter range for existence of solitary waves, qualitative and numerical analysis

Randrüüt & Braun 2009: On one-dimensional solitary waves in microstructured solids
approximate solution for small micro-nonlinearity

1D microstructured material



Governing equations

$$\rho u_{tt} = \alpha u_{xx} + A\varphi_x + \frac{1}{2}N(u_x^2)_x$$

$$I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi + \frac{1}{2}M(\varphi_x^2)_x$$

Reduction to Single Equation

$$\rho u_{tt} = \alpha u_{xx} + A\varphi_x \qquad + \frac{1}{2} N (u_x^2)_x$$

$$I\varphi_{tt} = C\varphi_{xx} - Au_x - B\varphi + \frac{1}{2} M (\varphi_x^2)_x$$

– introduce dimensionless variables X, T, U

– apply “slaving principle” to eliminate φ

$$U_{TT} = U_{XX} + \frac{1}{2}\epsilon\gamma_N^2 \left(U_X^2\right)_X + \delta^2 \left(U_{TT} - \gamma_1^2 U_{XX} + \frac{1}{2}\epsilon\gamma_M^2 U_{XX}^2\right)_{XX}$$

Evolution Equation

$$U_{TT} = U_{XX} + \frac{1}{2}\epsilon\gamma_N^2 \left(U_X^2\right)_X + \delta^2 \left(U_{TT} - \gamma_1^2 U_{XX} + \frac{1}{2}\epsilon\gamma_M^2 U_{XX}^2\right)_{XX}$$

– assume $U = U(\xi, \tau)$ with

$\xi = X - T$ moving length coordinate

$\tau = \epsilon T$ "slow" time

– introduce new dependent variable $\alpha = U_\xi$

$$\alpha_\tau + \frac{1}{2}\gamma_N^2 \left(\alpha^2\right)_\xi + \left(1 - \gamma_1^2\right)\alpha_{\xi\xi\xi} + \frac{1}{2}\epsilon\gamma_M^2 \left(\alpha_\xi^2\right)_{\xi\xi} = 0$$

CENS

Standardization

$$\alpha_\tau + \frac{1}{2}\gamma_N^2\left(\alpha^2\right)_\xi + \left(1-\gamma_1^2\right)\alpha_{\xi\xi\xi} + \frac{1}{2}\epsilon\gamma_M^2\left(\alpha_\xi^2\right)_{\xi\xi} = 0$$

Transformation

$$\alpha = \frac{6}{\gamma_N^2}\left(1-\gamma_1^2\right)^{1/3} y \qquad \tau = t$$

$$\xi = \left(1-\gamma_1^2\right)^{1/3} x \qquad \varepsilon = \frac{\epsilon\gamma_M^2}{(1-\gamma_1^2)\gamma_N^2}$$

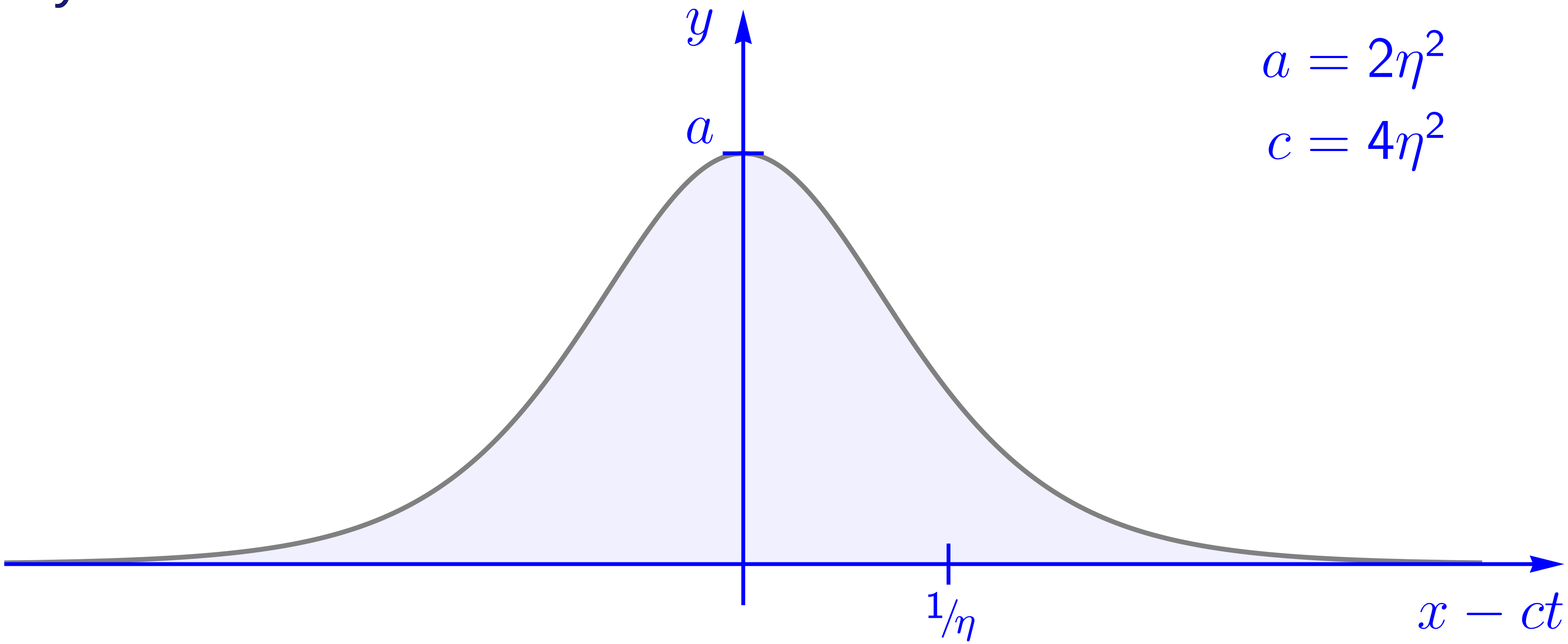
$$y_t + 3\left(y^2\right)_x + y_{xxx} + 3\varepsilon\left(y_x^2\right)_{xx} = 0$$

Solitary Waves

Korteweg-de Vries equation

$$y_t + 3\left(y^2\right)_x + y_{xxx} = 0$$

Solitary waves



Korteweg-de Vries equation — extended

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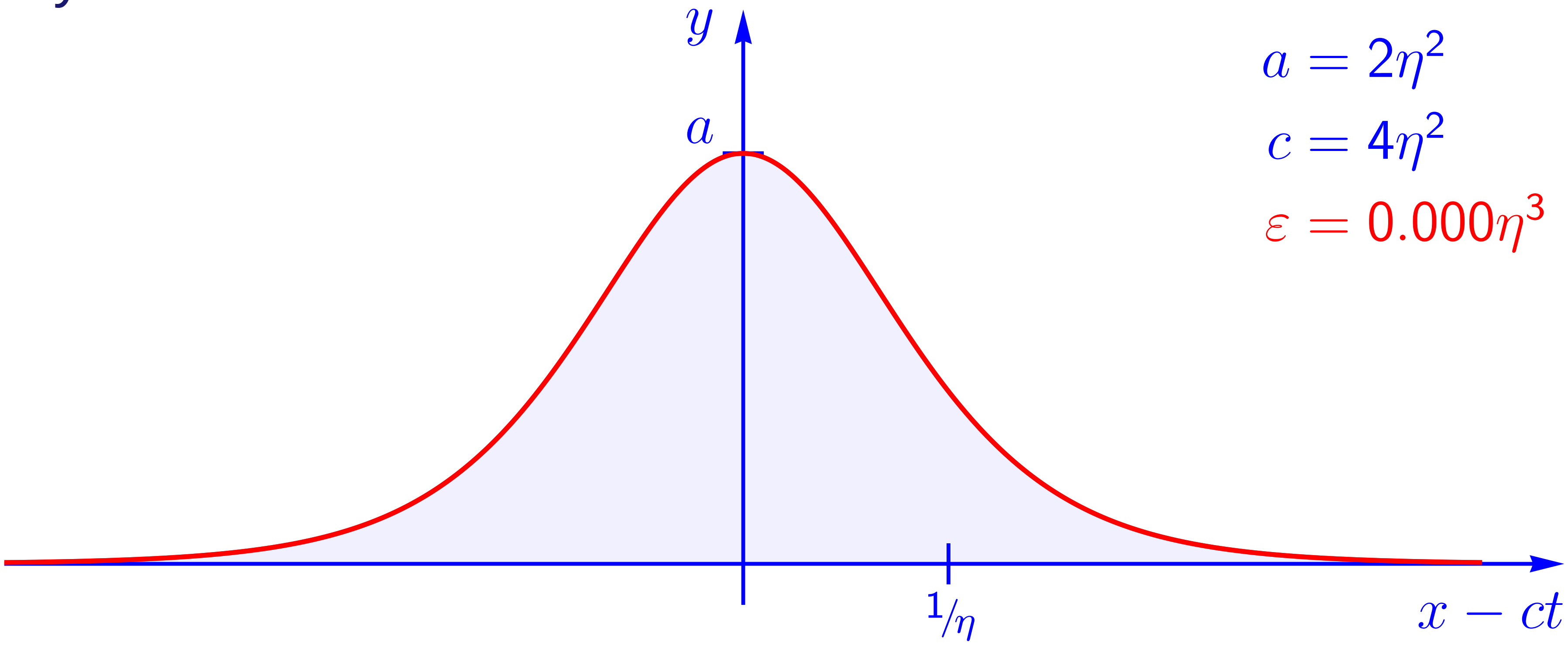

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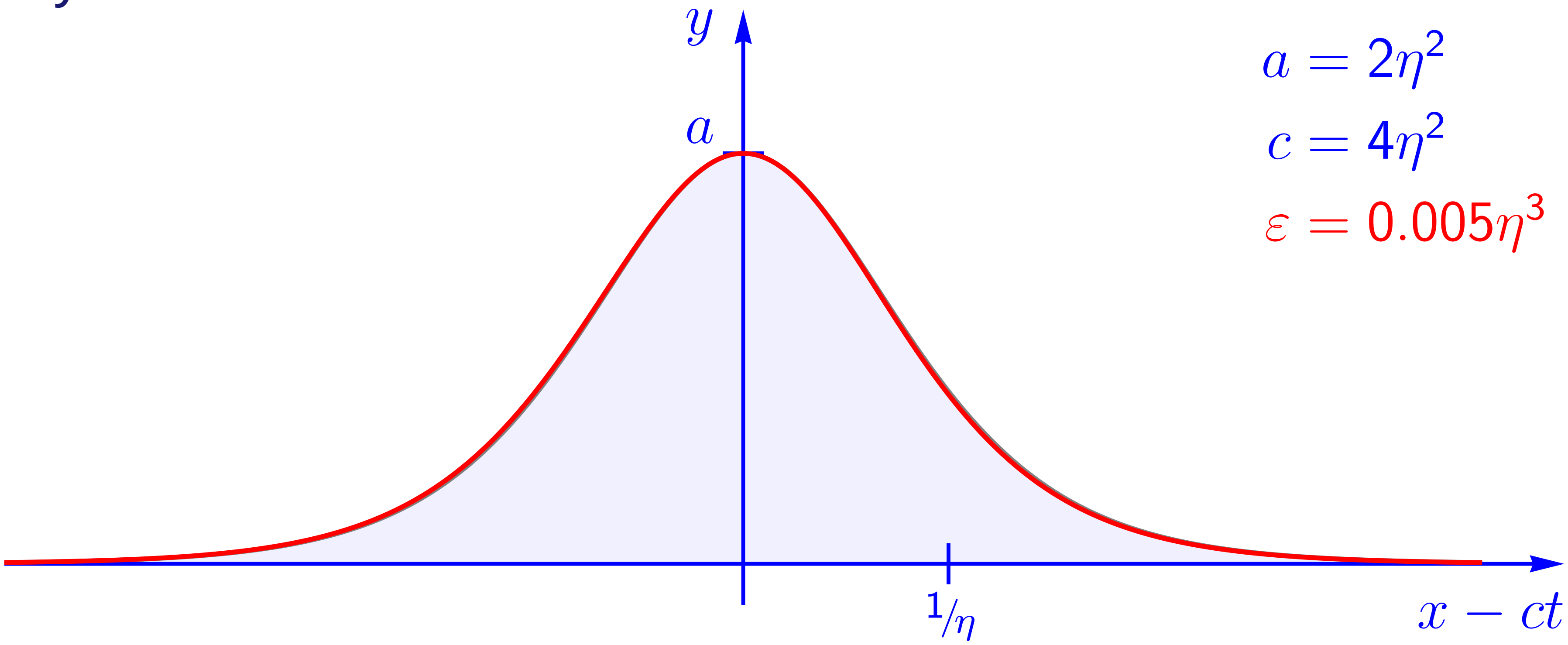

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$c = 4\eta^2$
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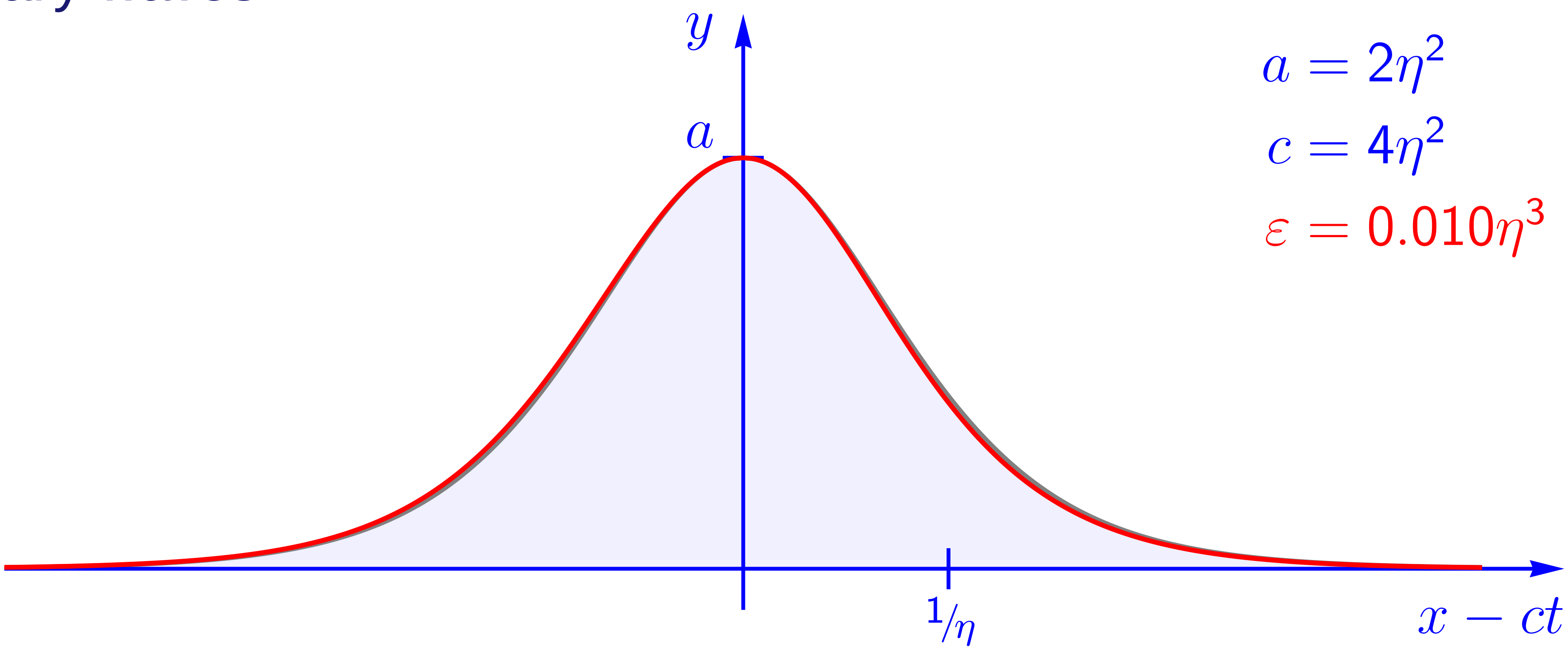

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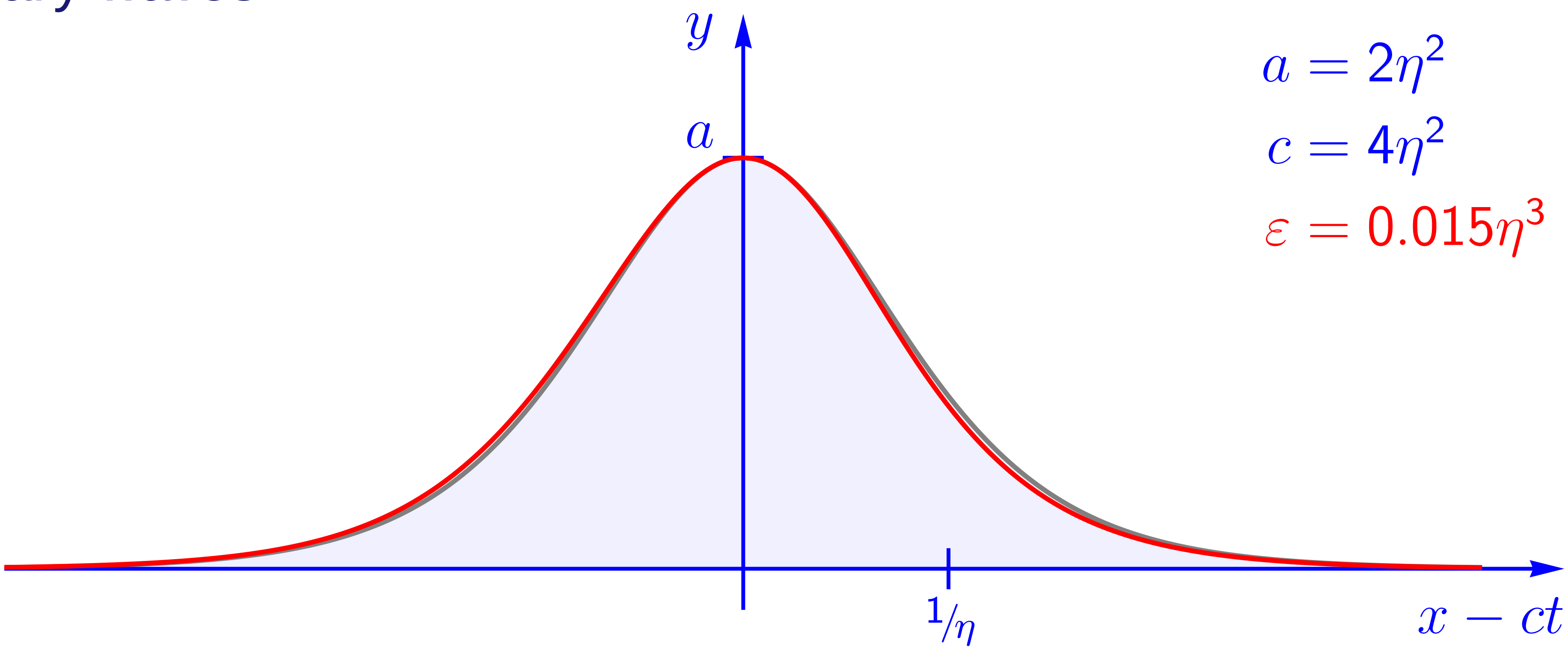

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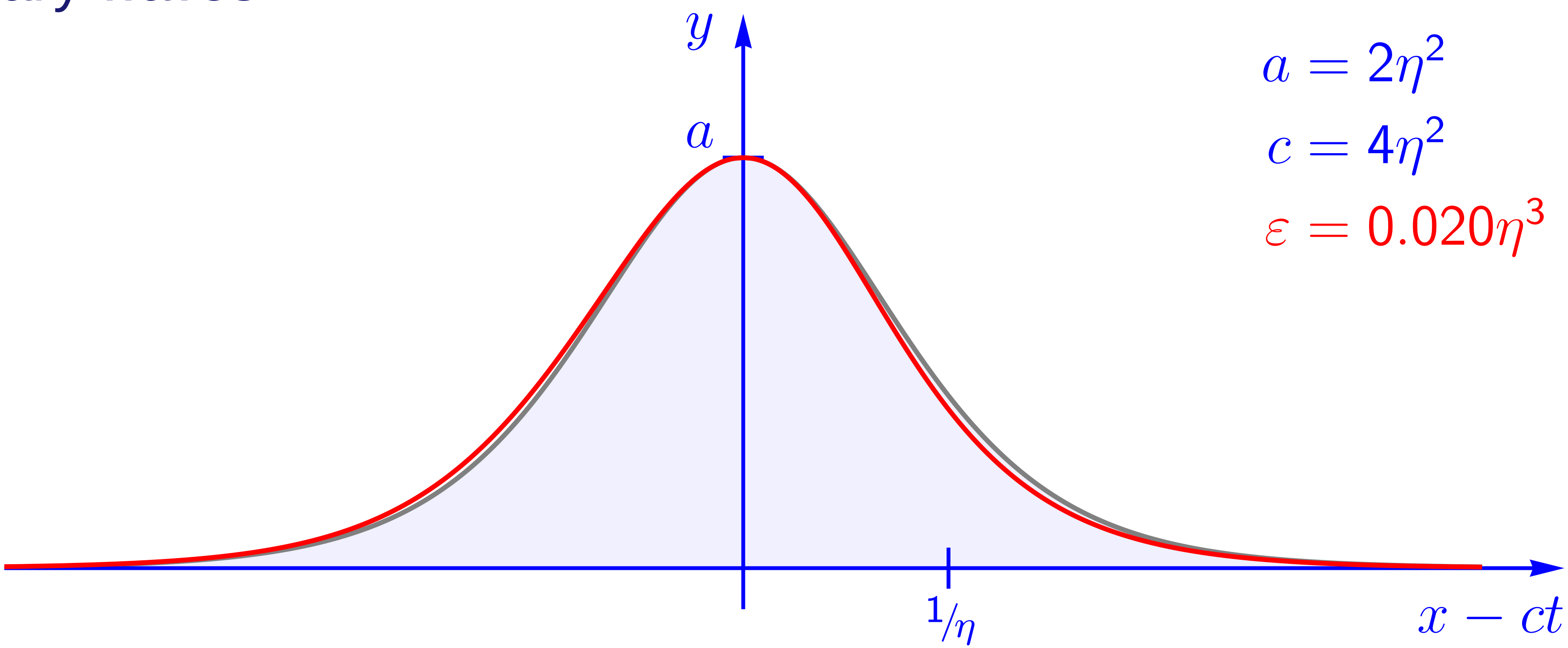

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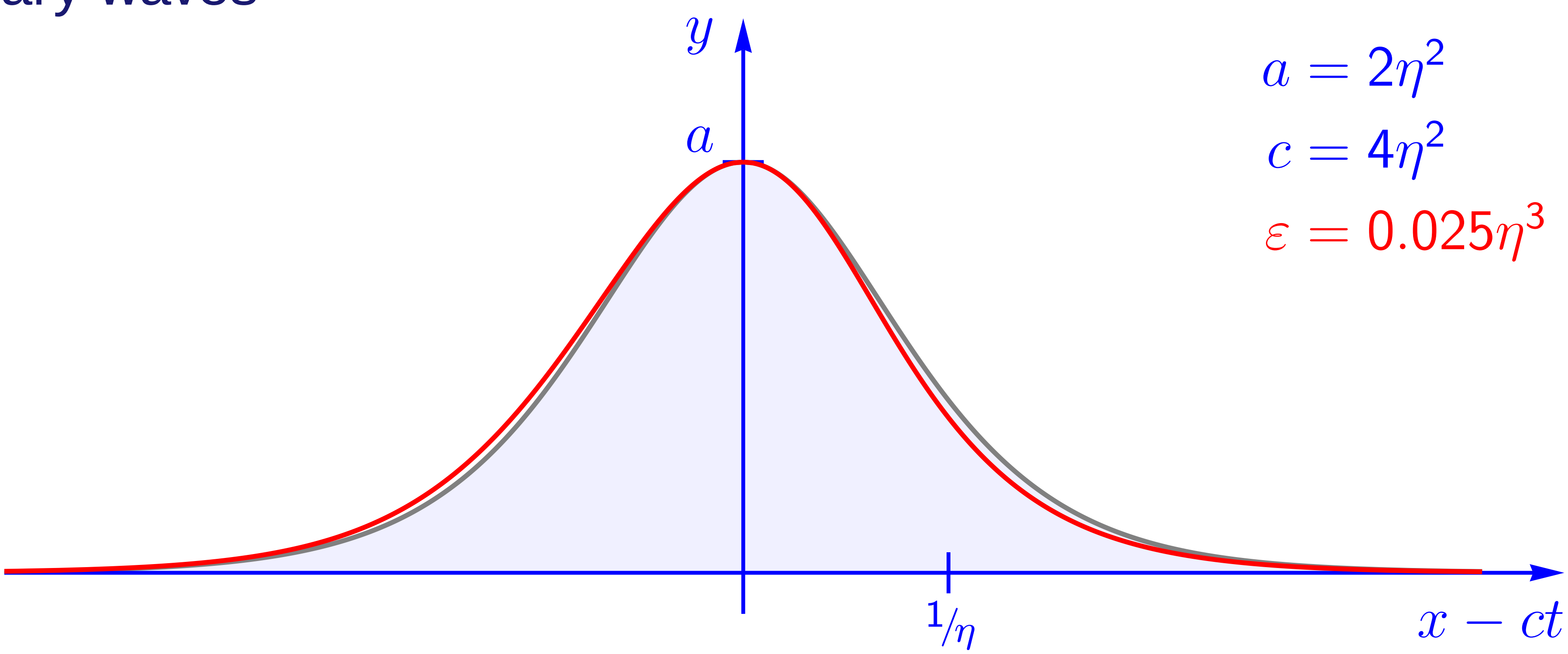

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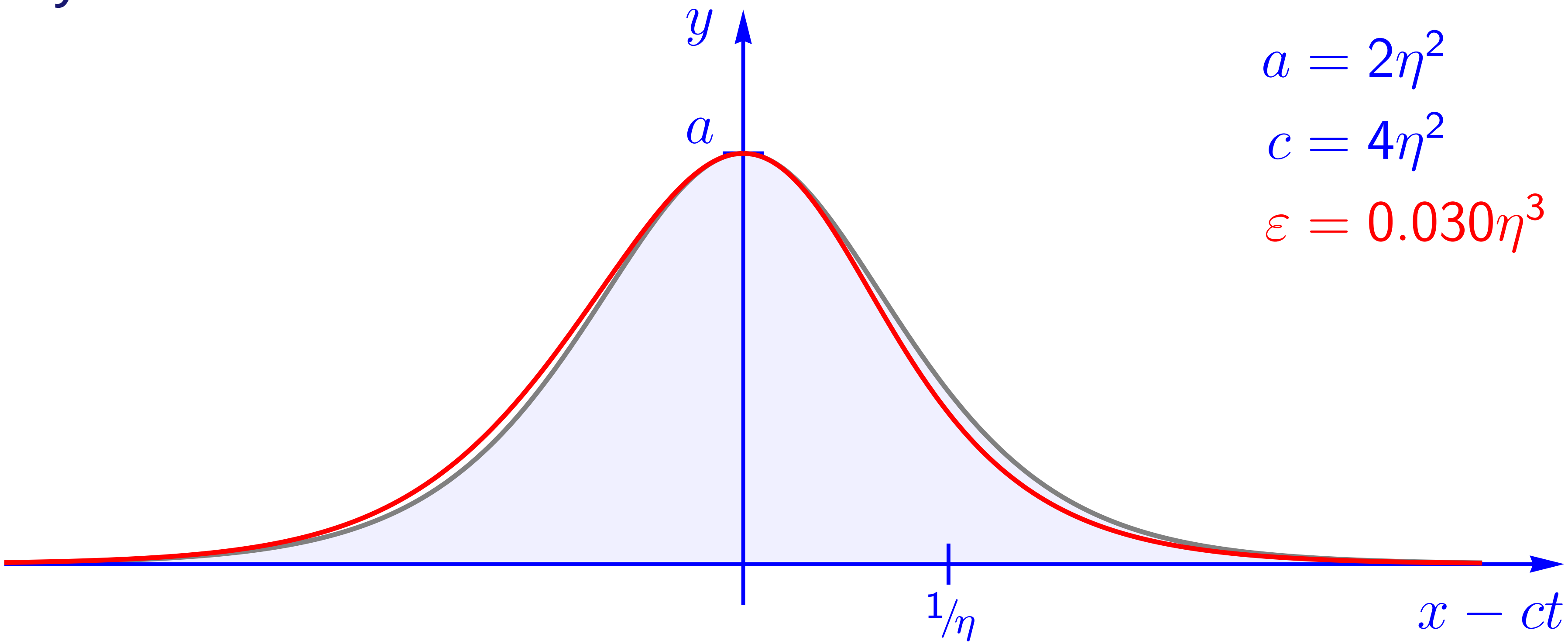

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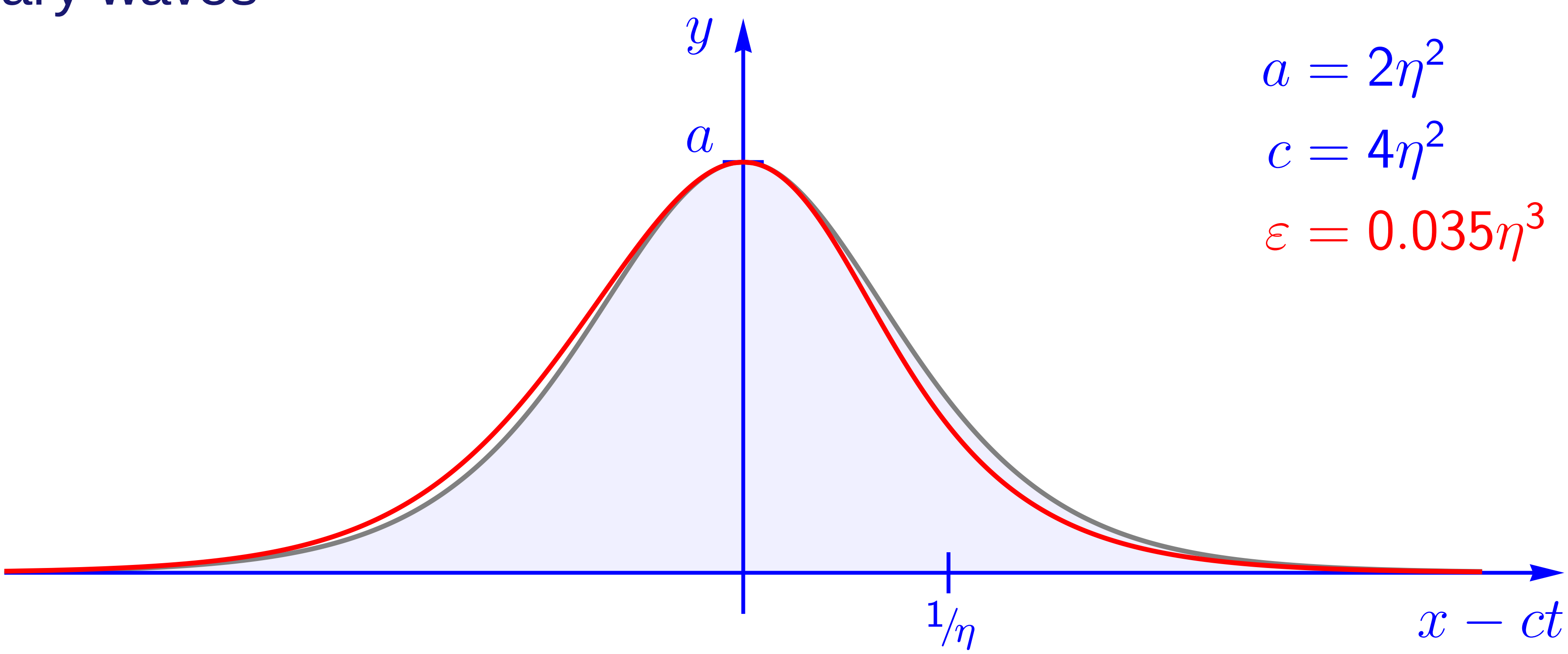

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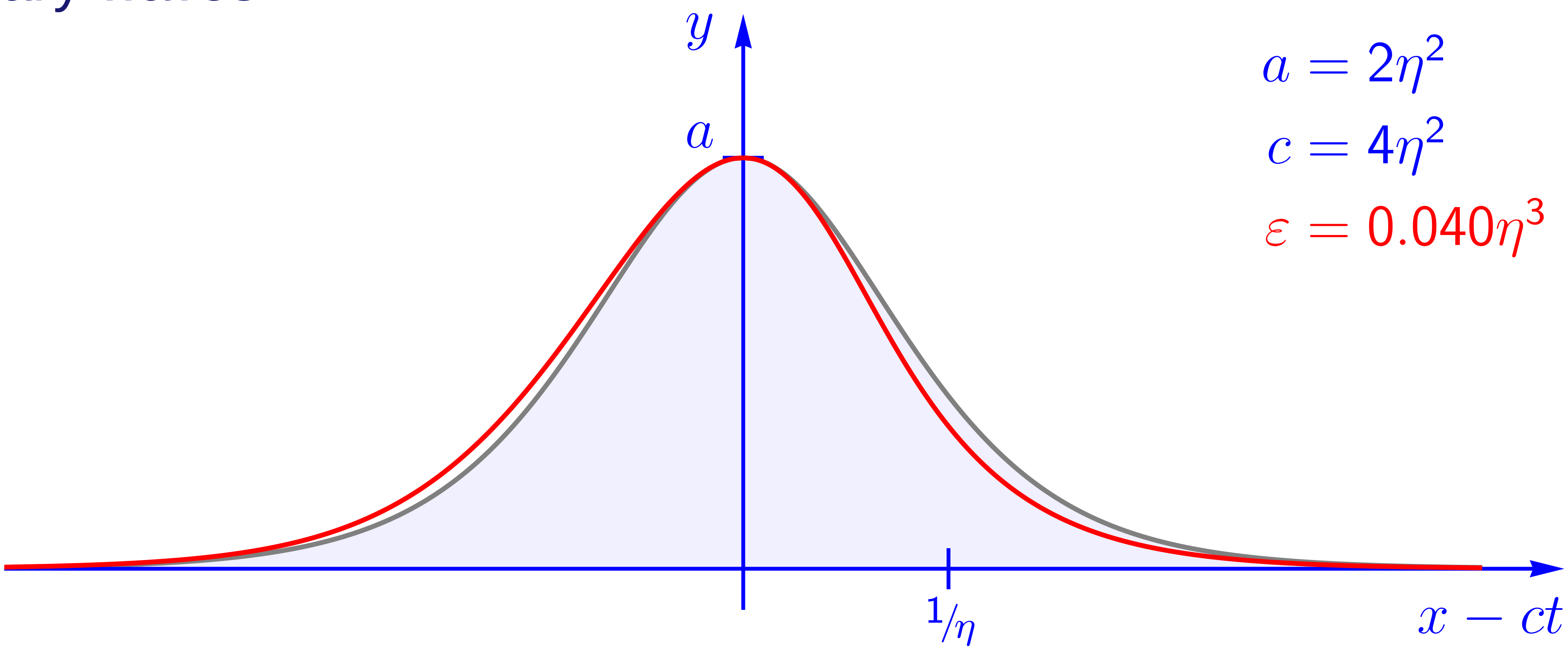

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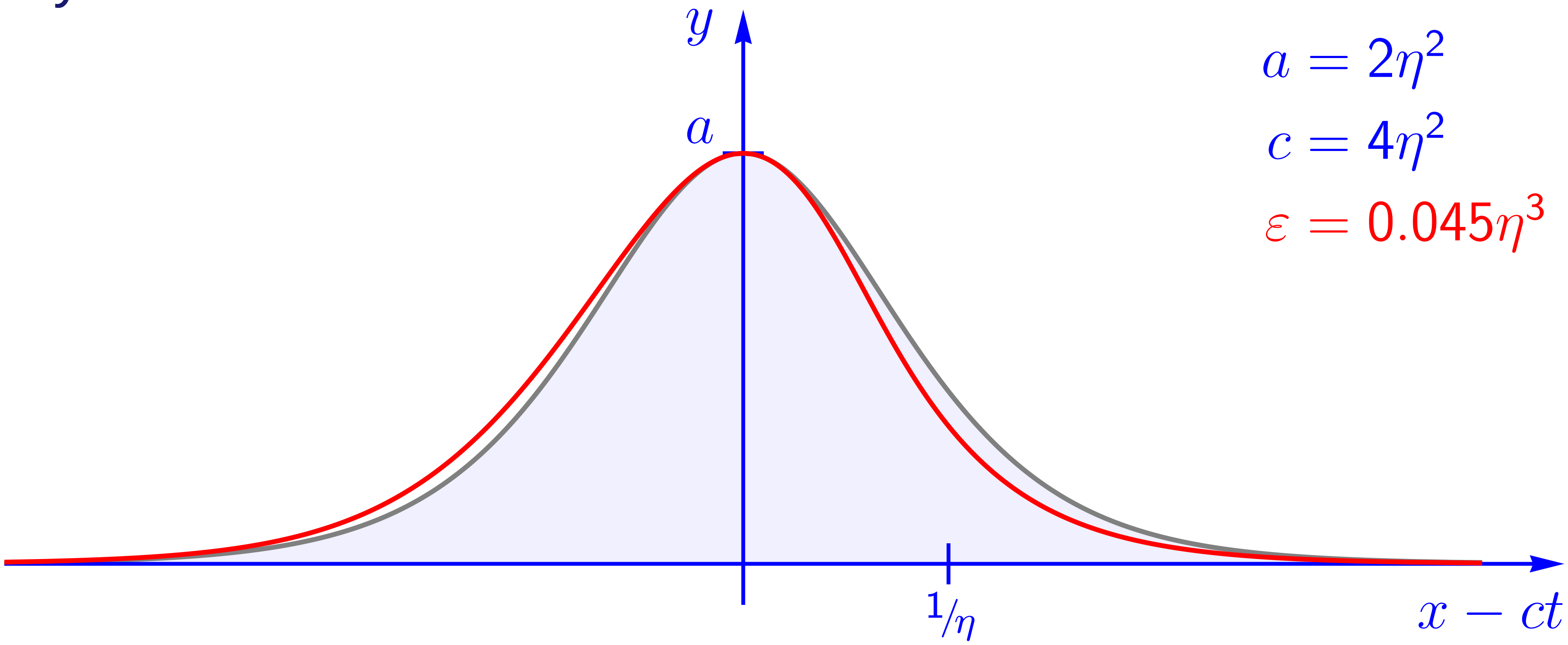

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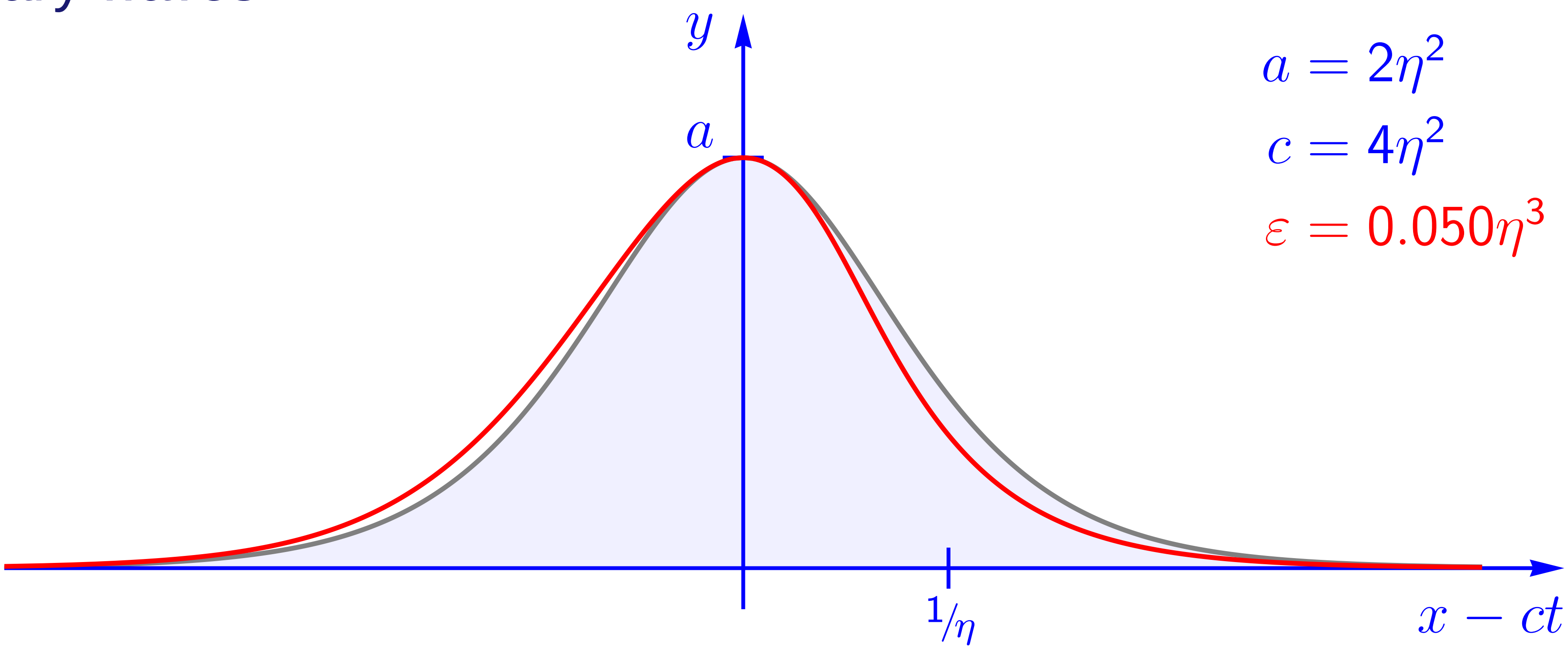

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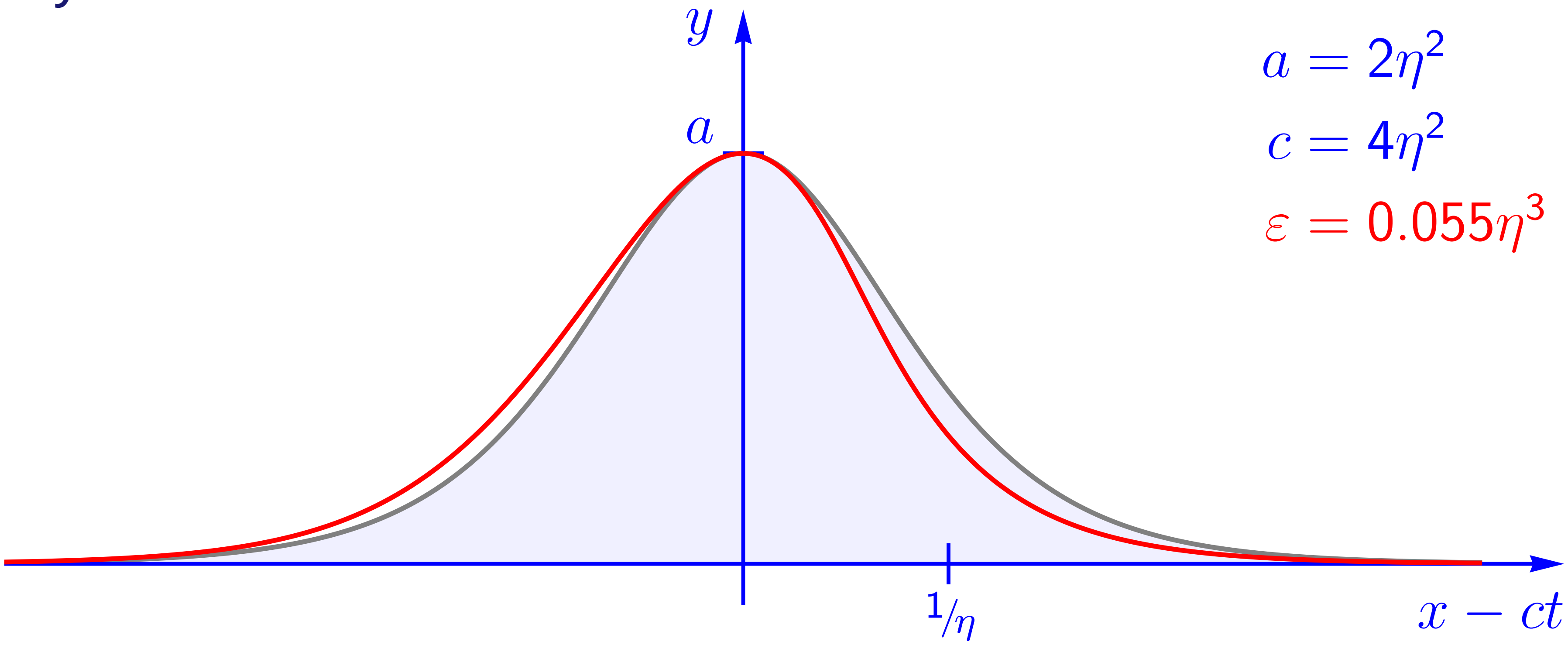

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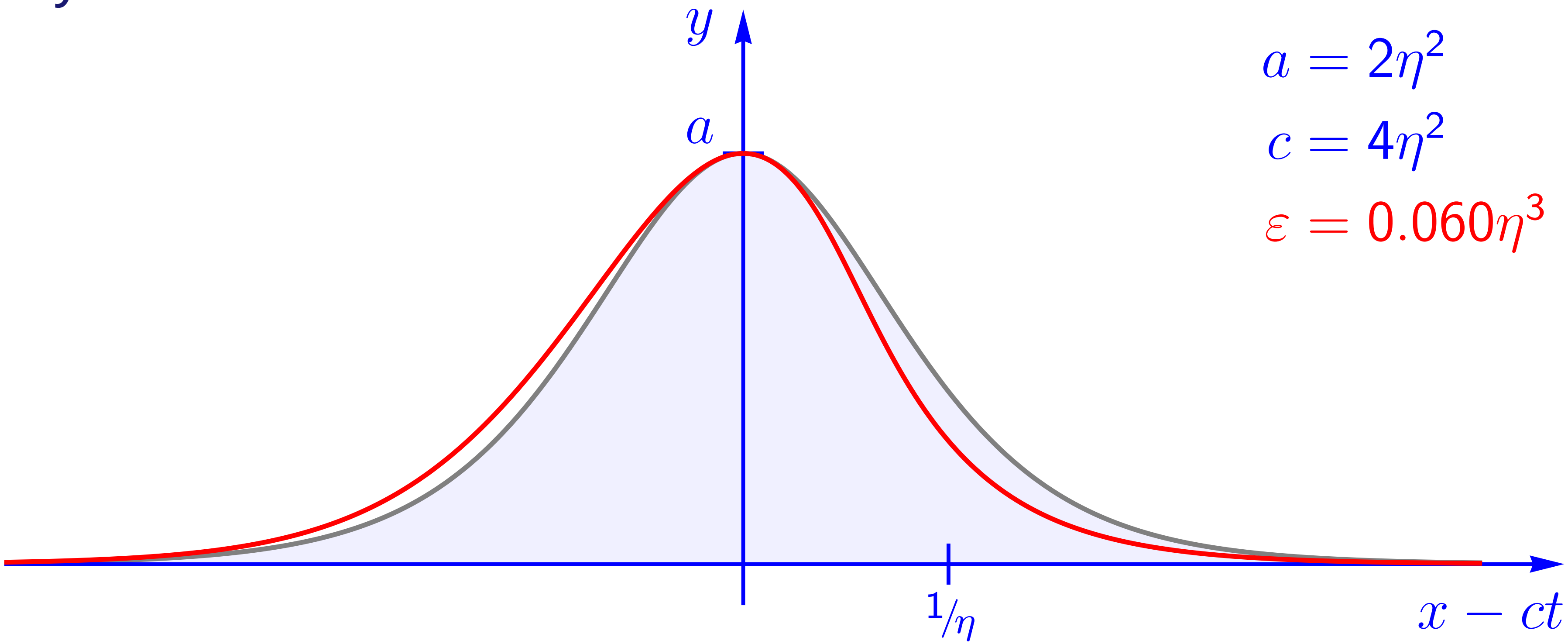

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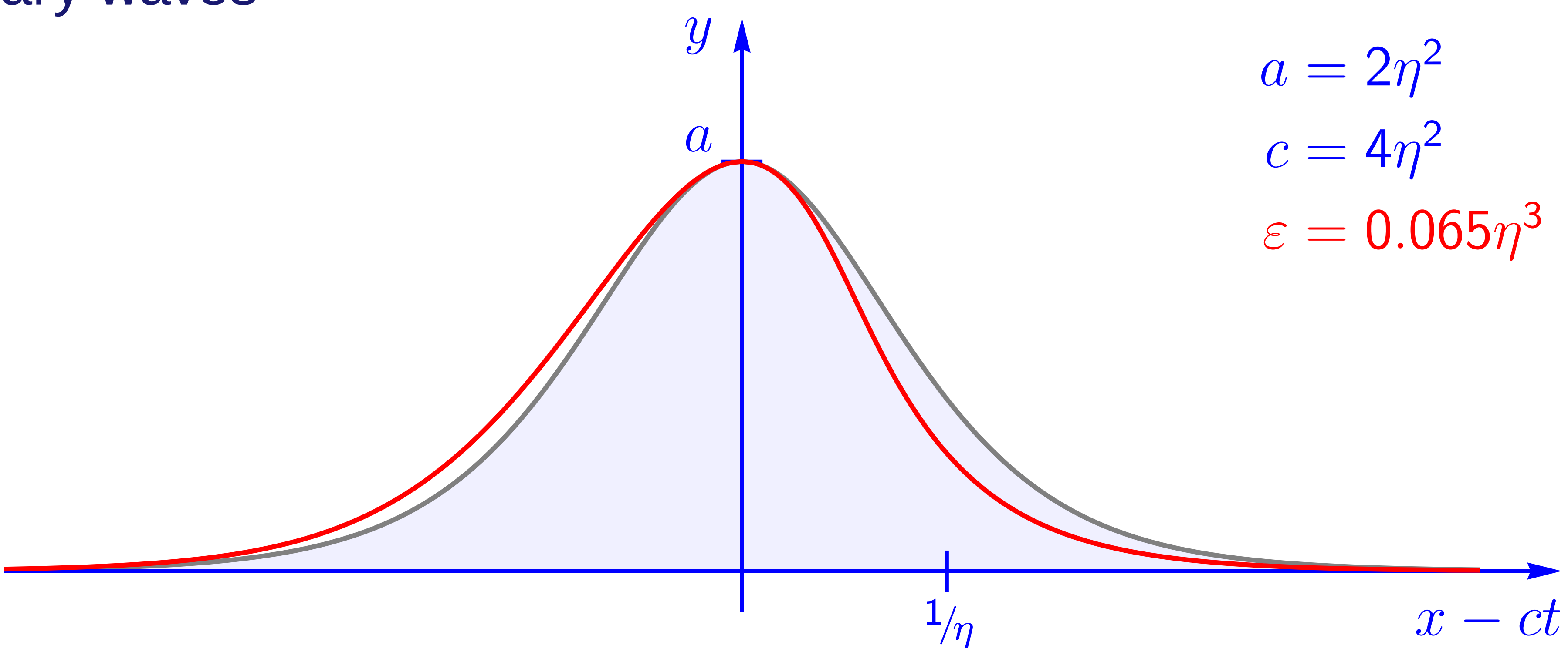

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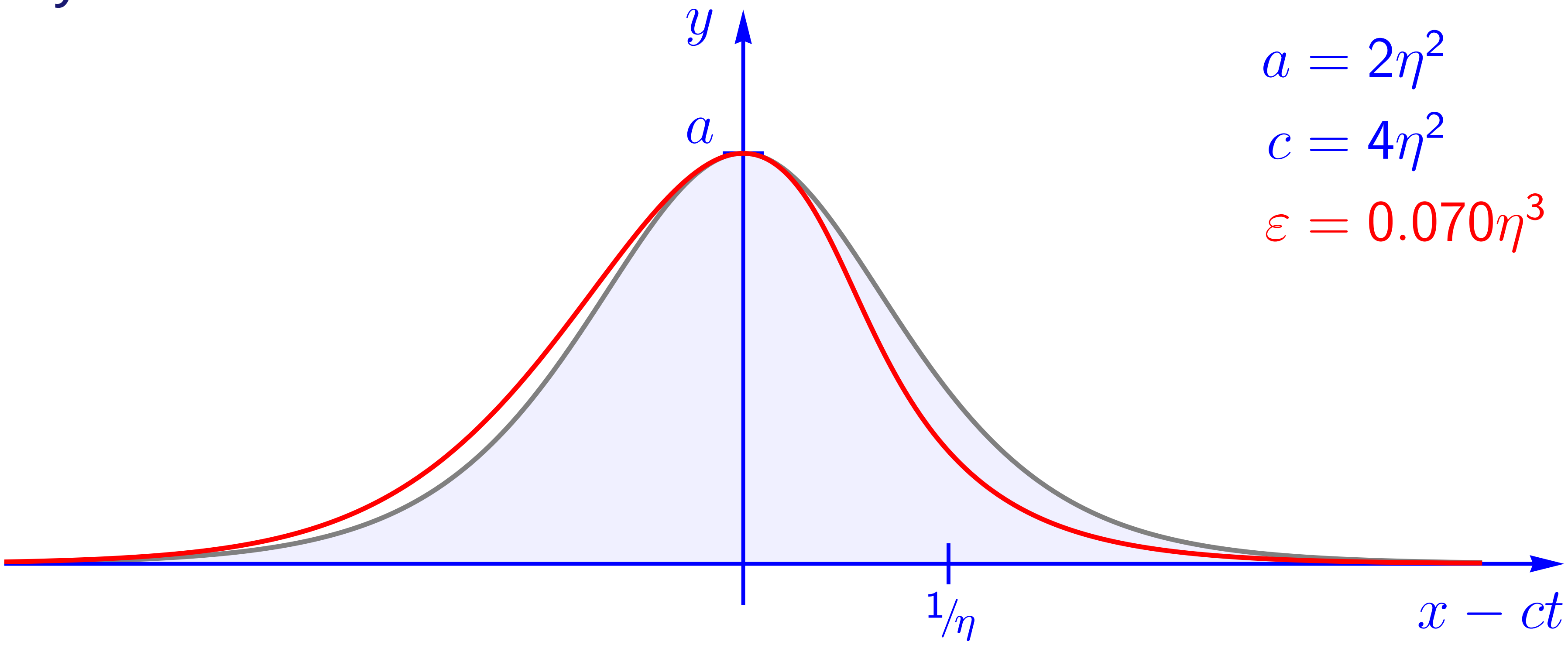

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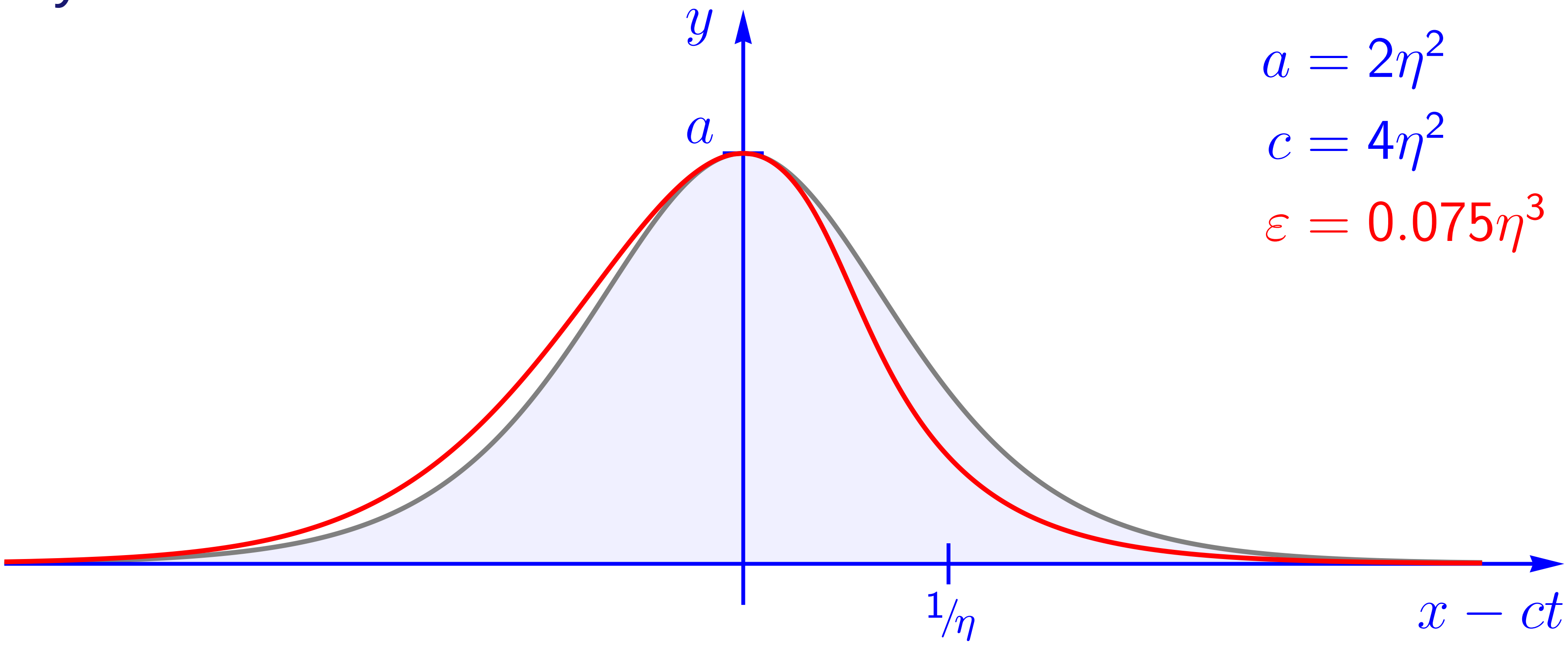

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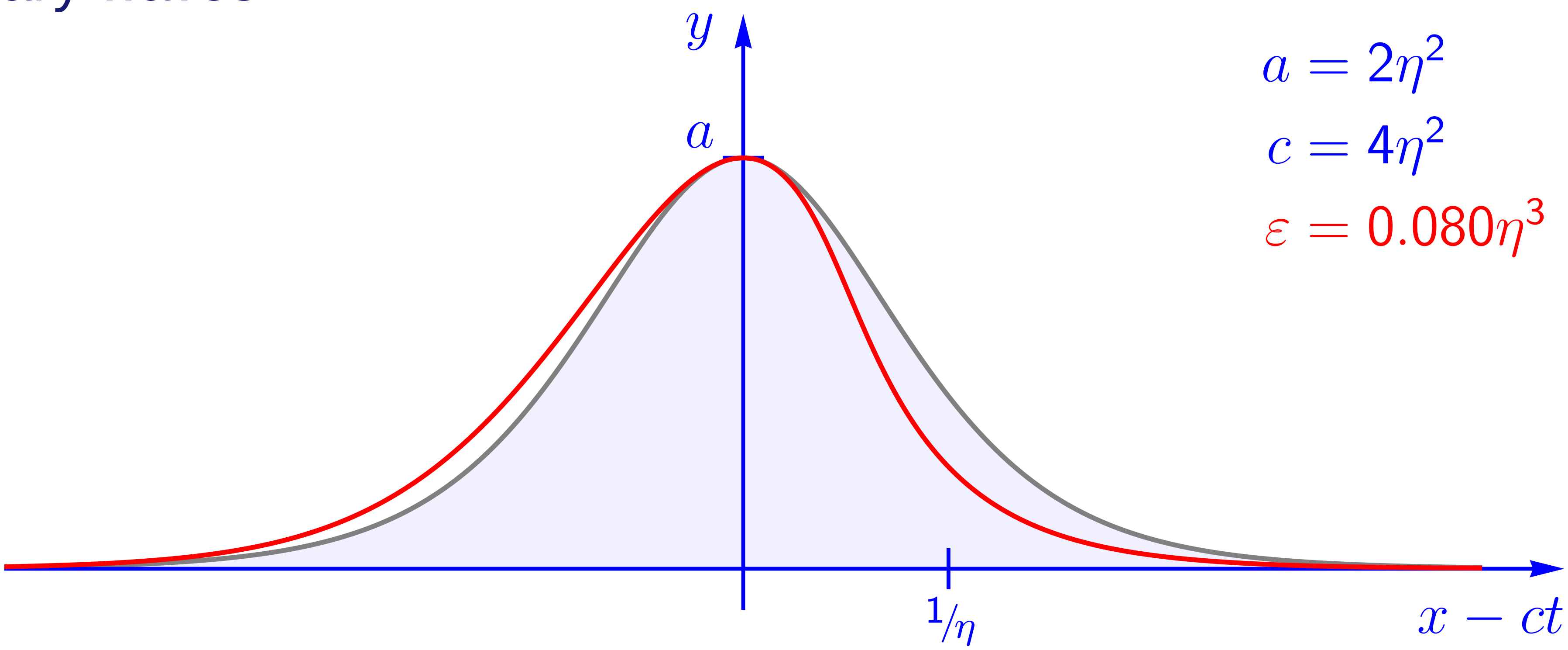

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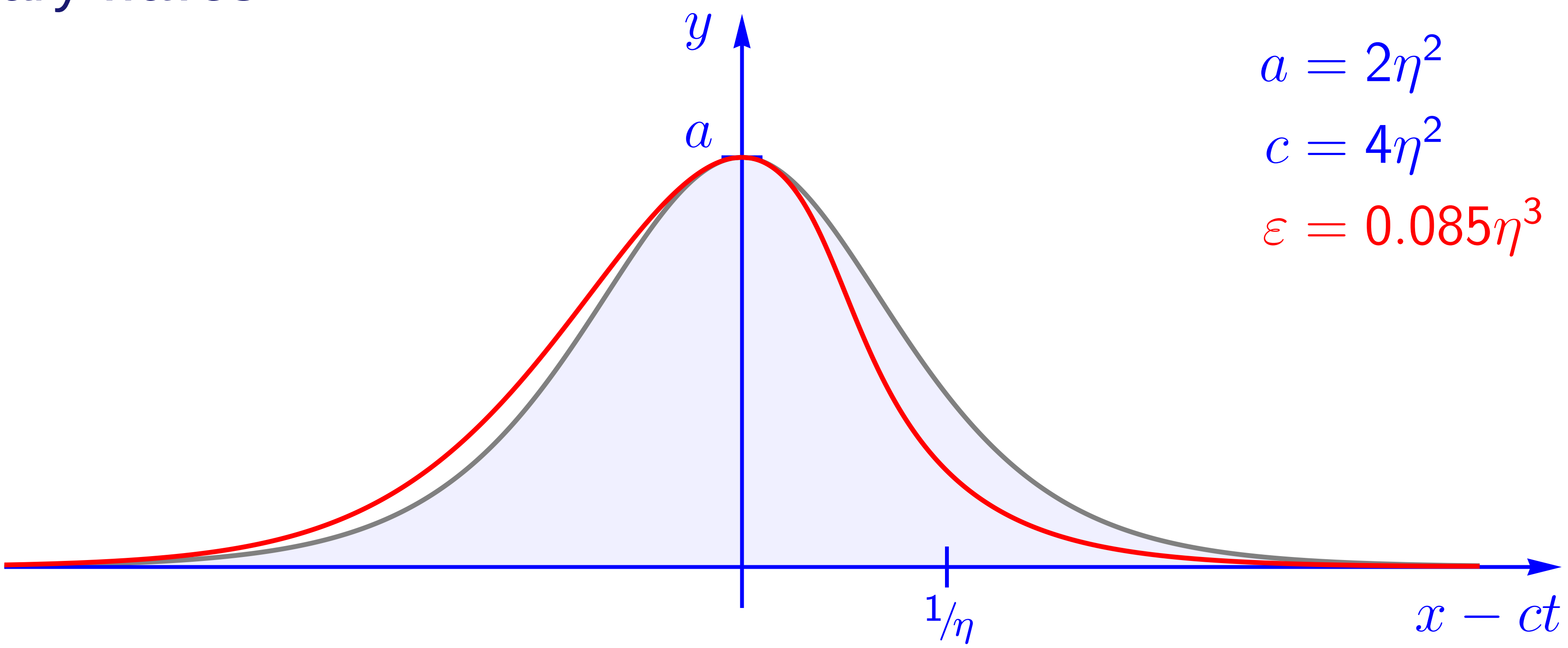

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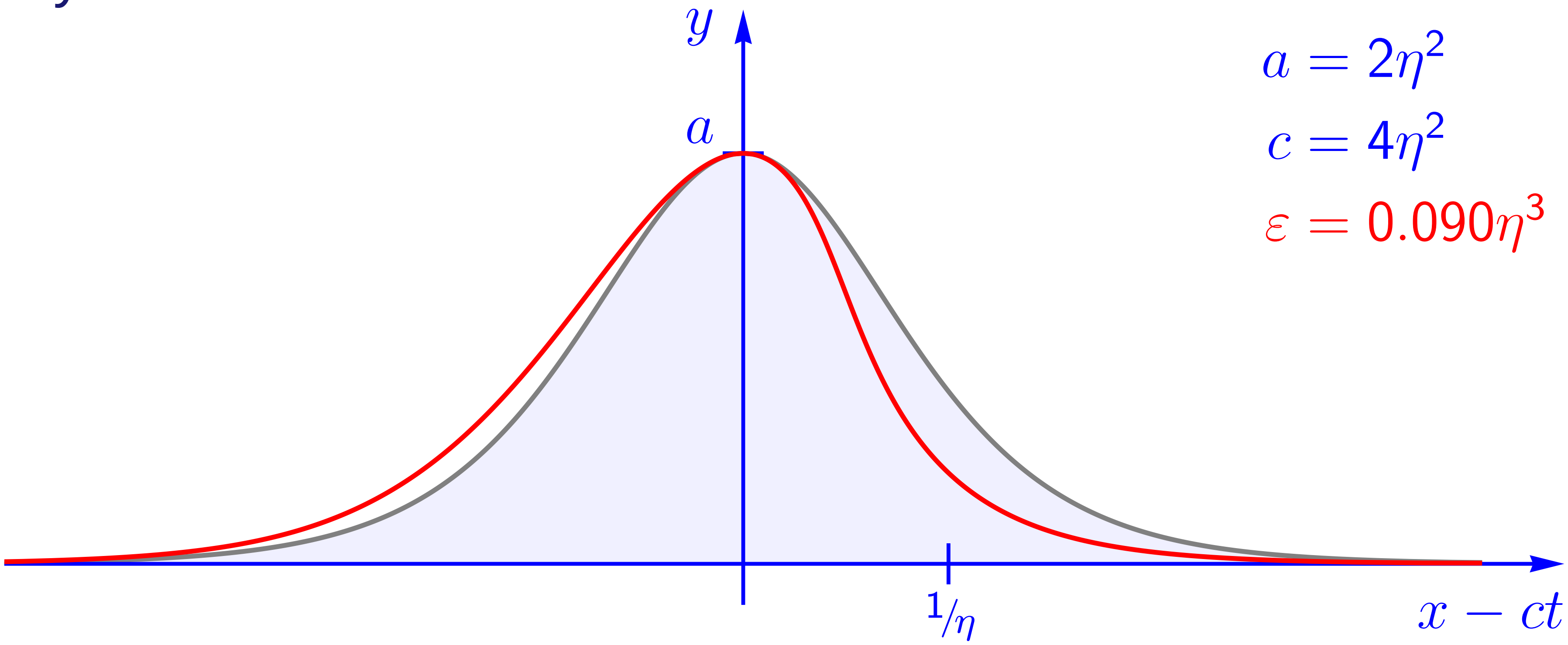

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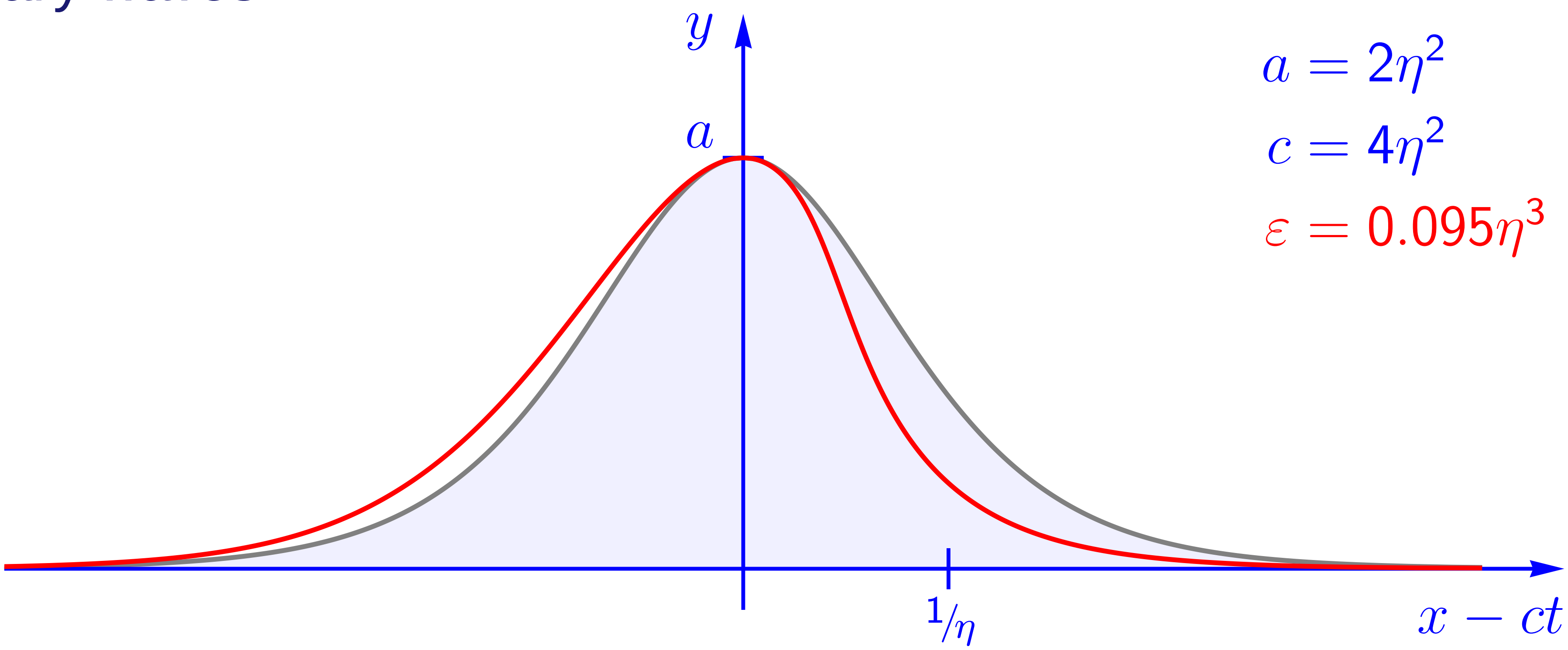

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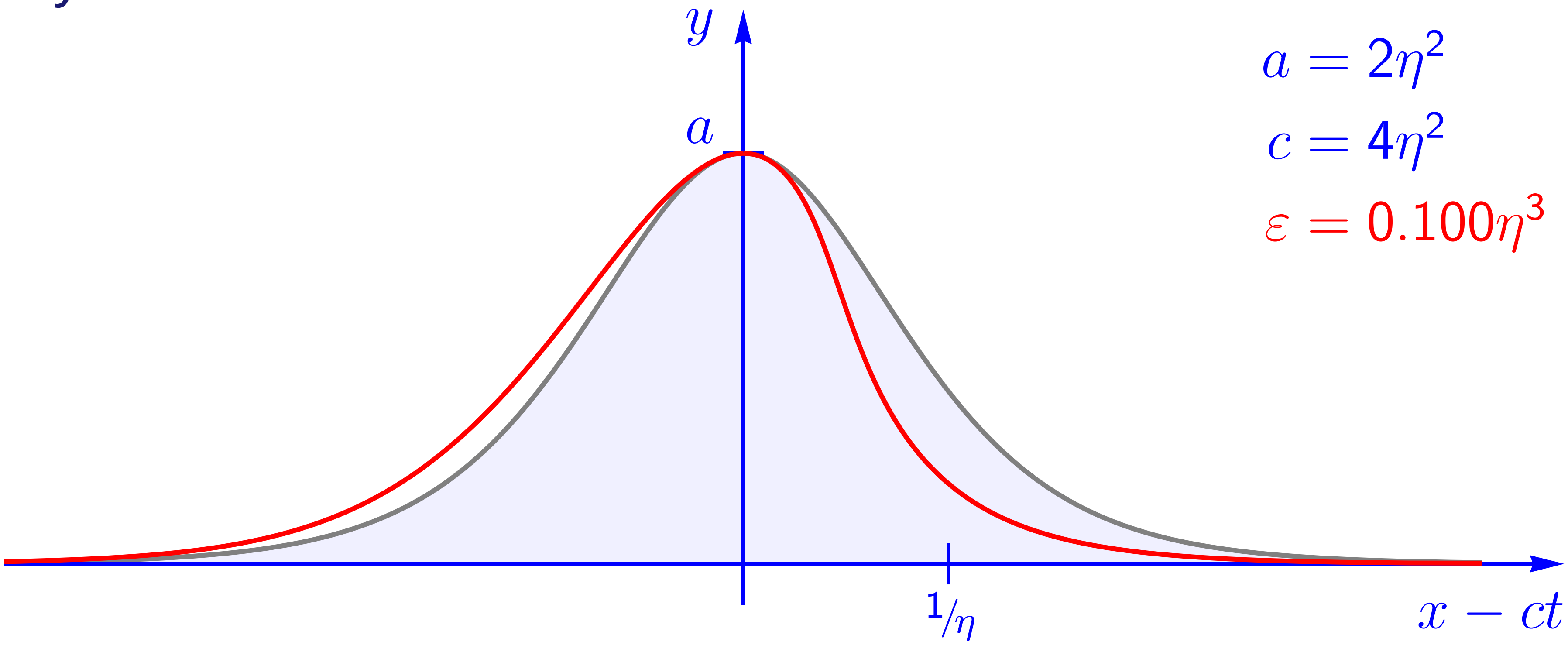
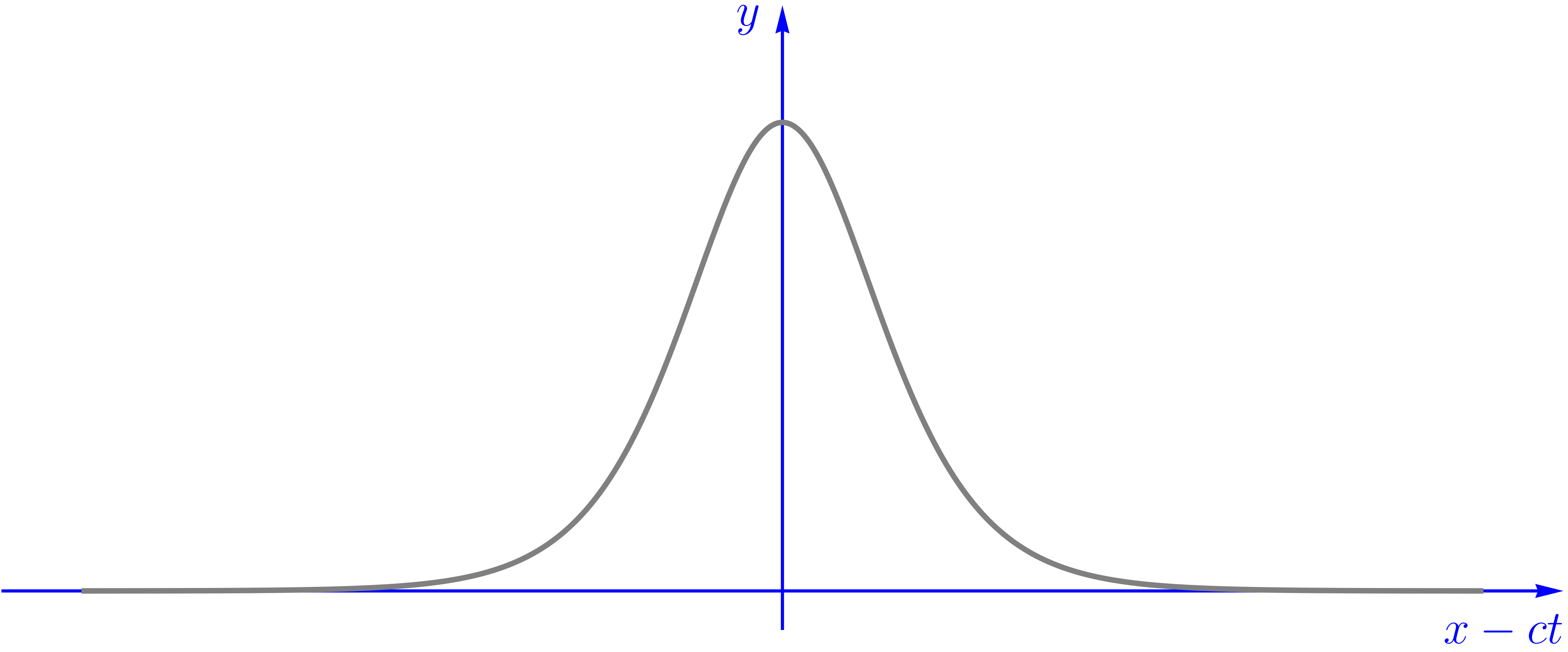
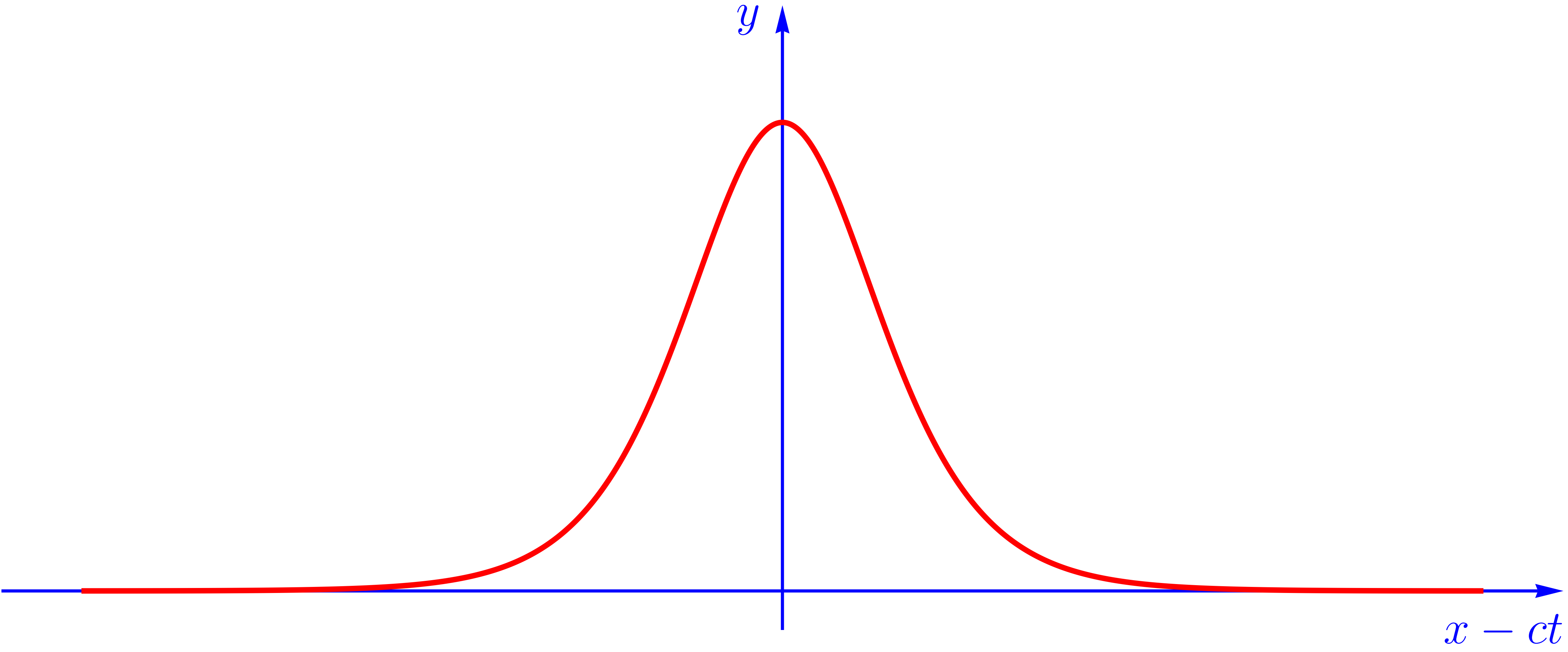
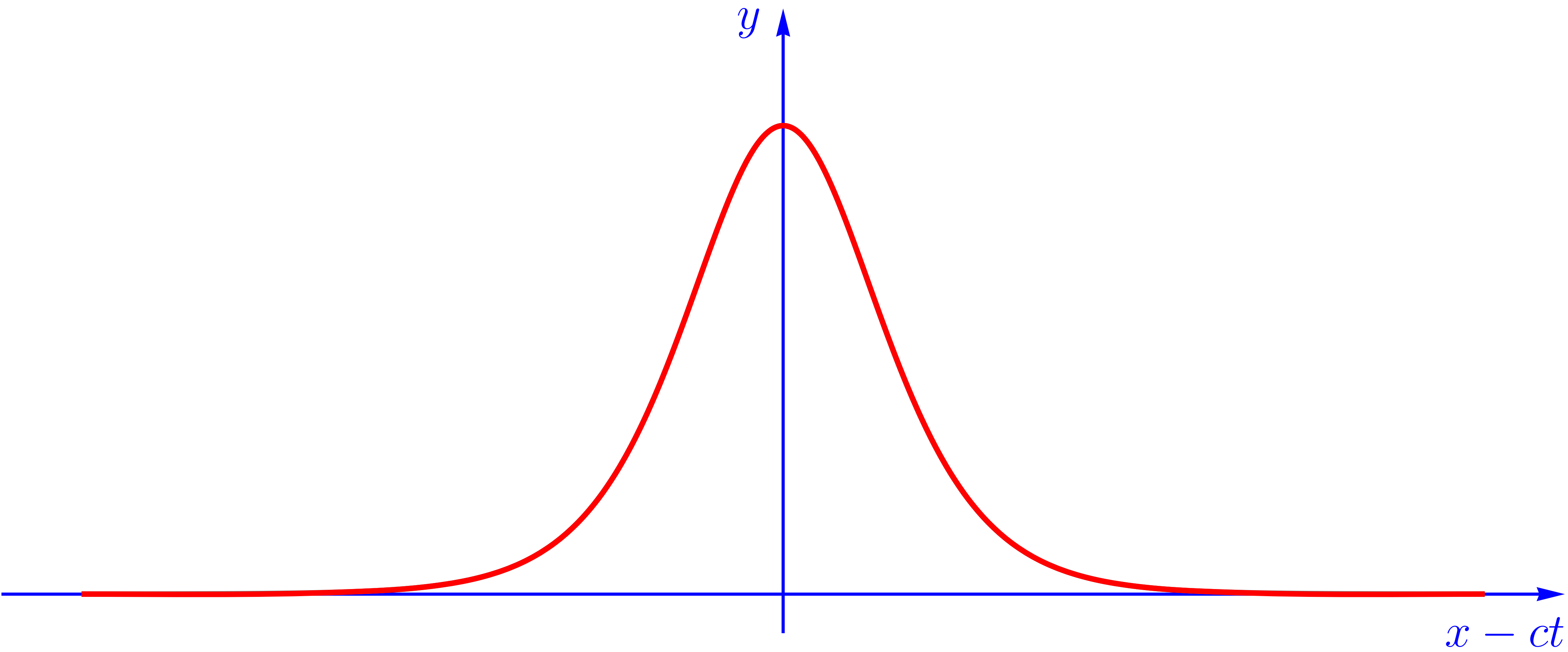


KdV Soliton




y
$x - ct$

Cnoidal Waves



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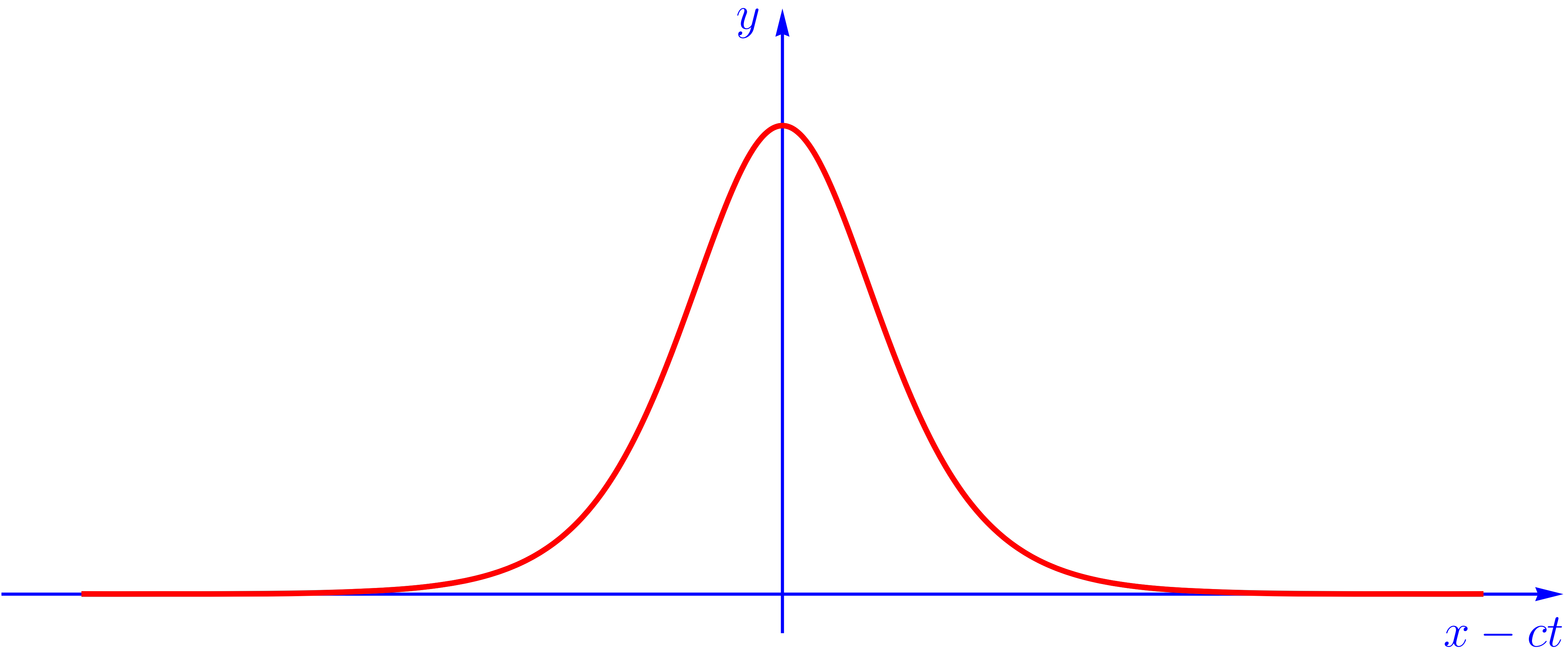

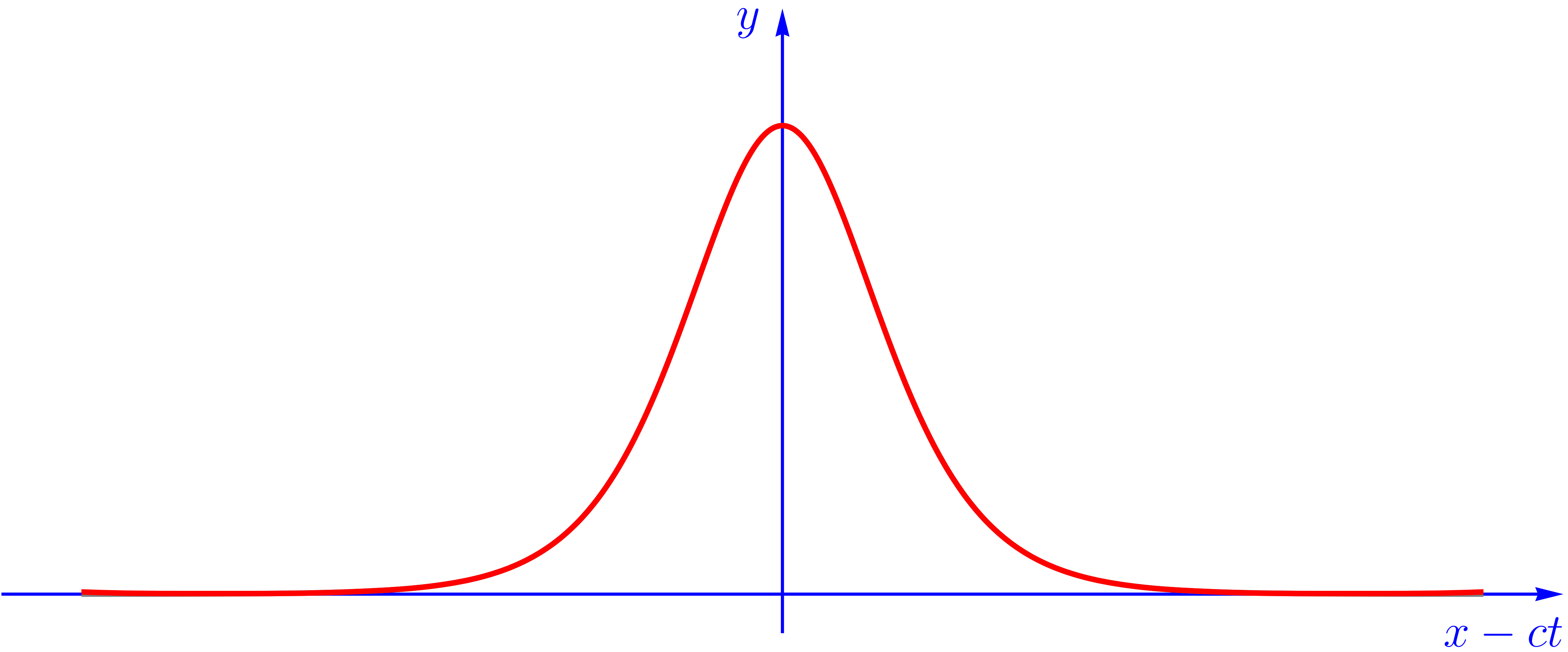
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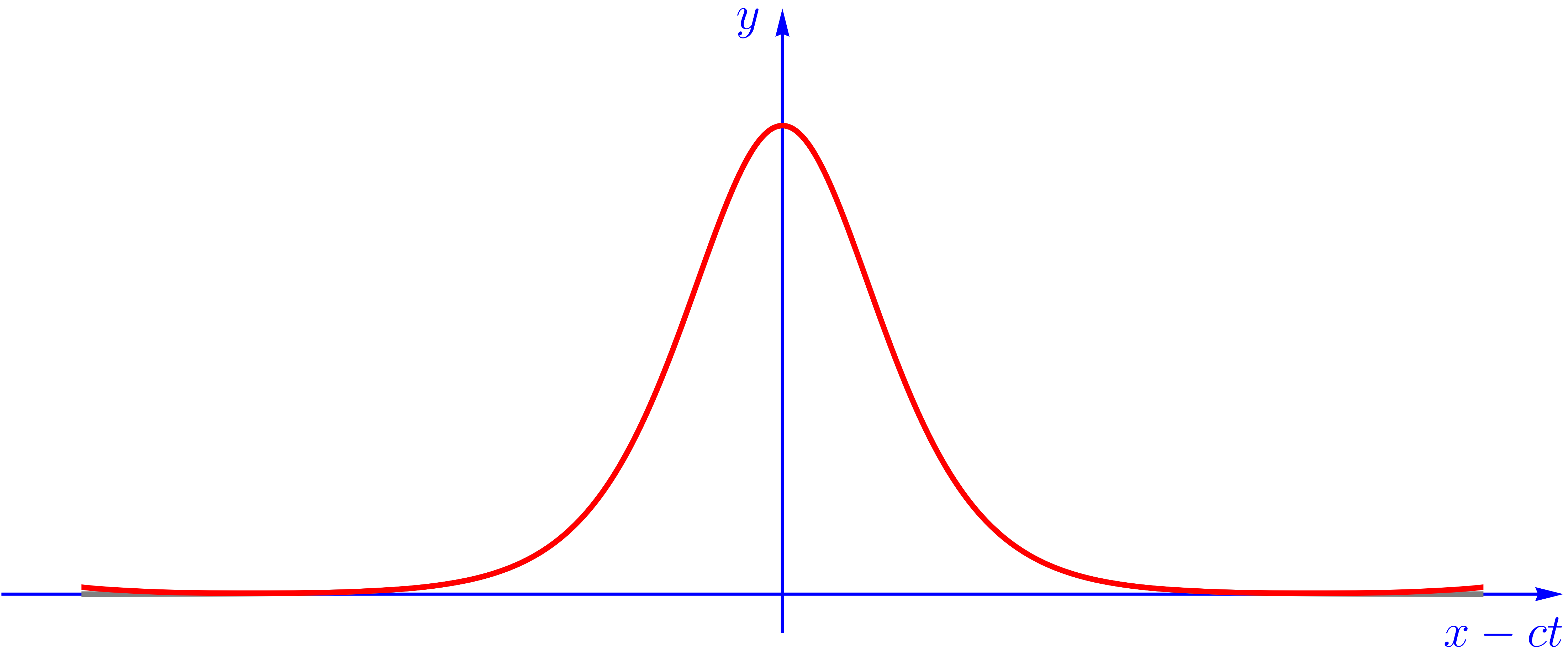
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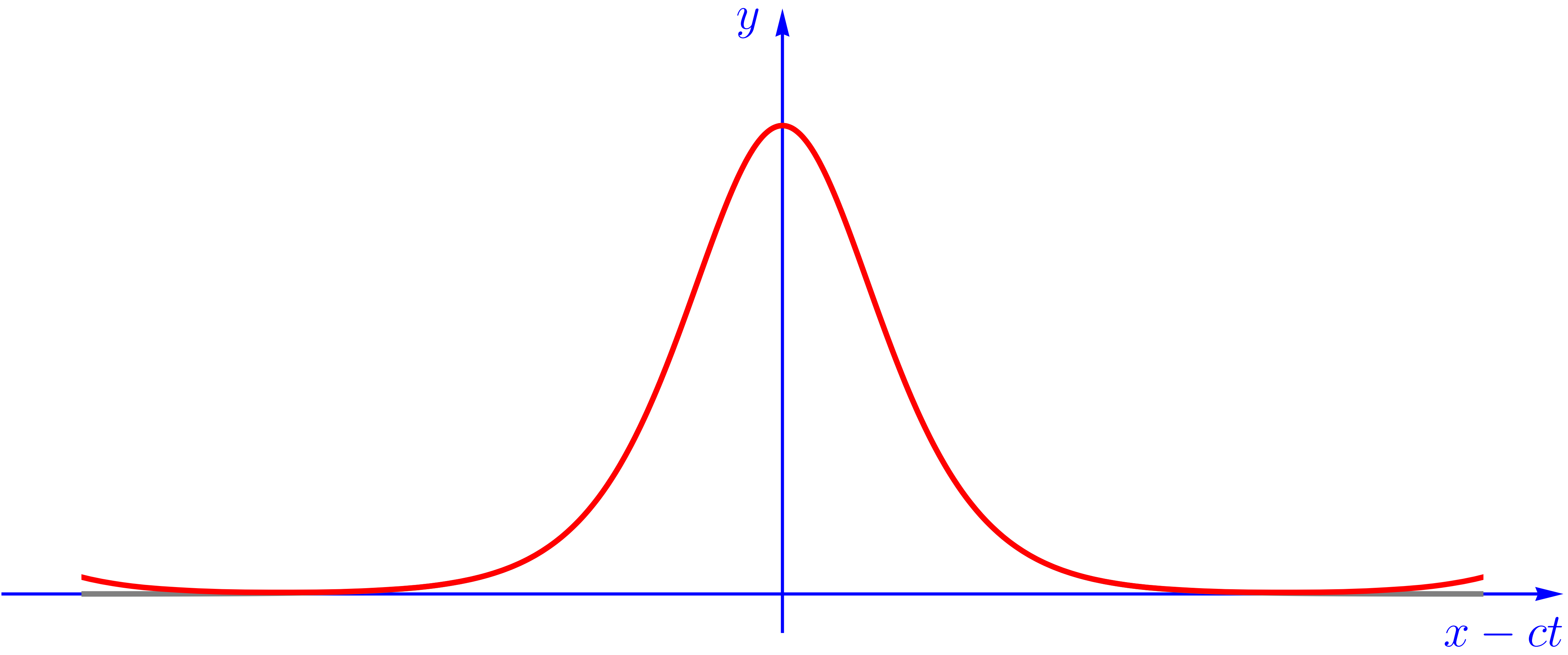
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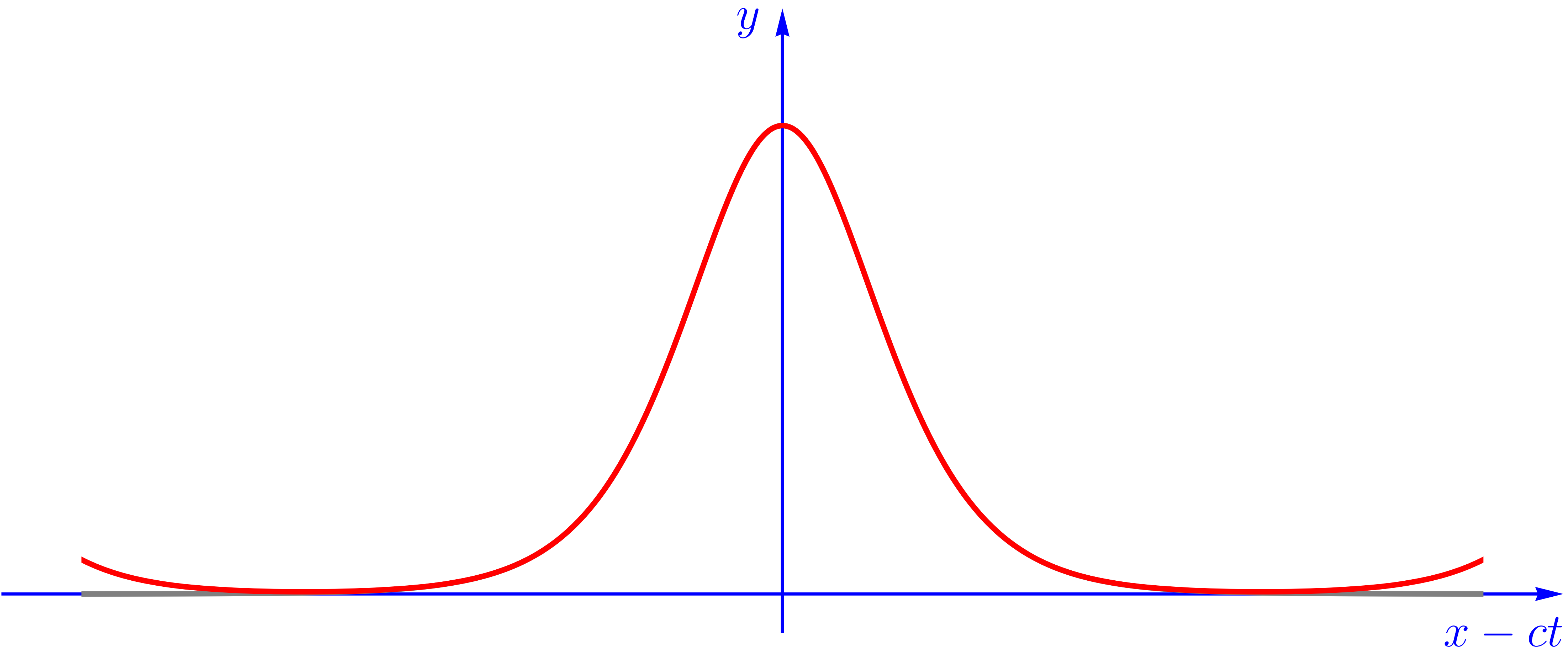
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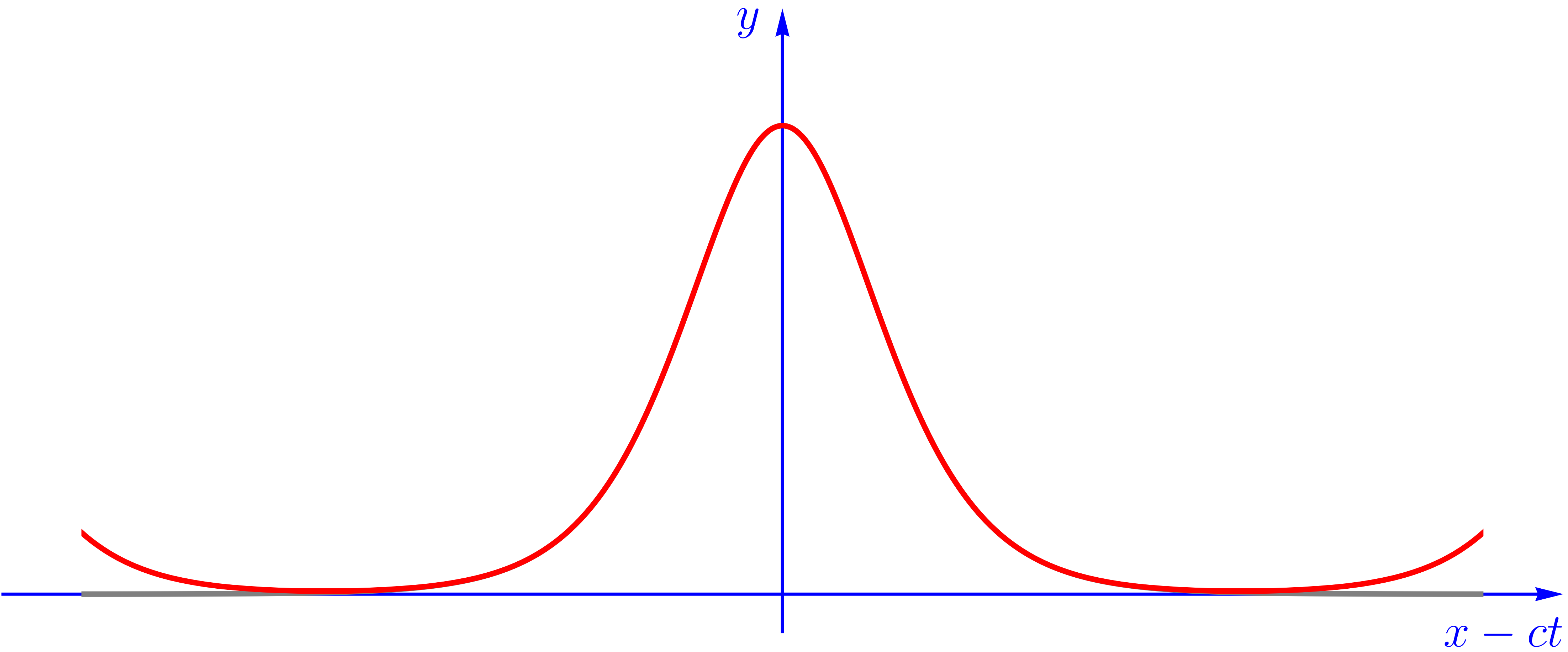

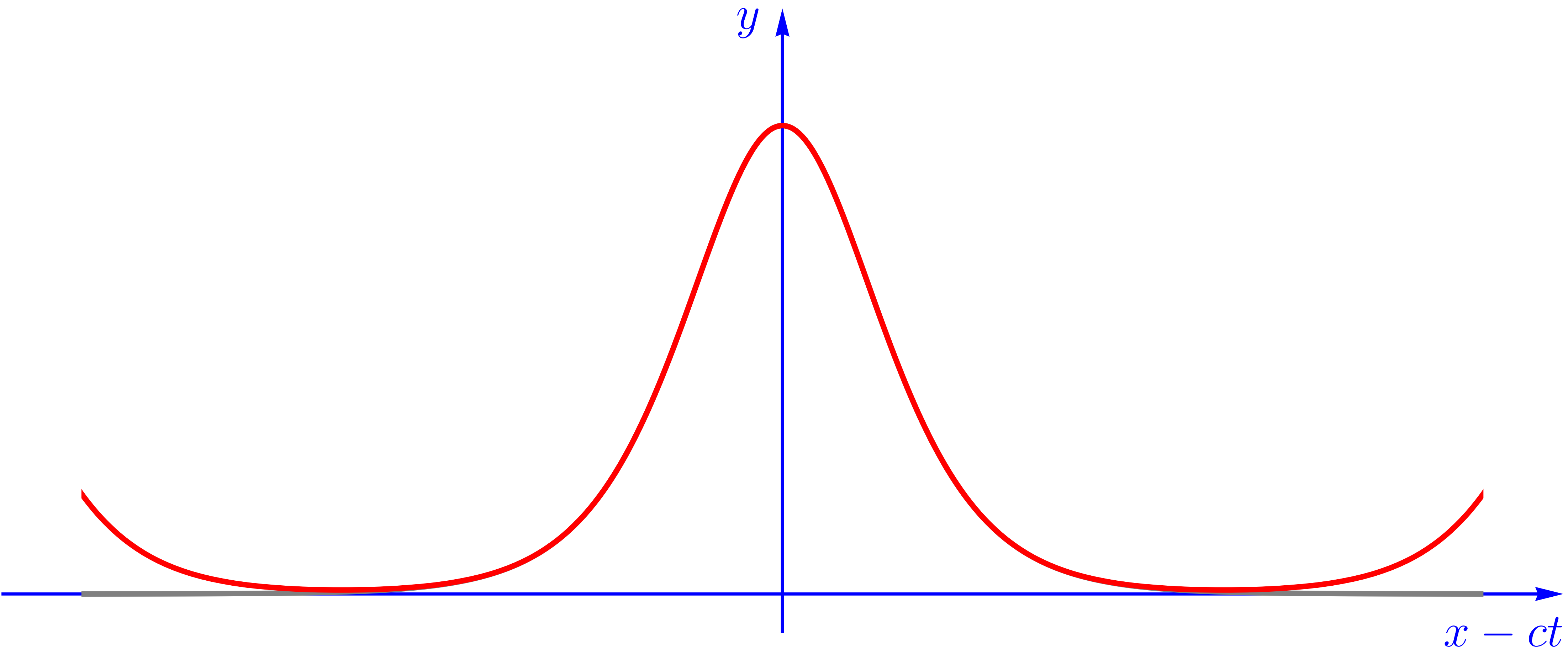
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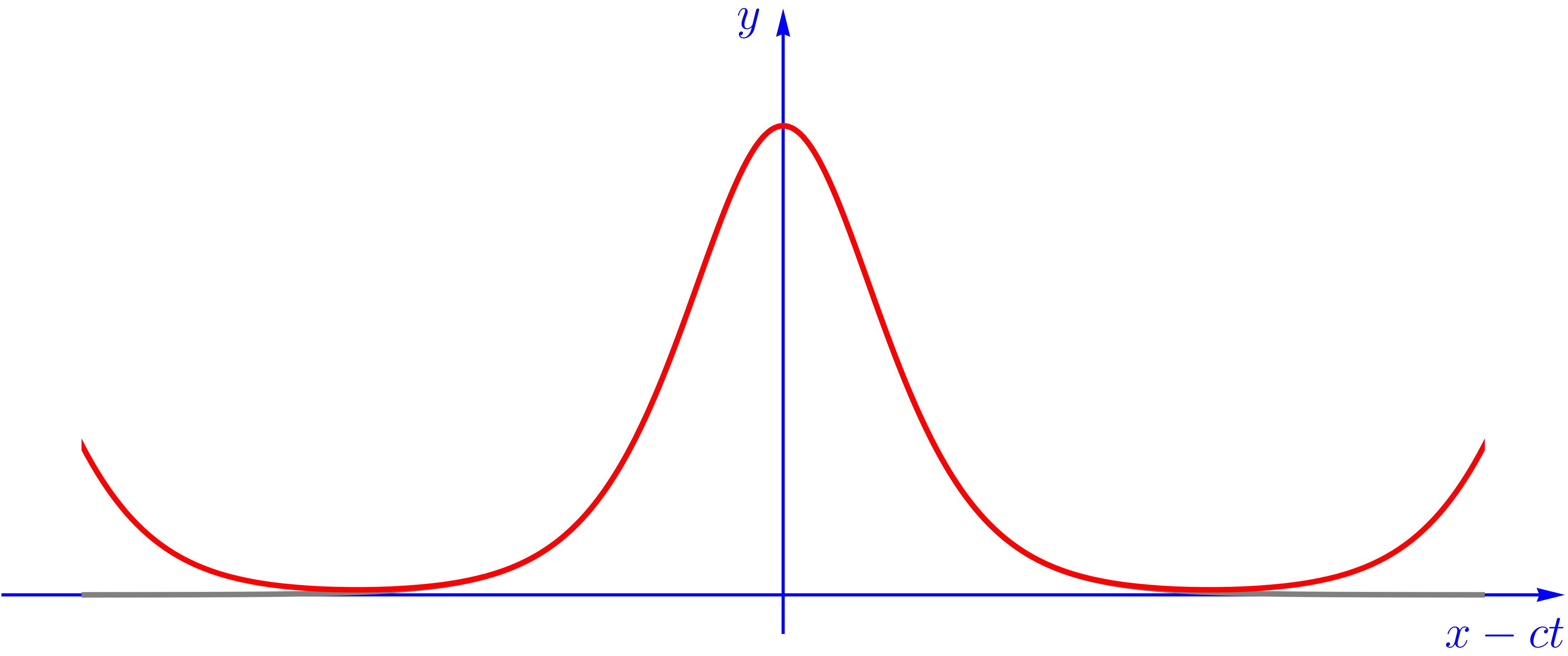
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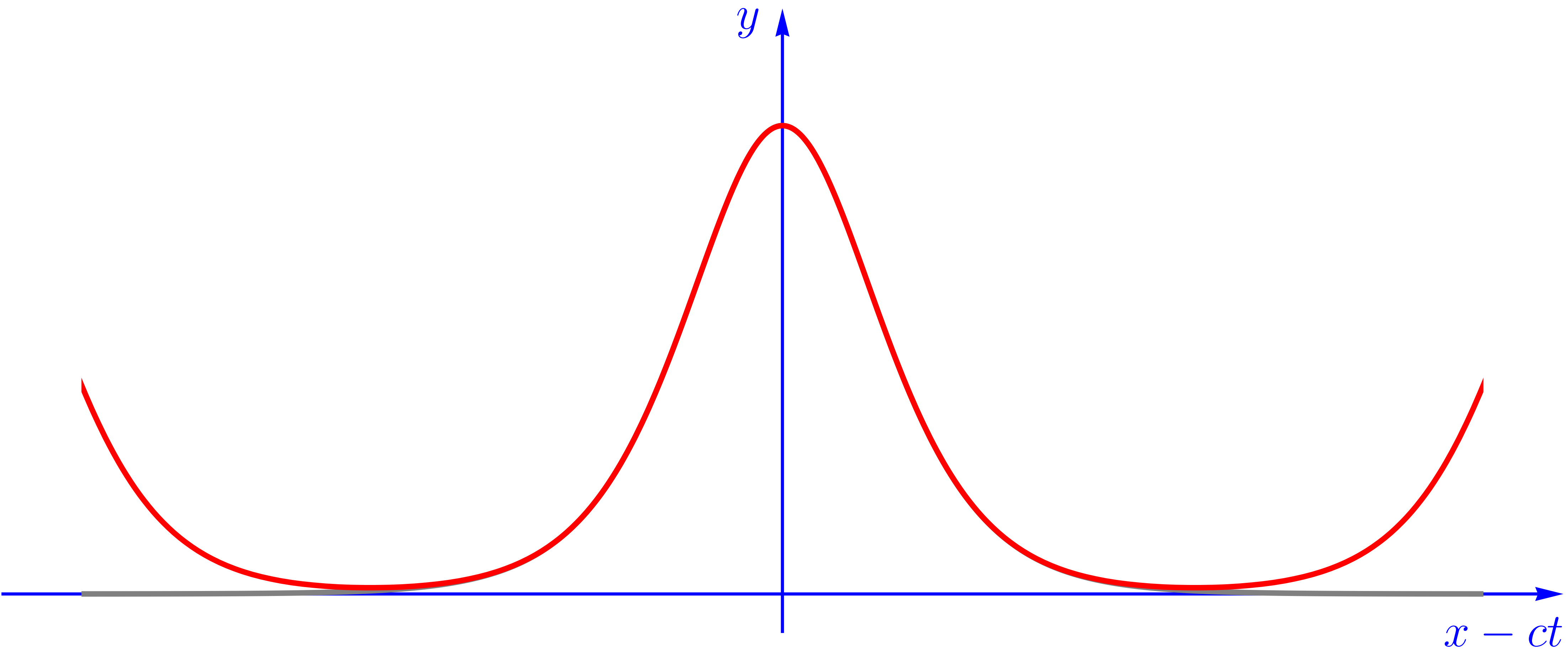

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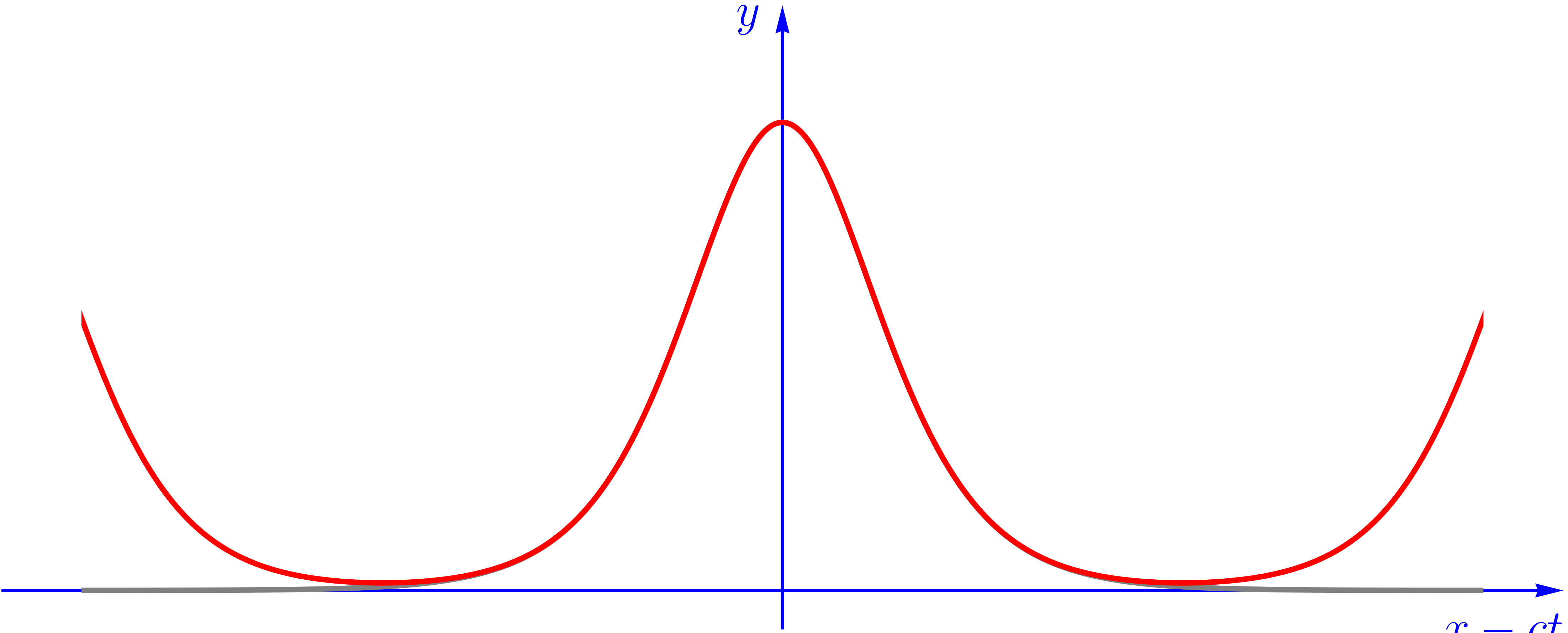

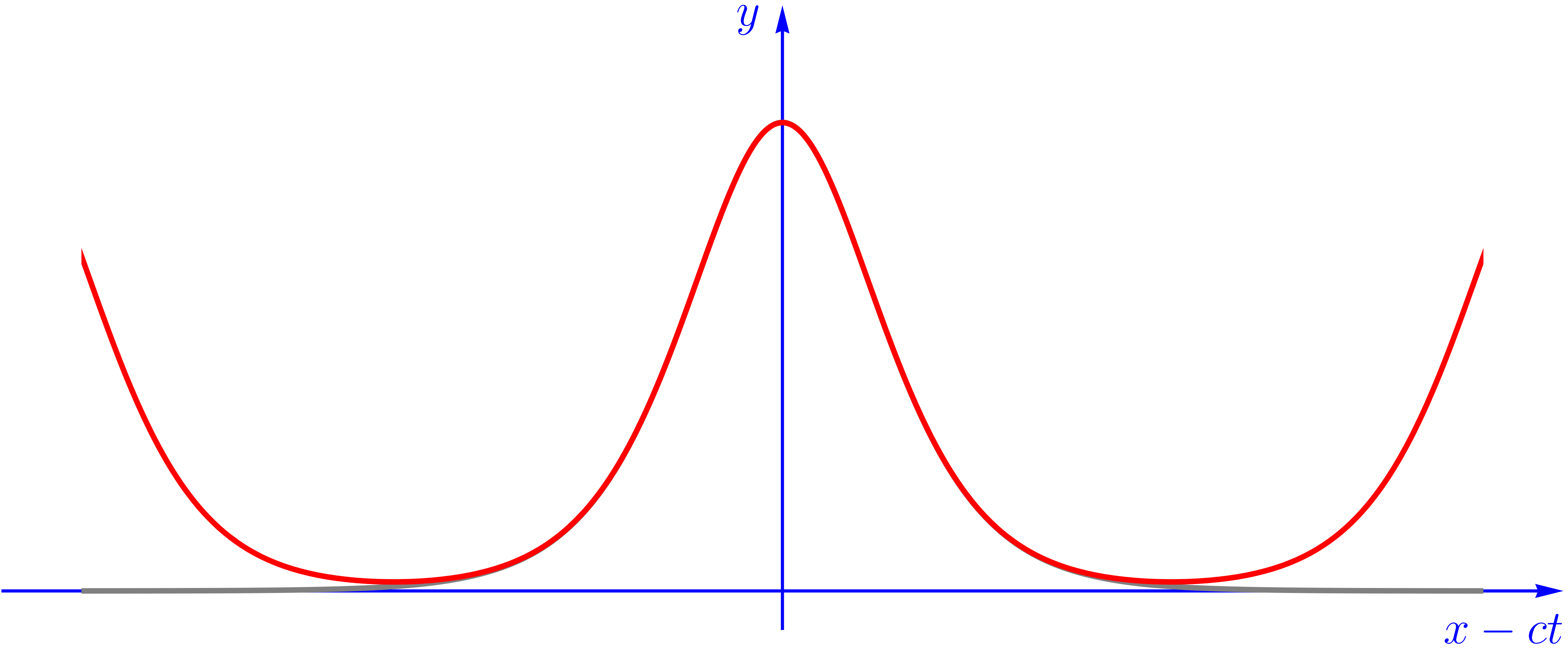
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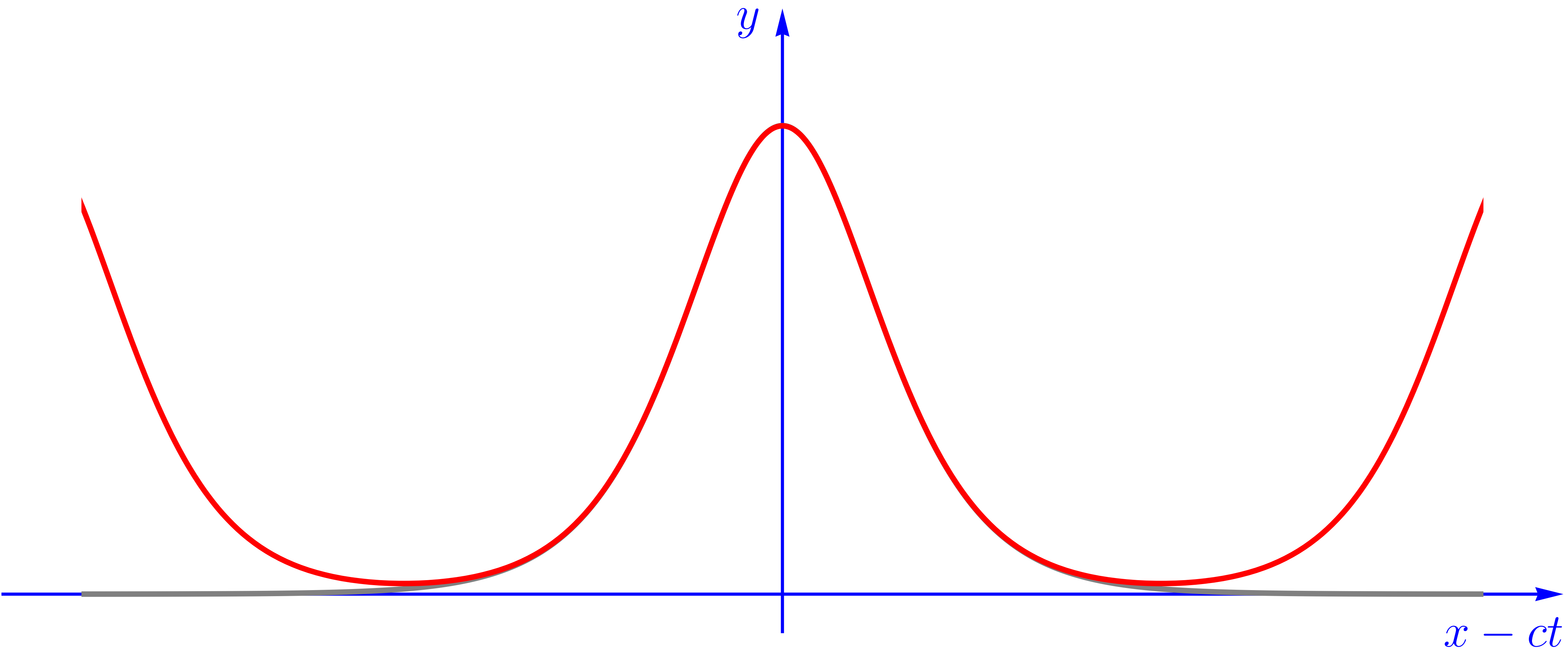

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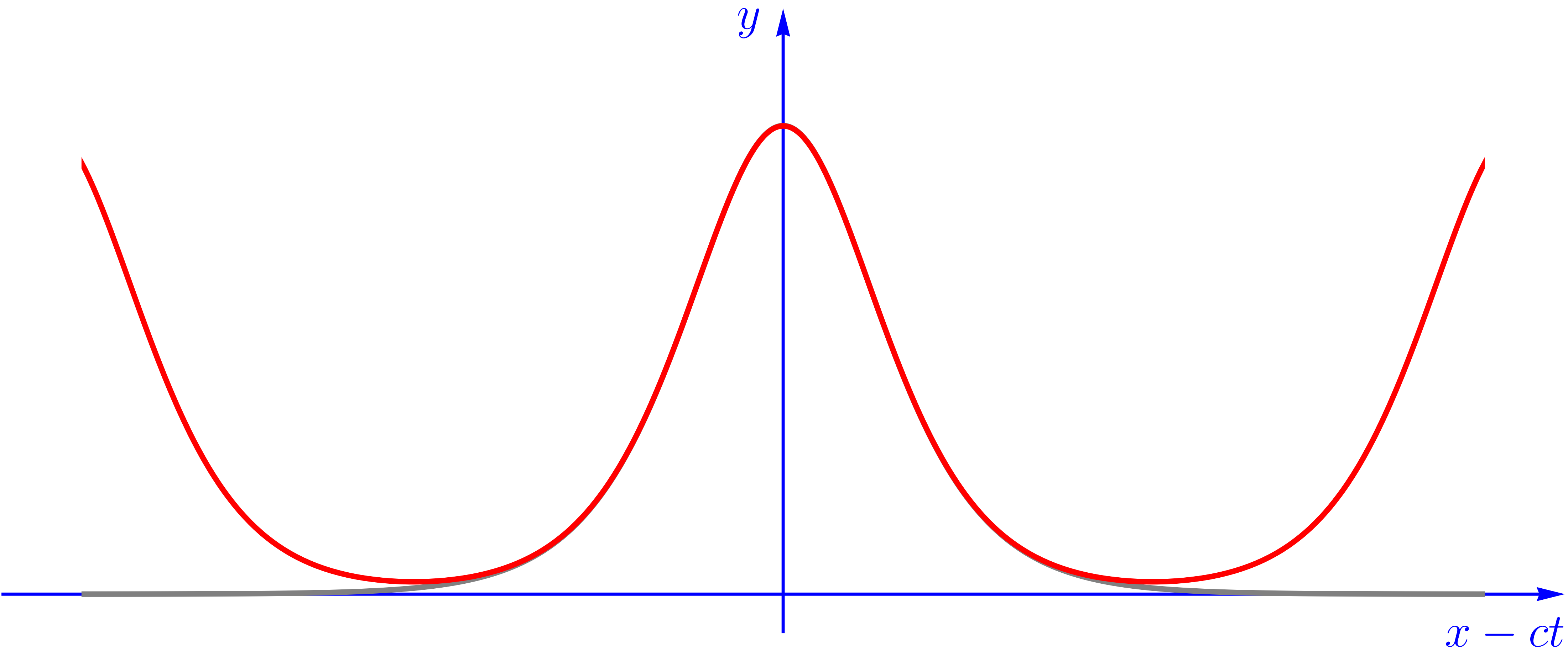

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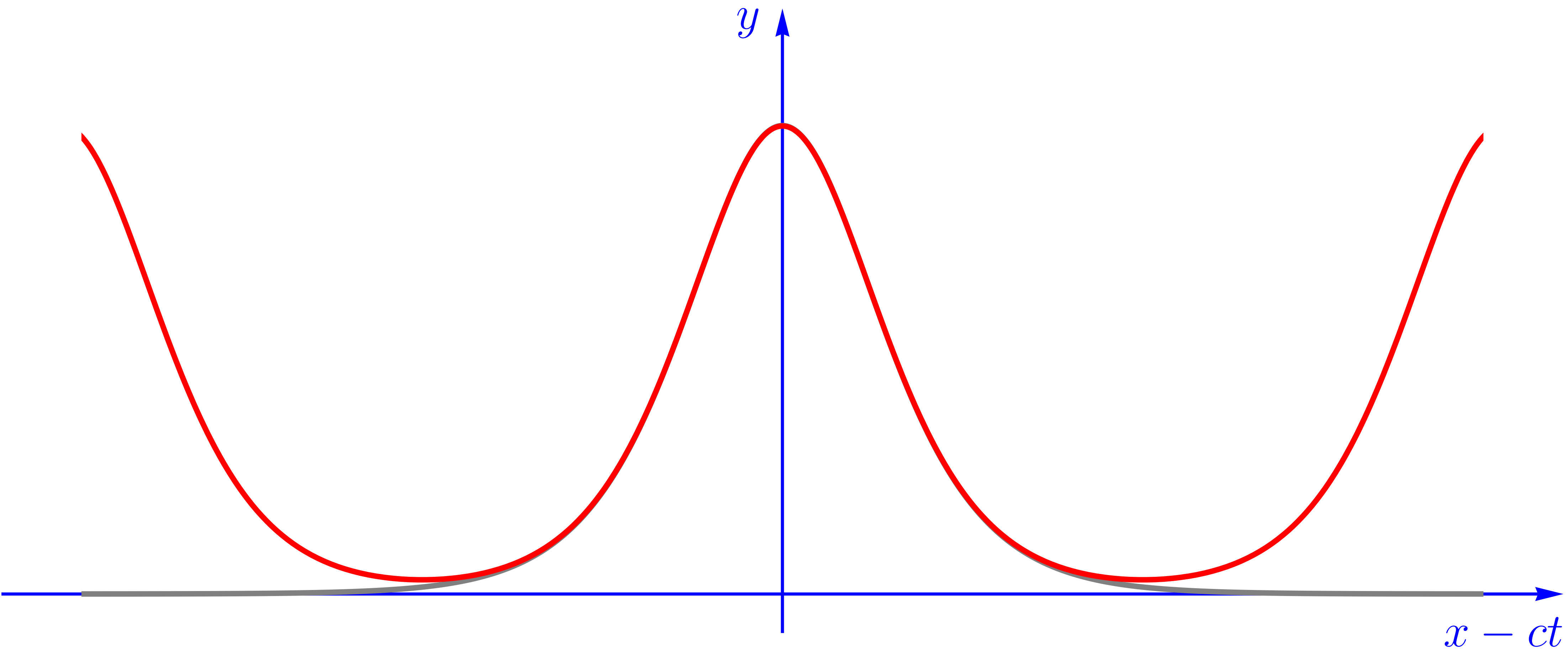

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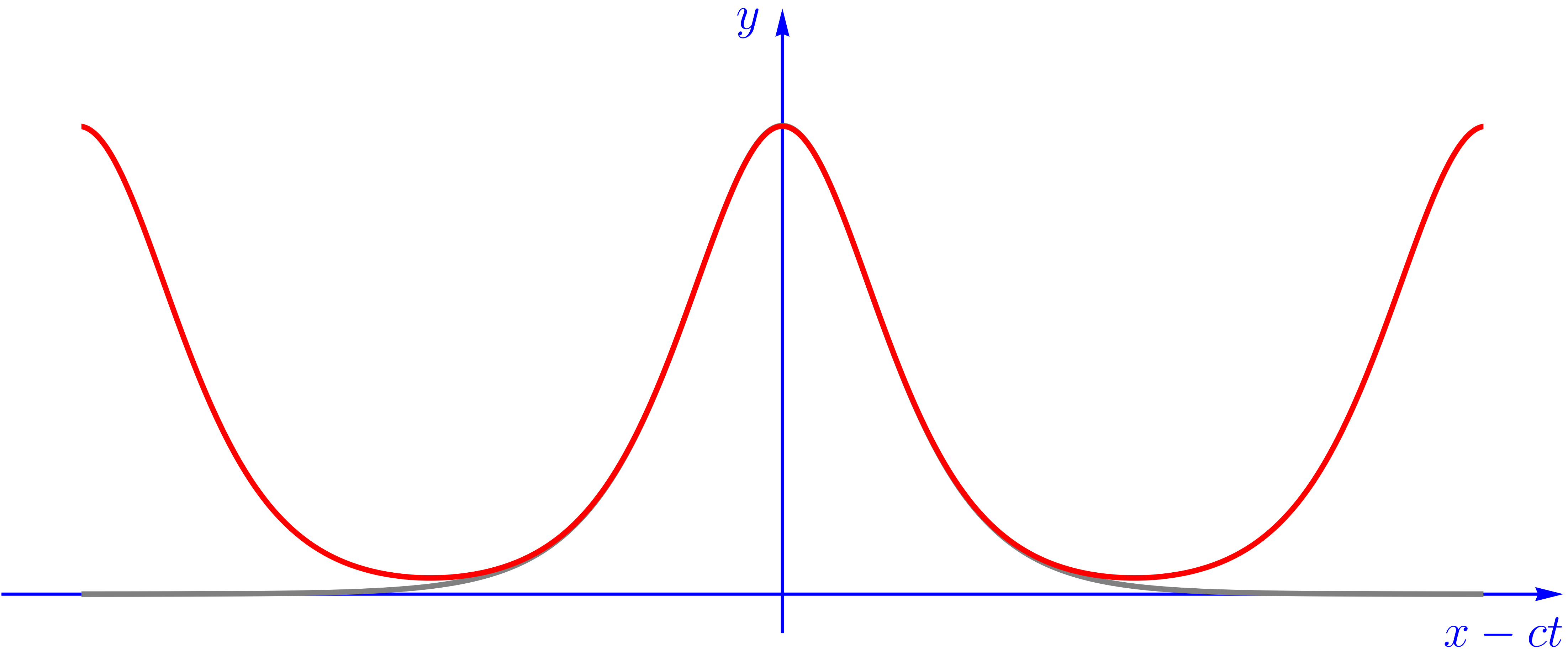

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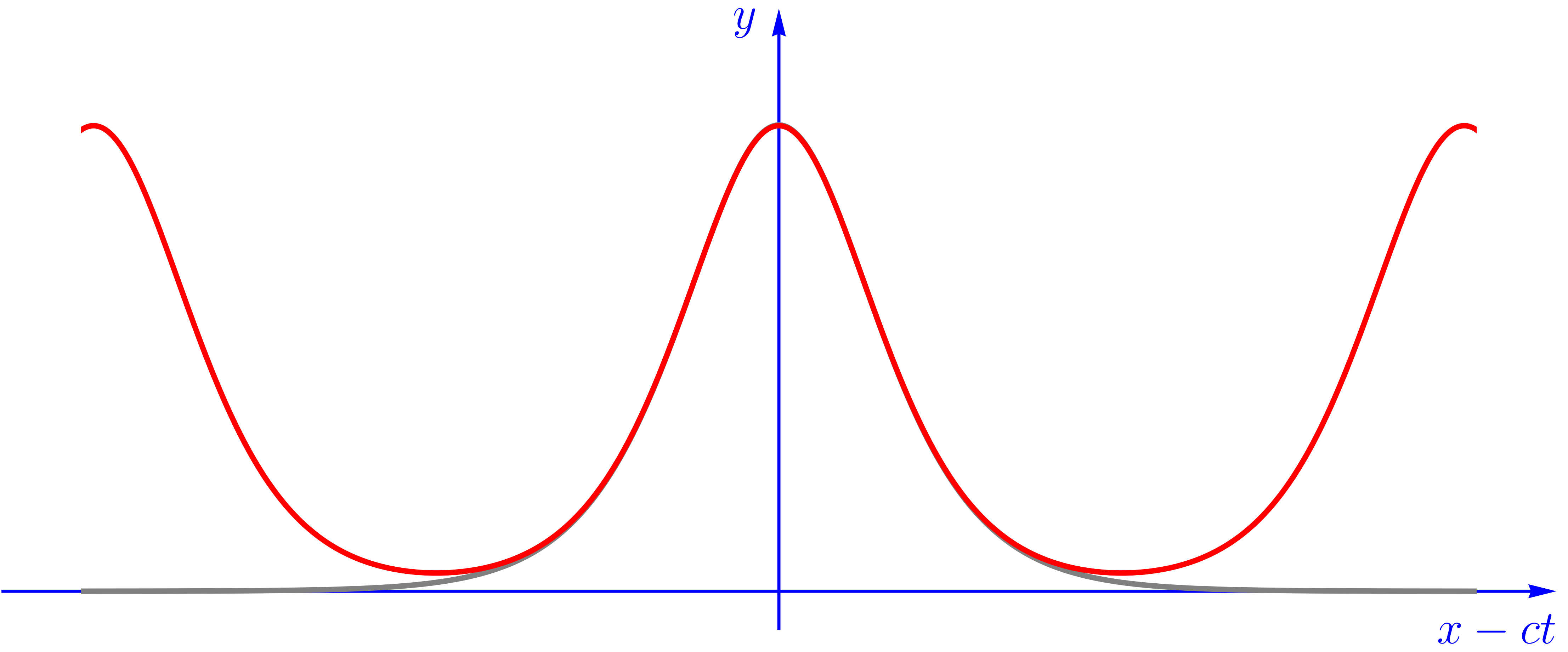

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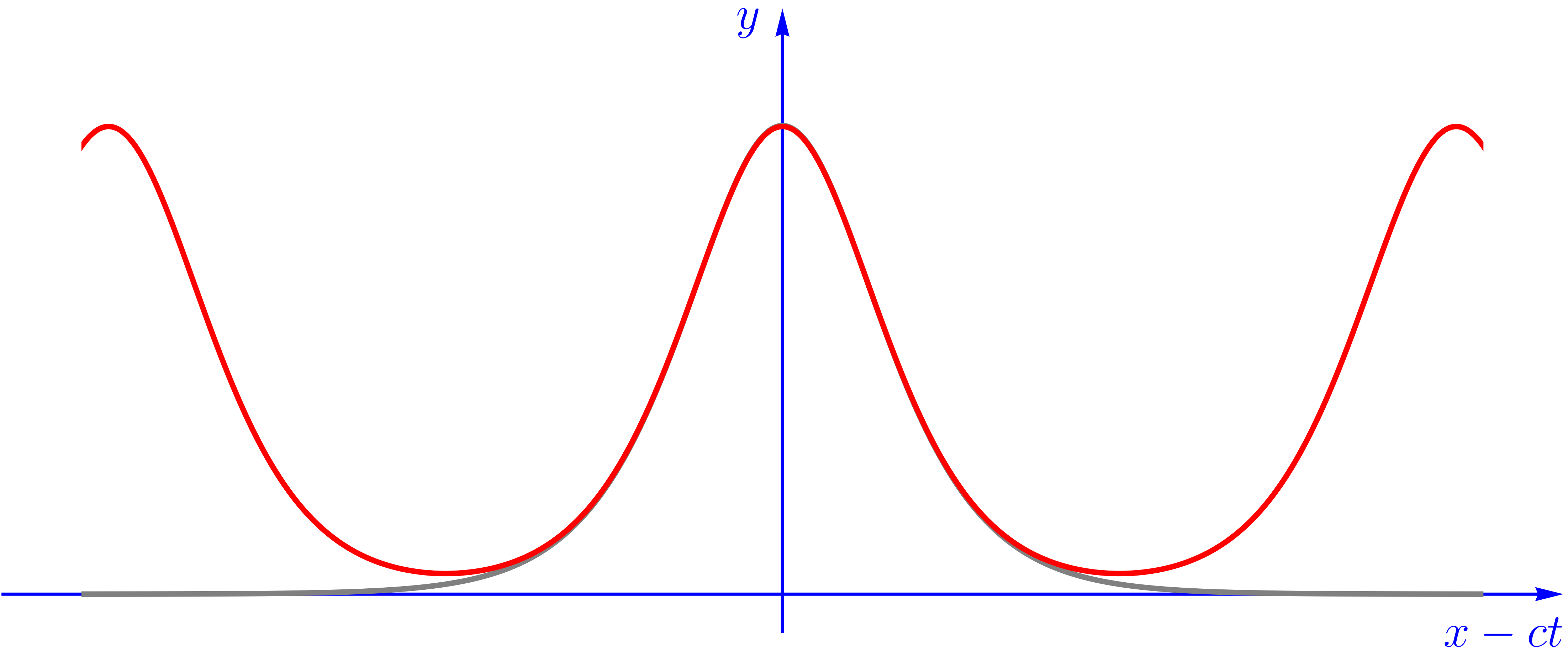

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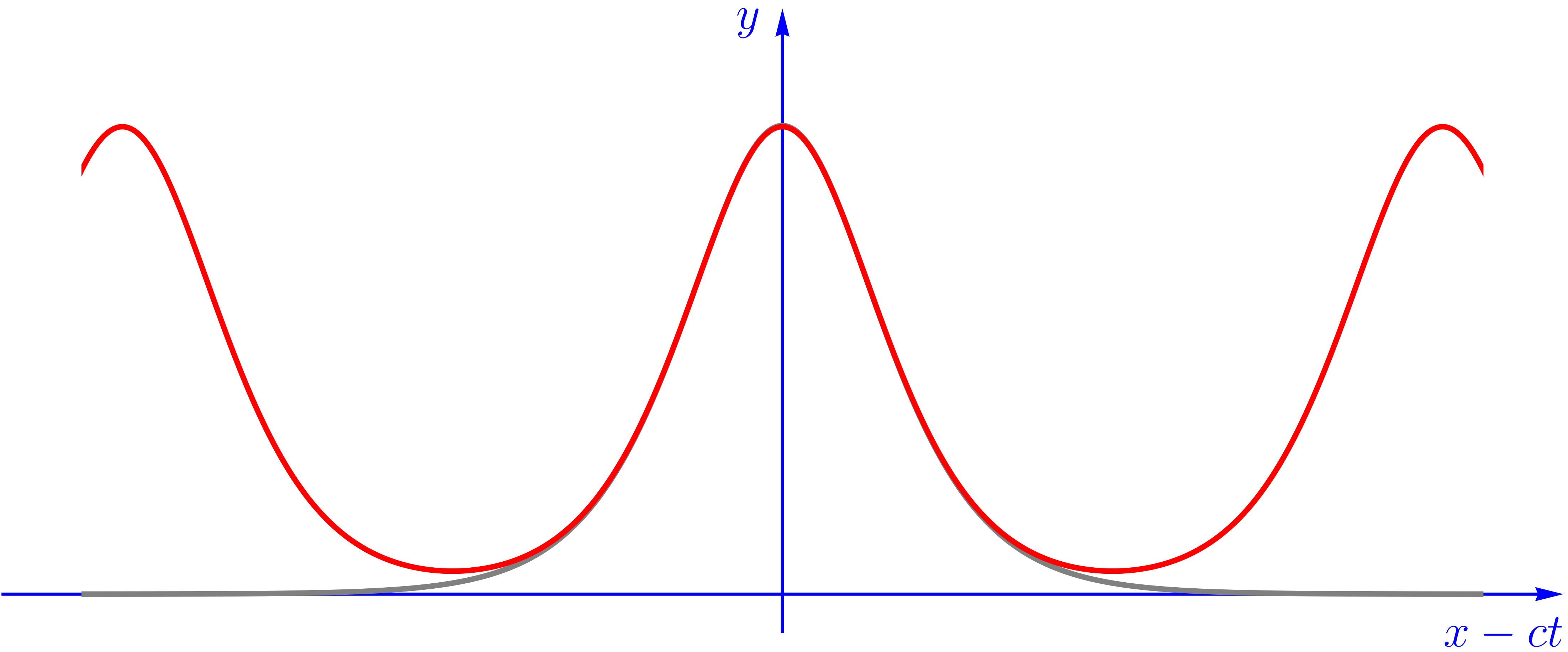

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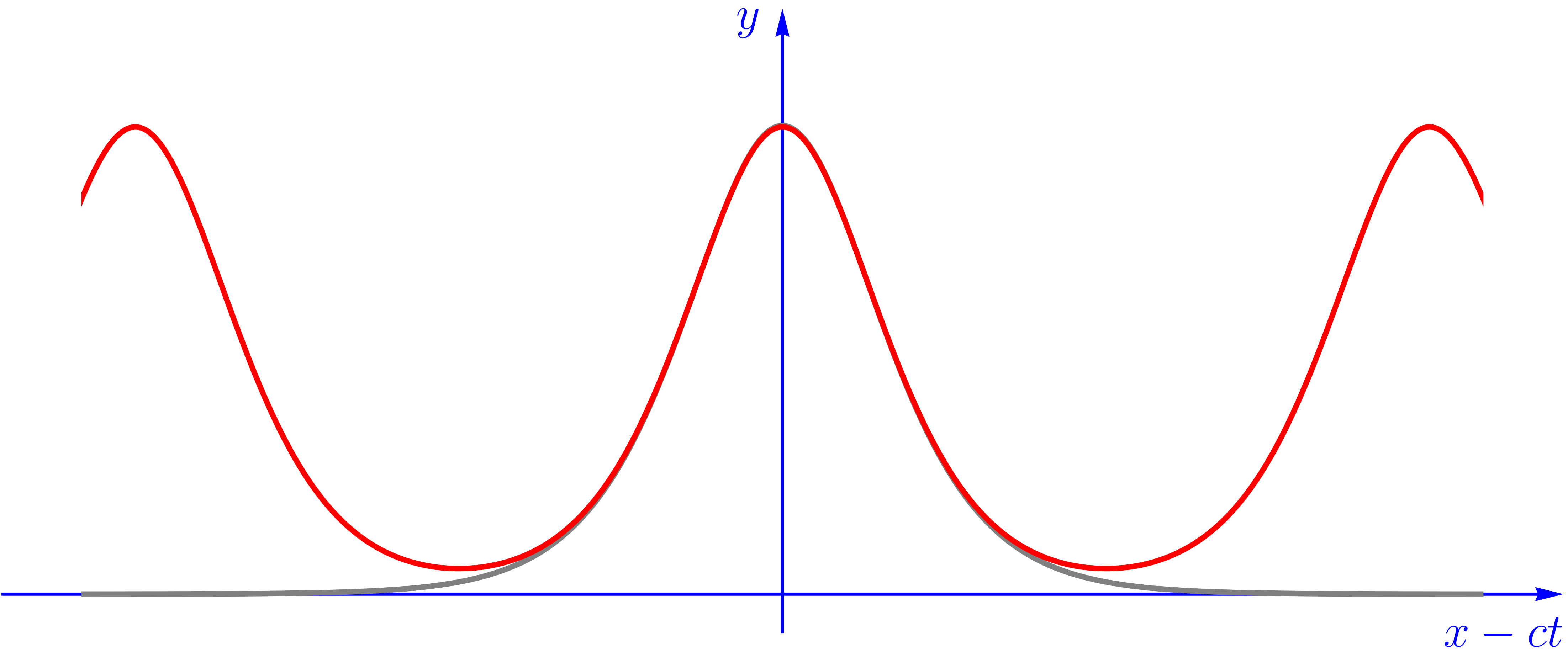

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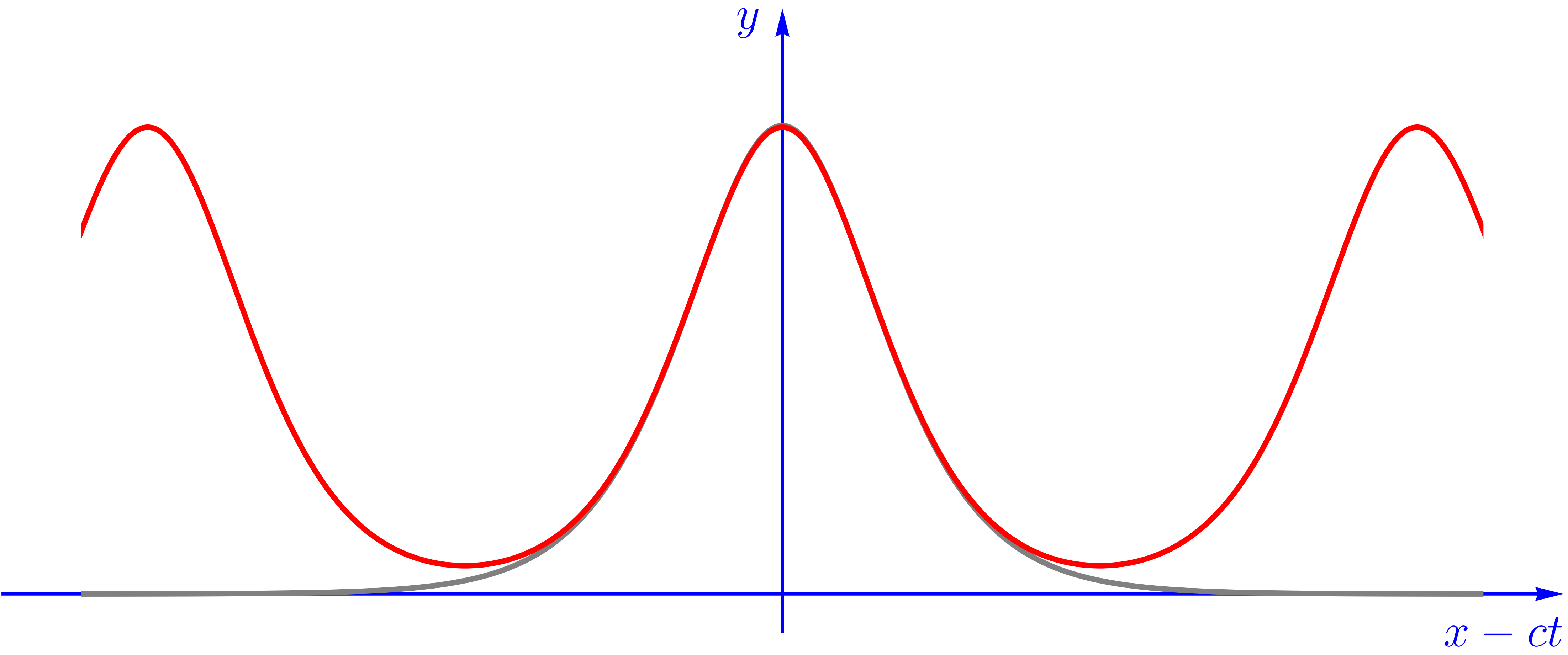

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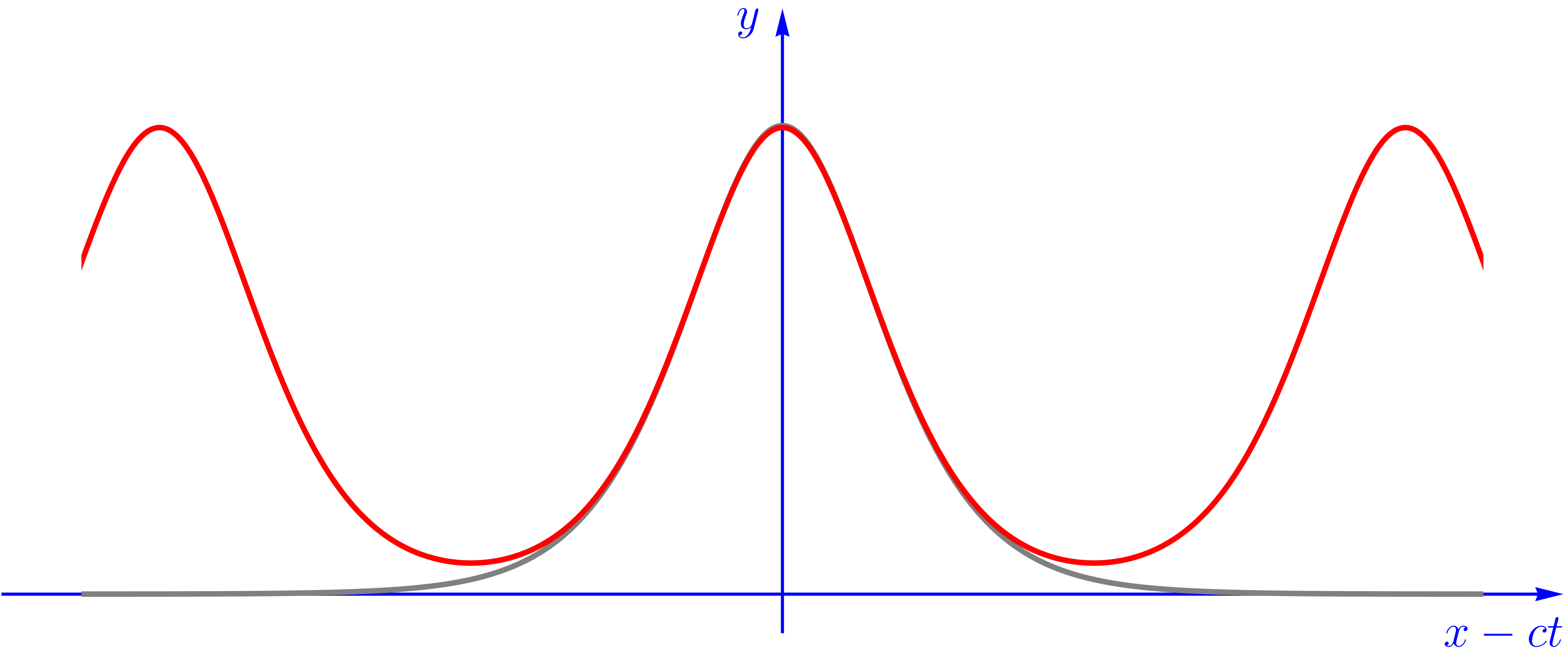

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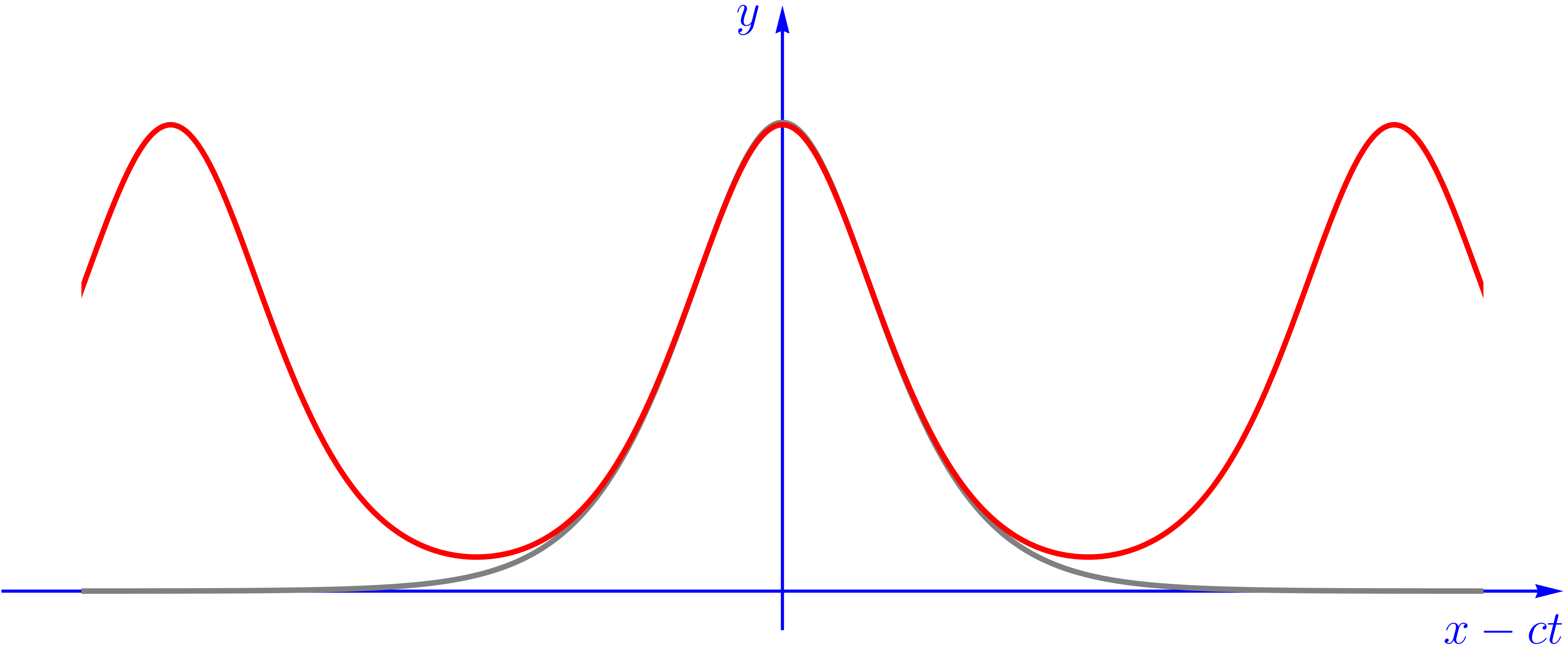

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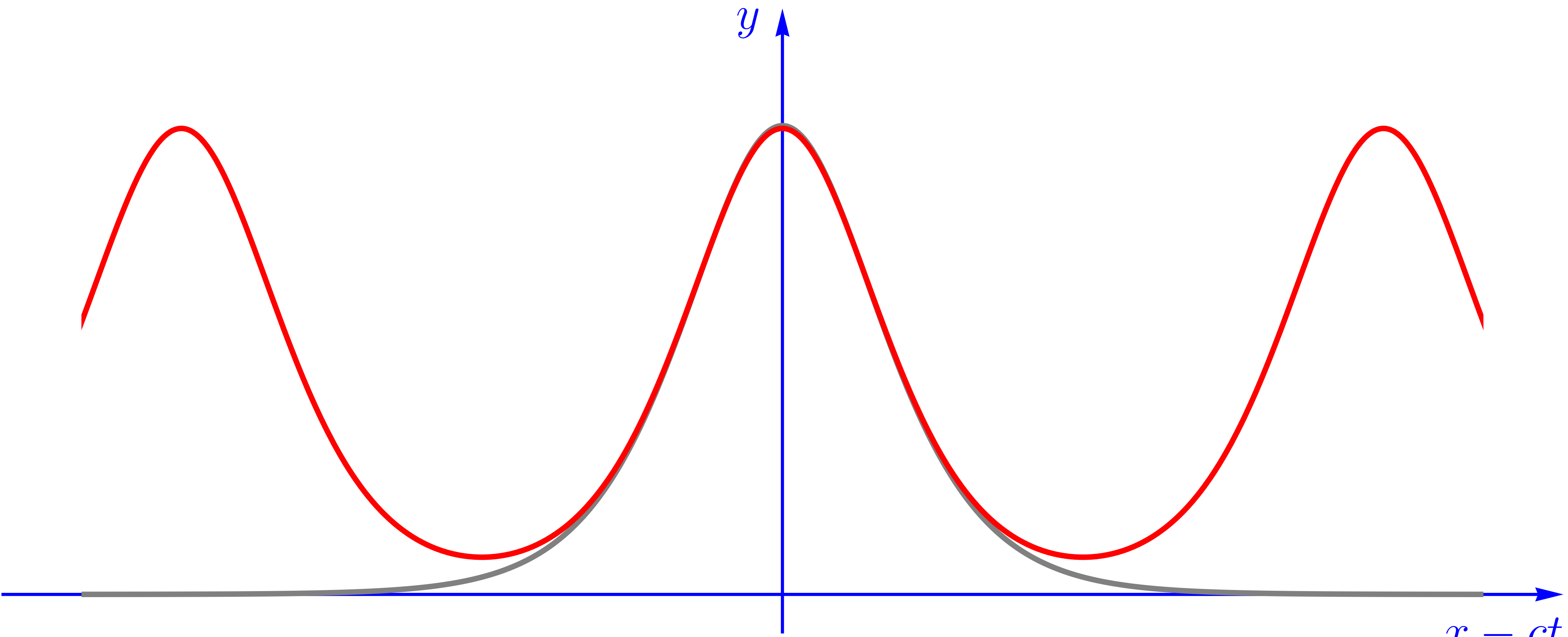

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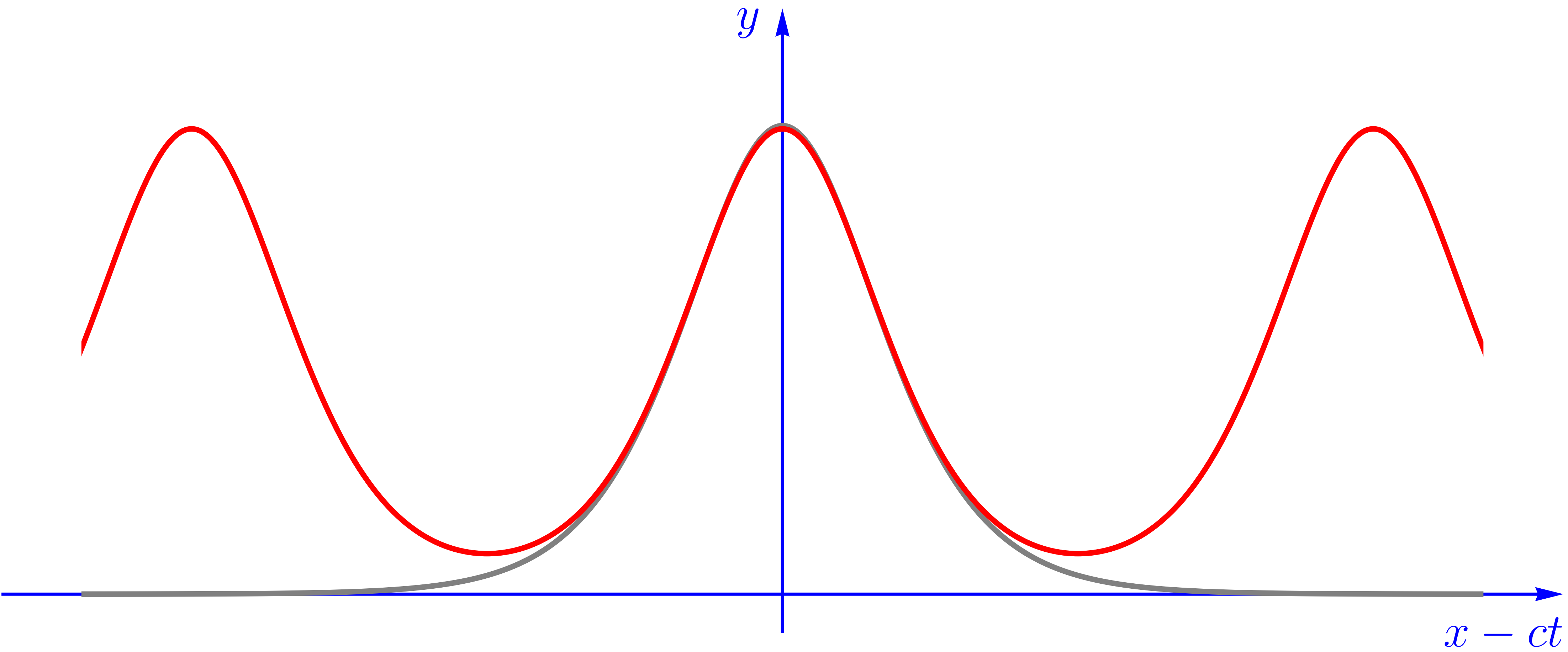

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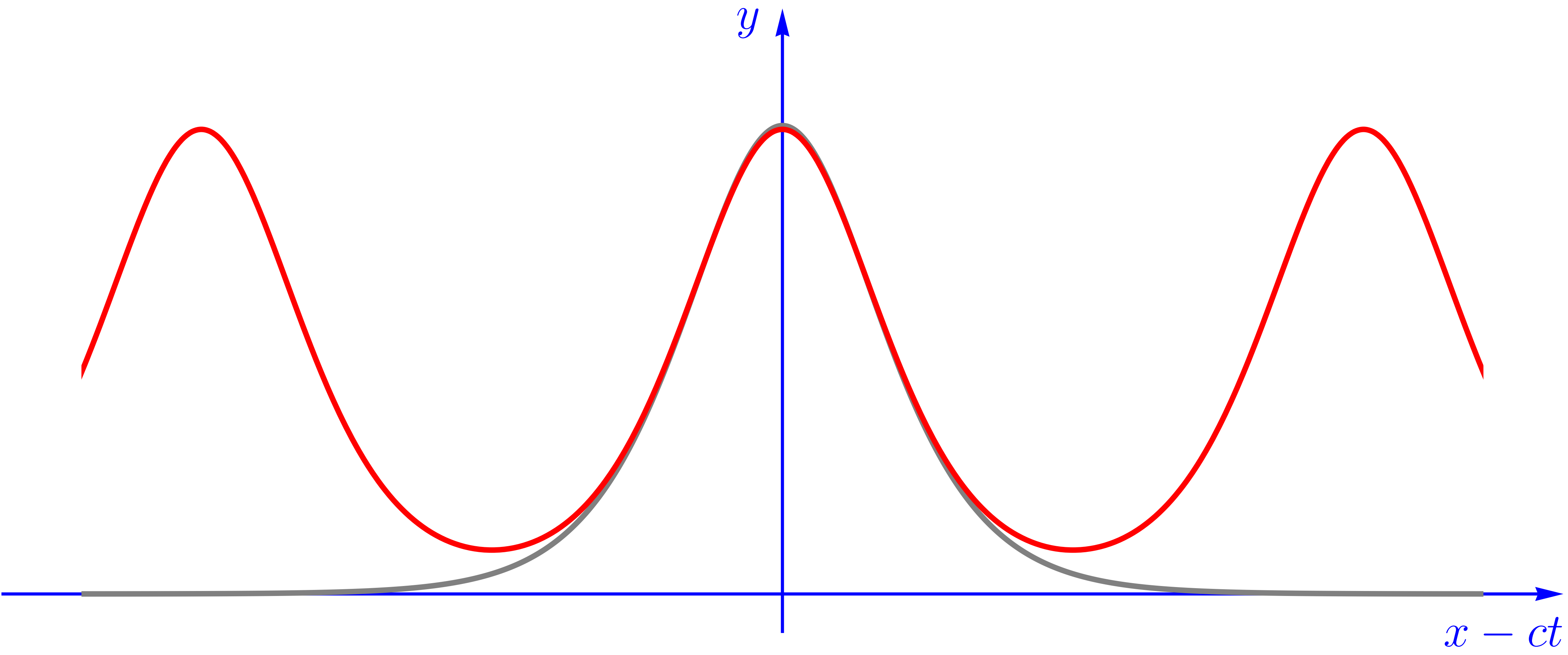

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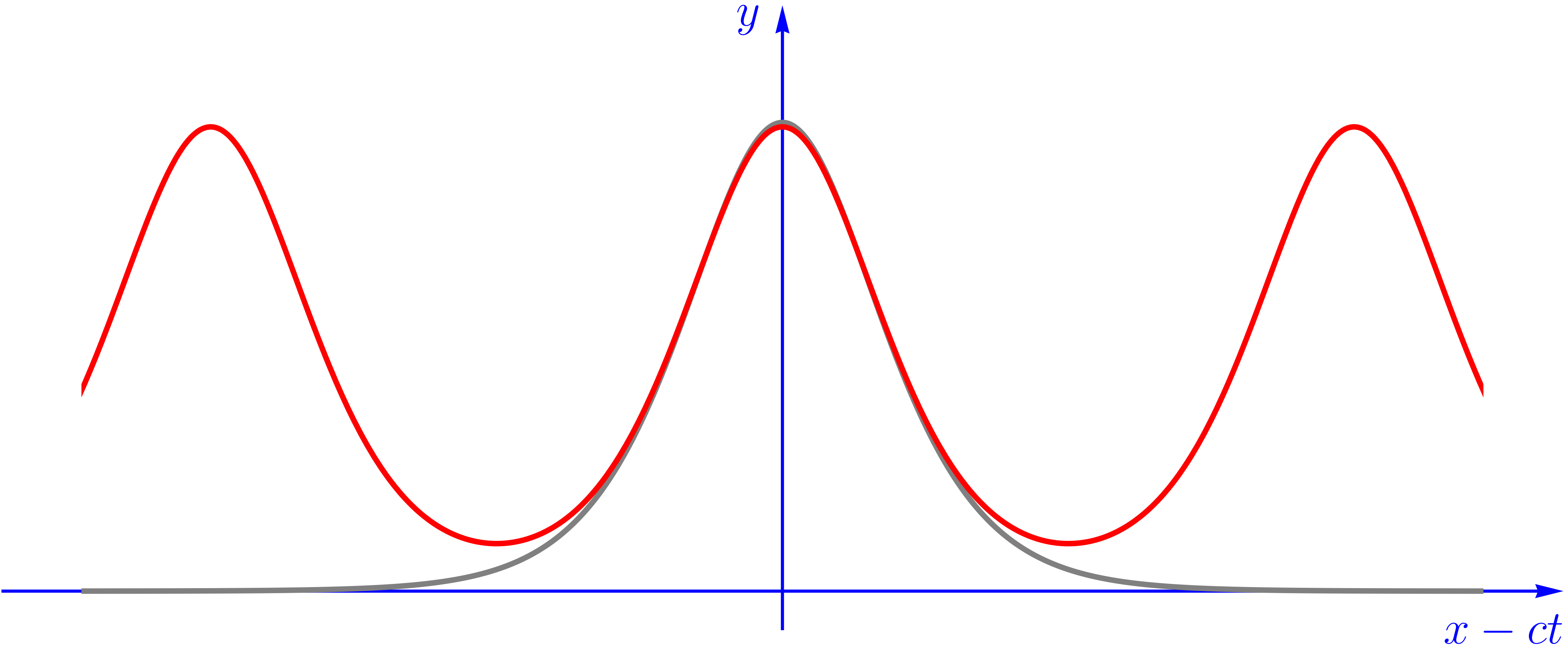

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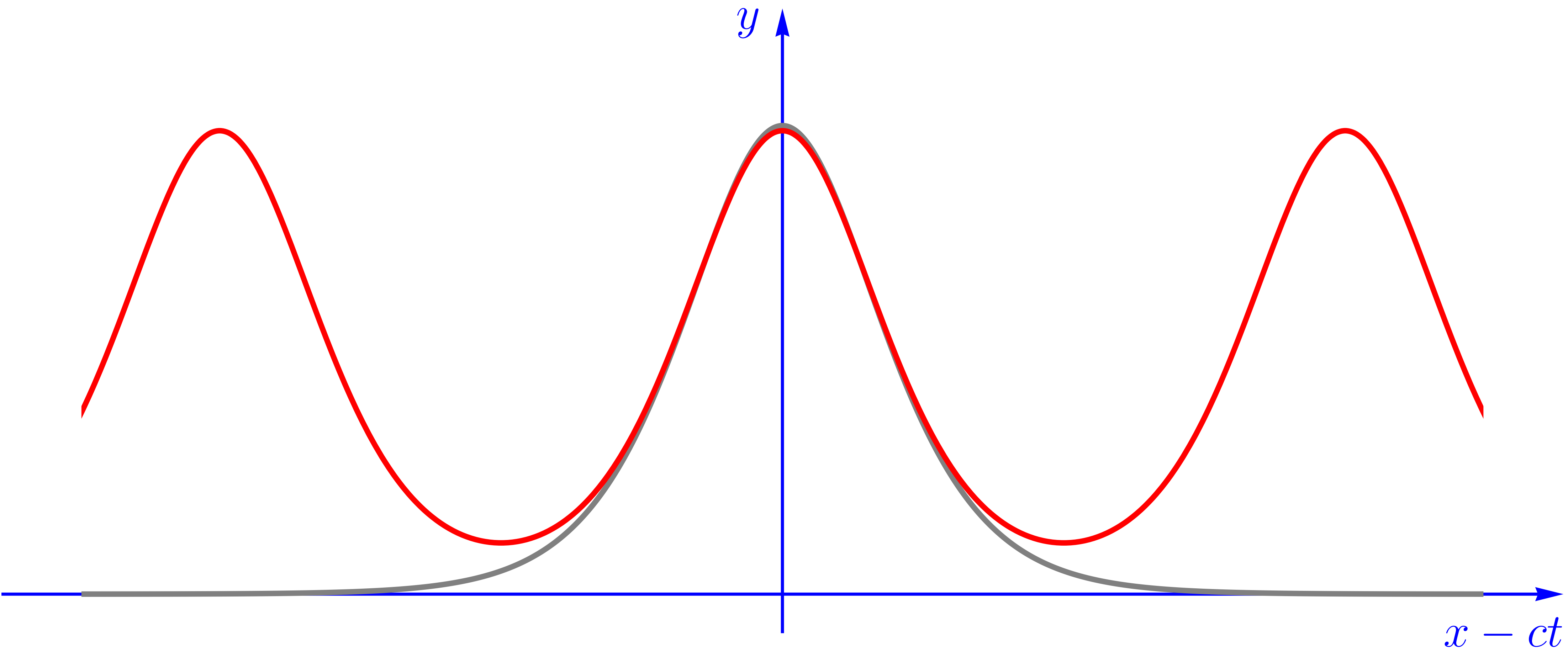
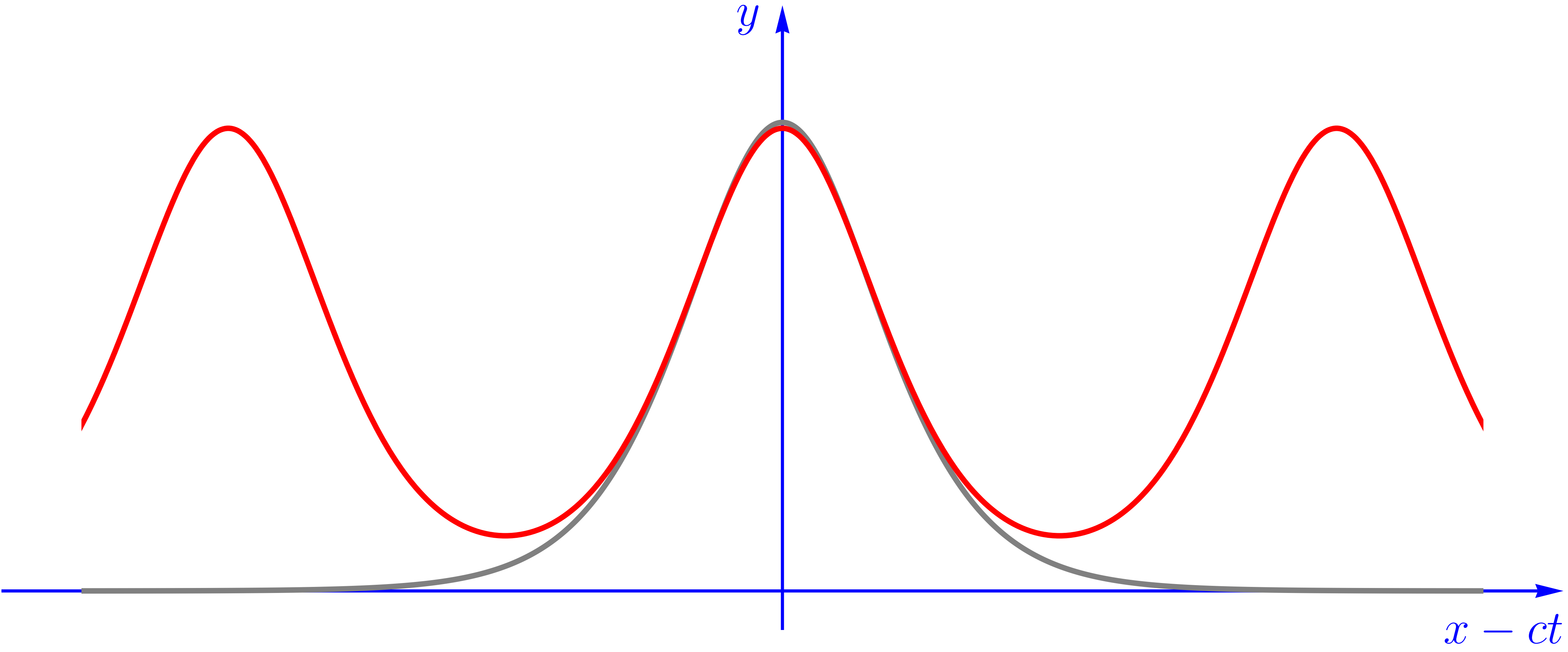
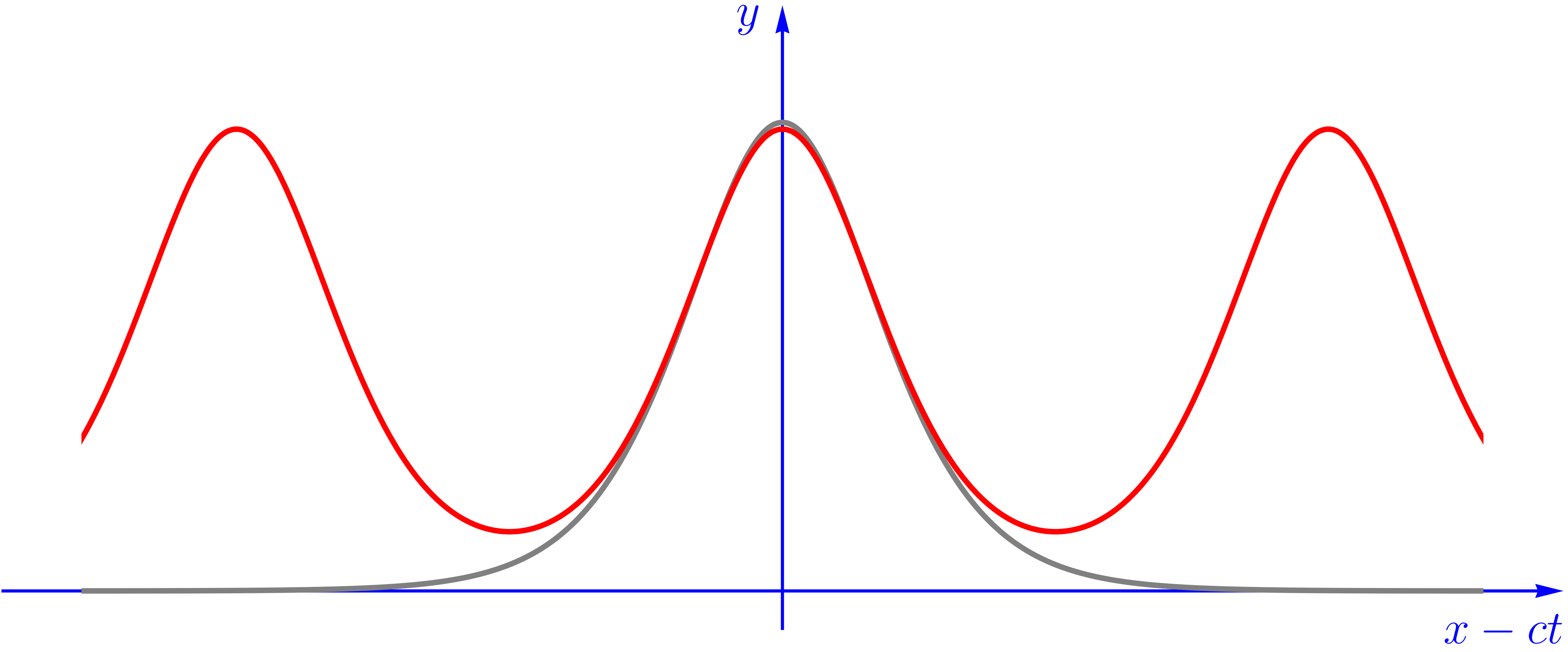


Cnoidal Waves



Cnoidal Waves



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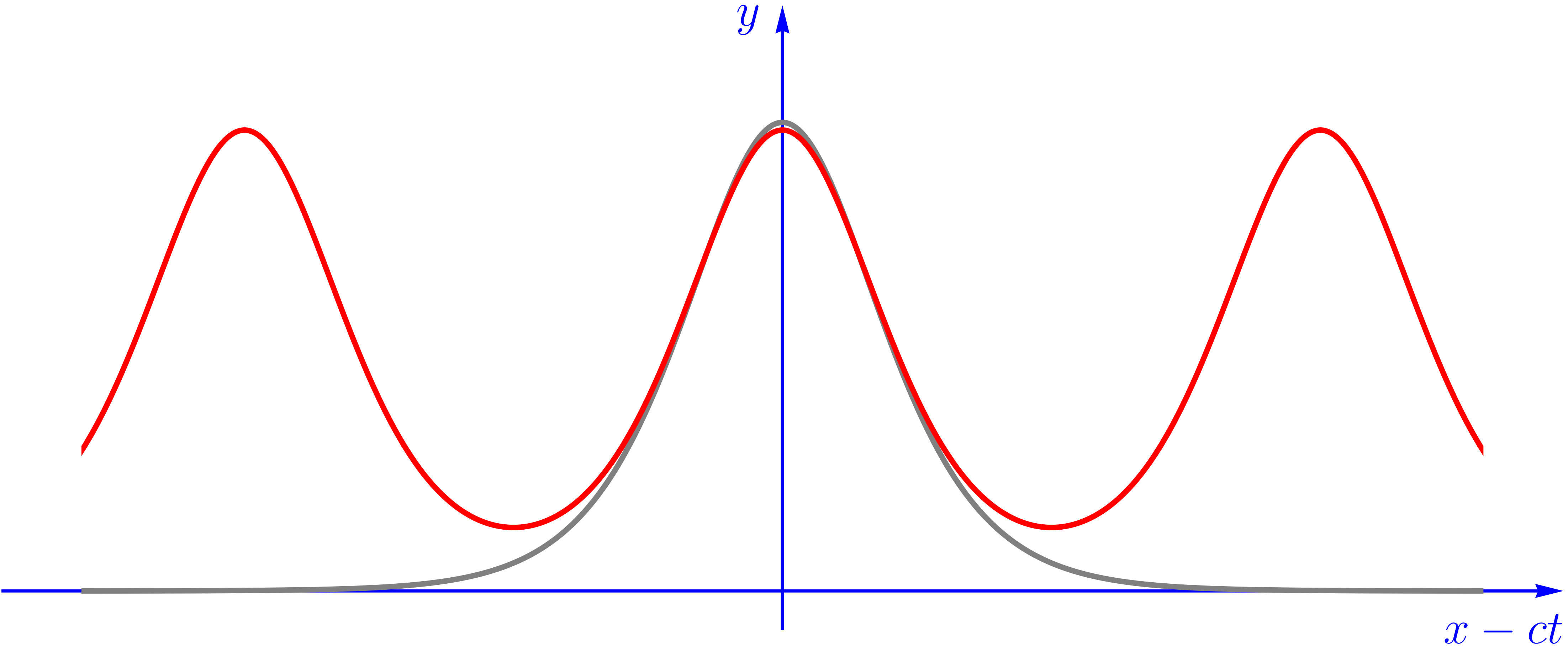

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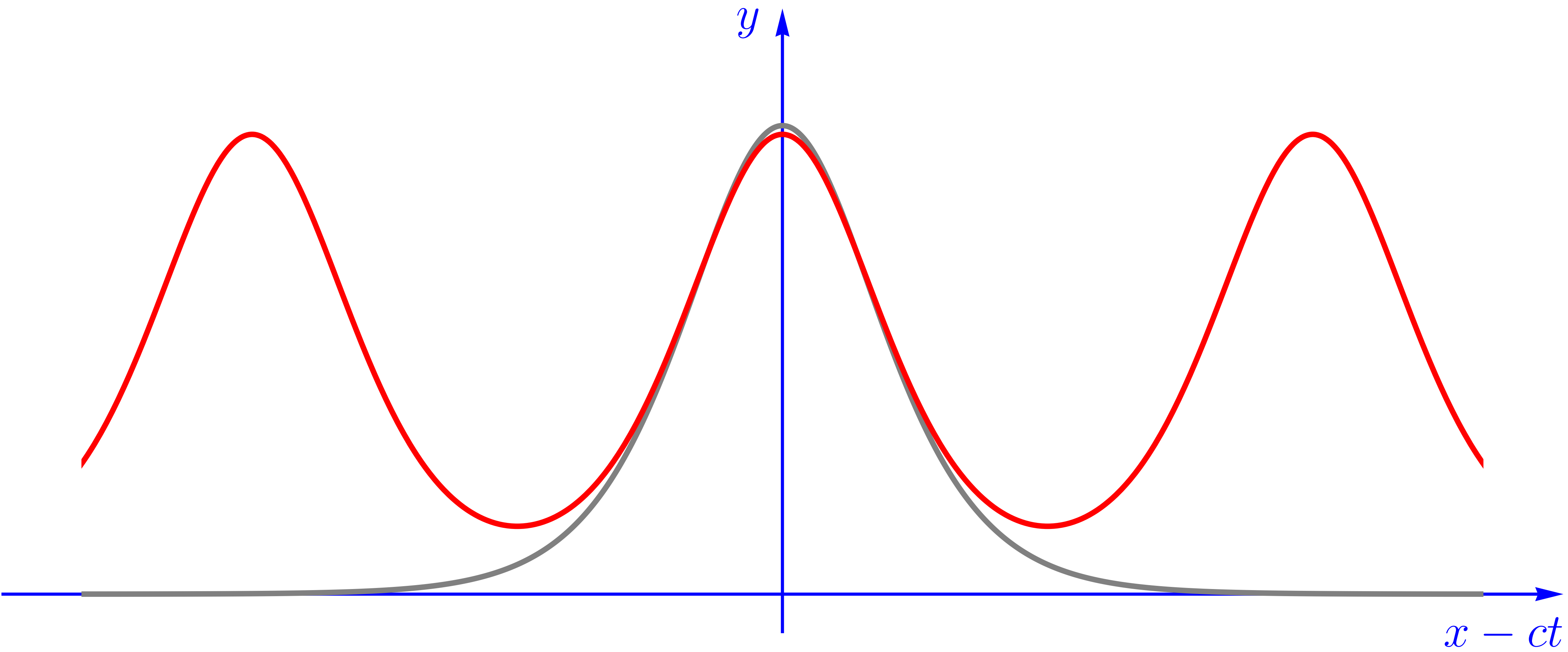

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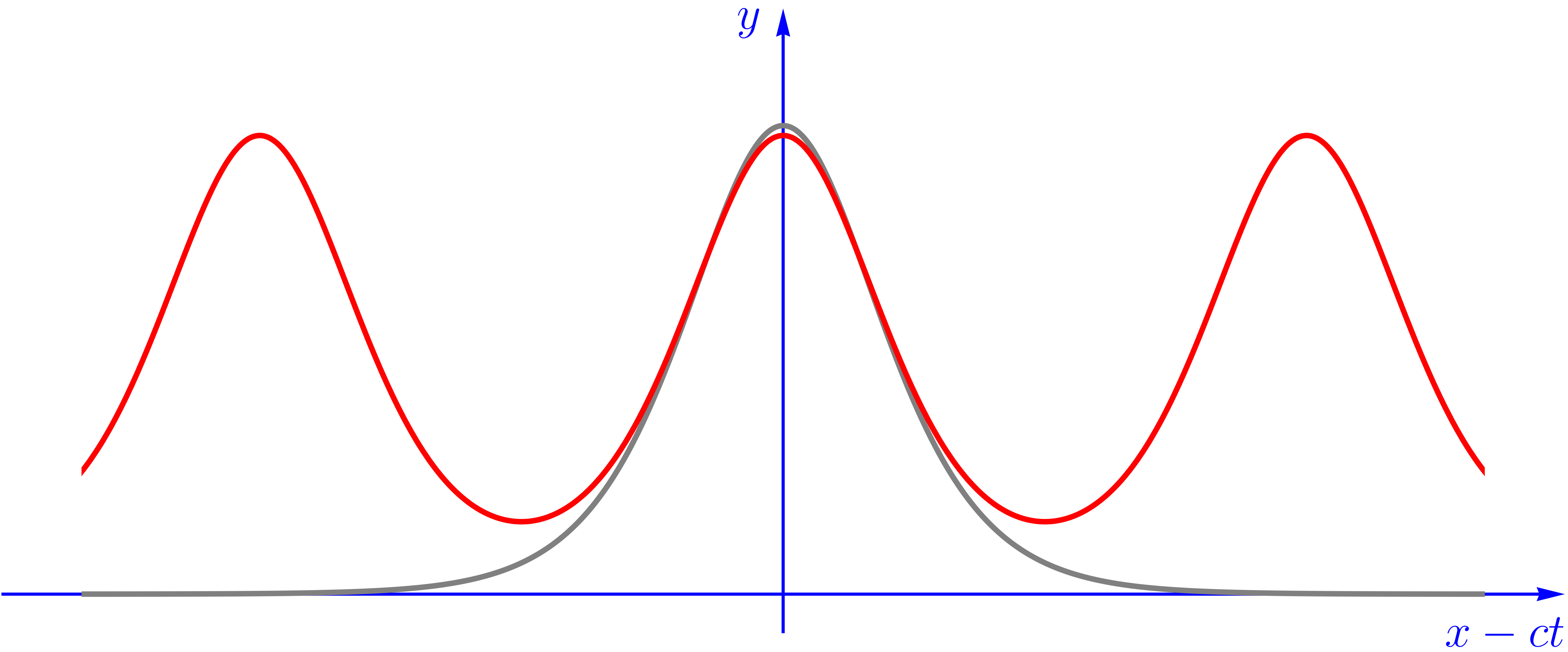

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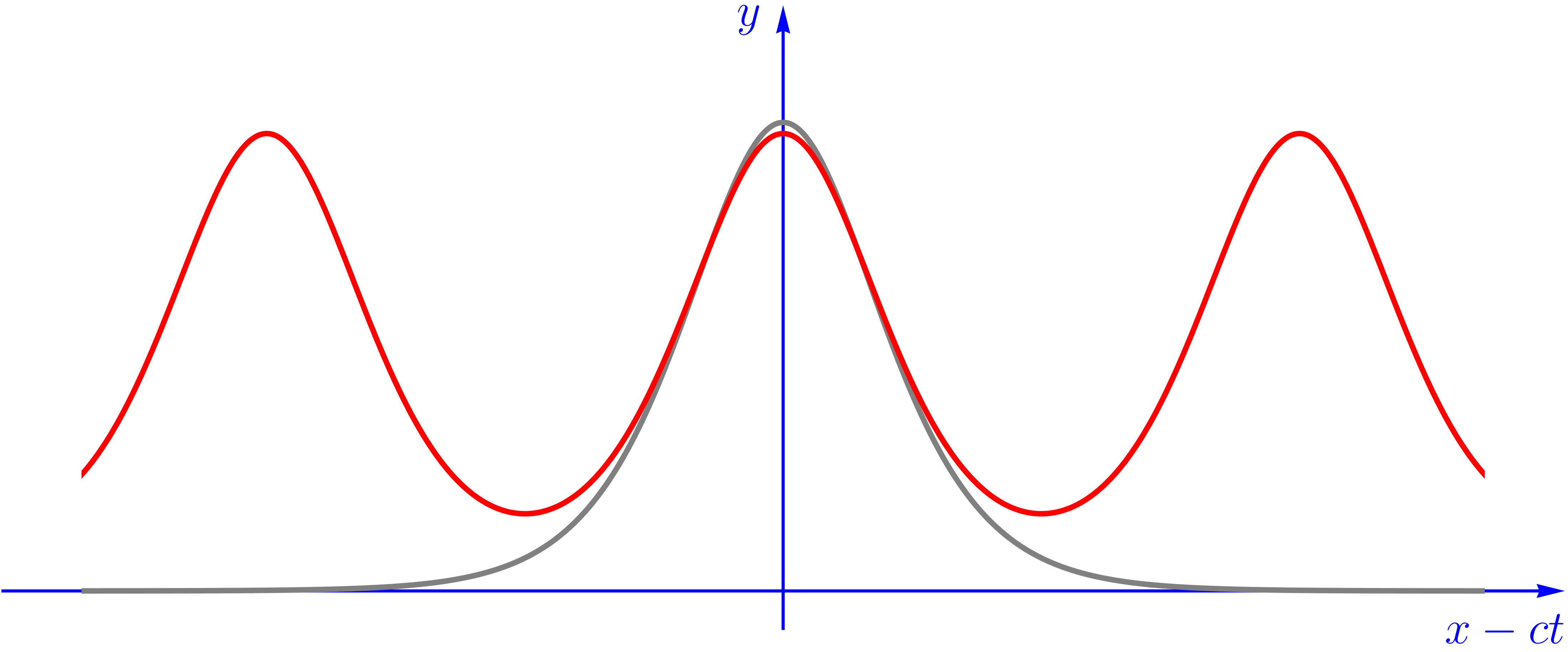

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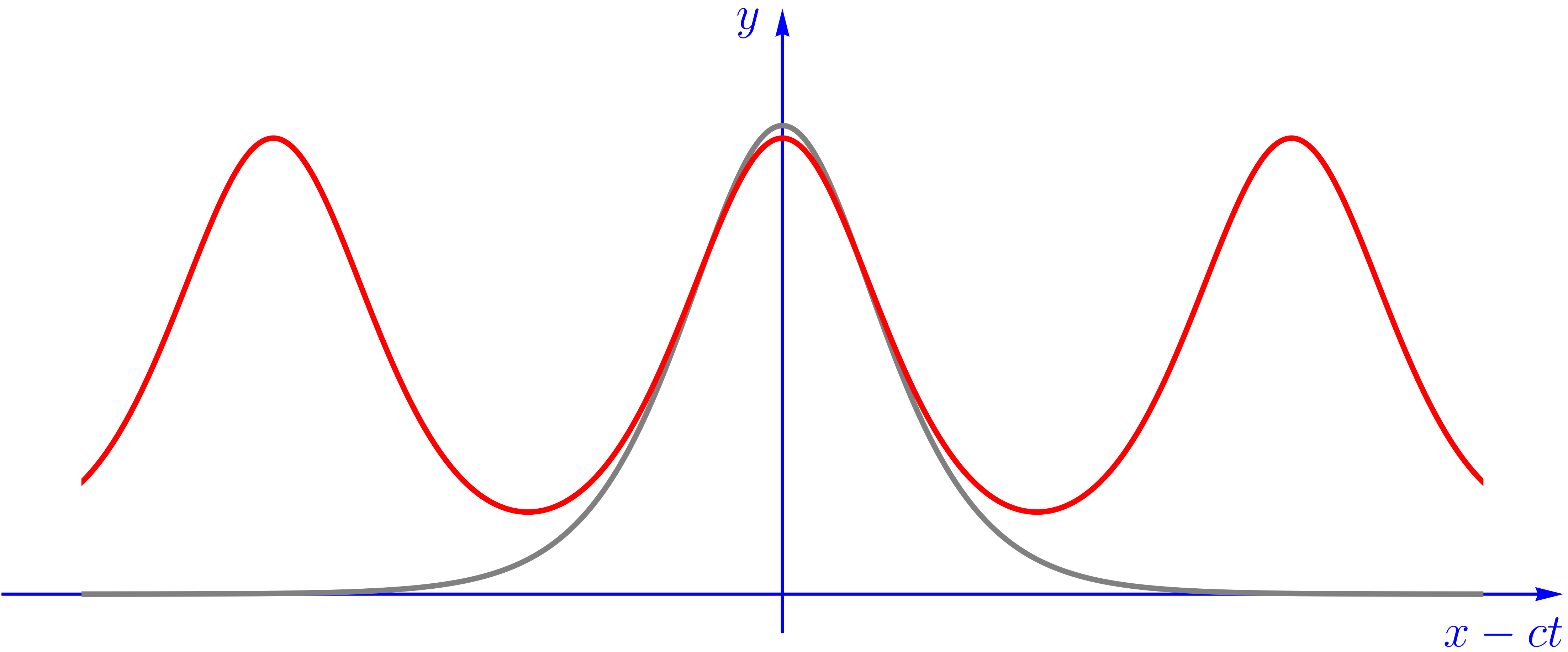

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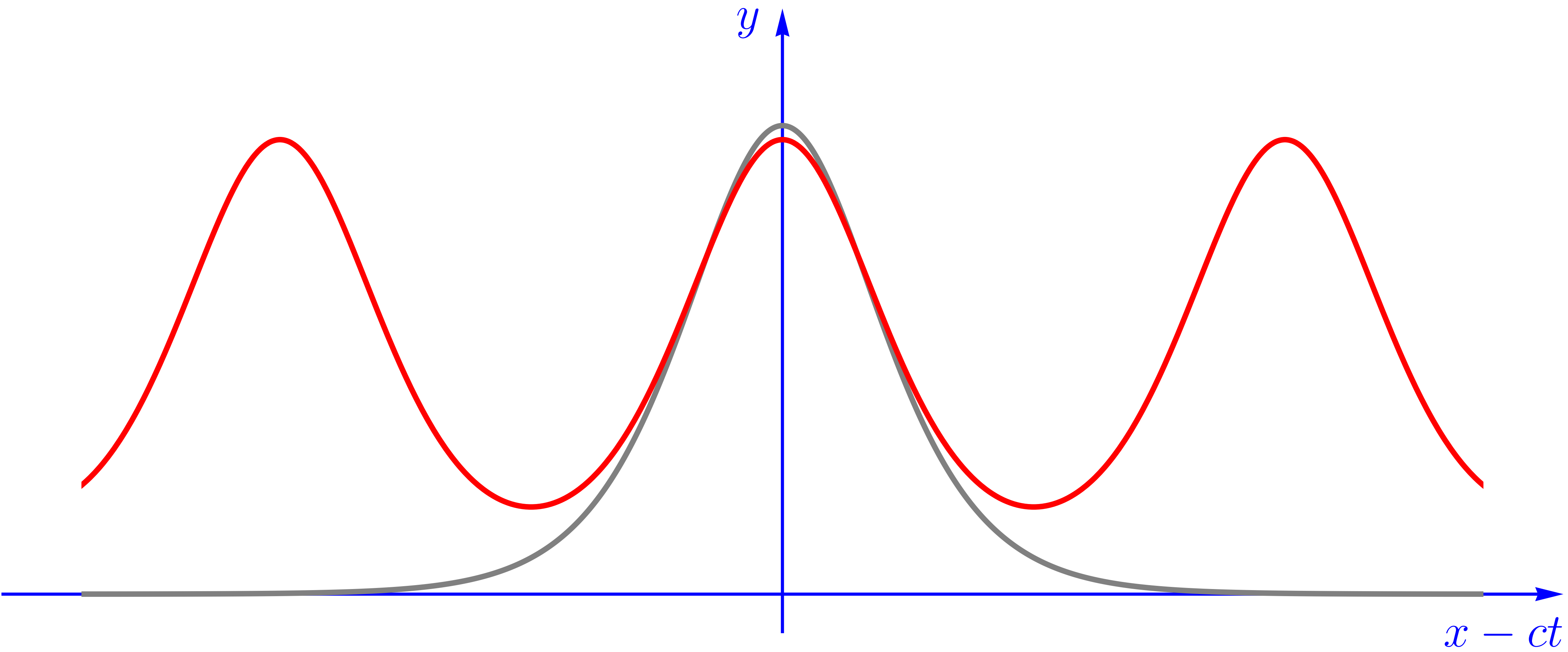

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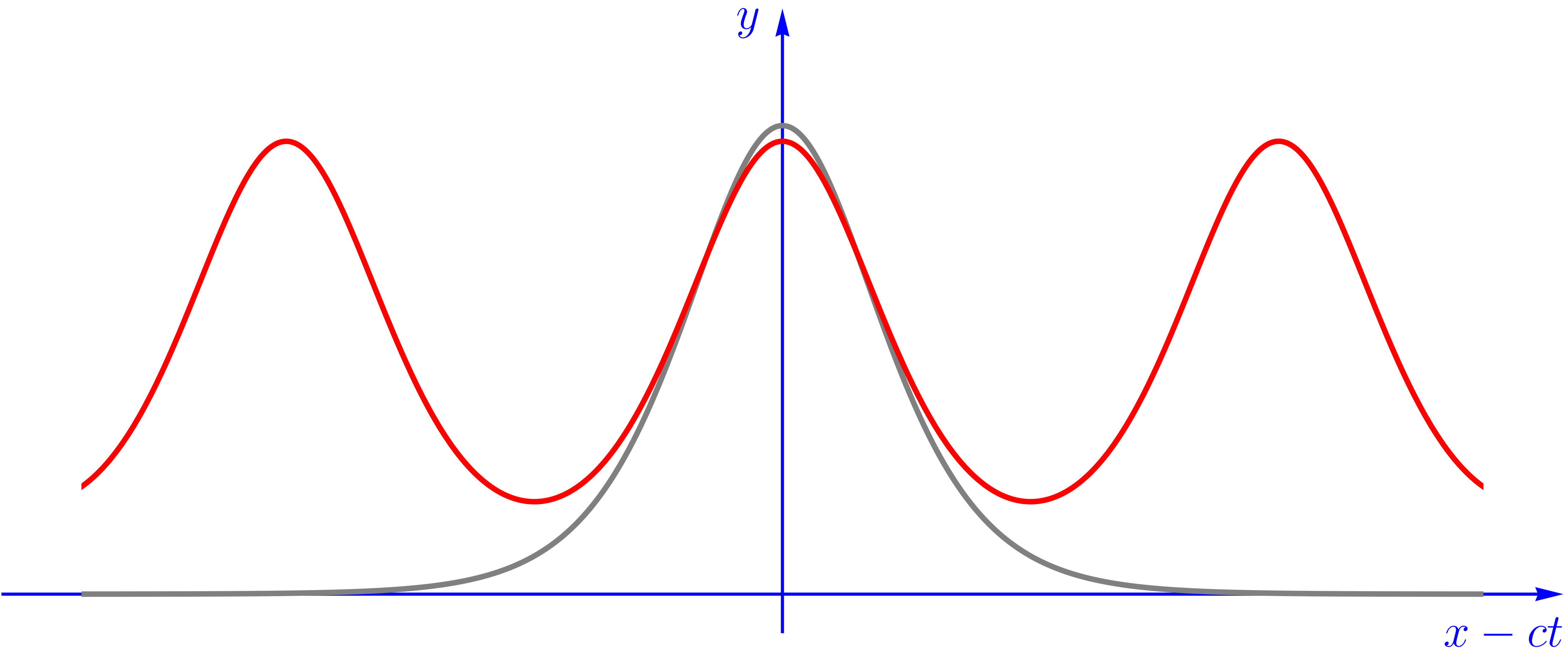

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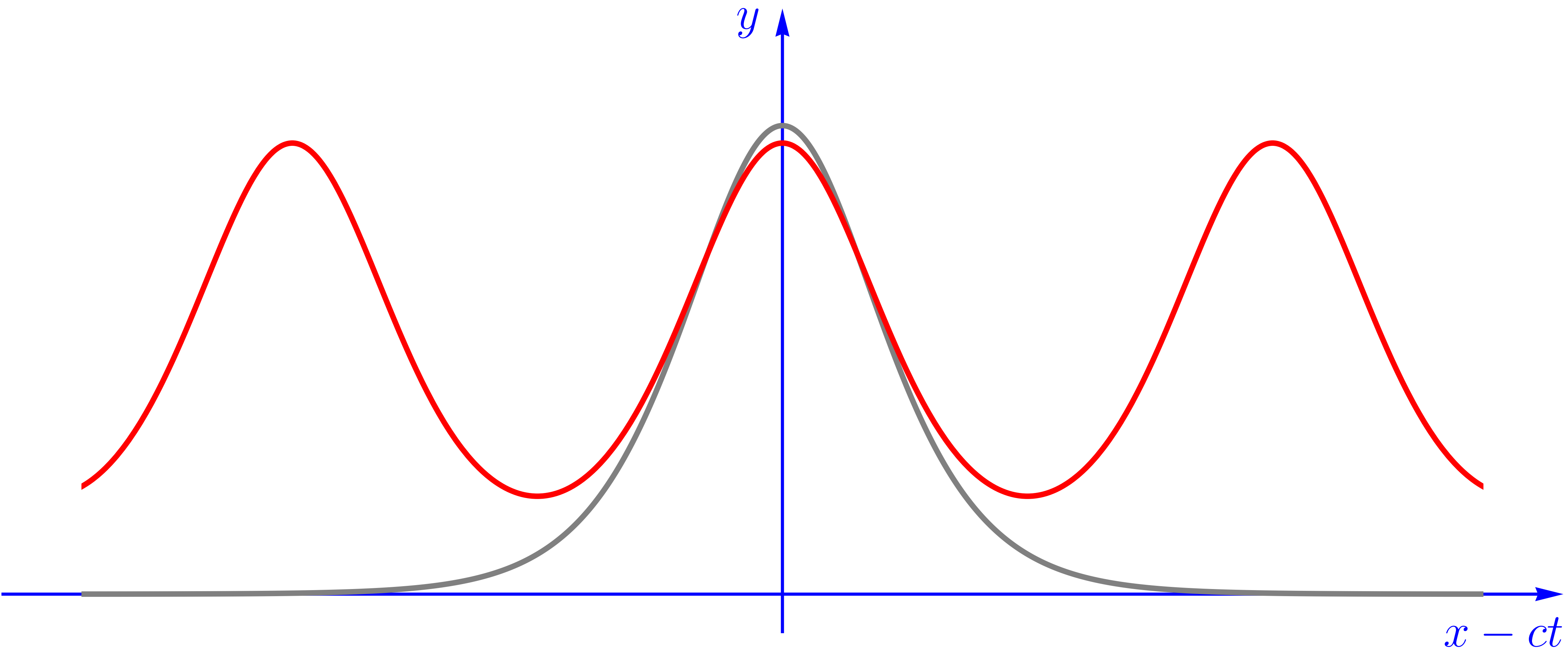

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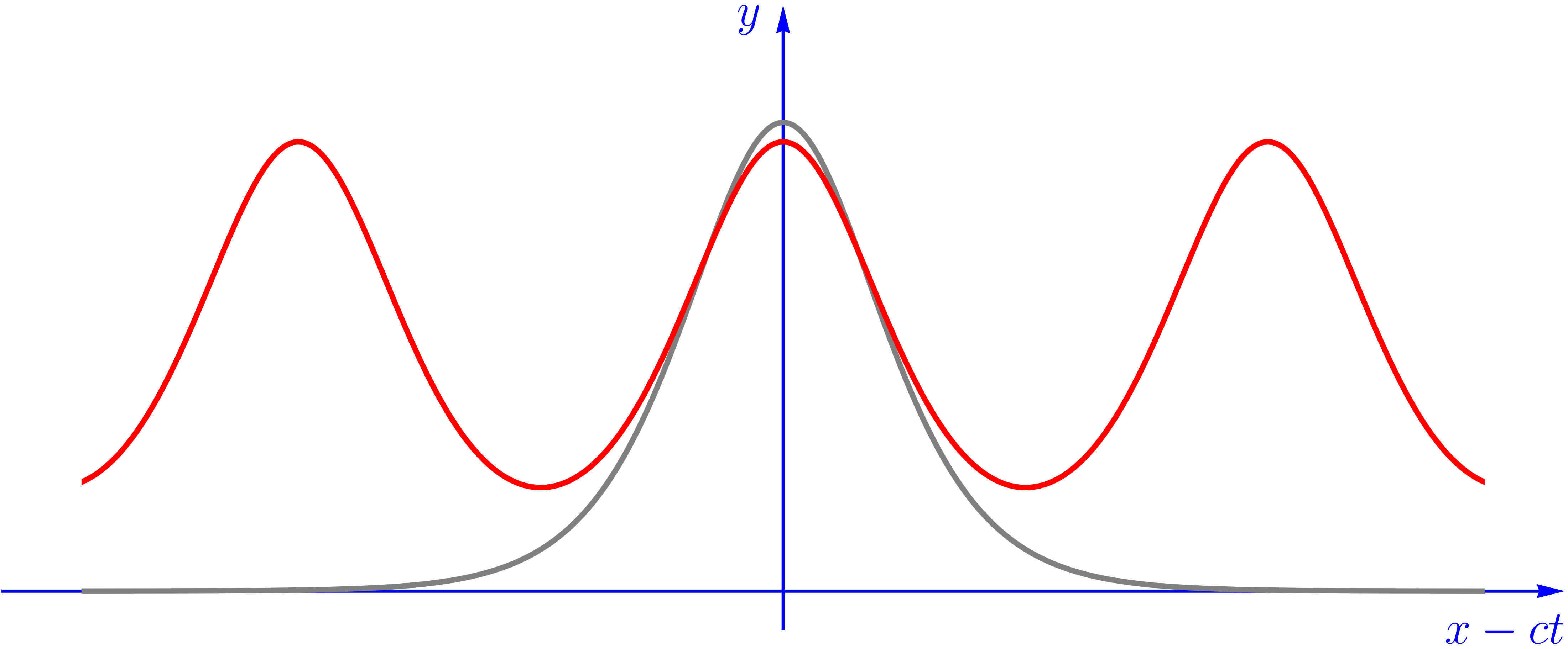

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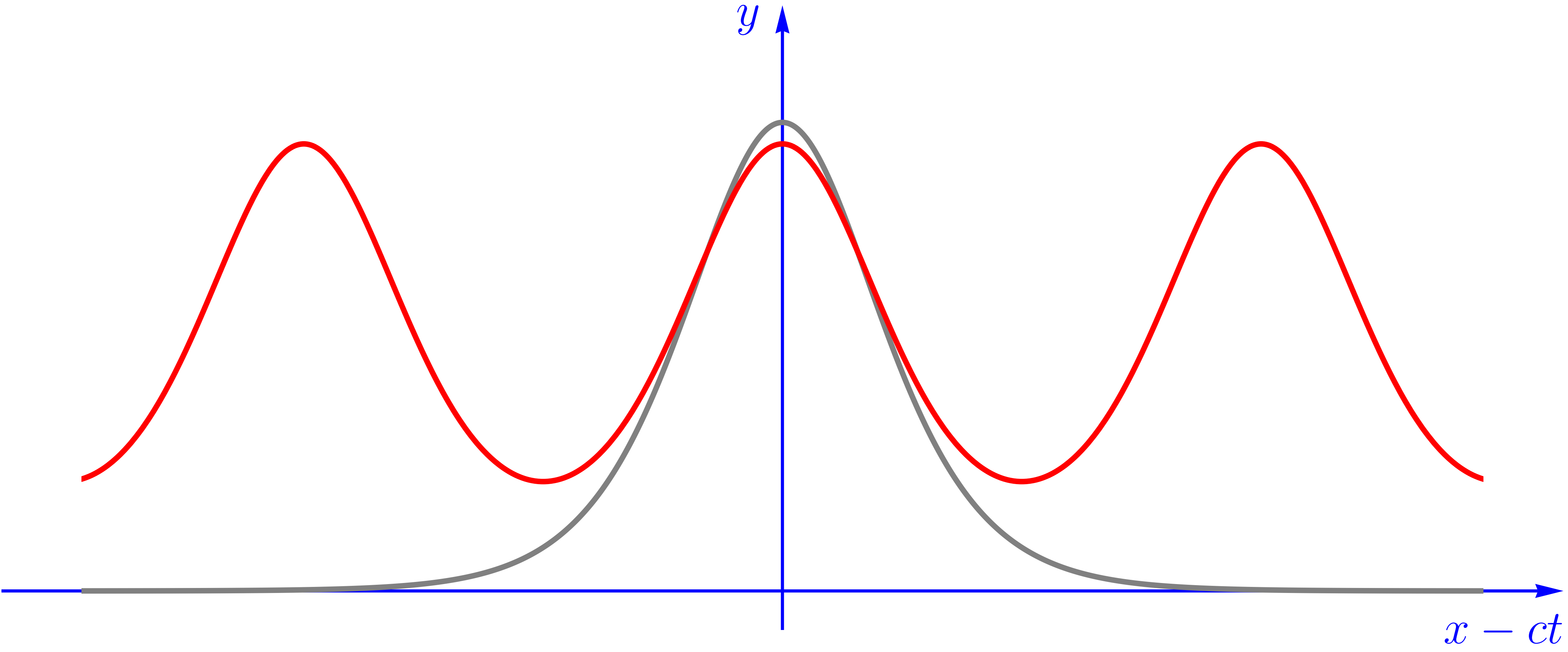

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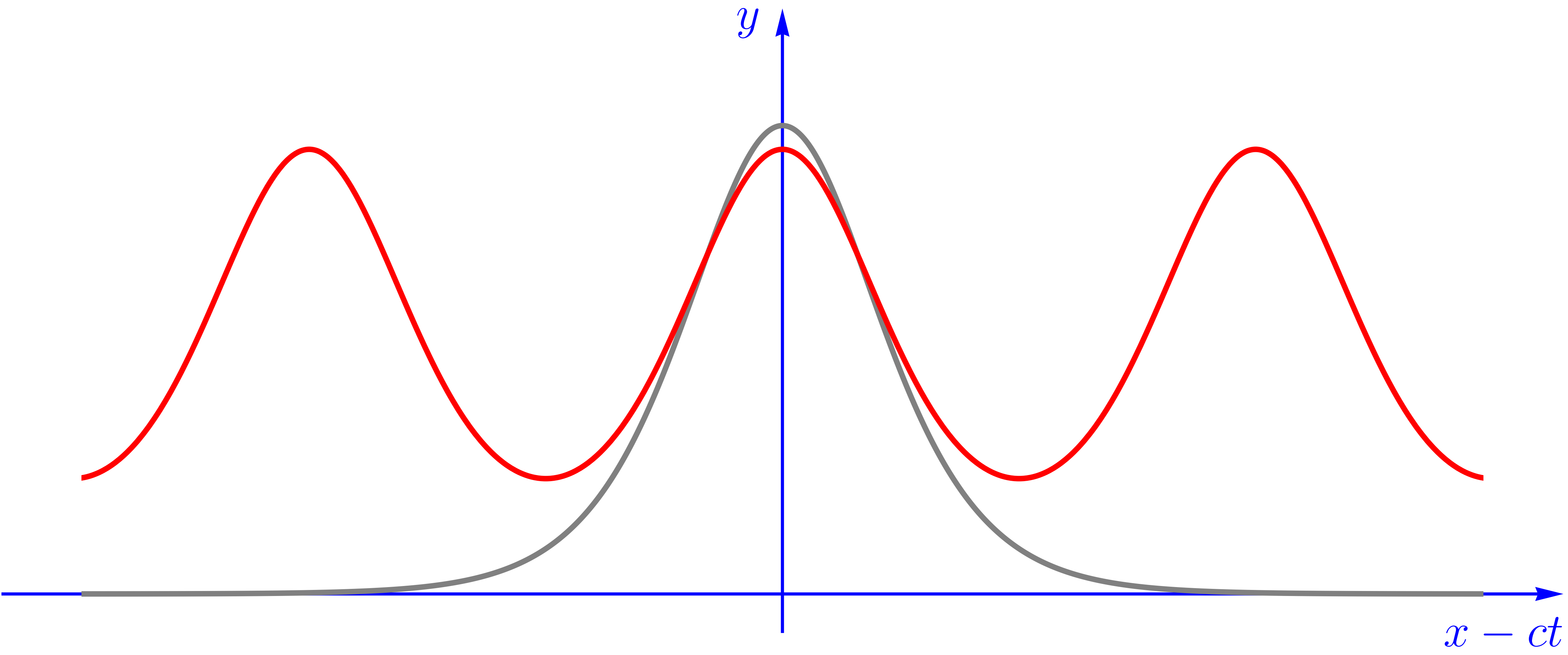

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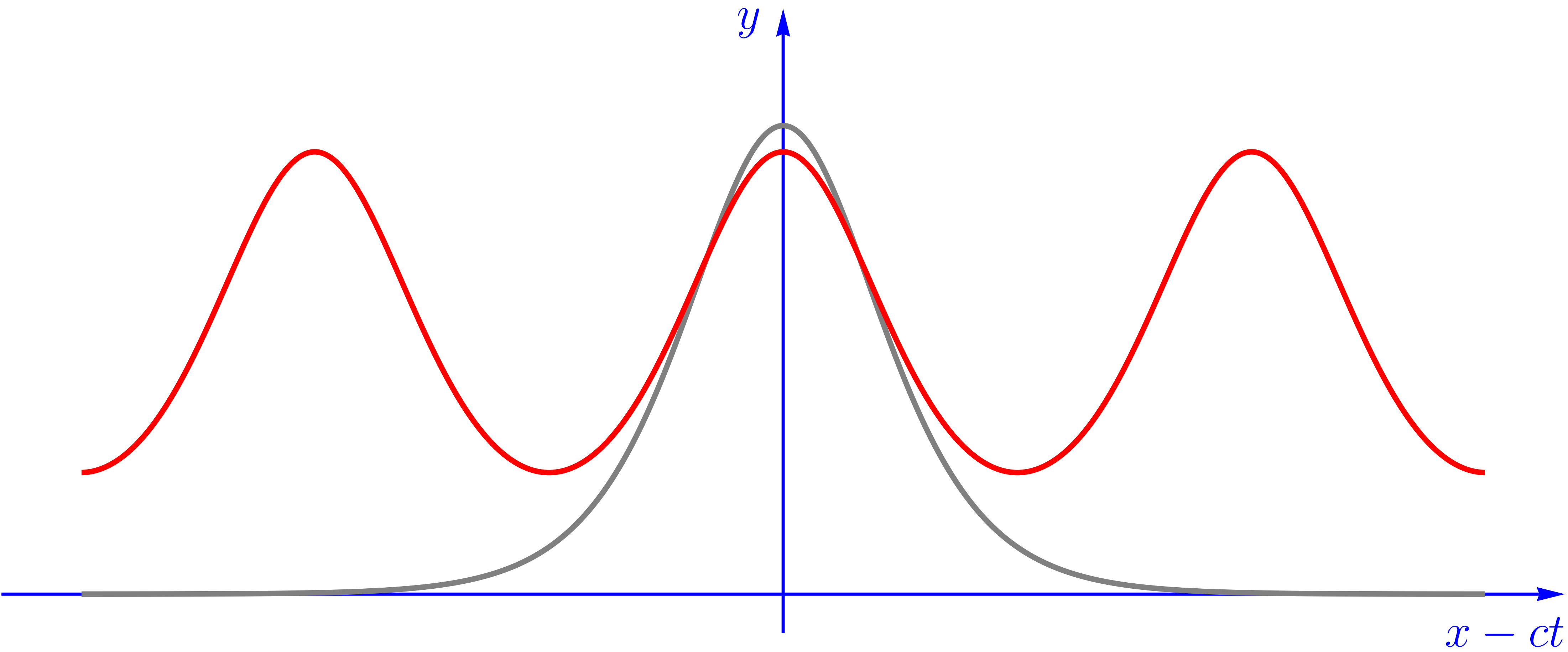

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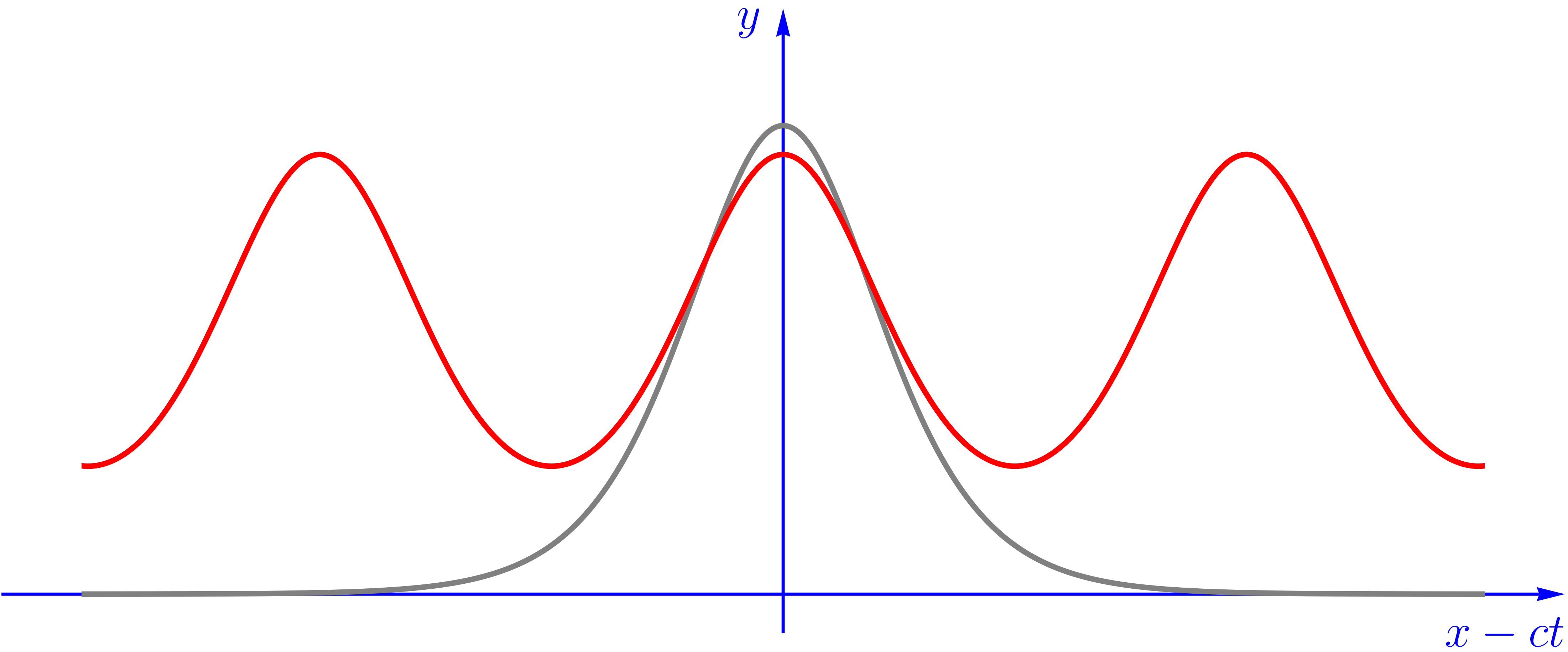

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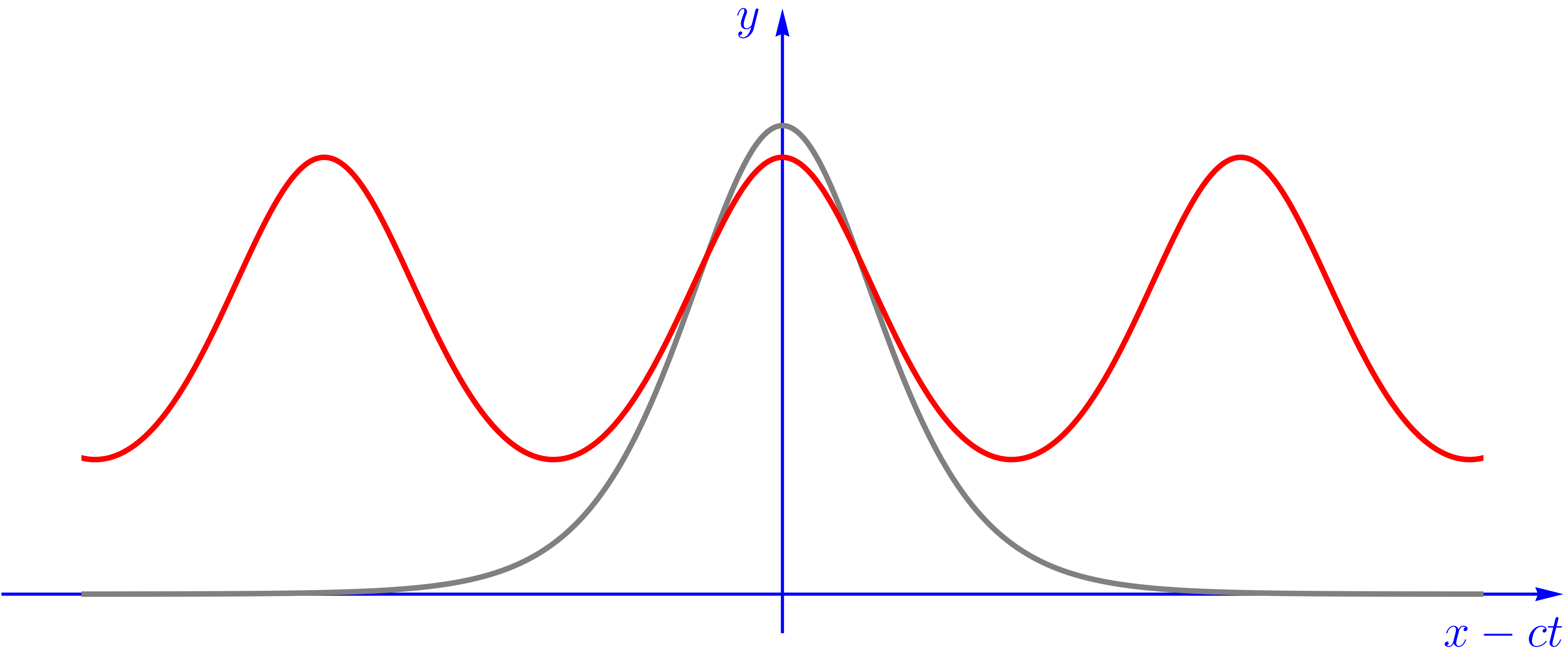

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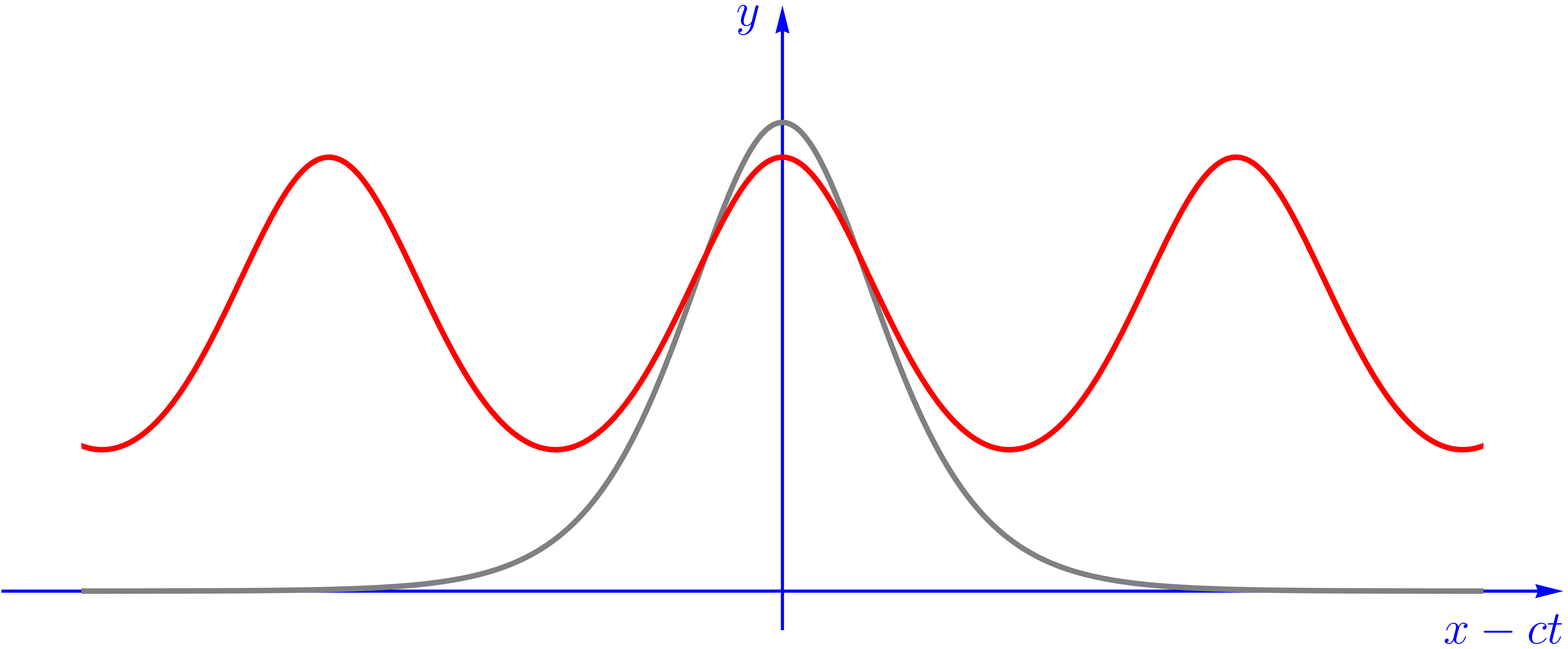
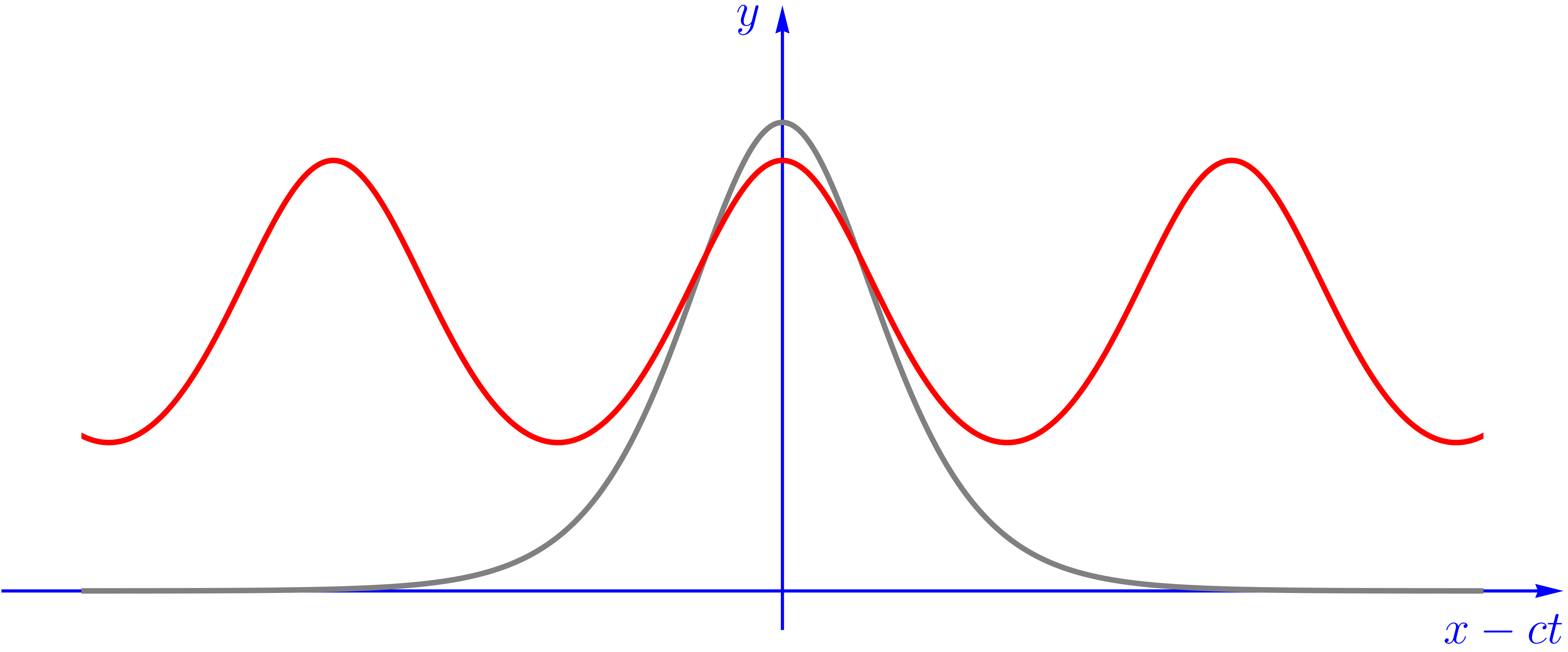
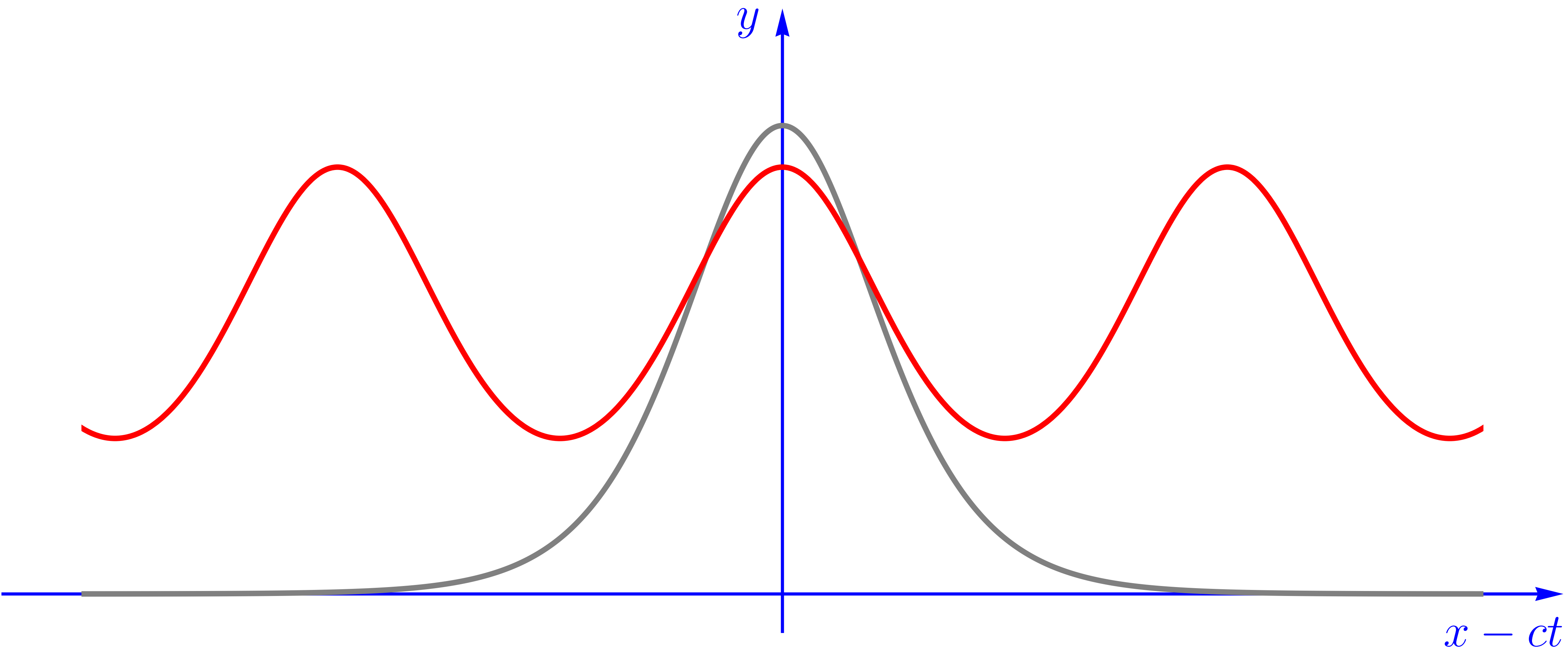
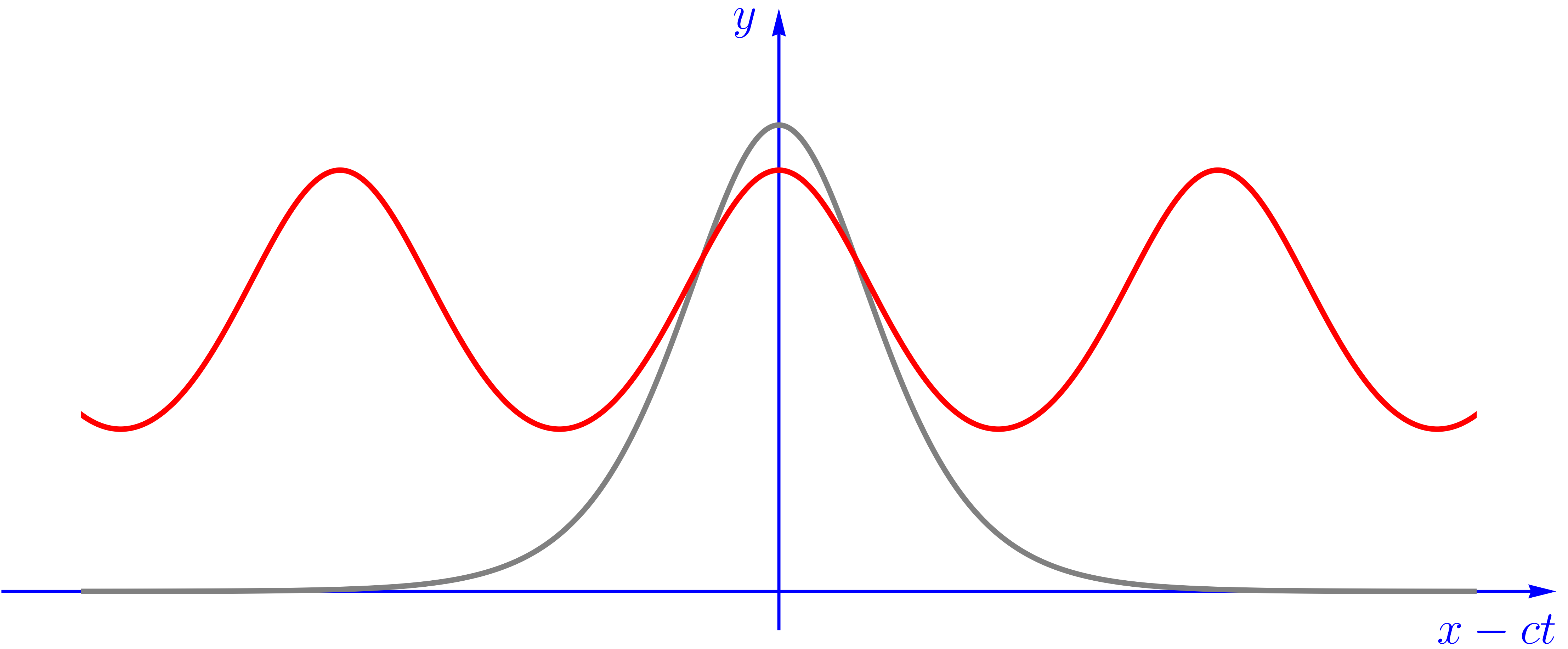



y
$x - ct$

Cnoidal Waves

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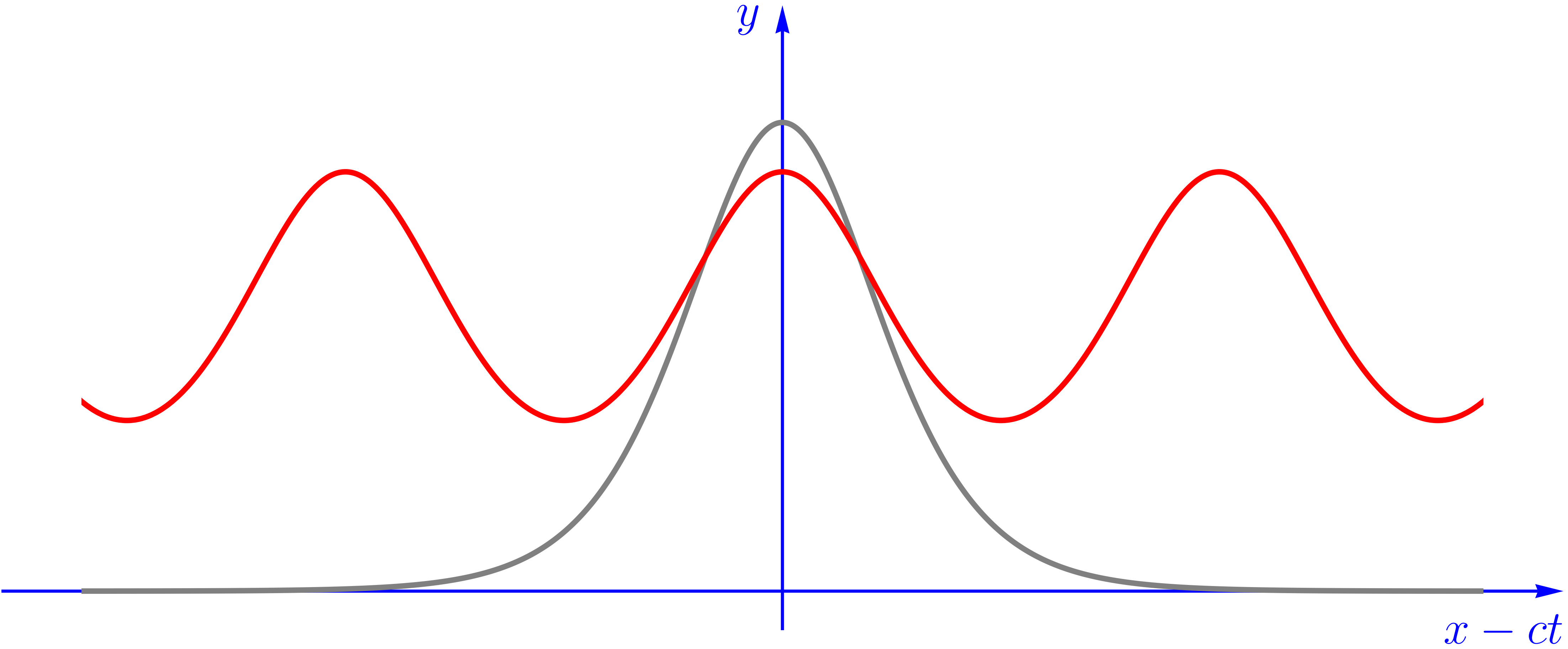
Cnoidal Waves



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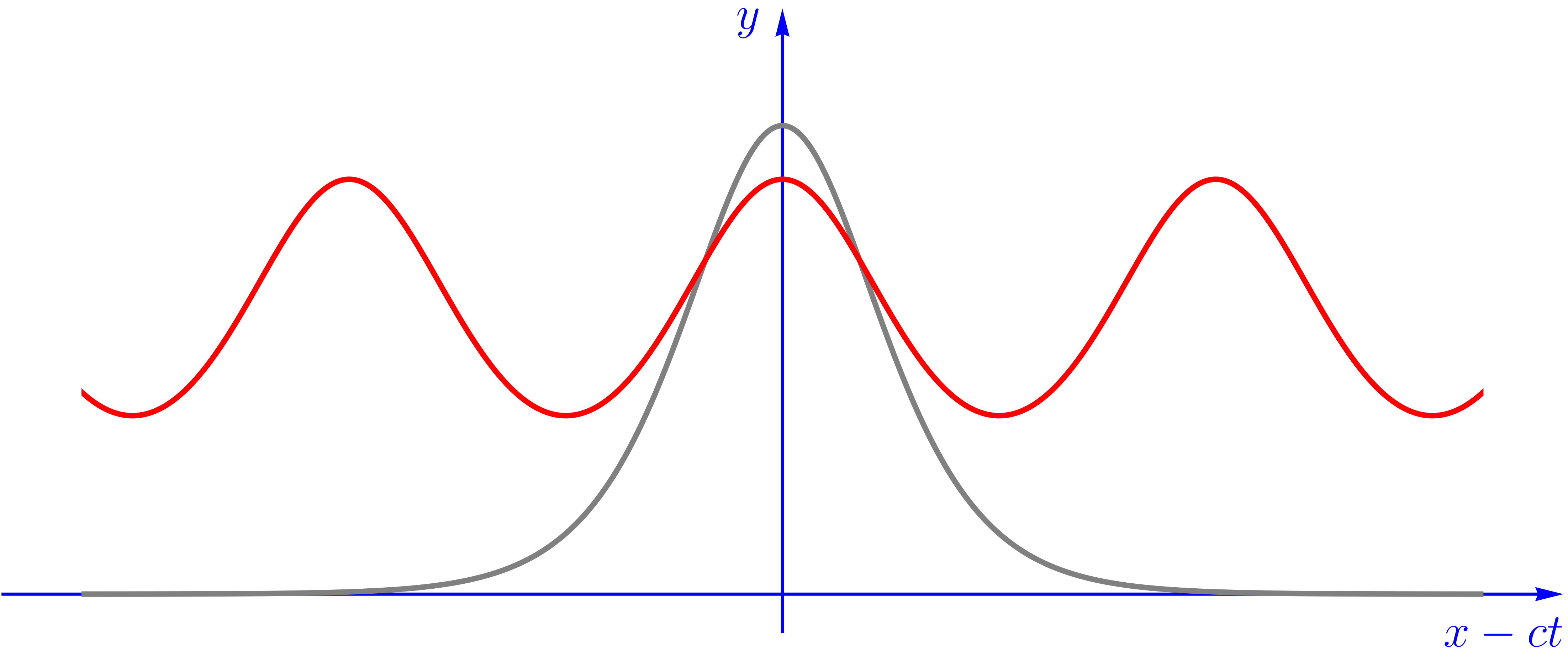

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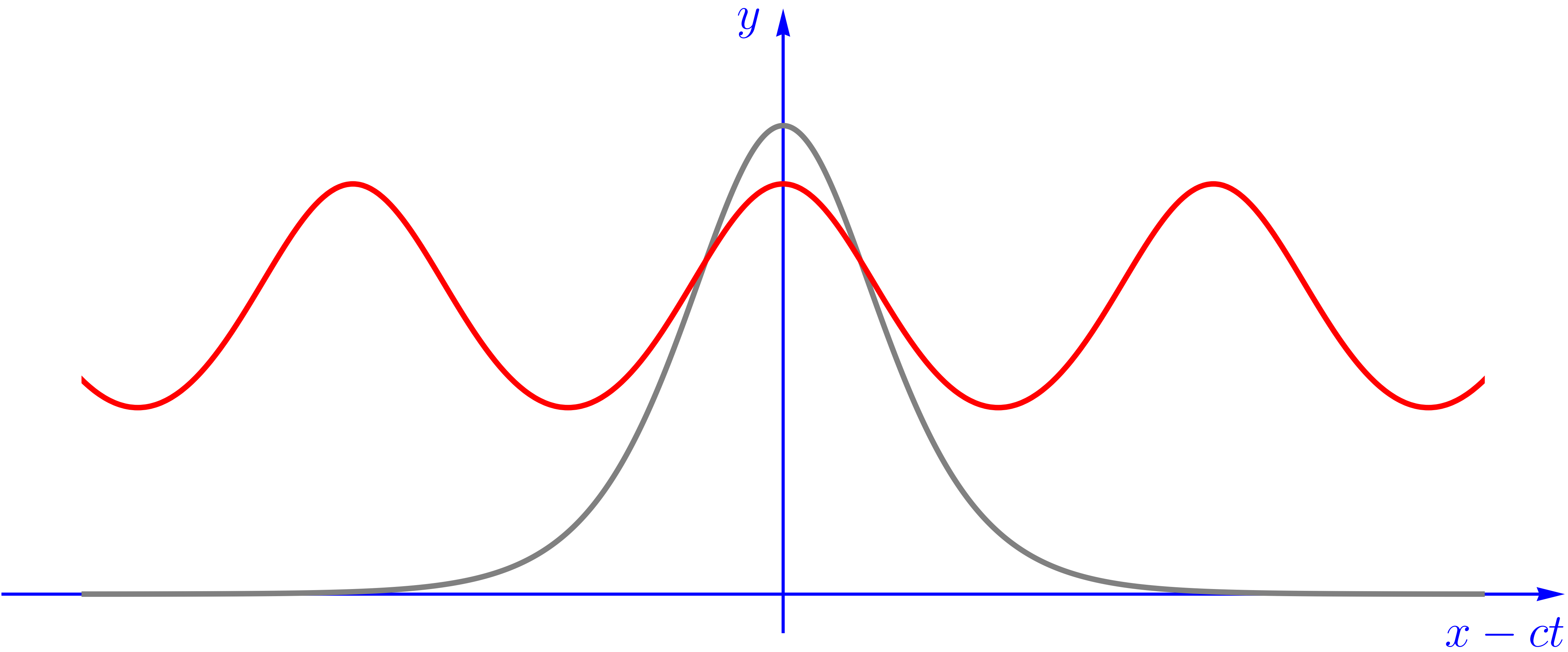

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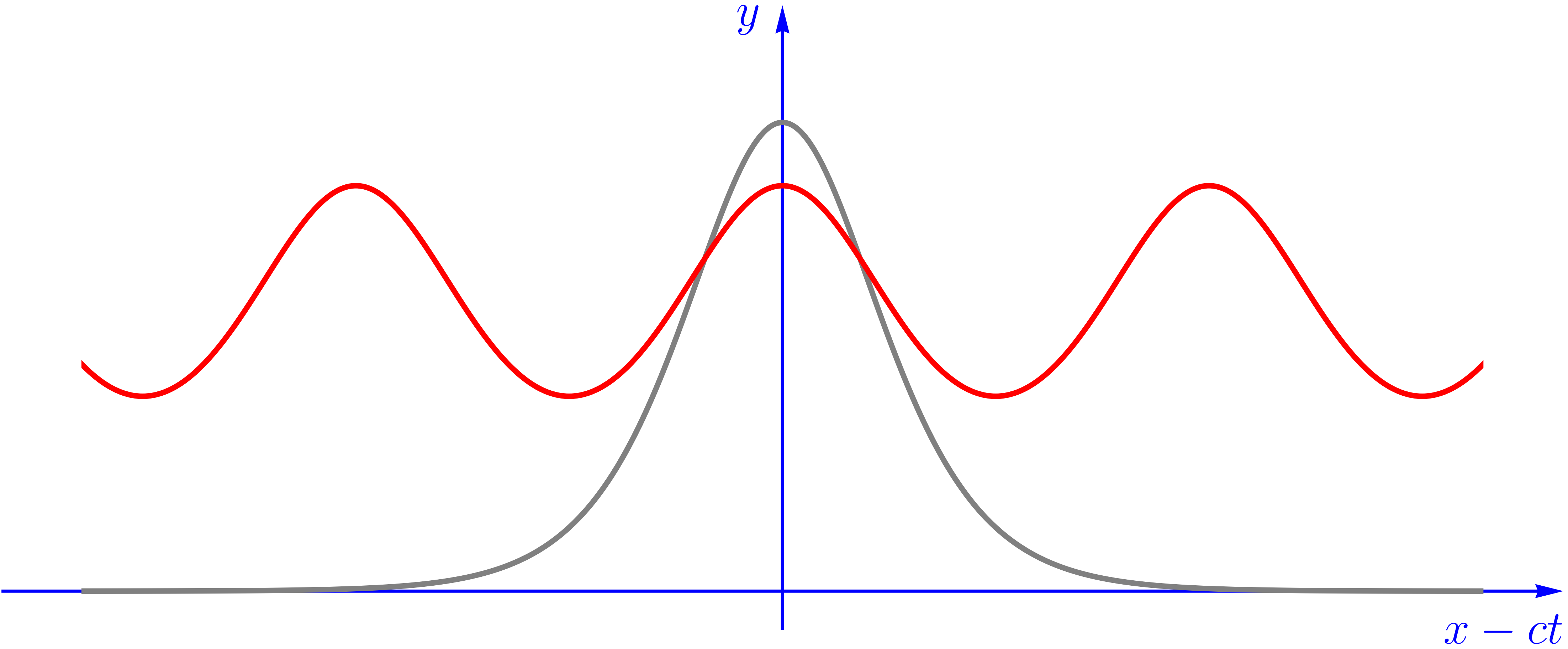

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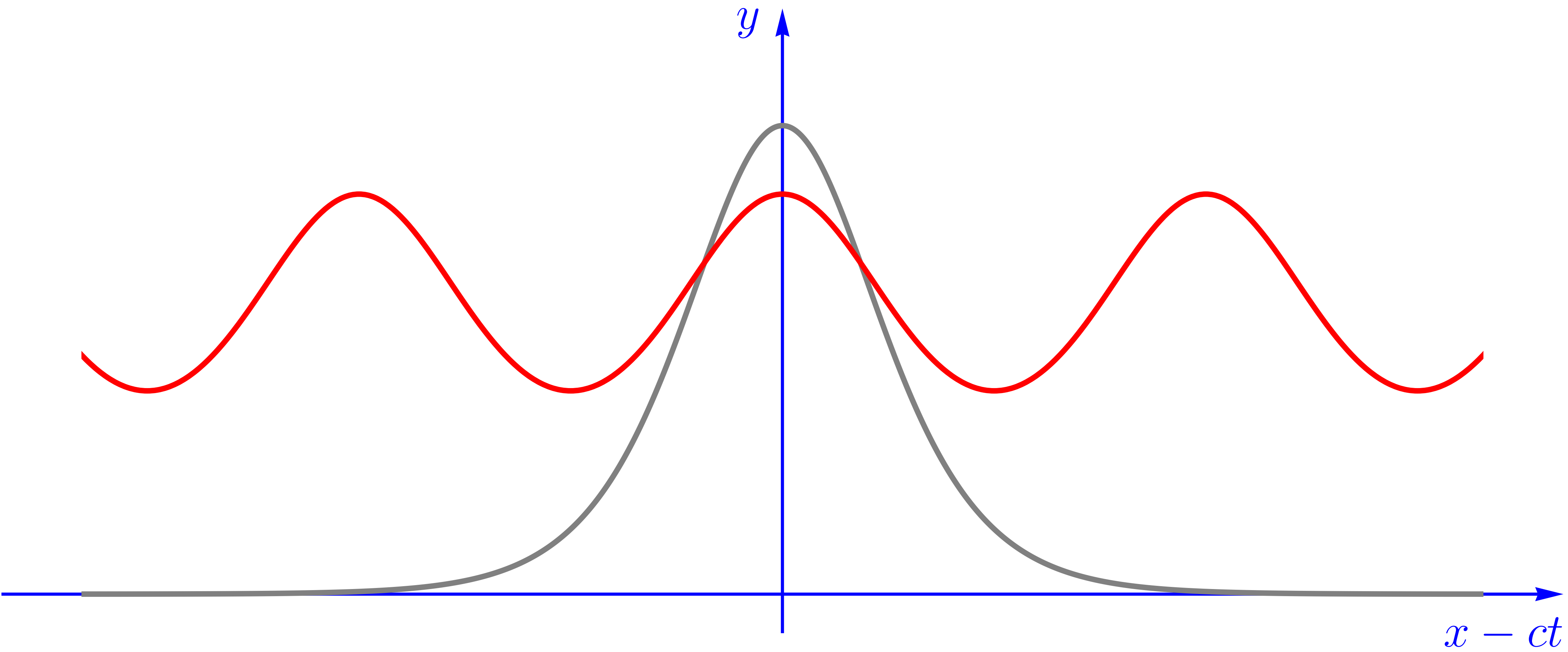

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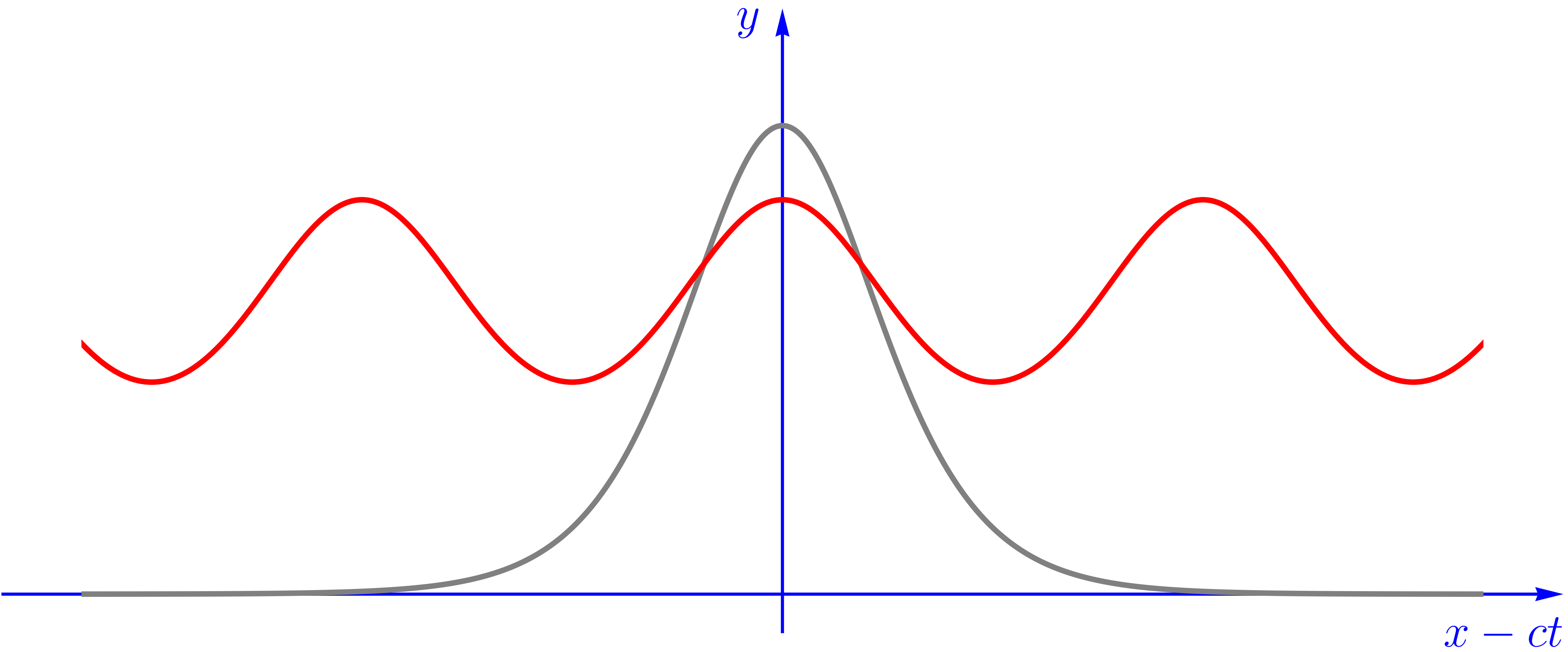
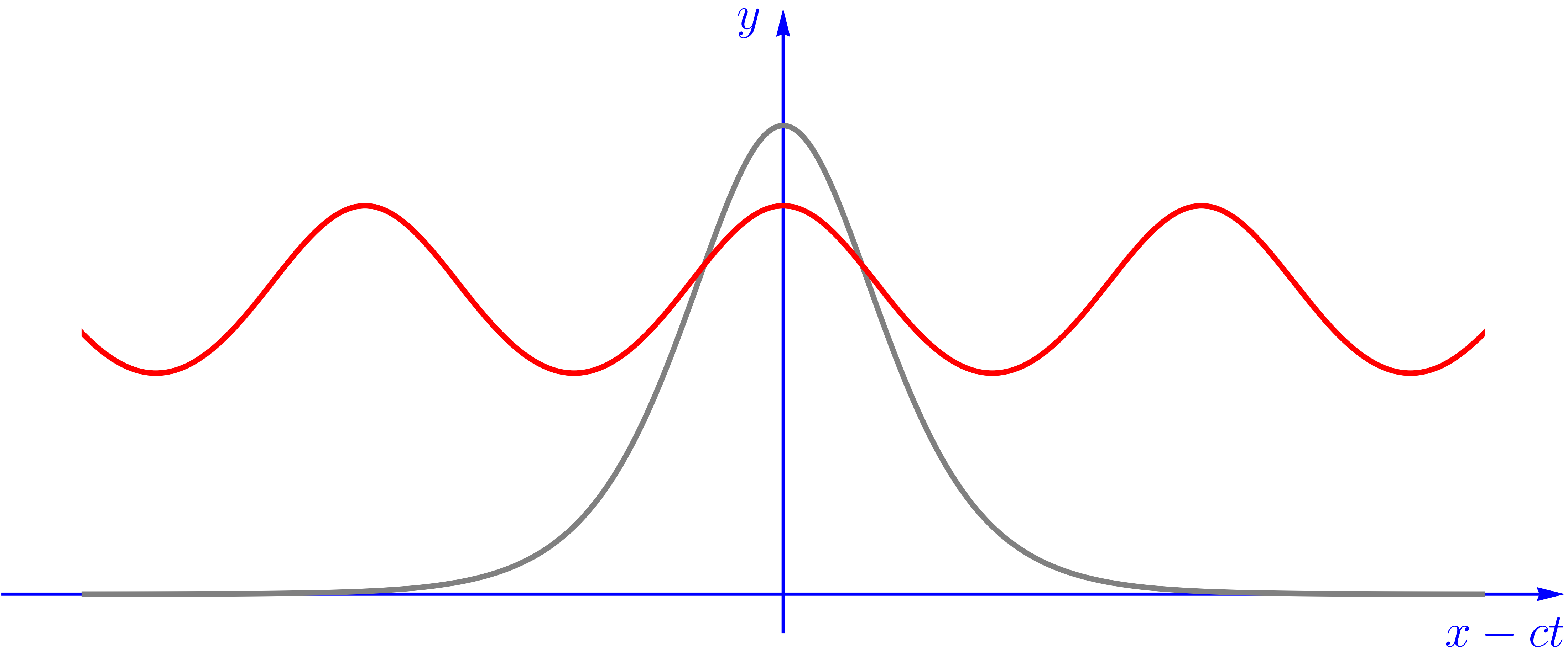
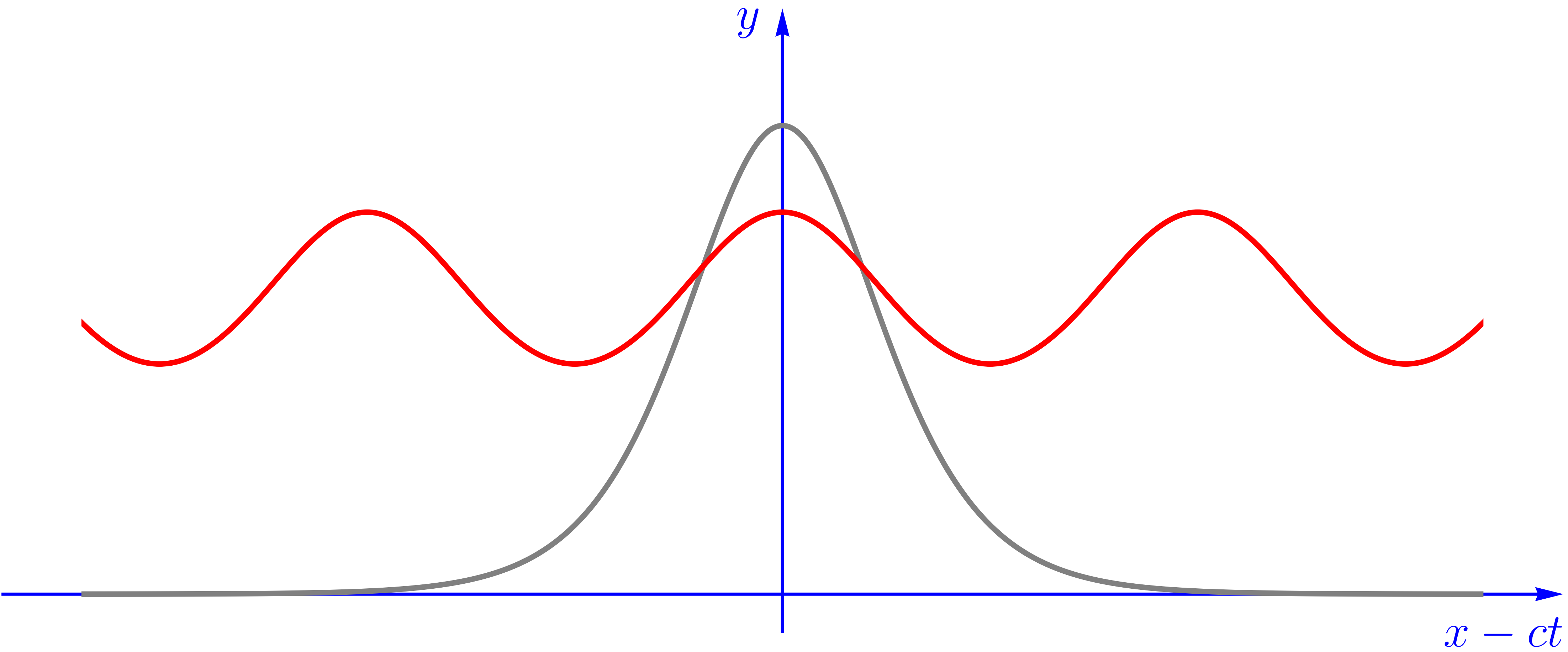




y
$x - ct$

Cnoidal Waves



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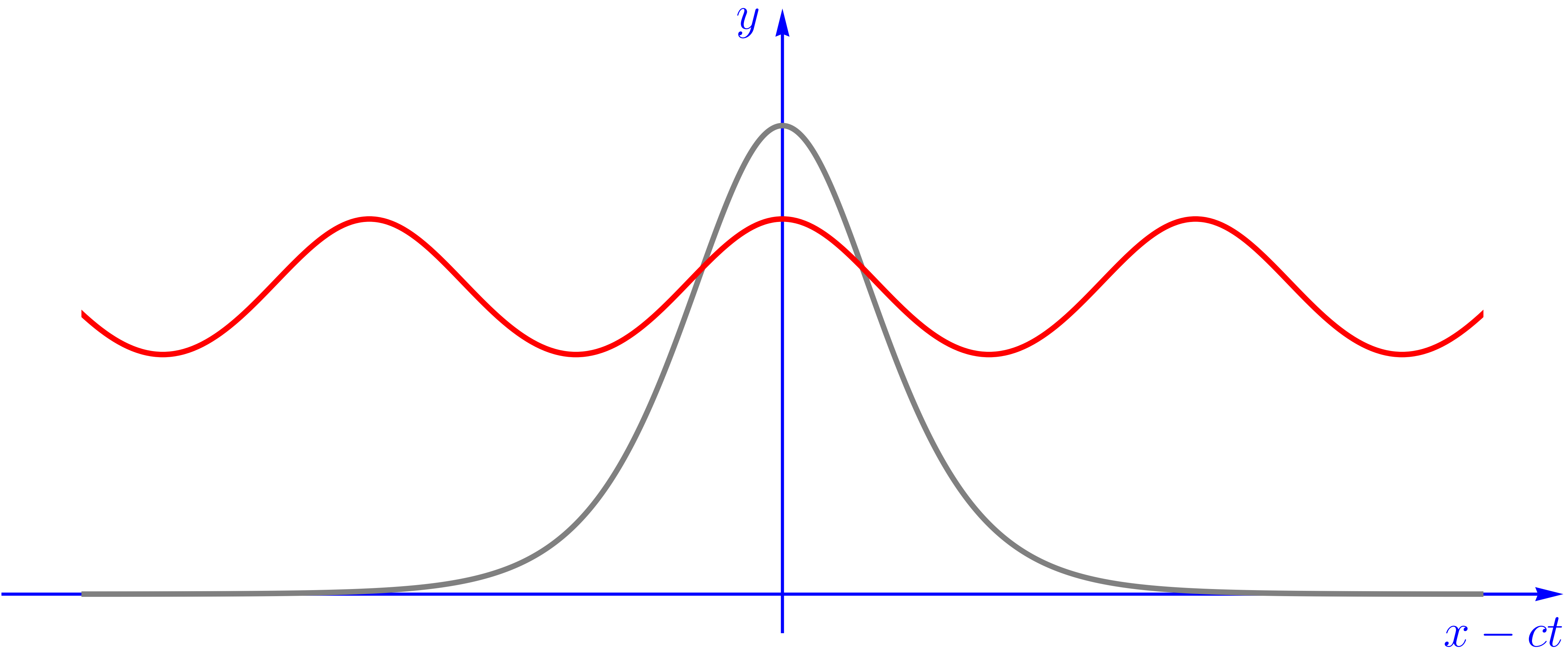

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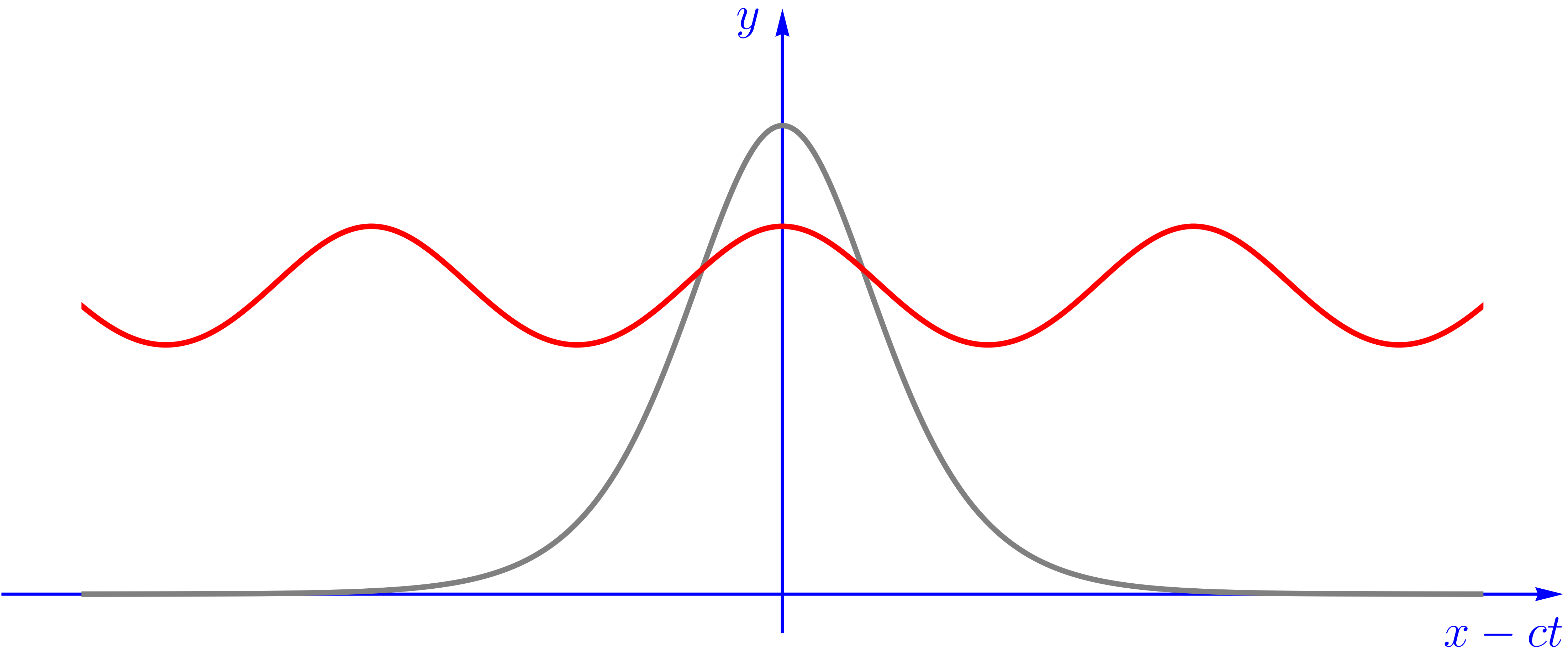

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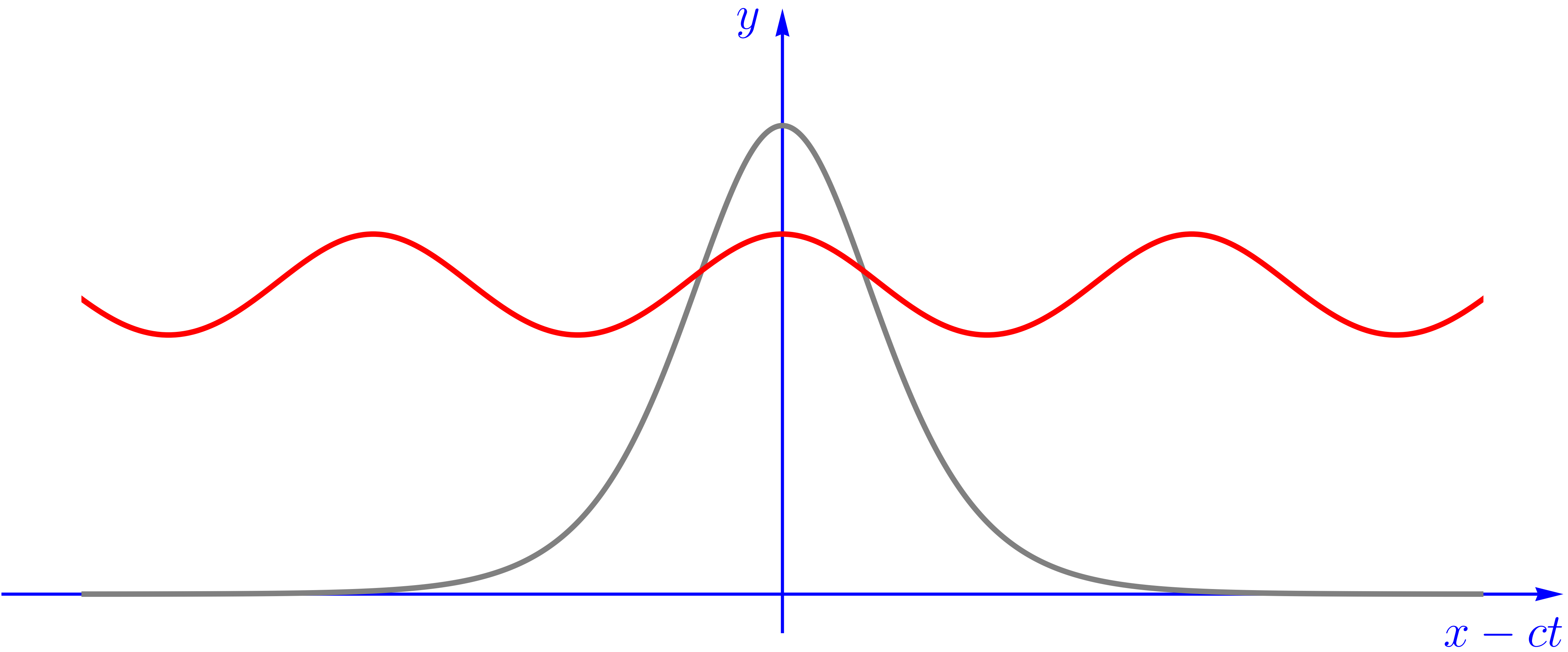

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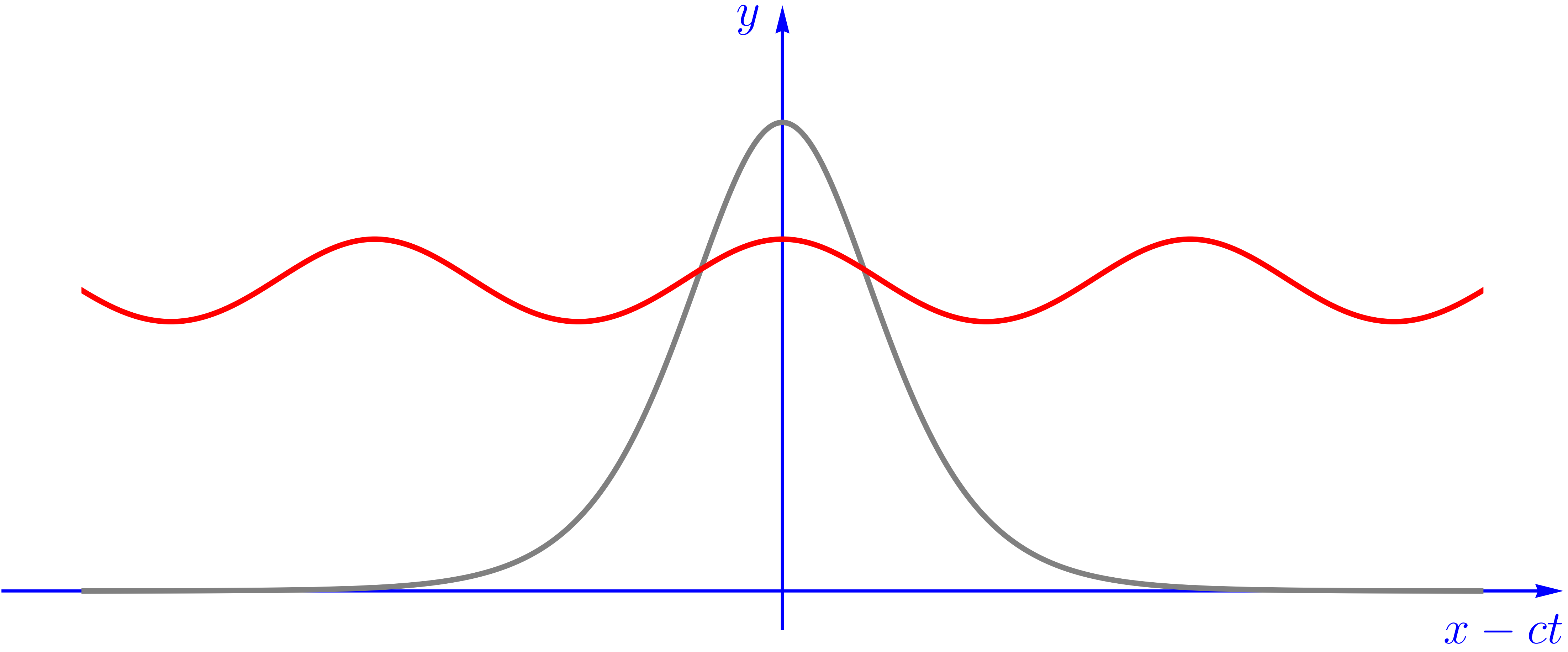

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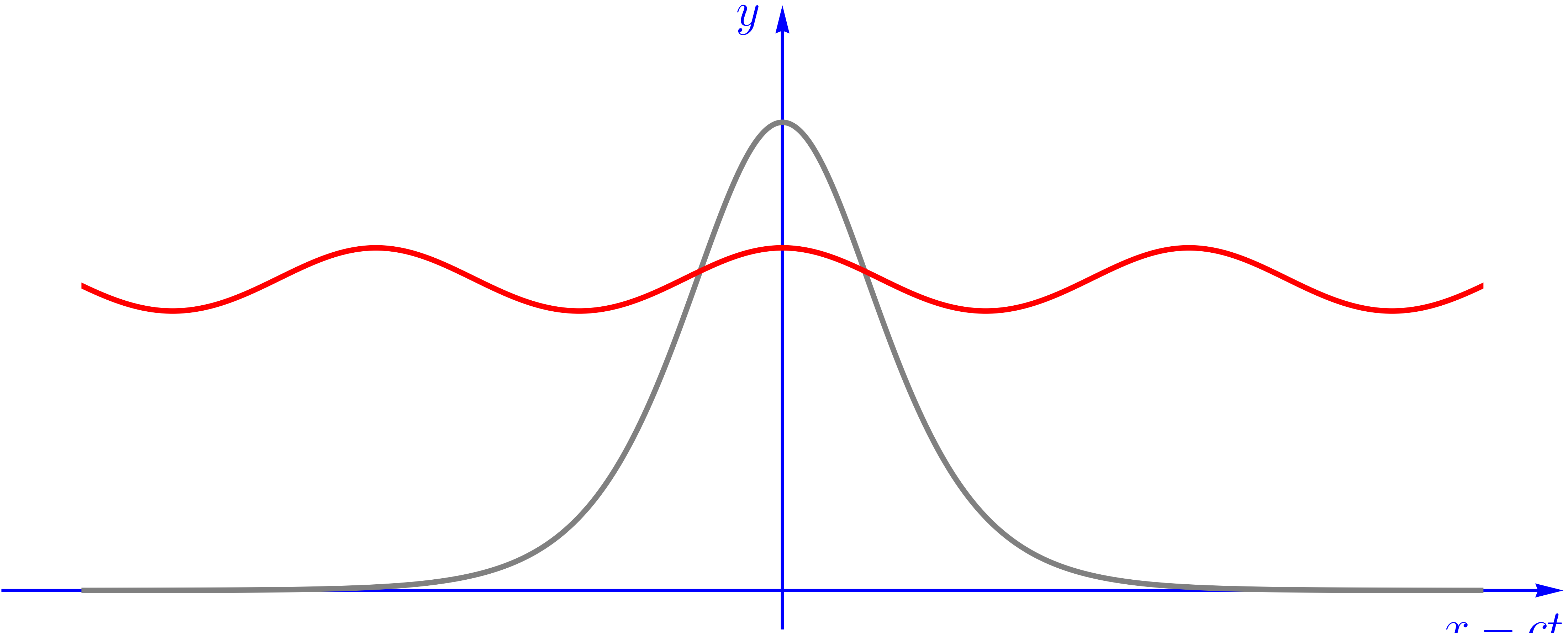

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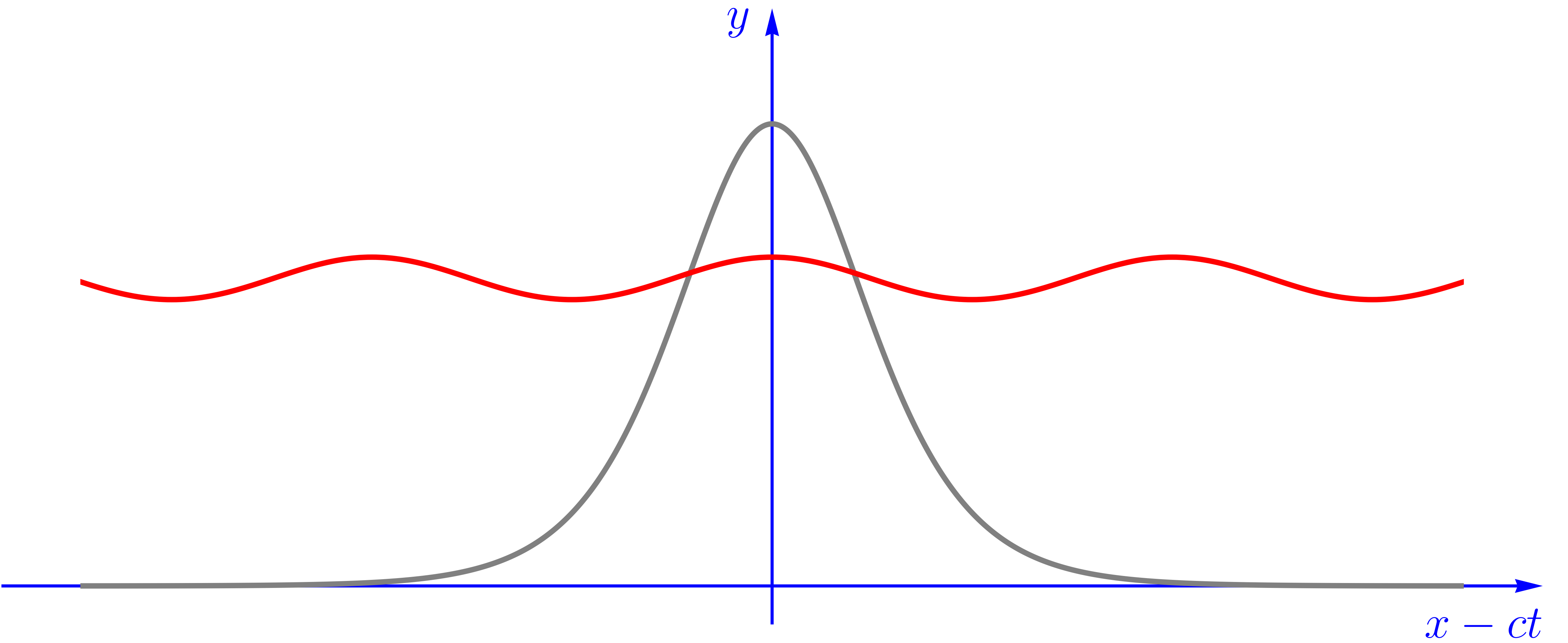

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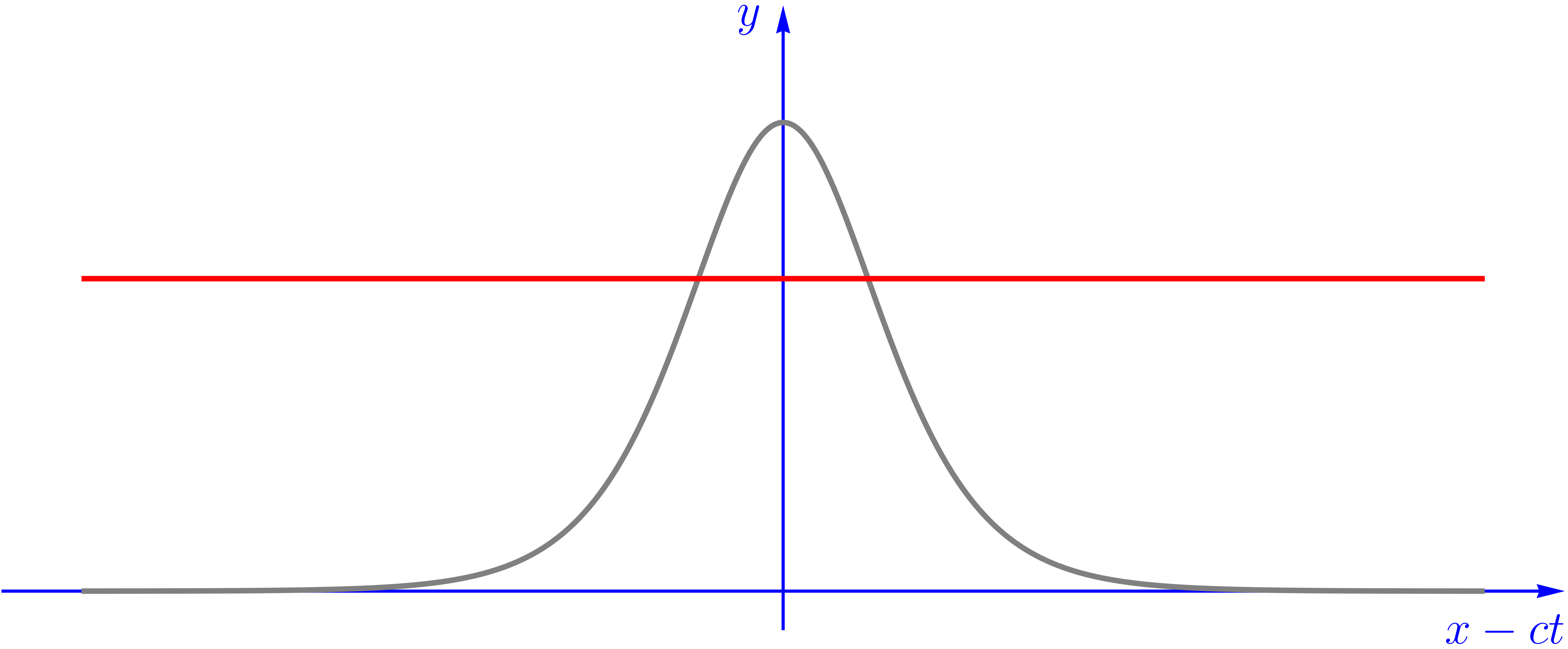


Korteweg-de Vries equation

$$y_t + 3\left(y^2\right)_x + y_{xxx} = 0$$

Waves of constant shape

$$y(x,t) = q(\theta) \qquad \theta = x - ct$$

Ordinary differential equation

$$-cq' + 3\left(q^2\right)' + q''' = 0 \qquad q' \equiv \frac{\mathrm{d}q}{\mathrm{d}\theta}$$

First integration

$$q'' = A + cq - 3q^2$$

Integration (Continued)

$$q' \frac{\mathrm{d}q'}{\mathrm{d}q} = q'' = A + cq - 3q^2$$

Second integration

$$q'^2 = 2B + 2Aq + cq^2 - 2q^3 \equiv f(q)$$

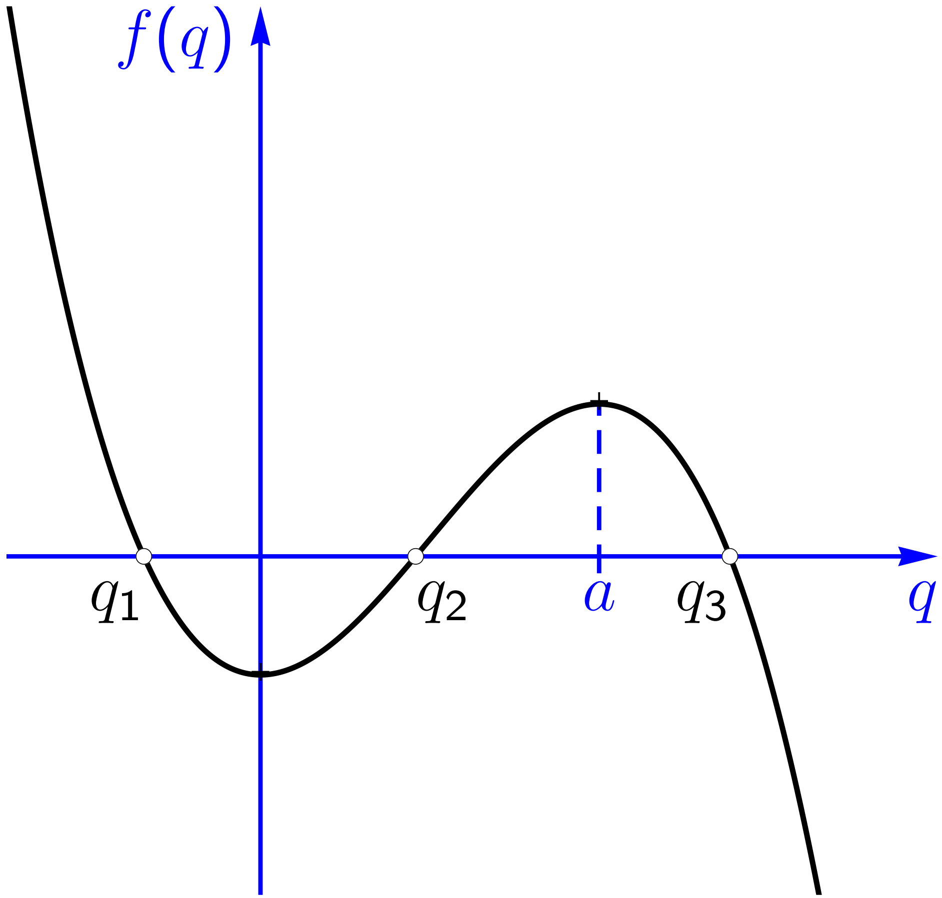
Phase curves

$$\boxed{\frac{\mathrm{d}q}{\mathrm{d}\theta} = \pm\sqrt{f(q)}}$$

Representation of the function f by its (real) roots $q_1 \leq q_2 < q_3$

$$f(q) = 2(q - q_1)(q - q_2)(q_3 - q)$$

Function f and Phase Curves



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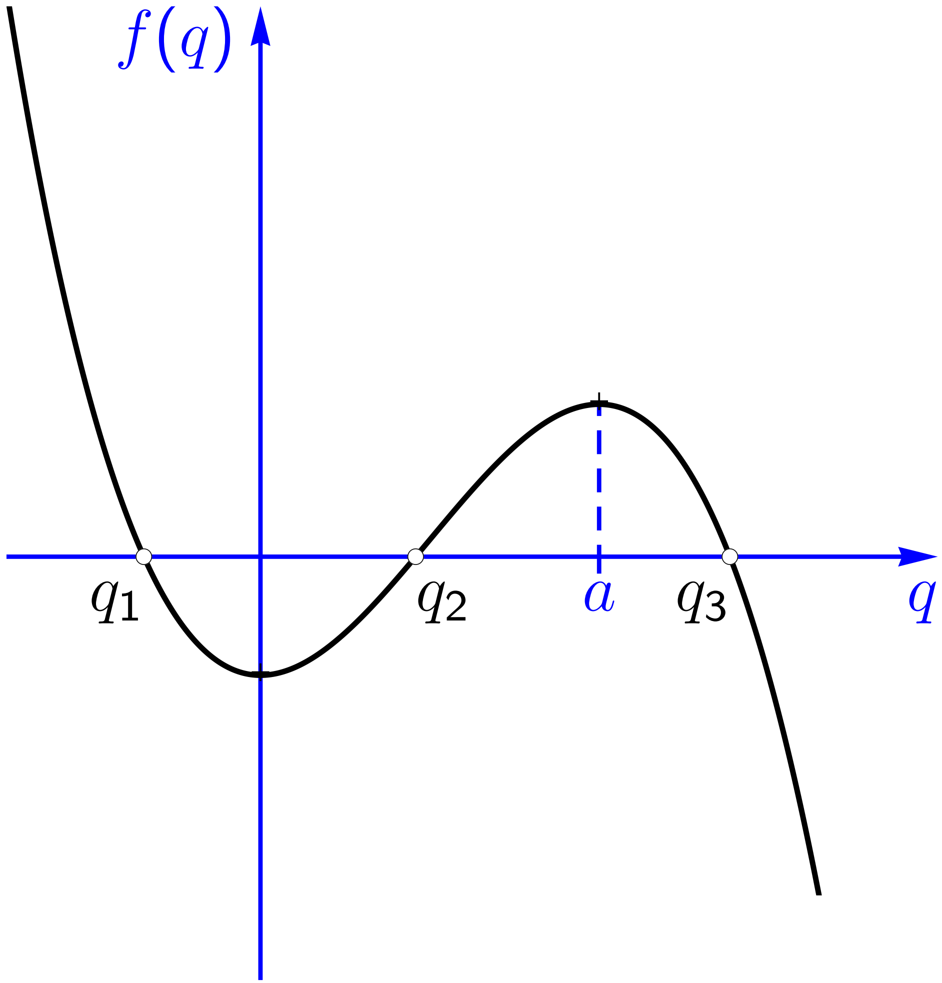

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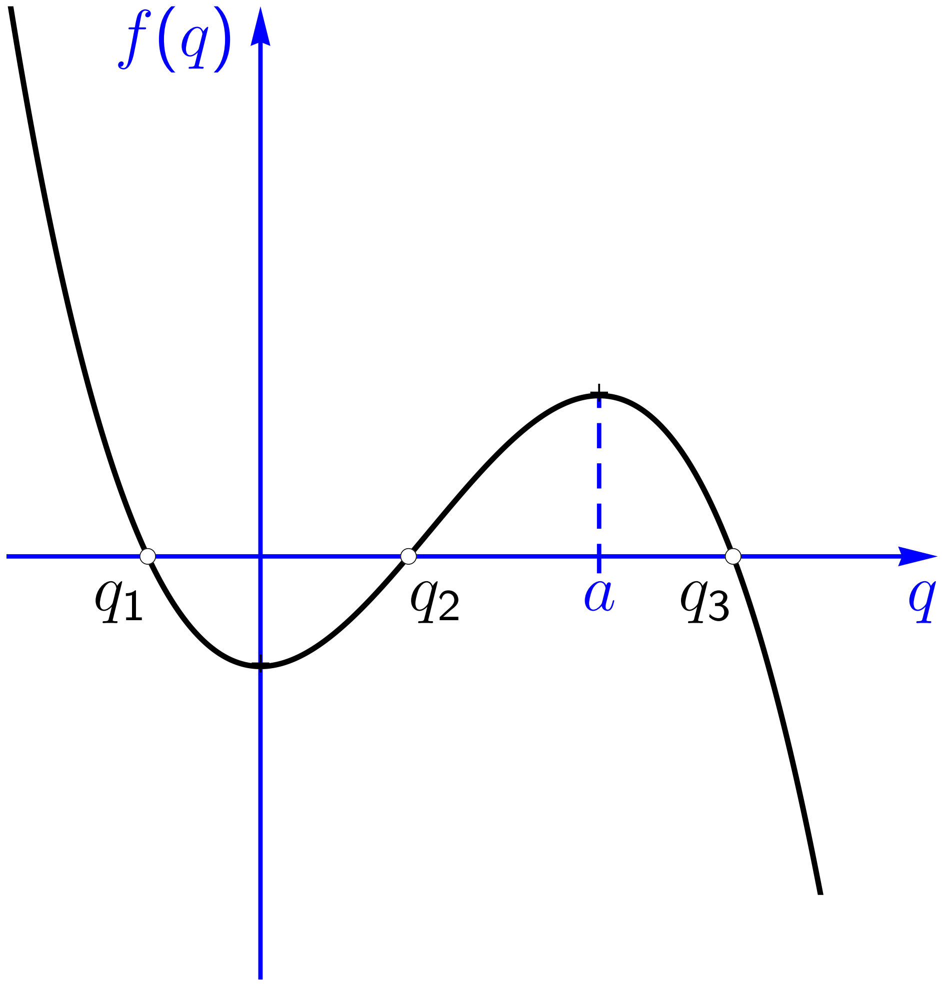

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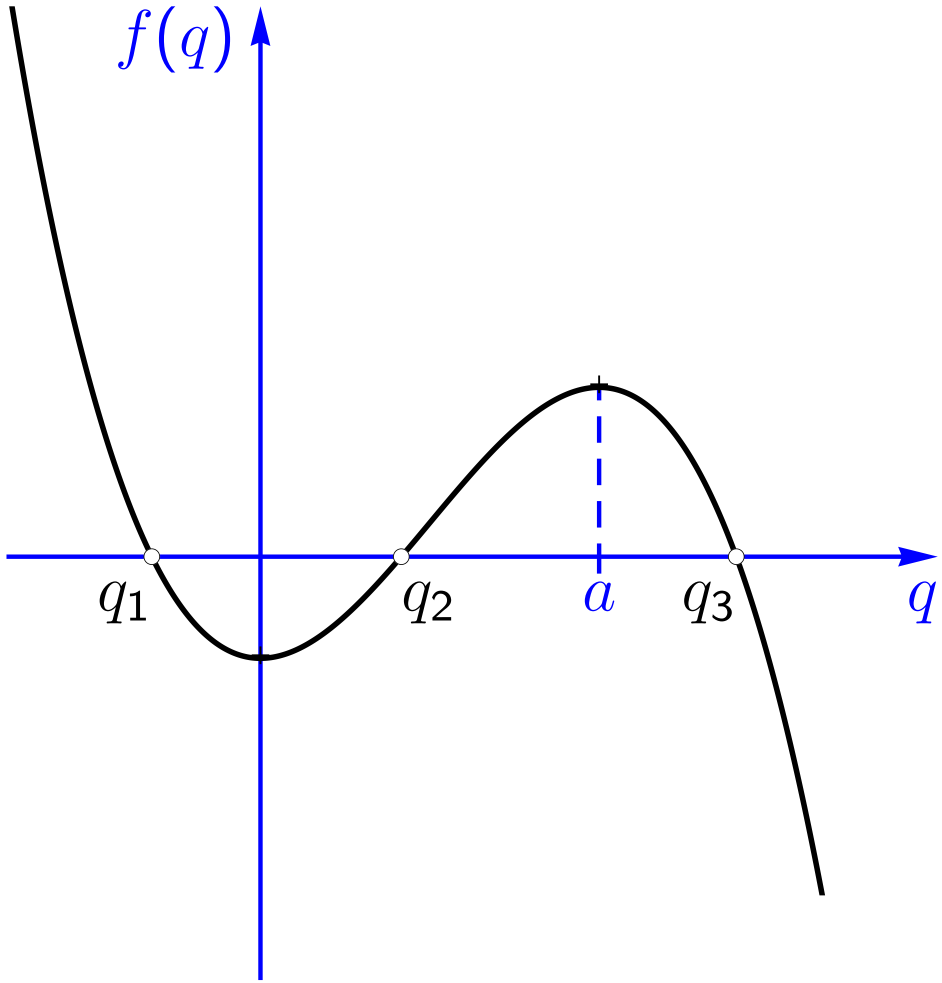

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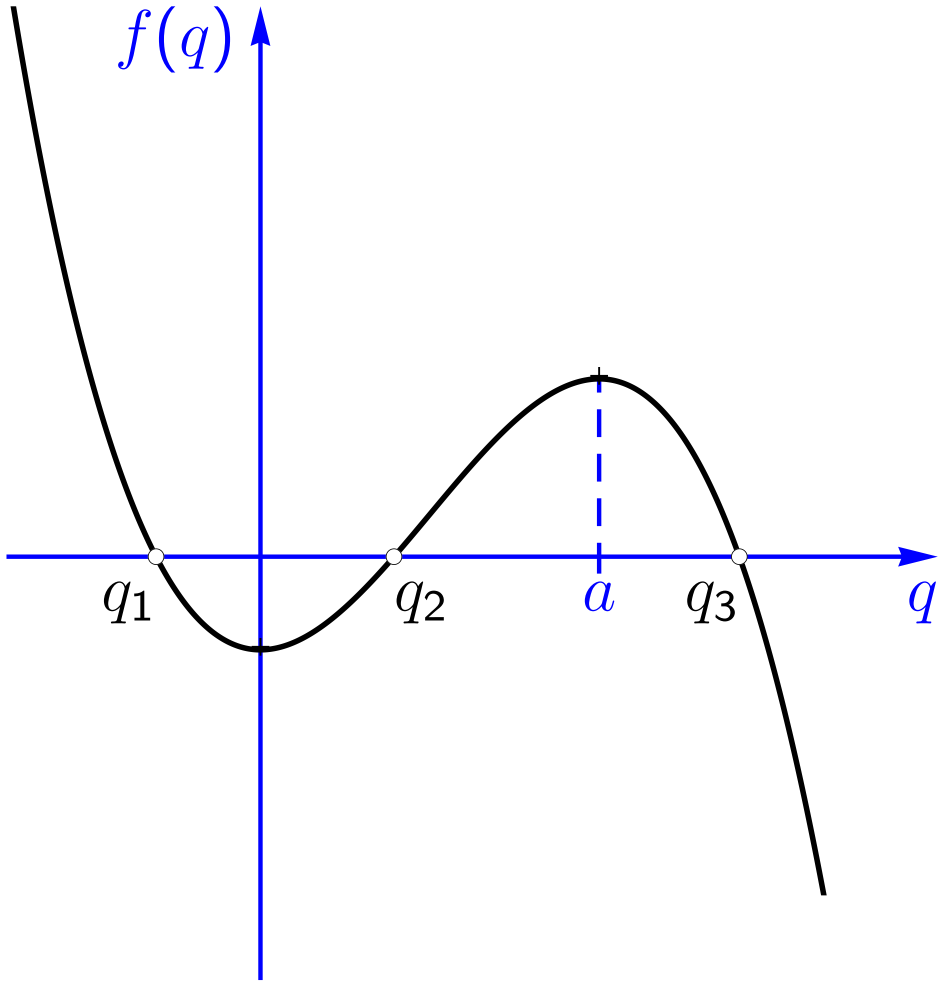

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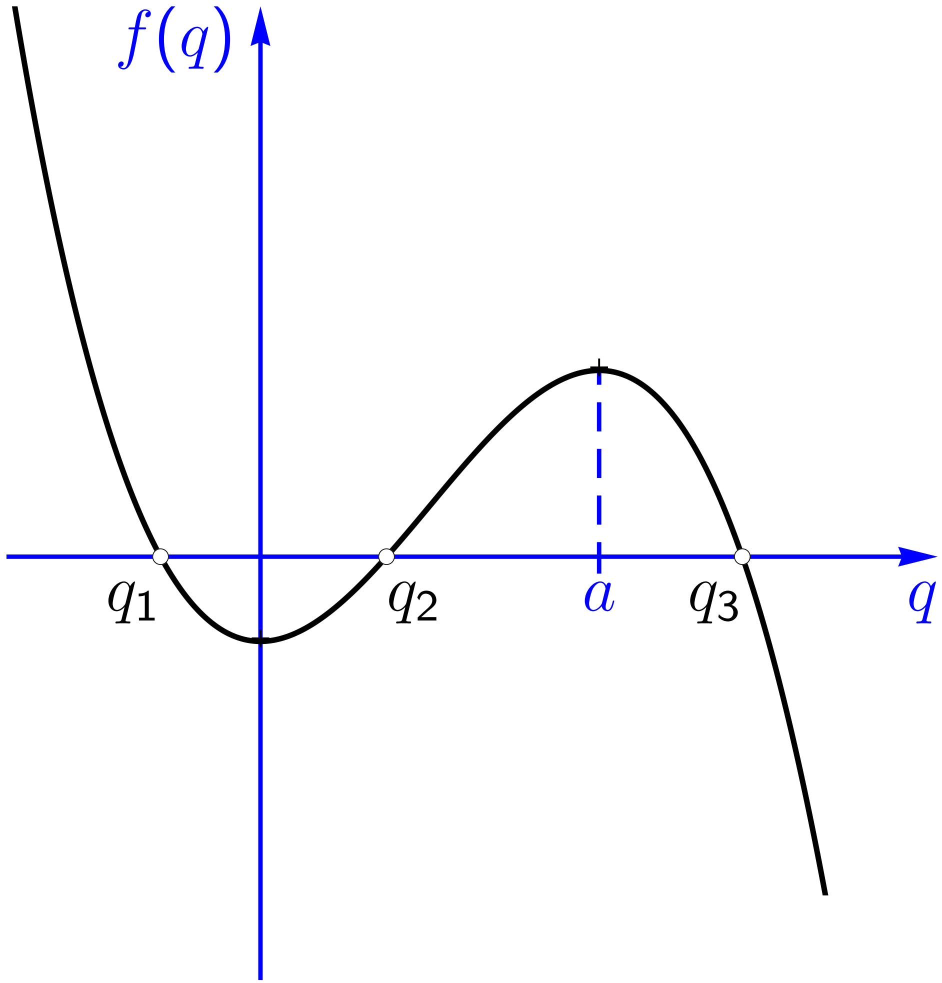

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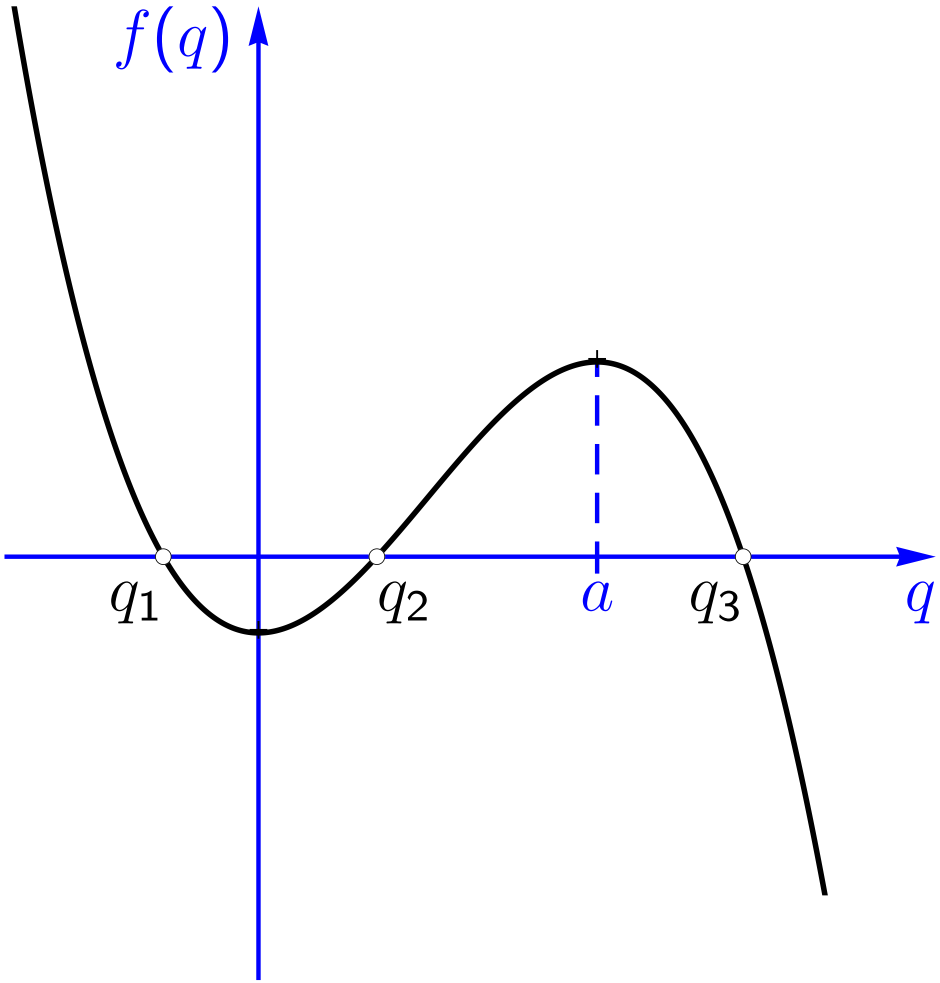

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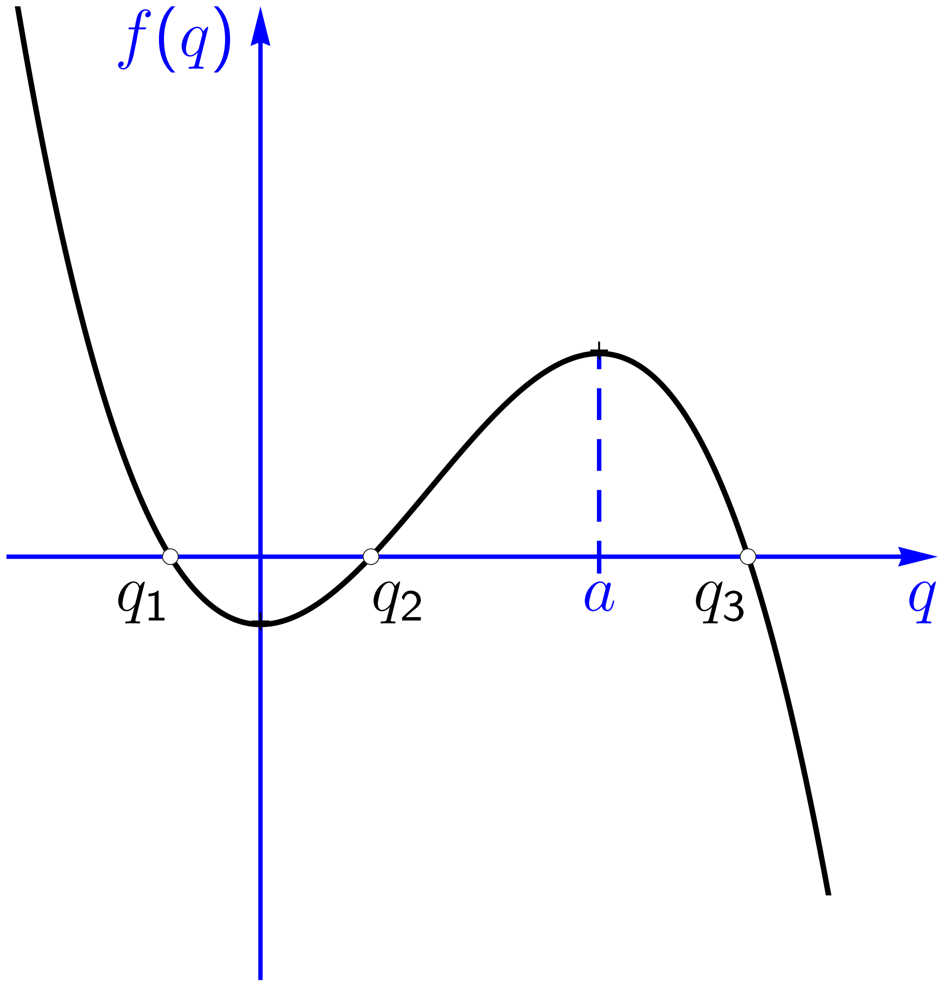

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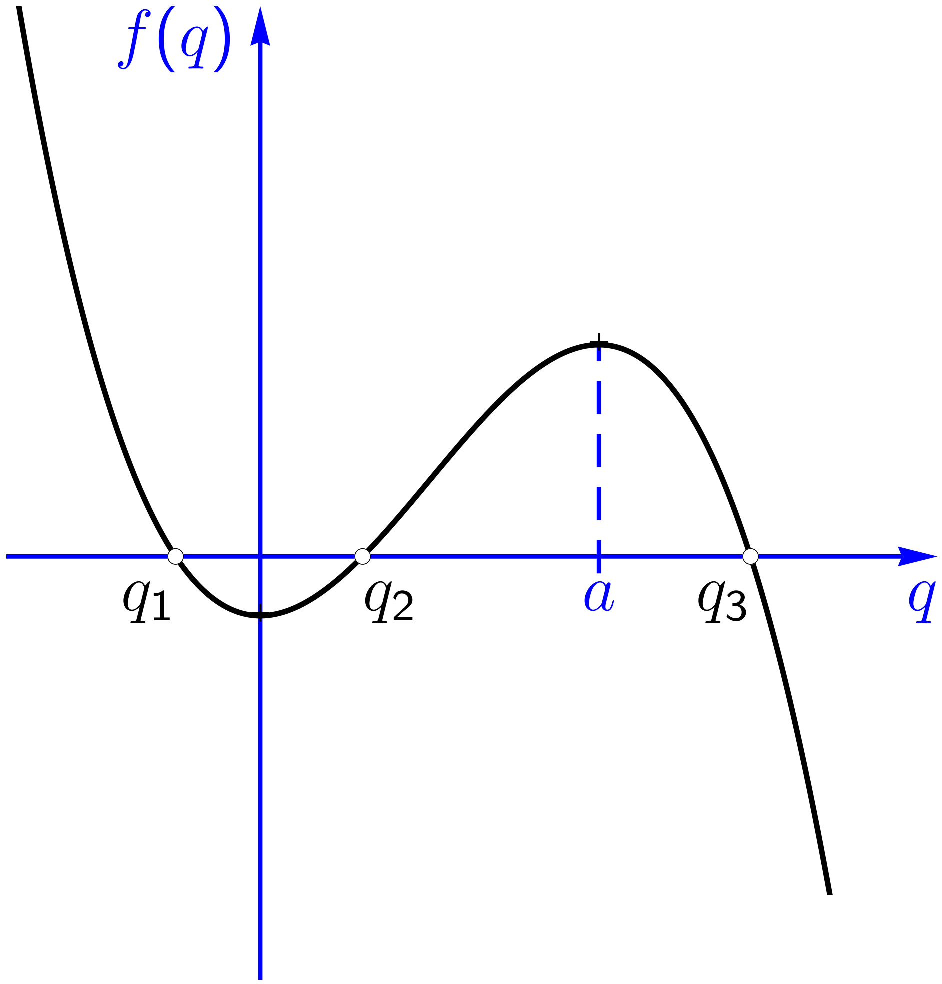

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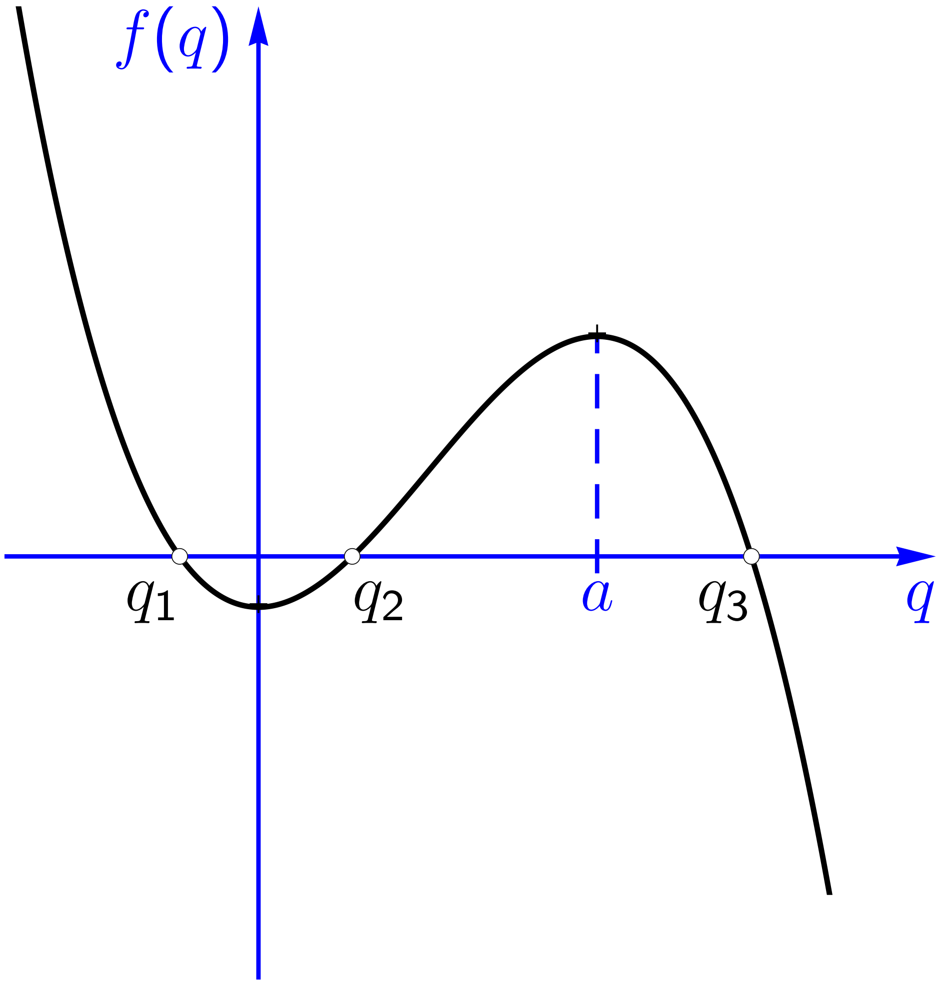

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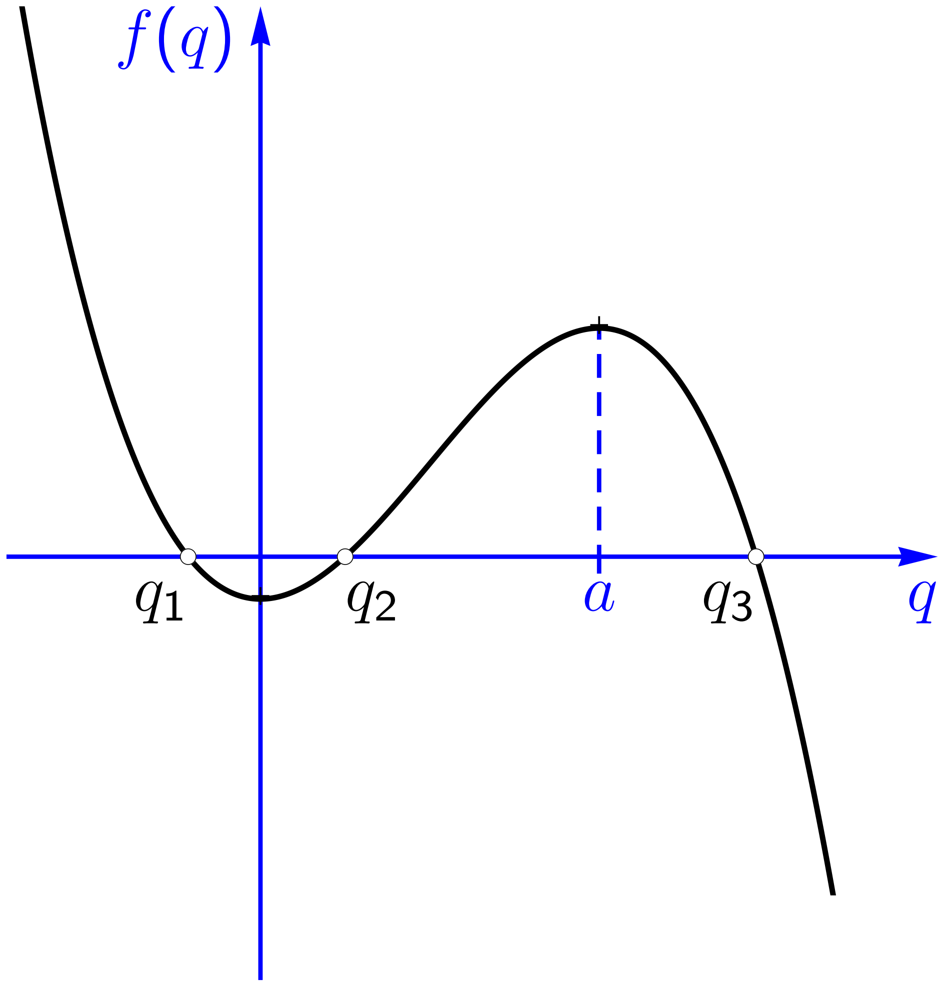

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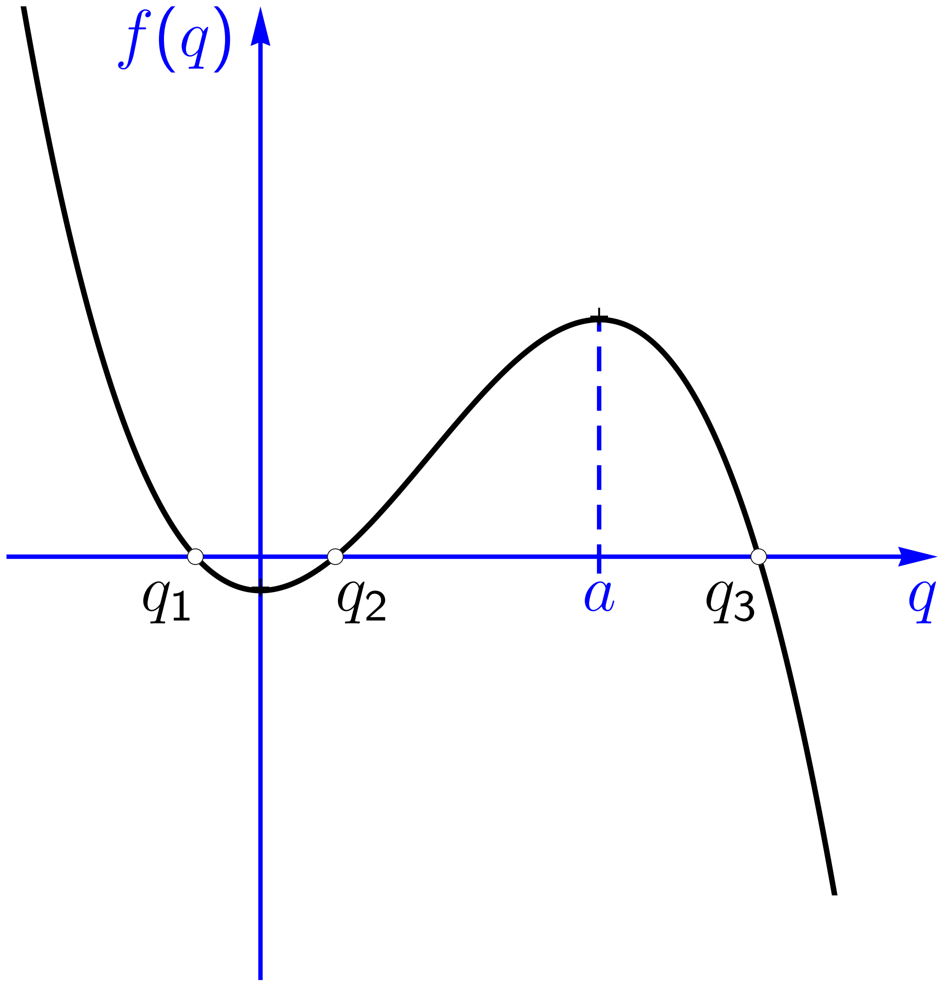

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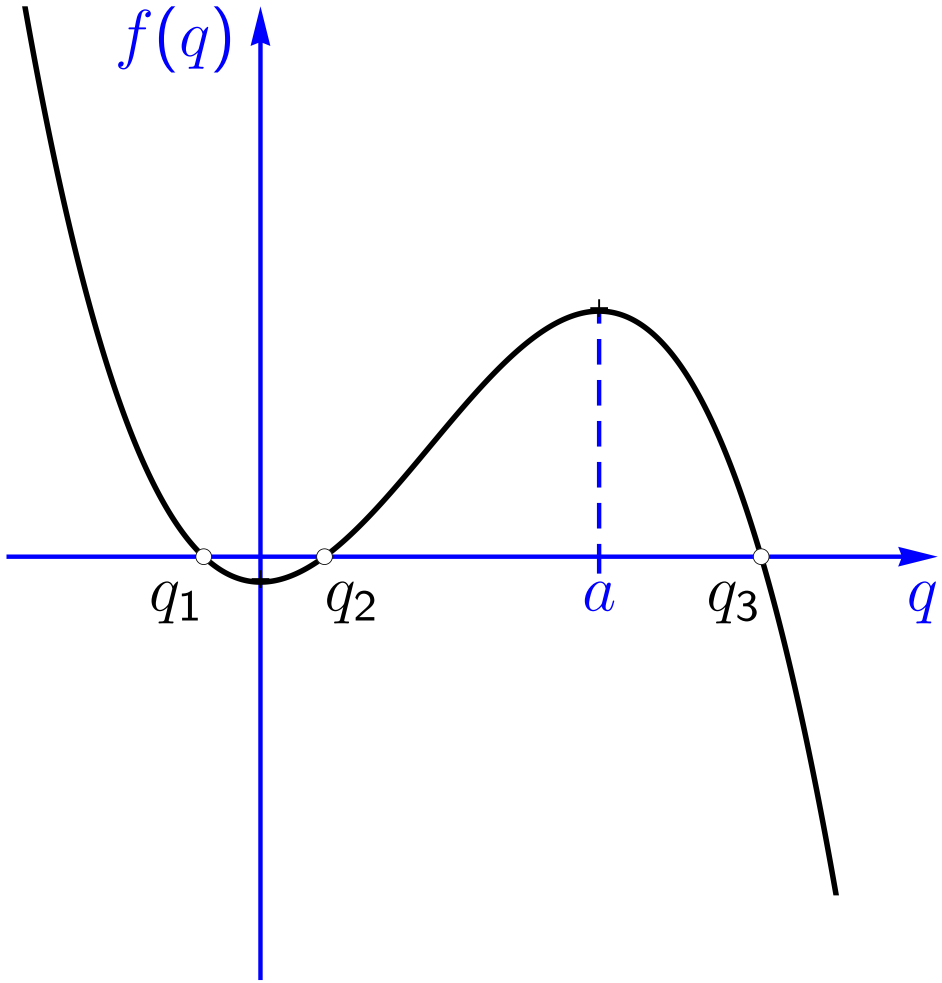

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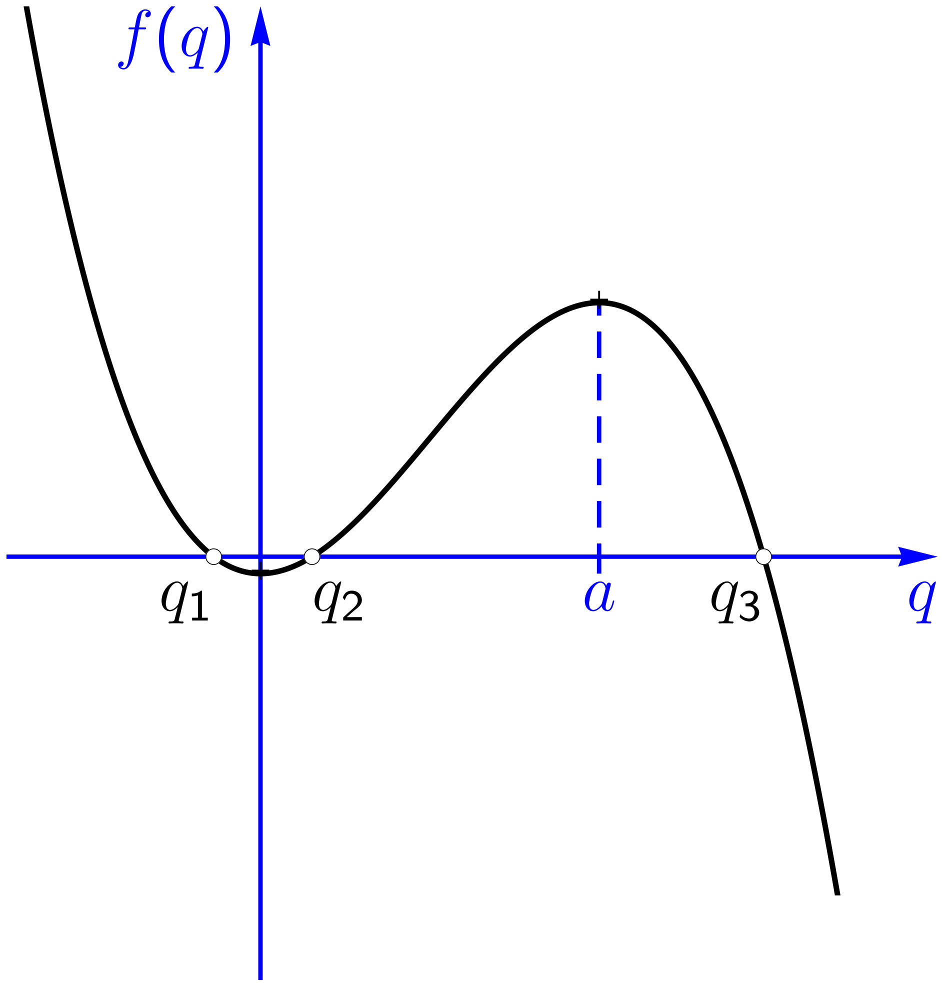

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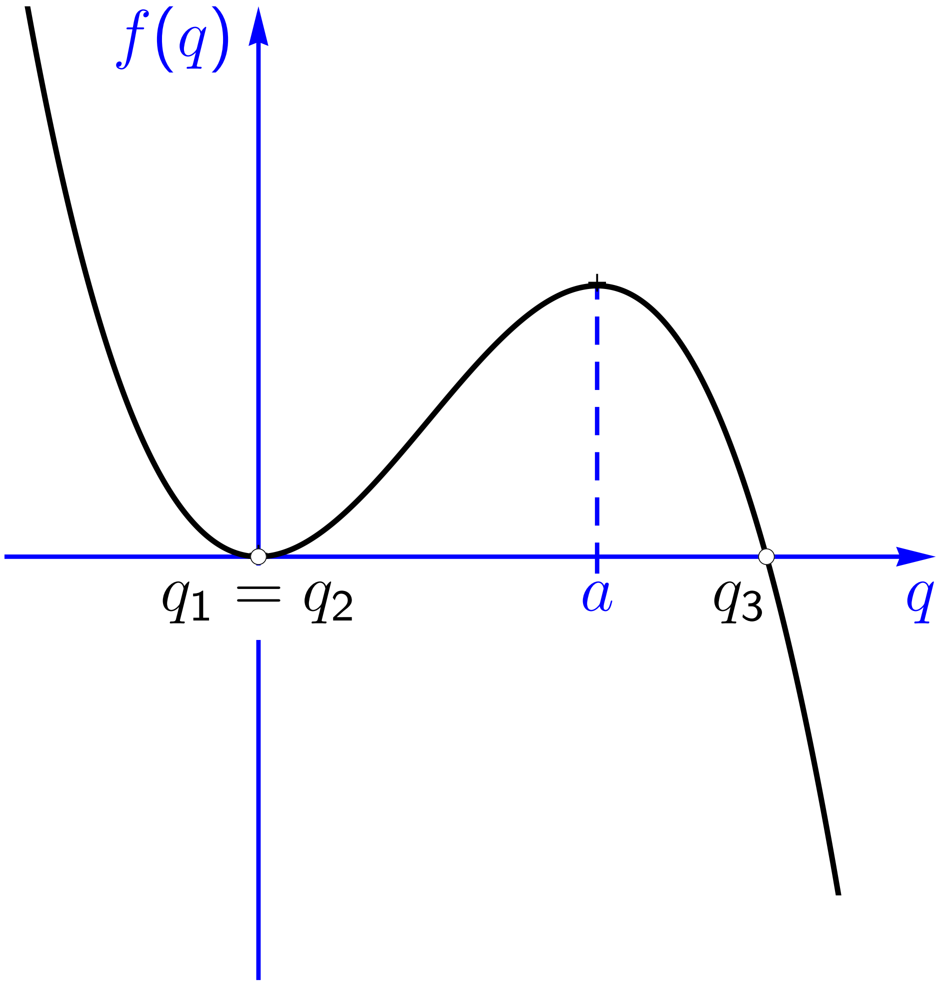

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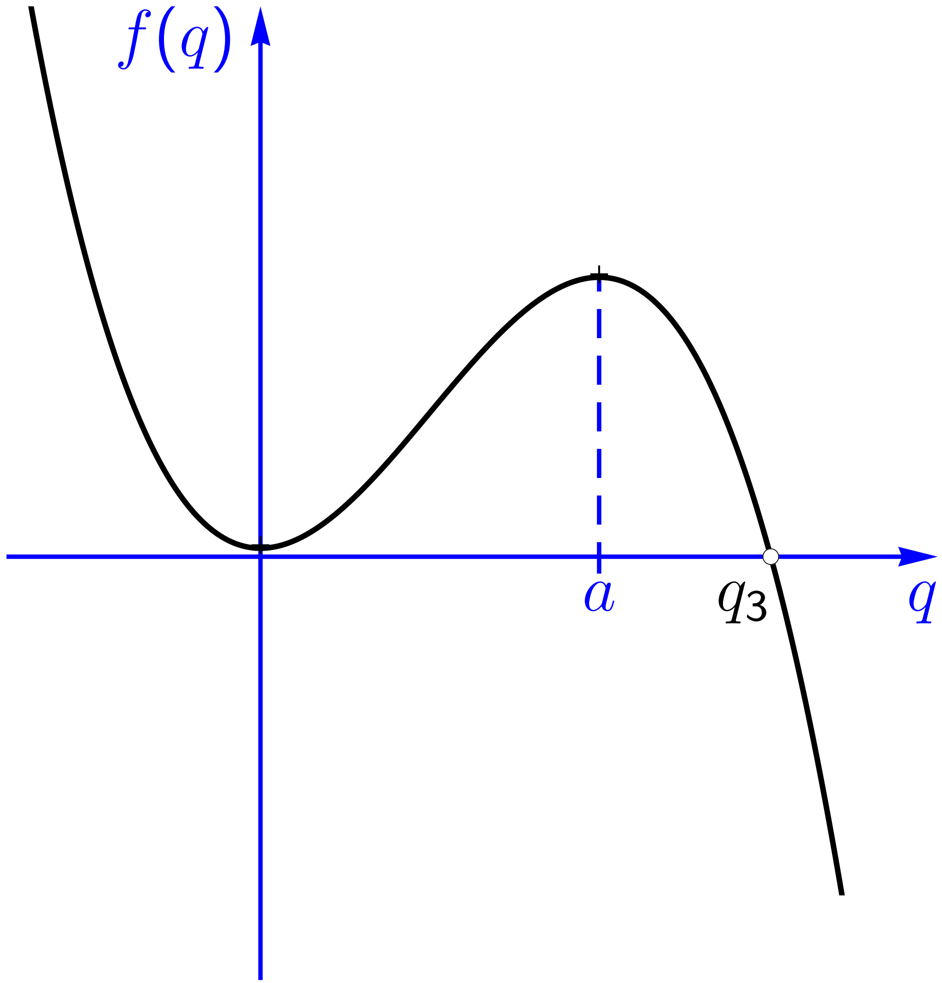

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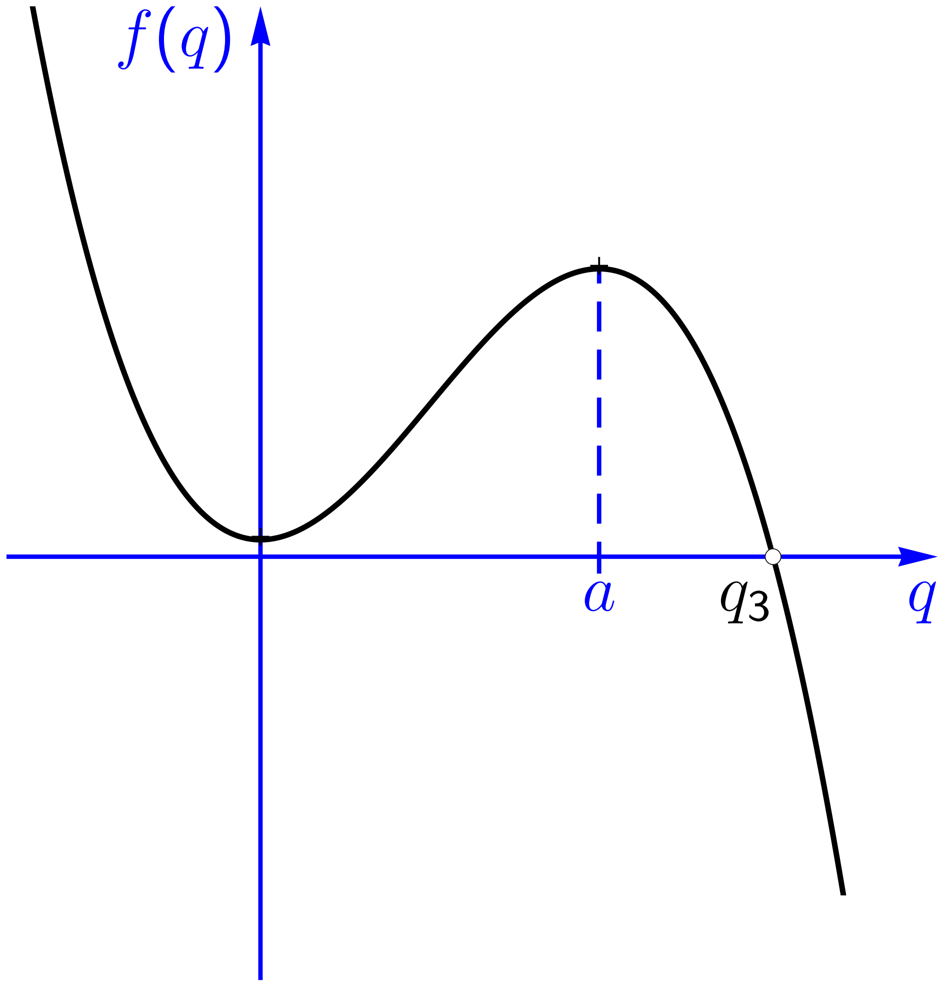

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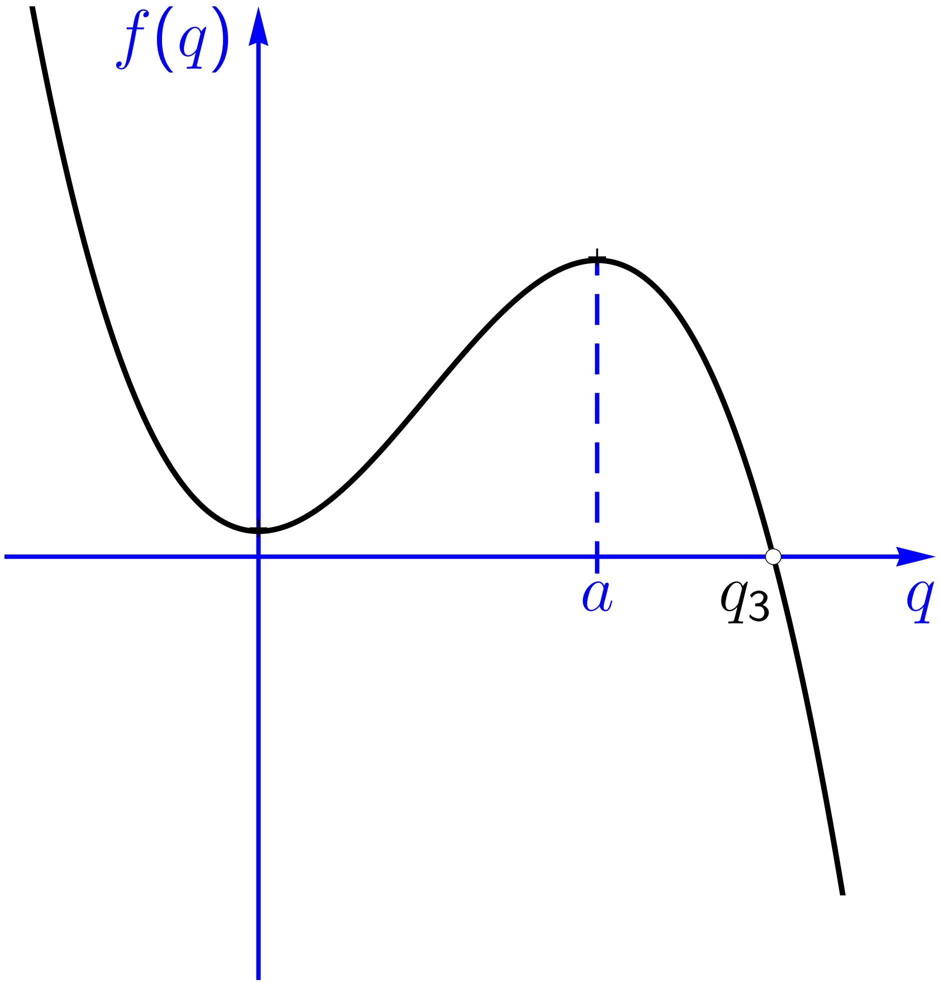

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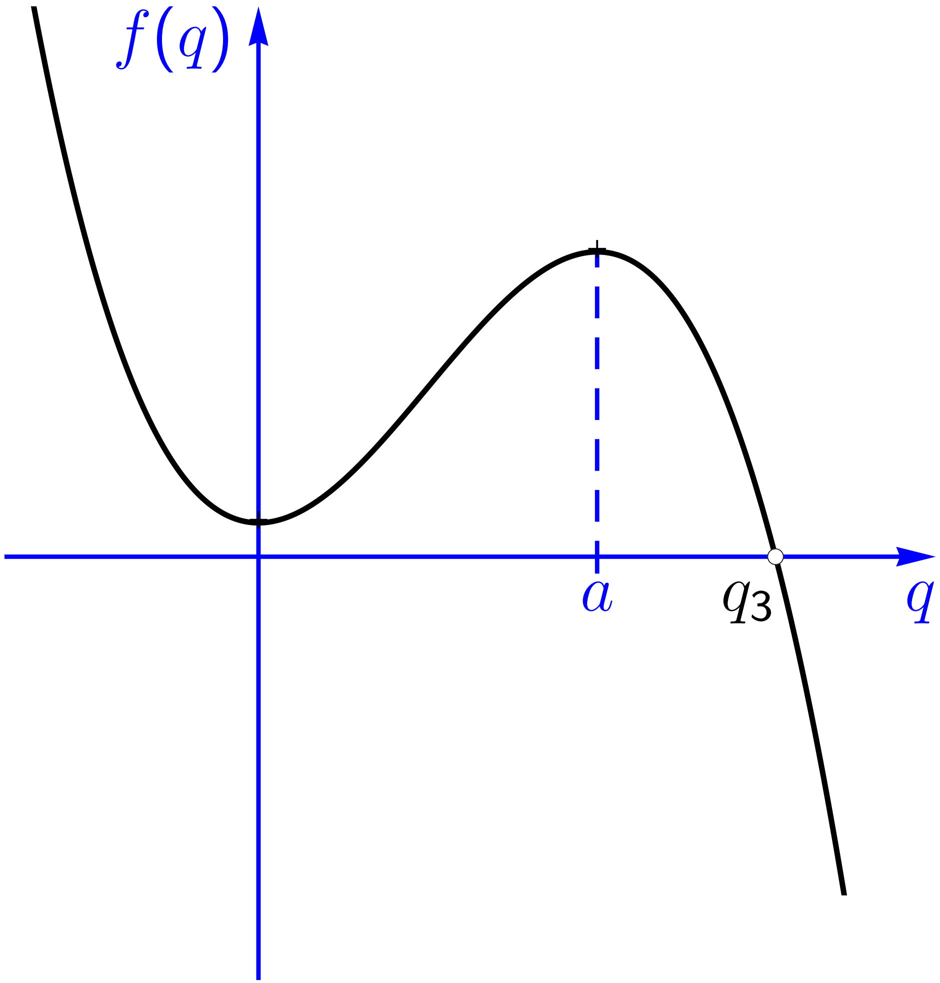

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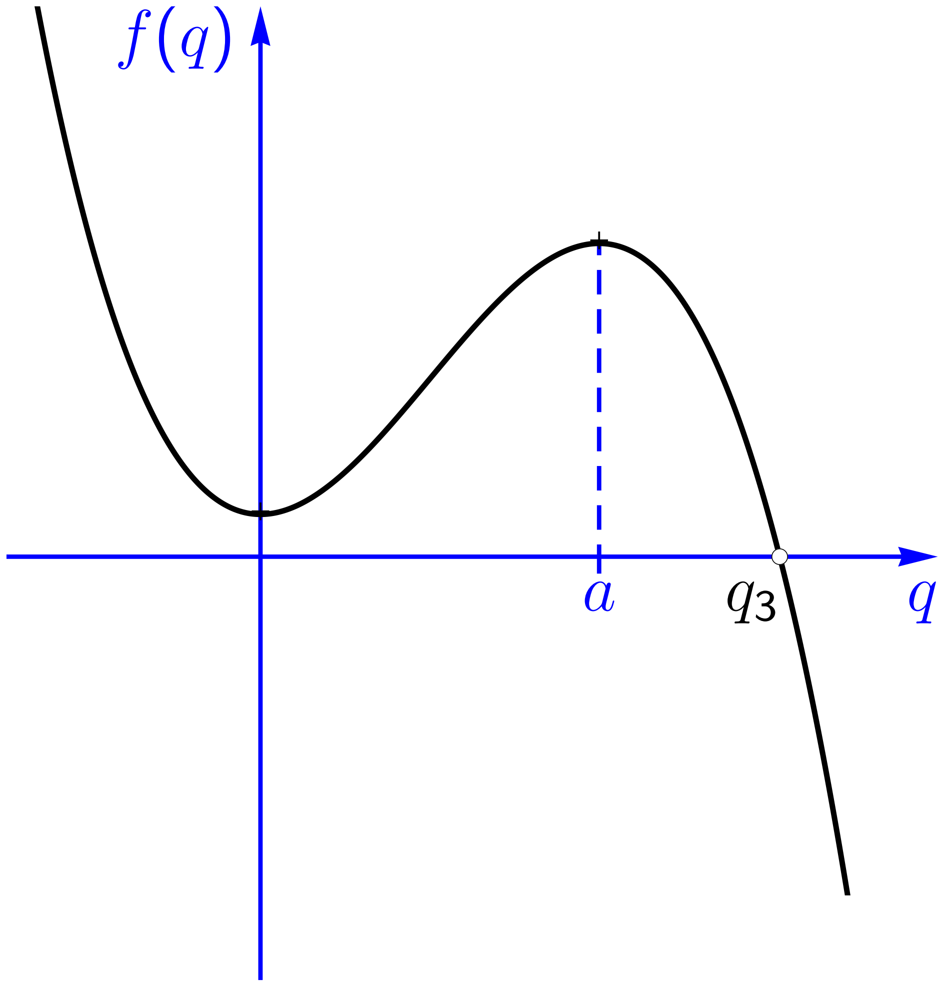

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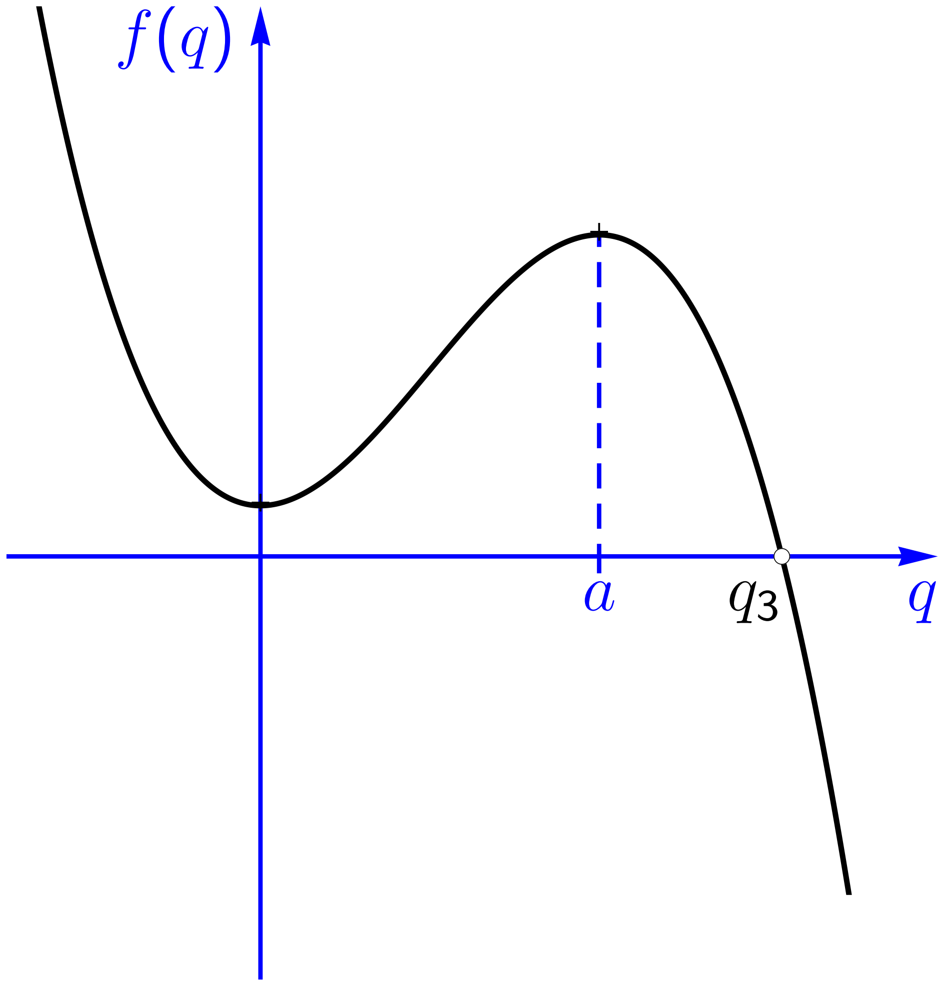

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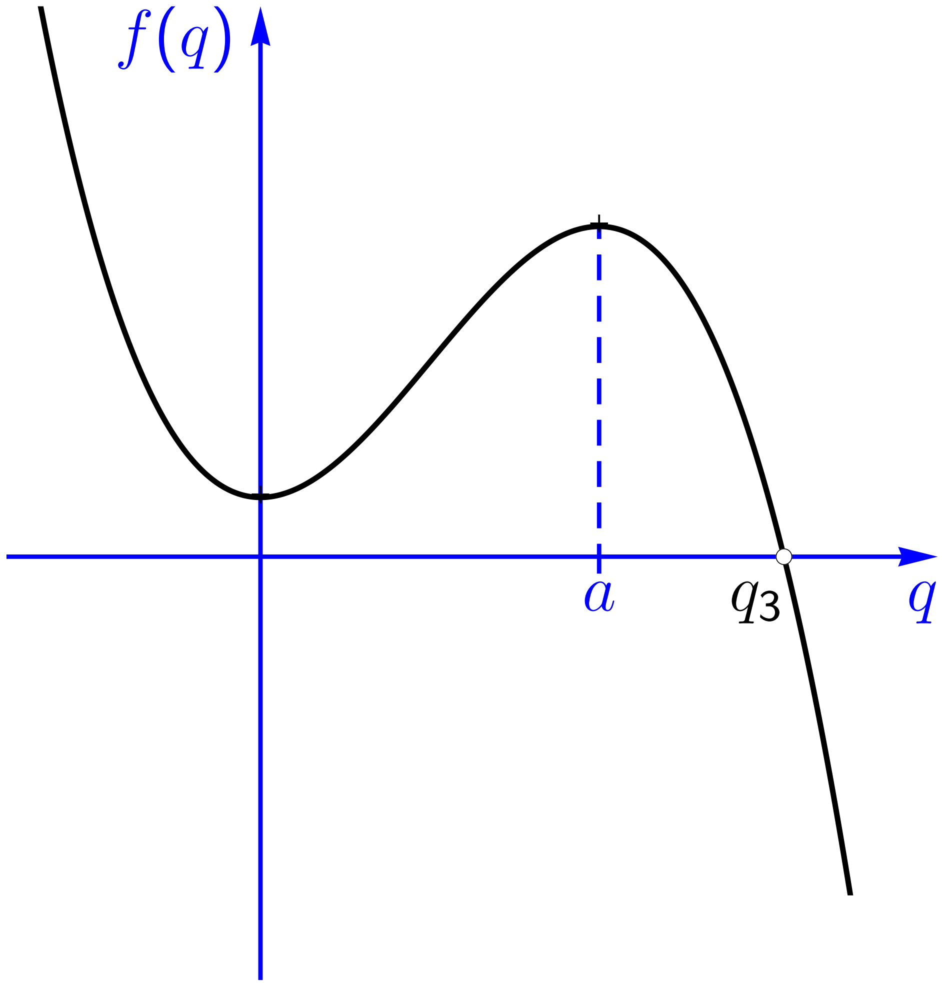

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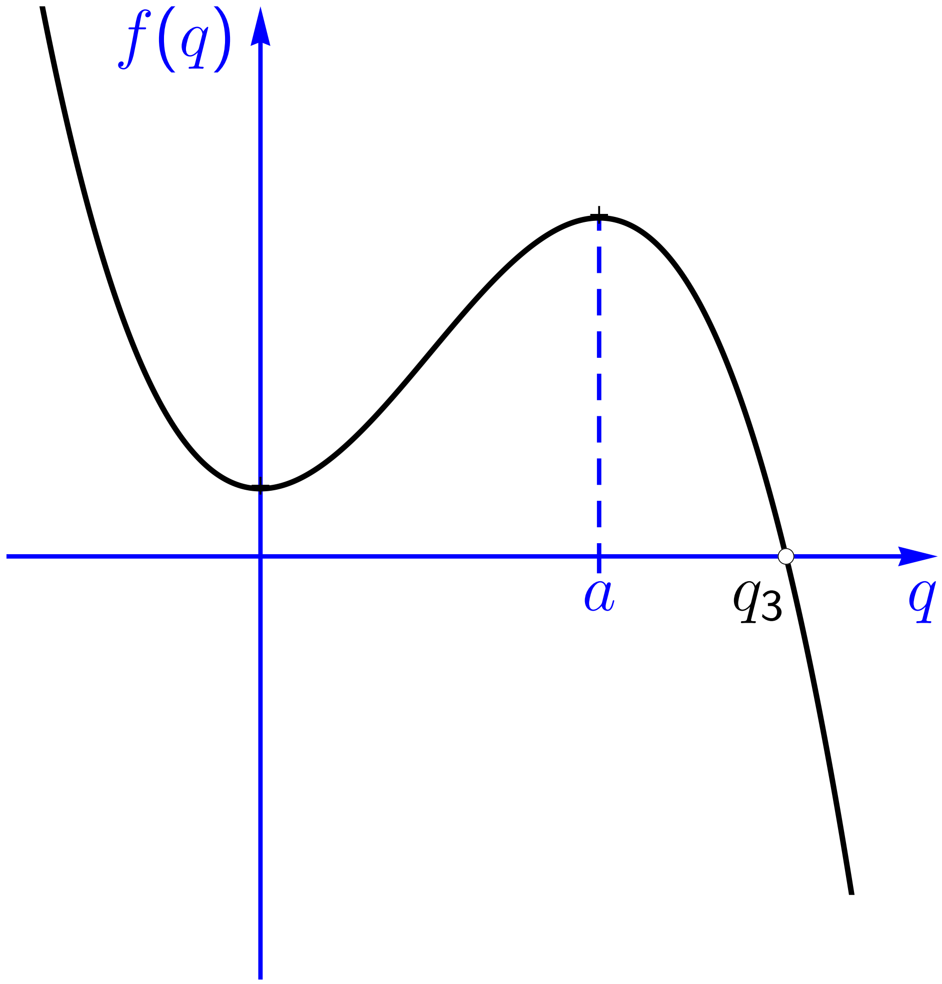

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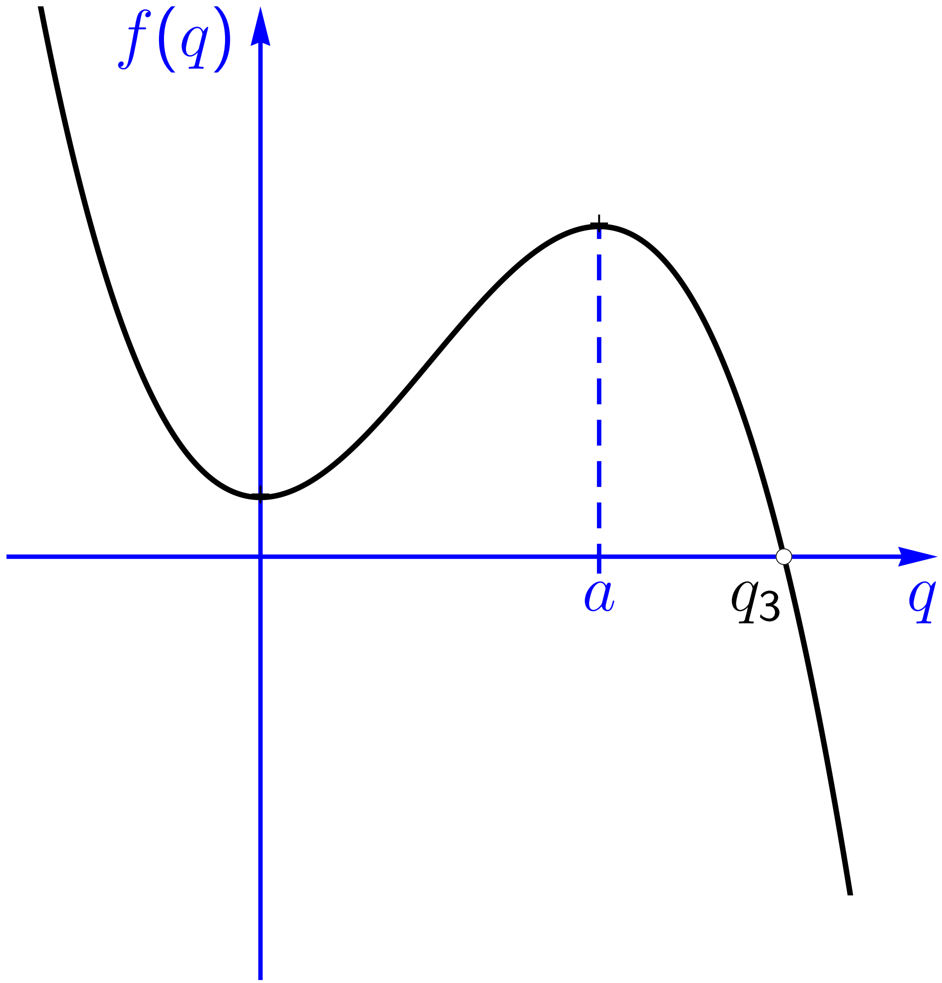

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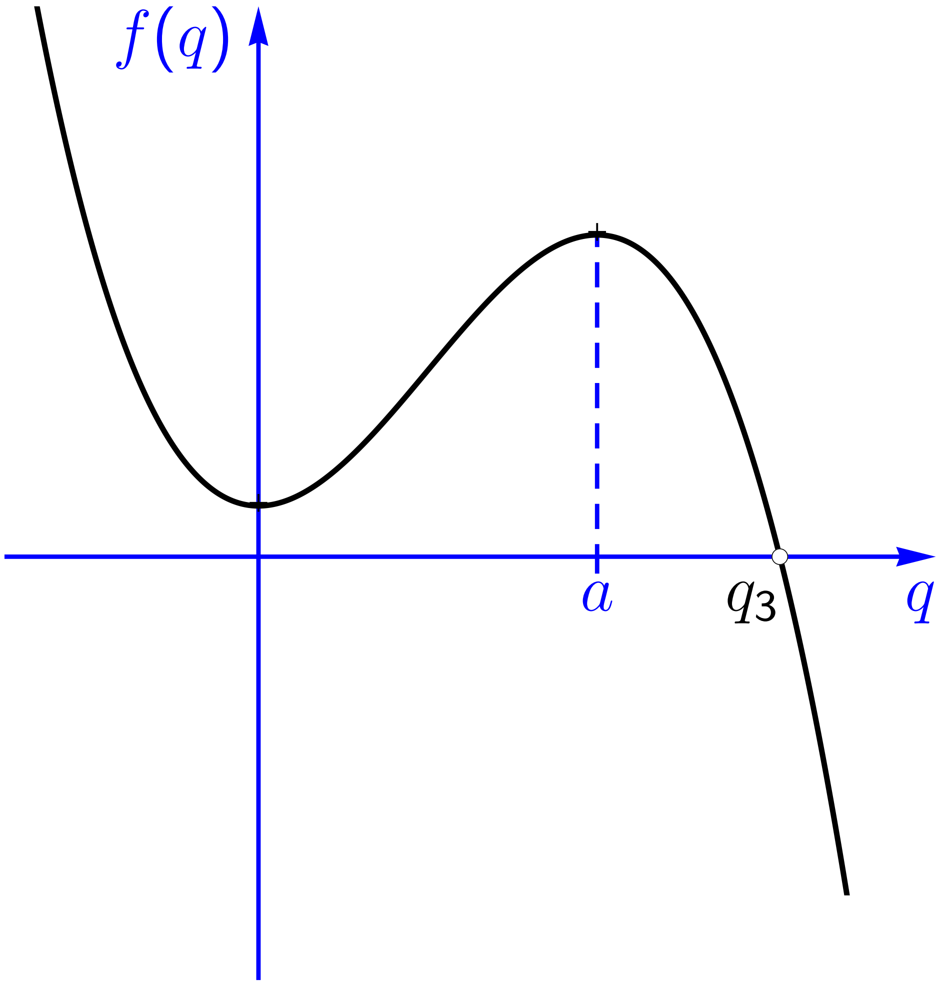

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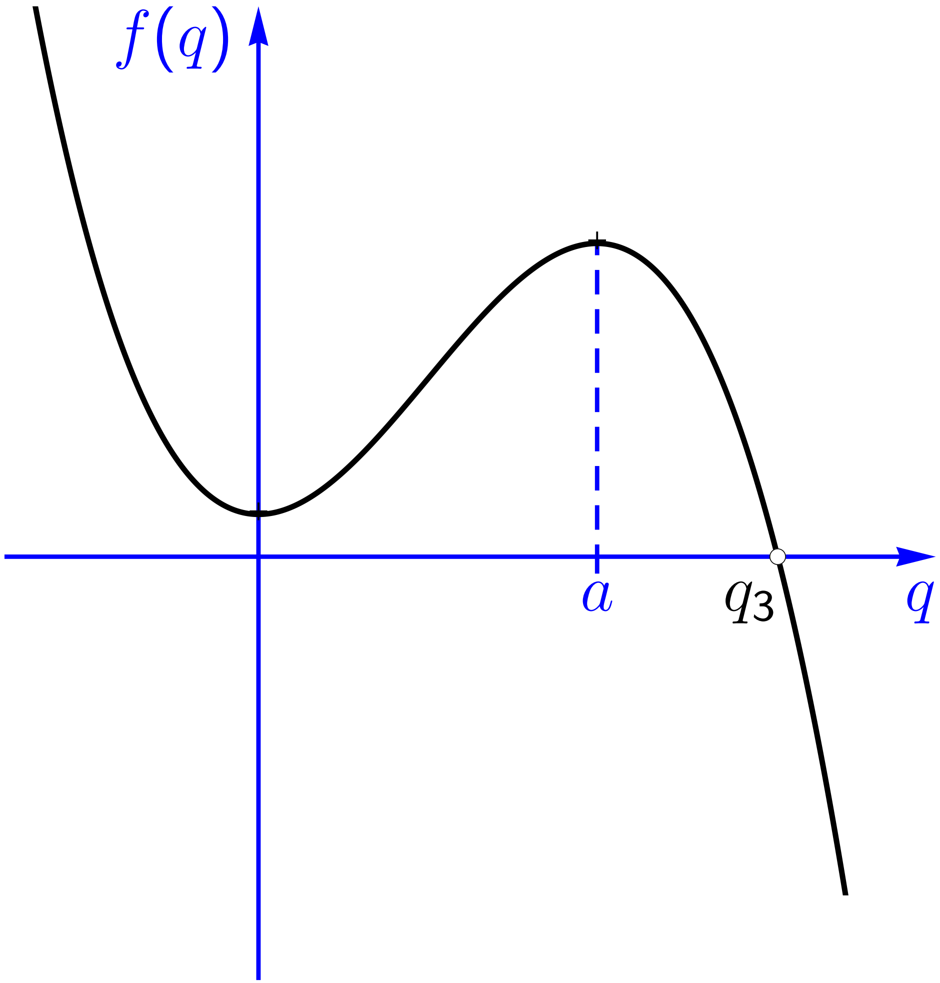

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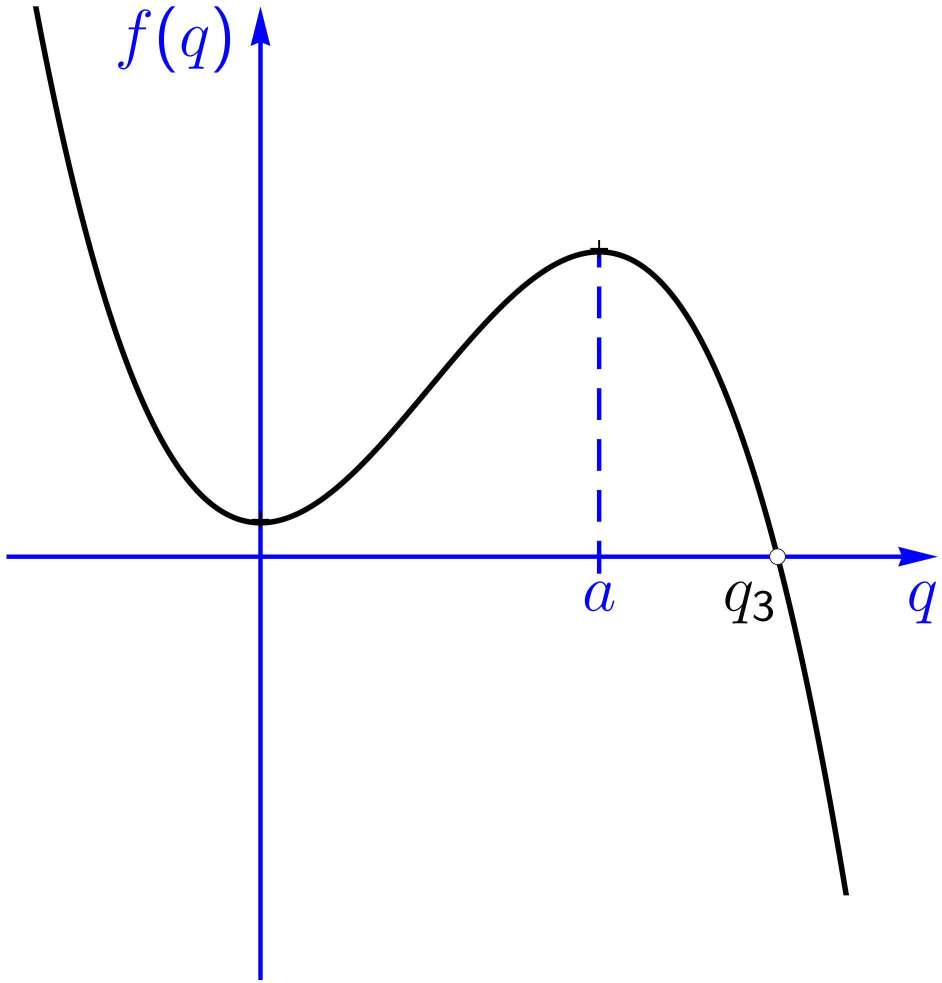

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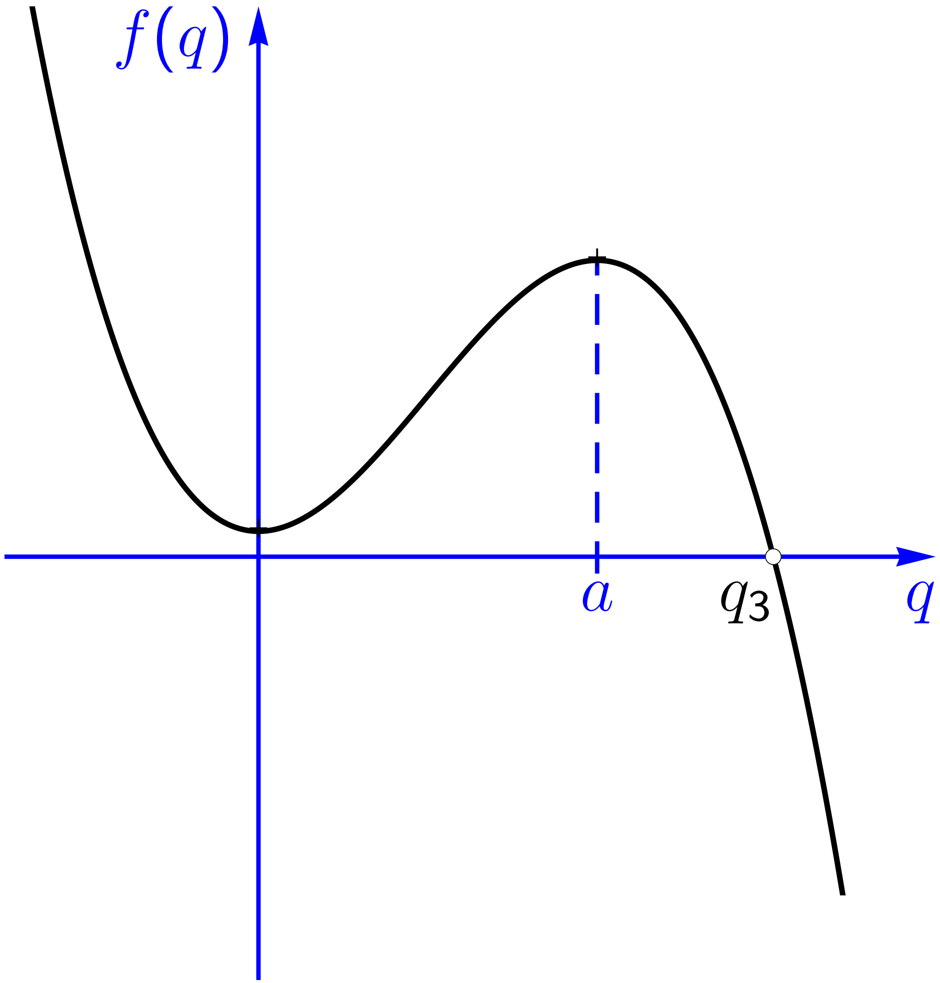

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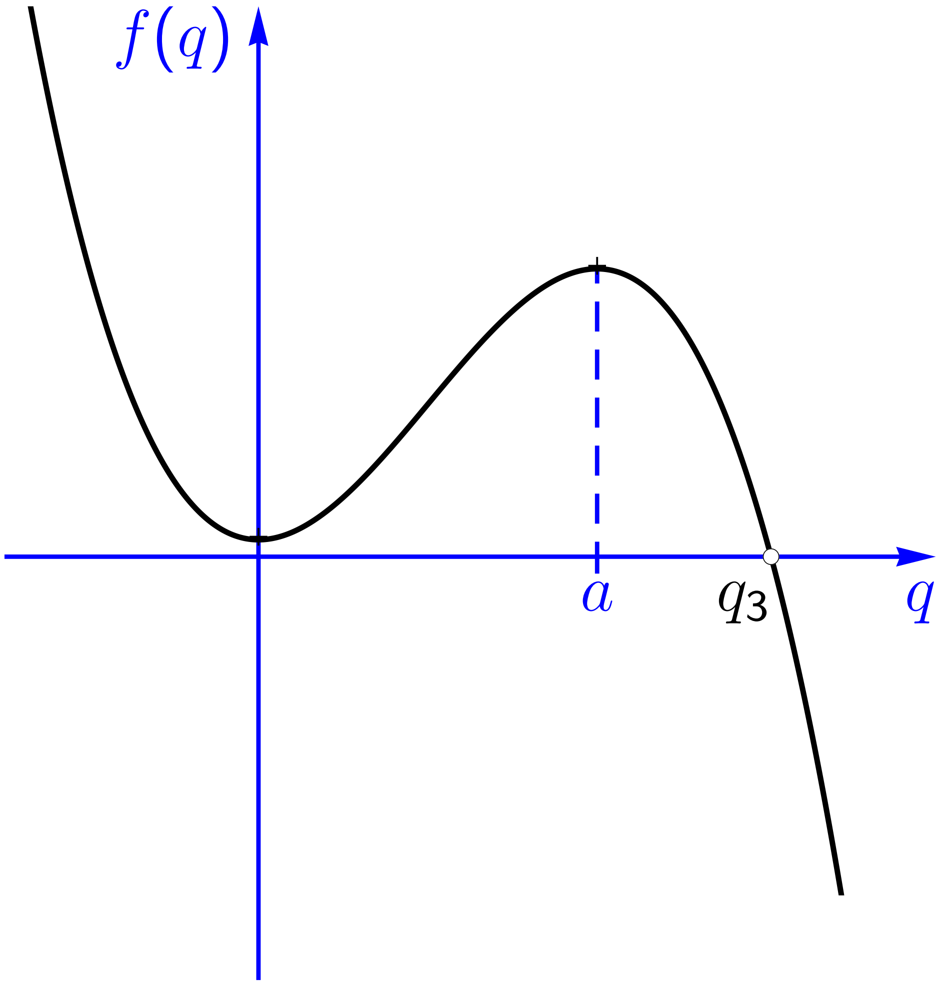

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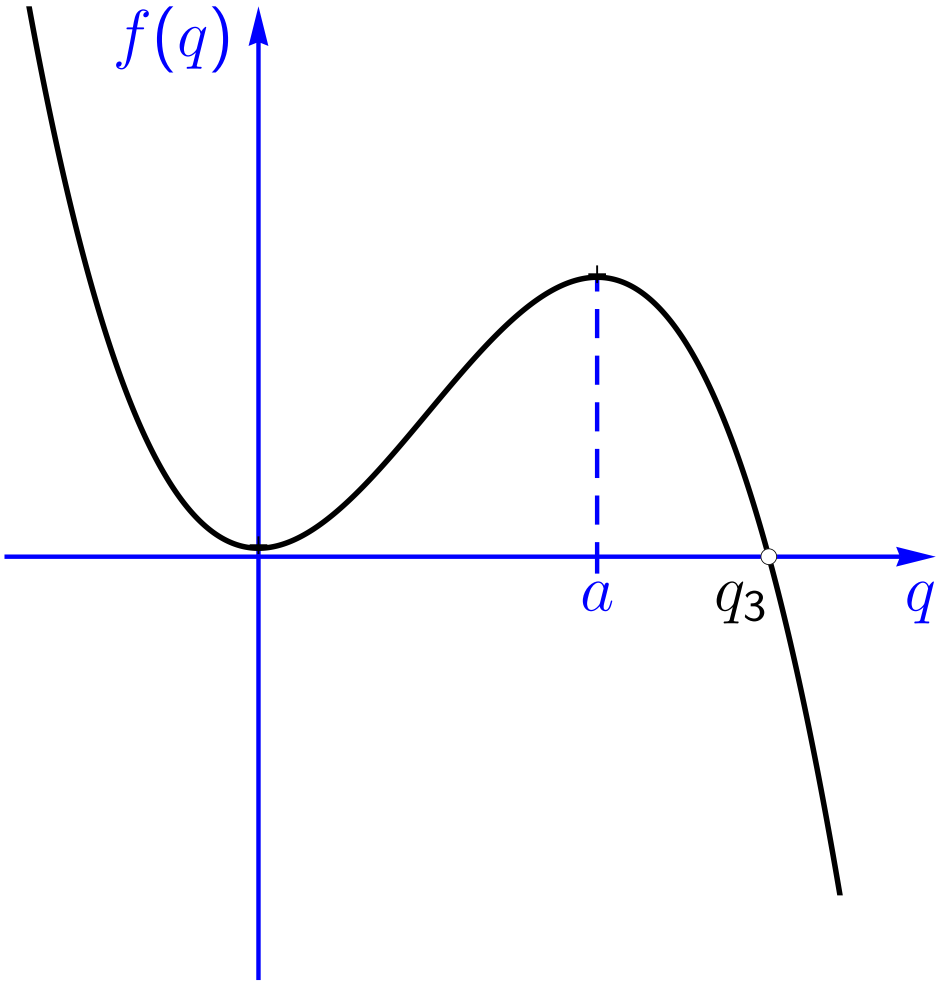

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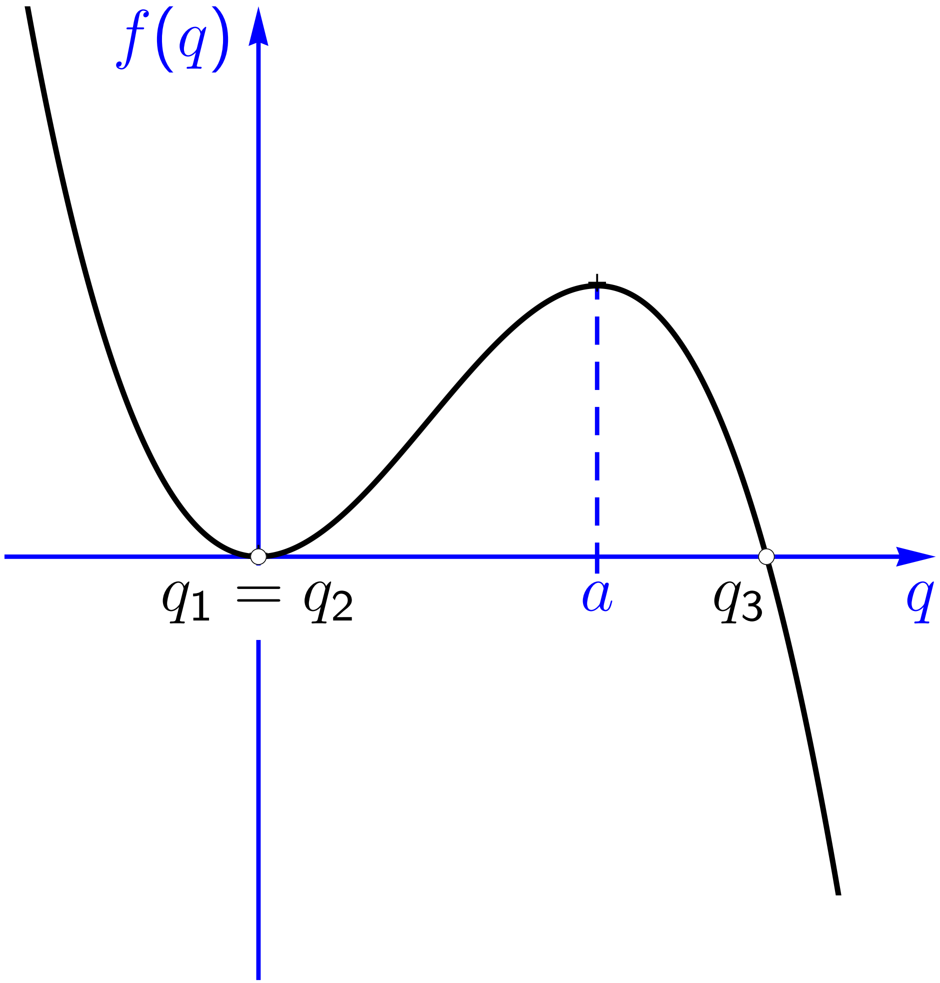

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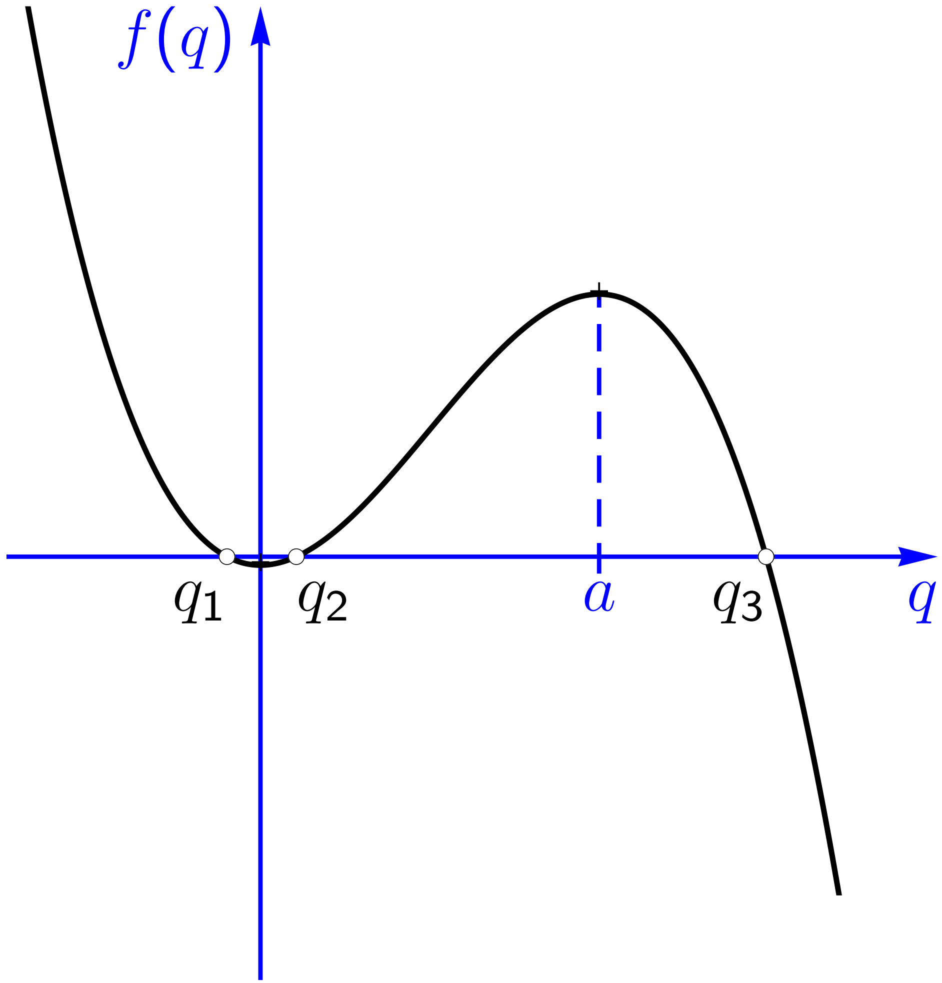

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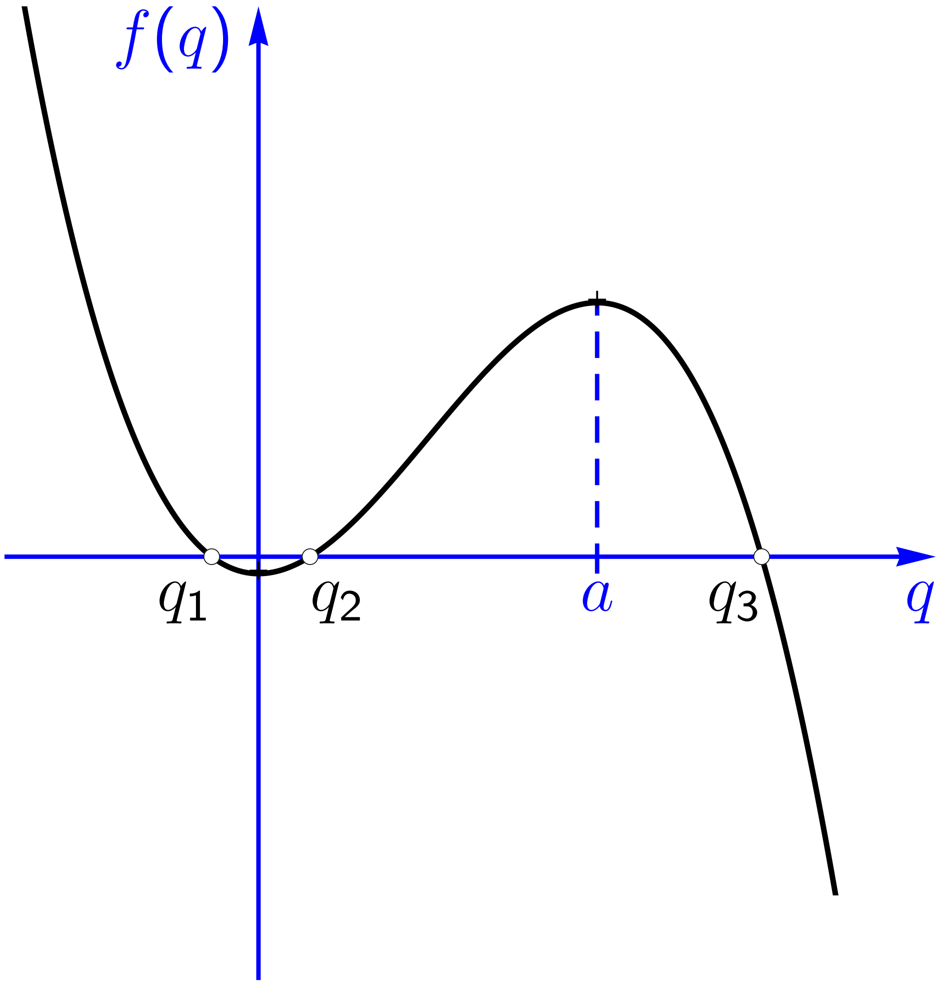

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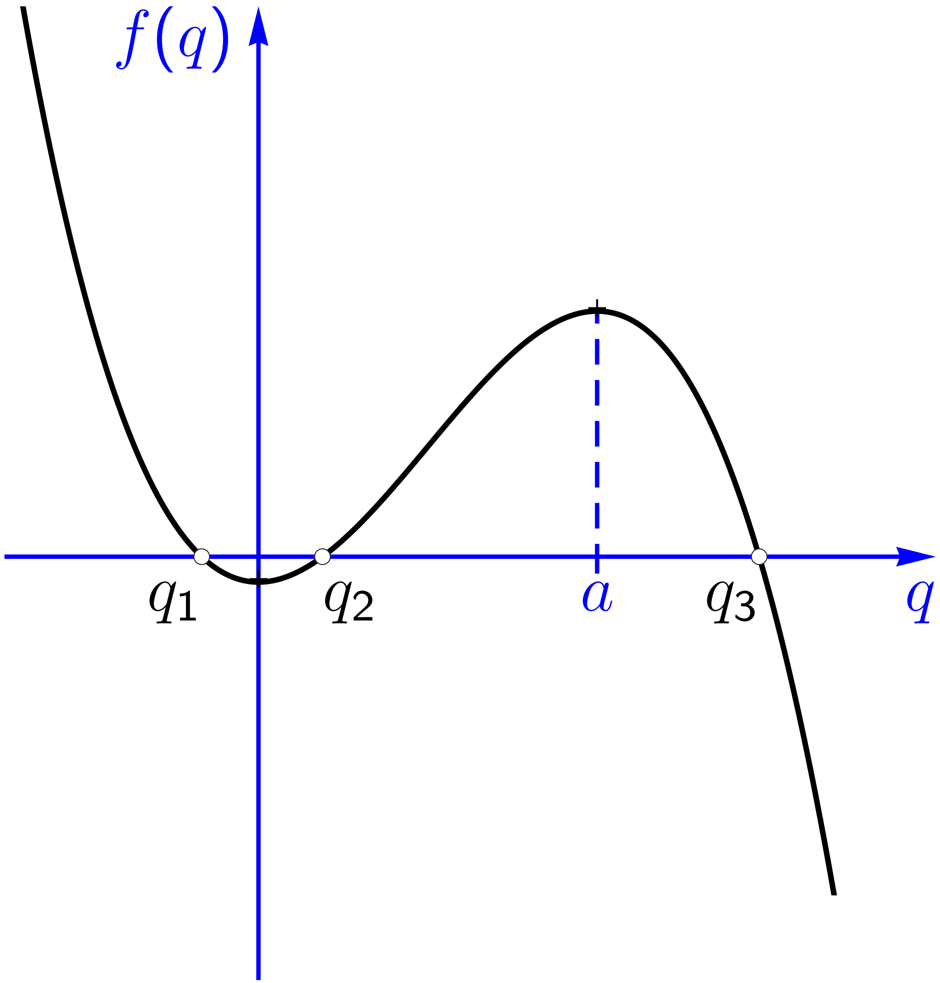
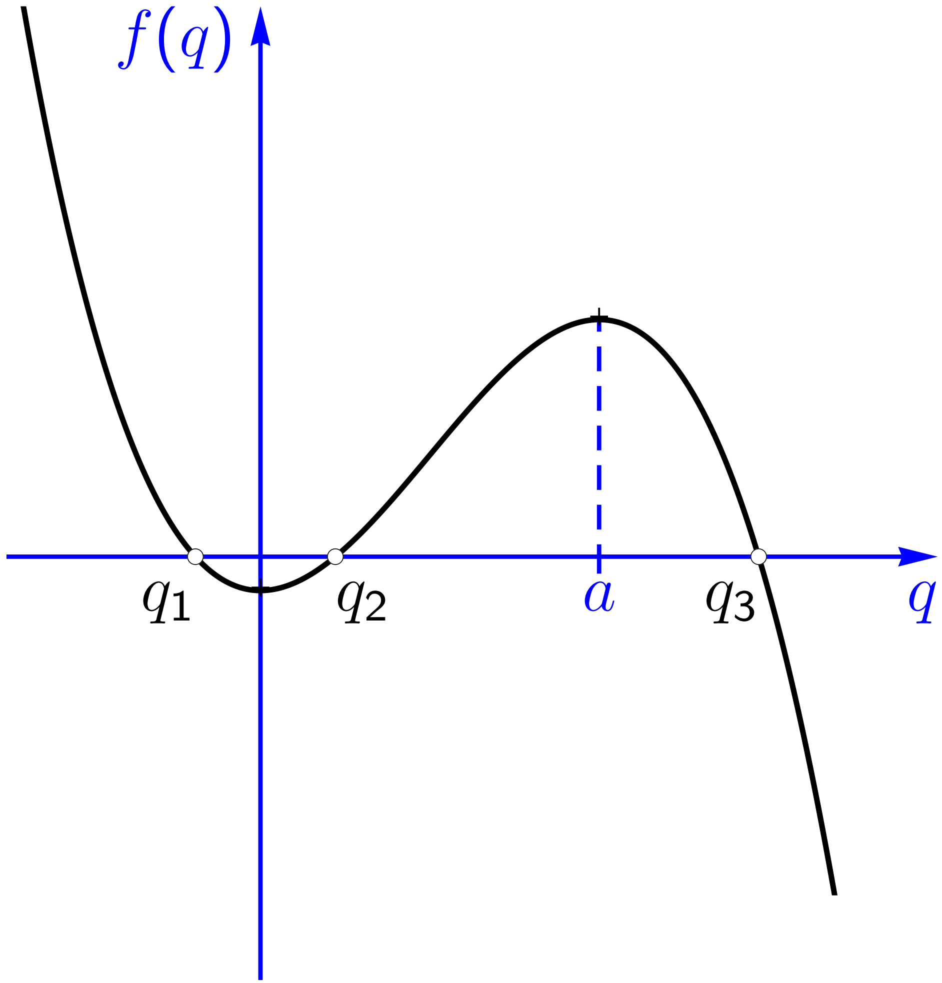
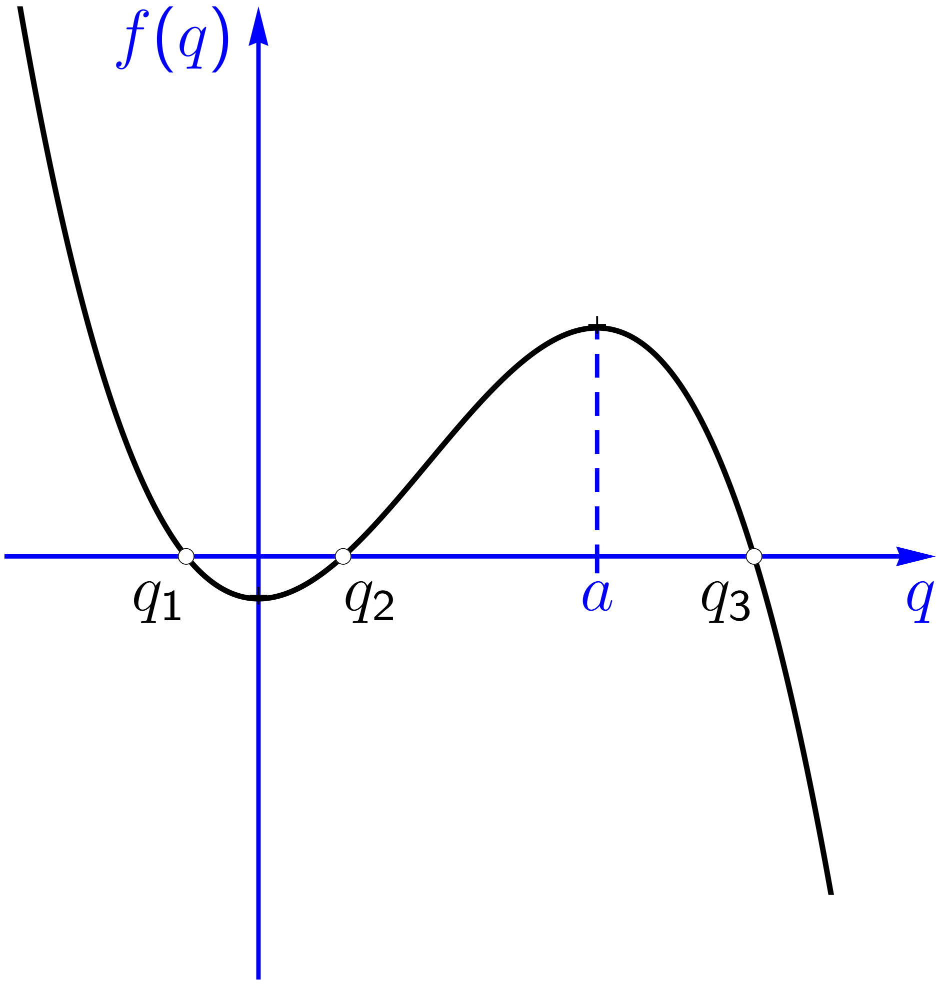
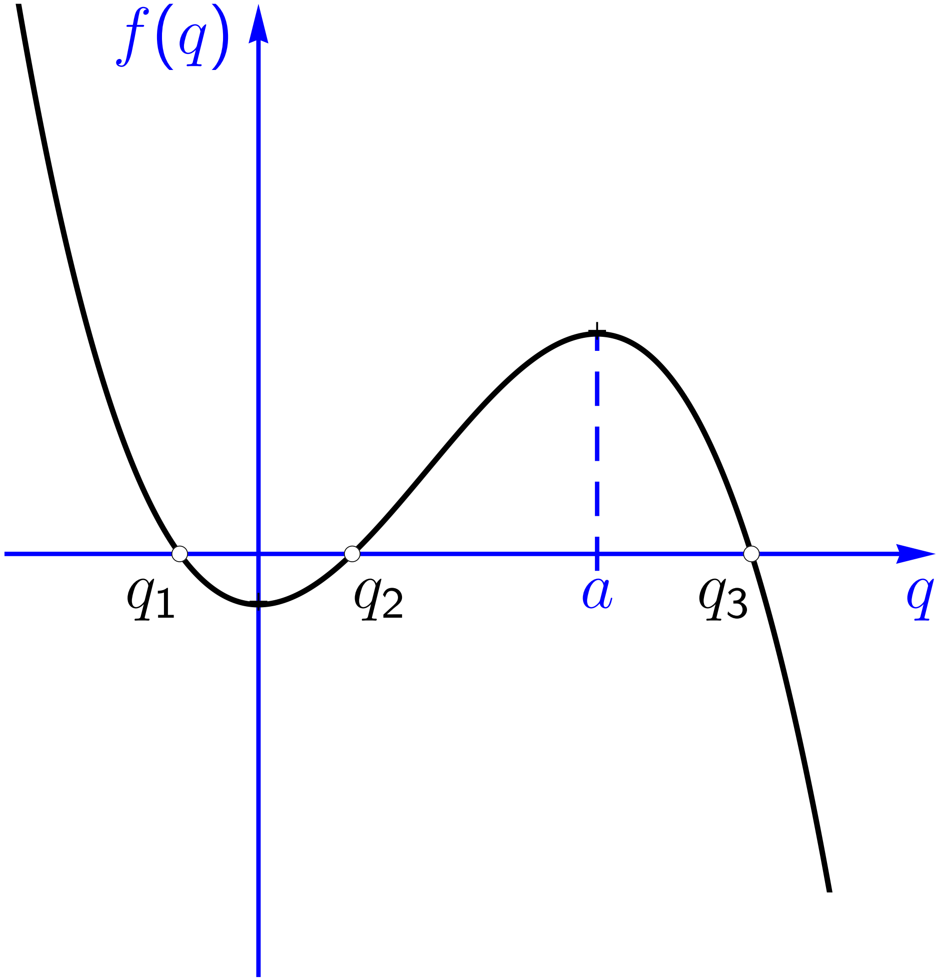


Function f and Phase Curves




$f(q)$
q_1
q_2
a
q_3
q

Function f and Phase Curves



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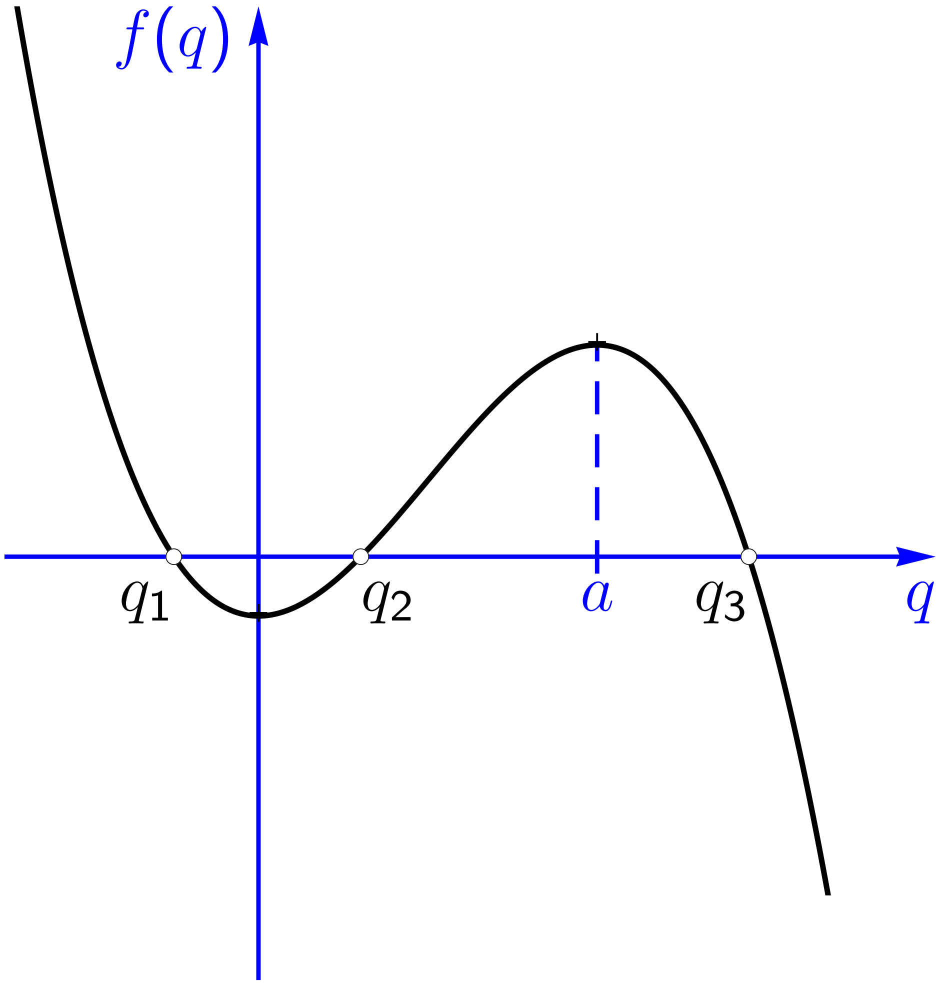

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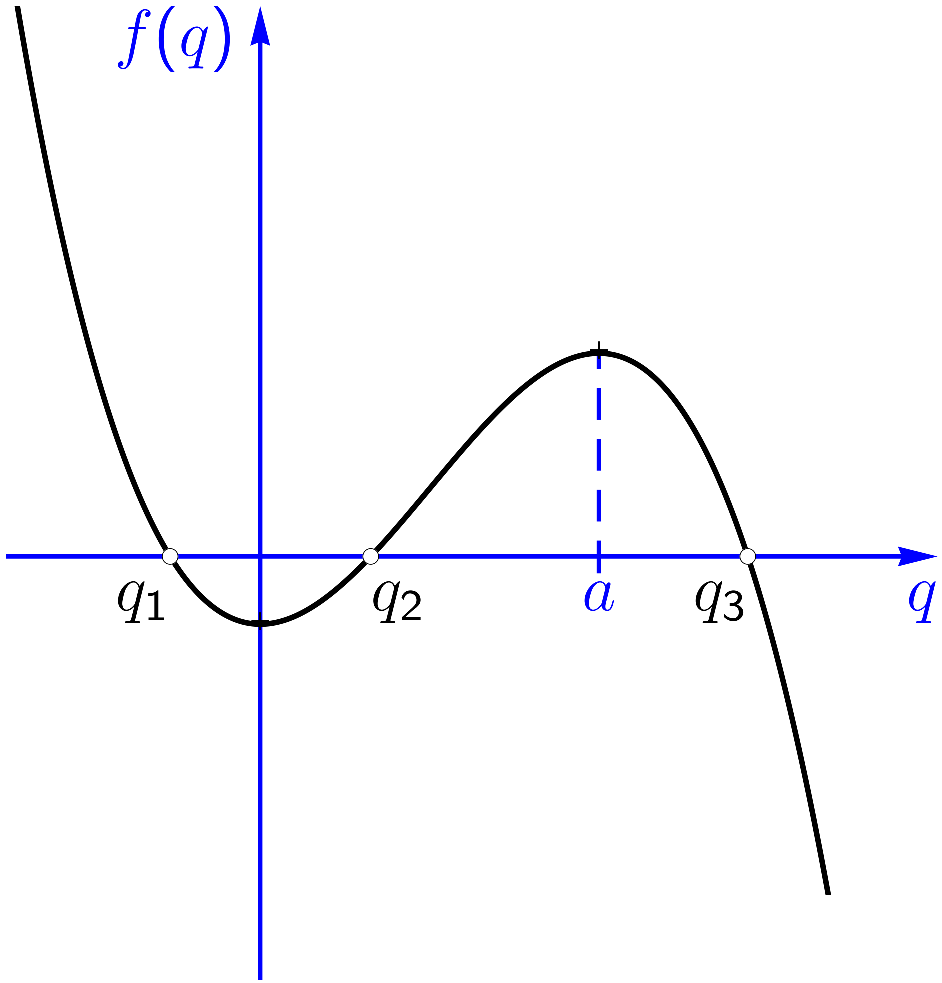

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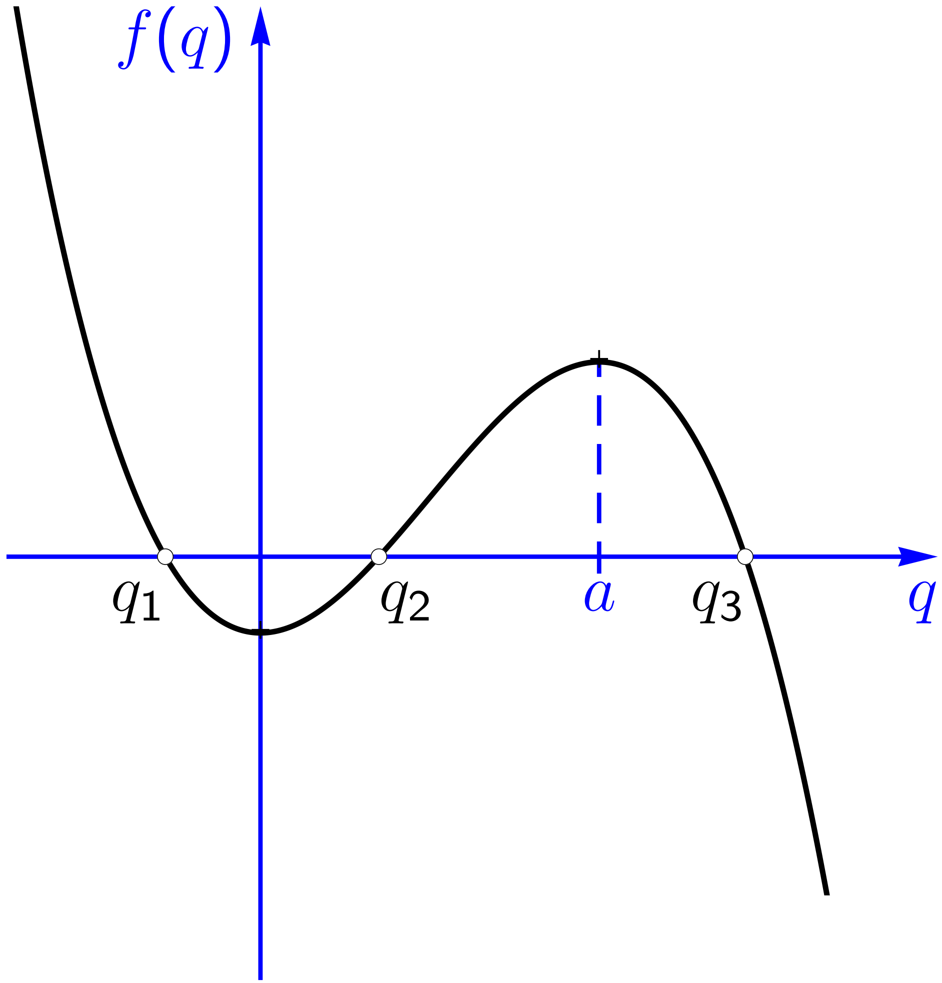

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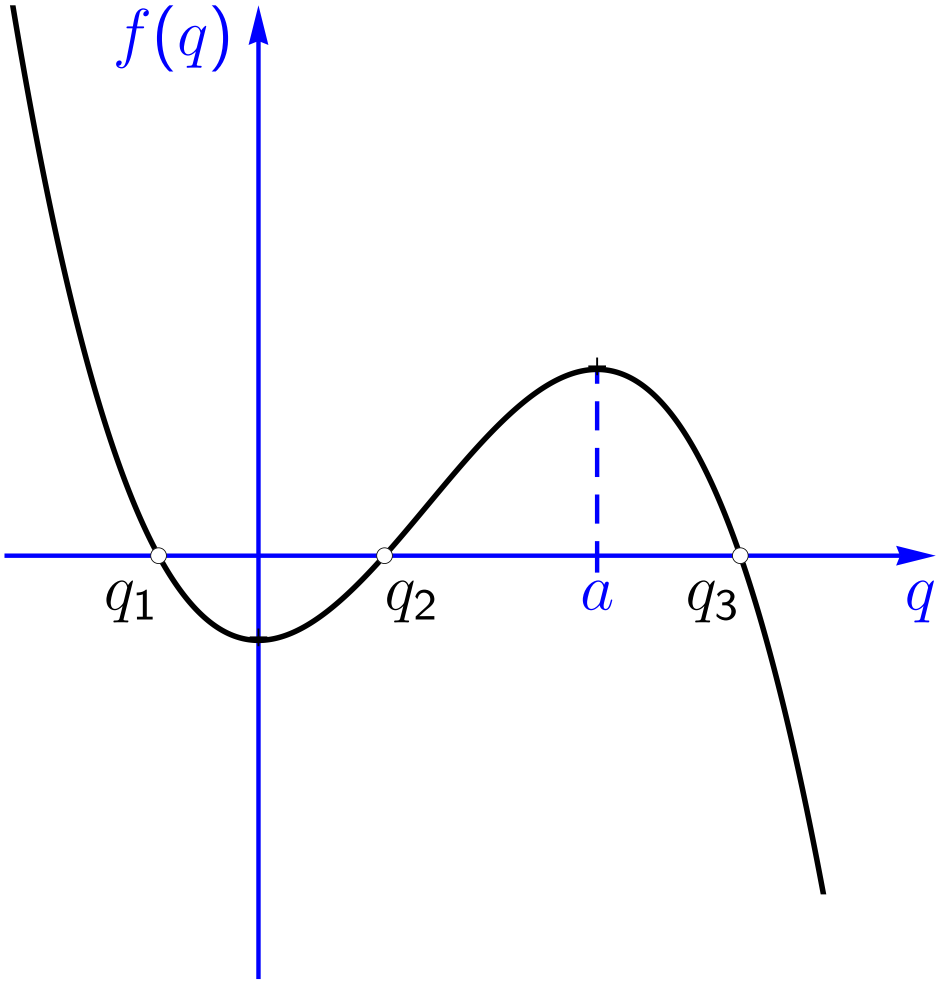

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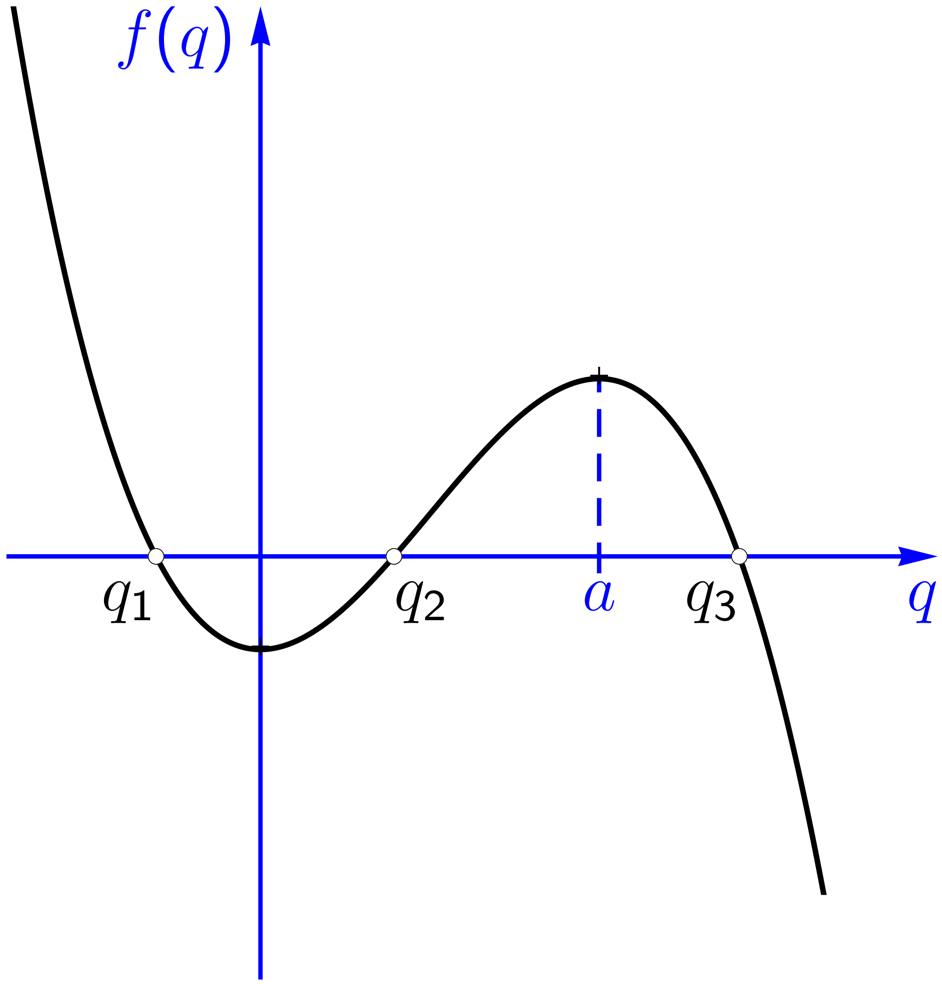

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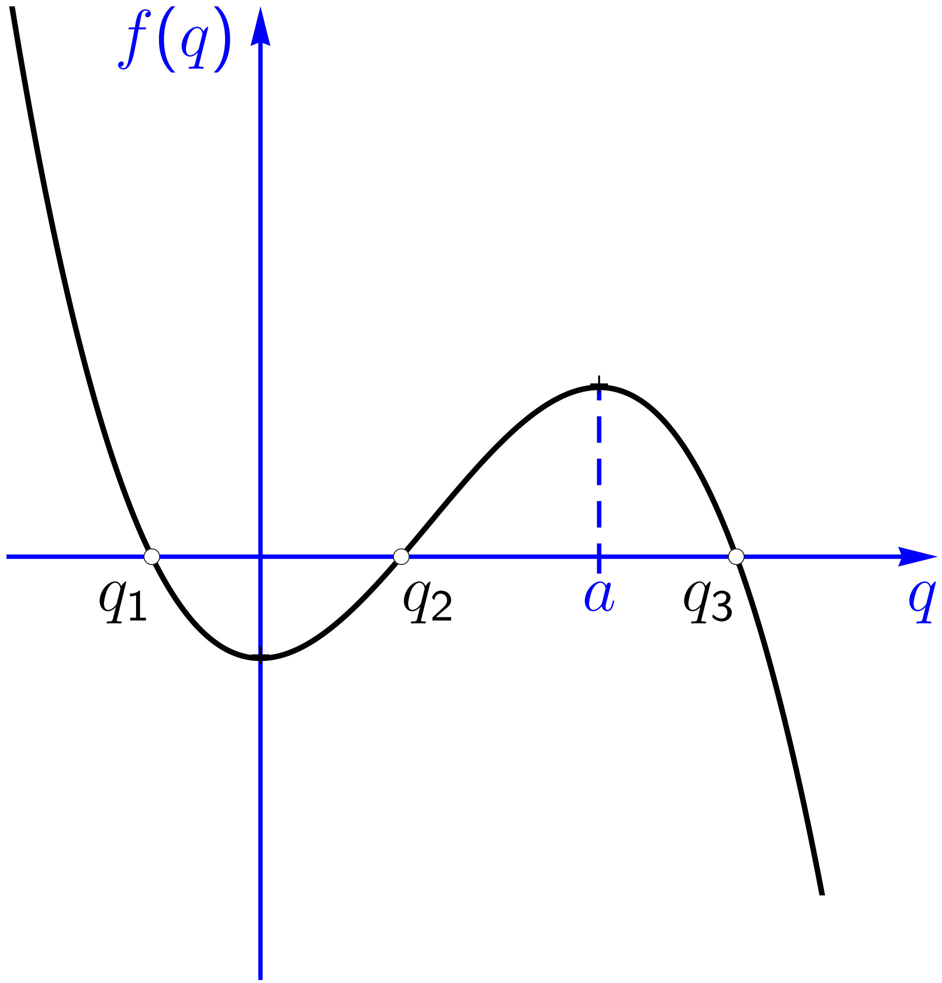

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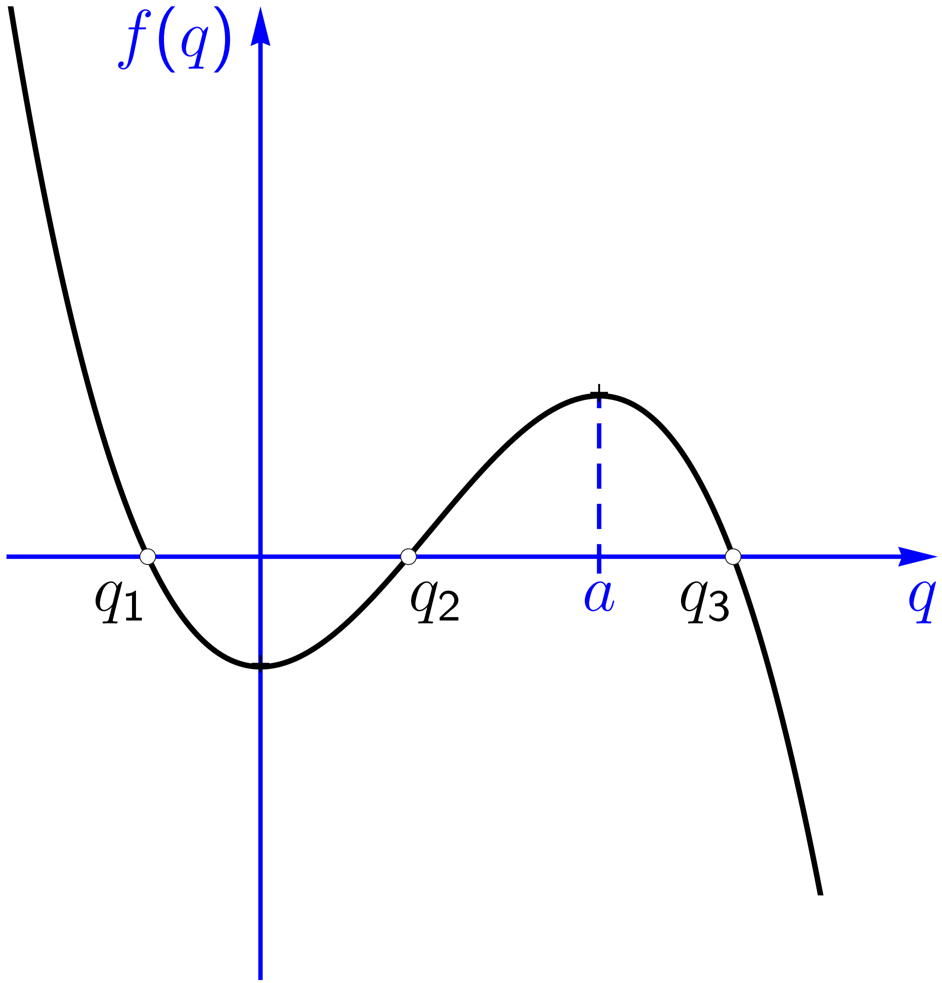

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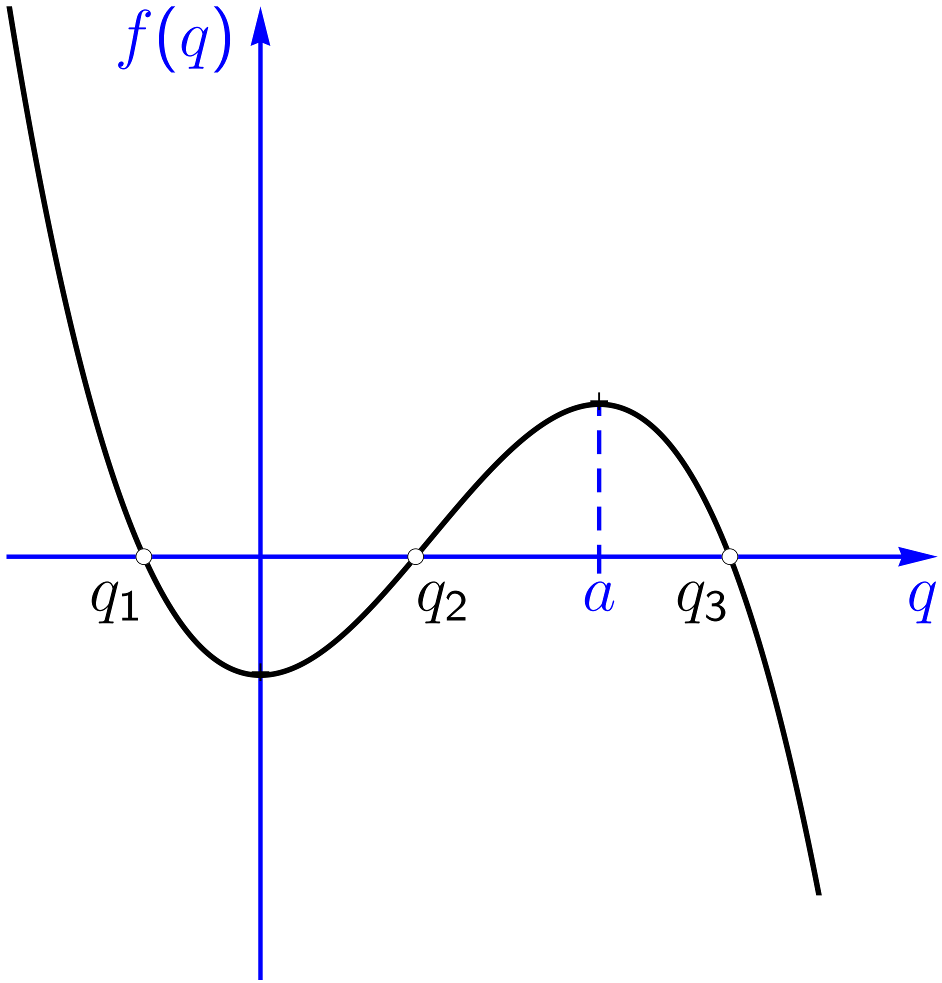

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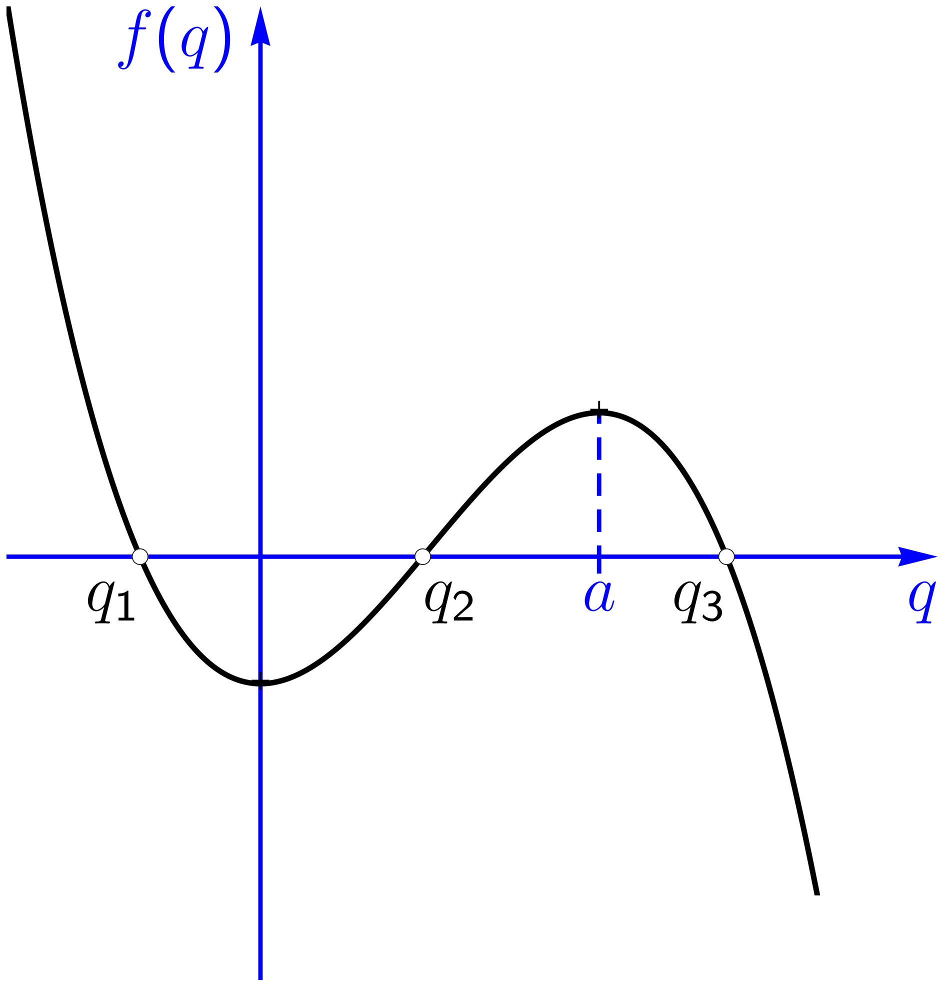

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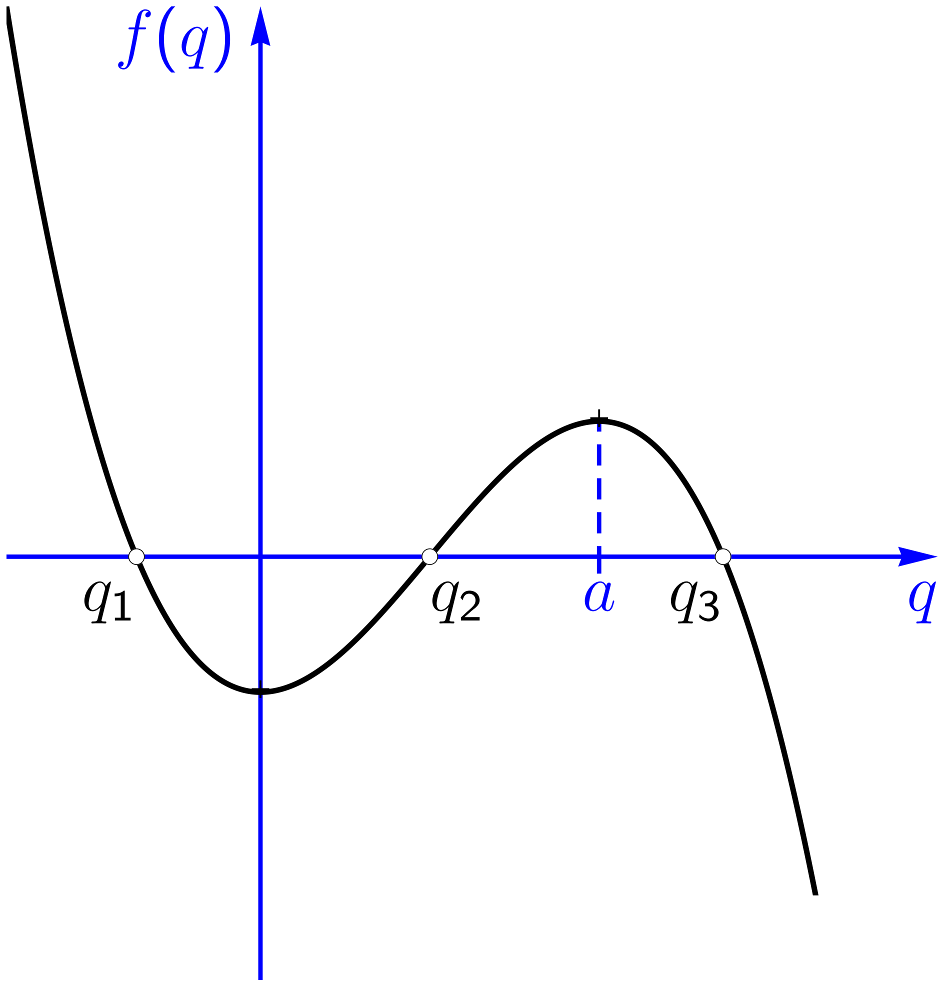

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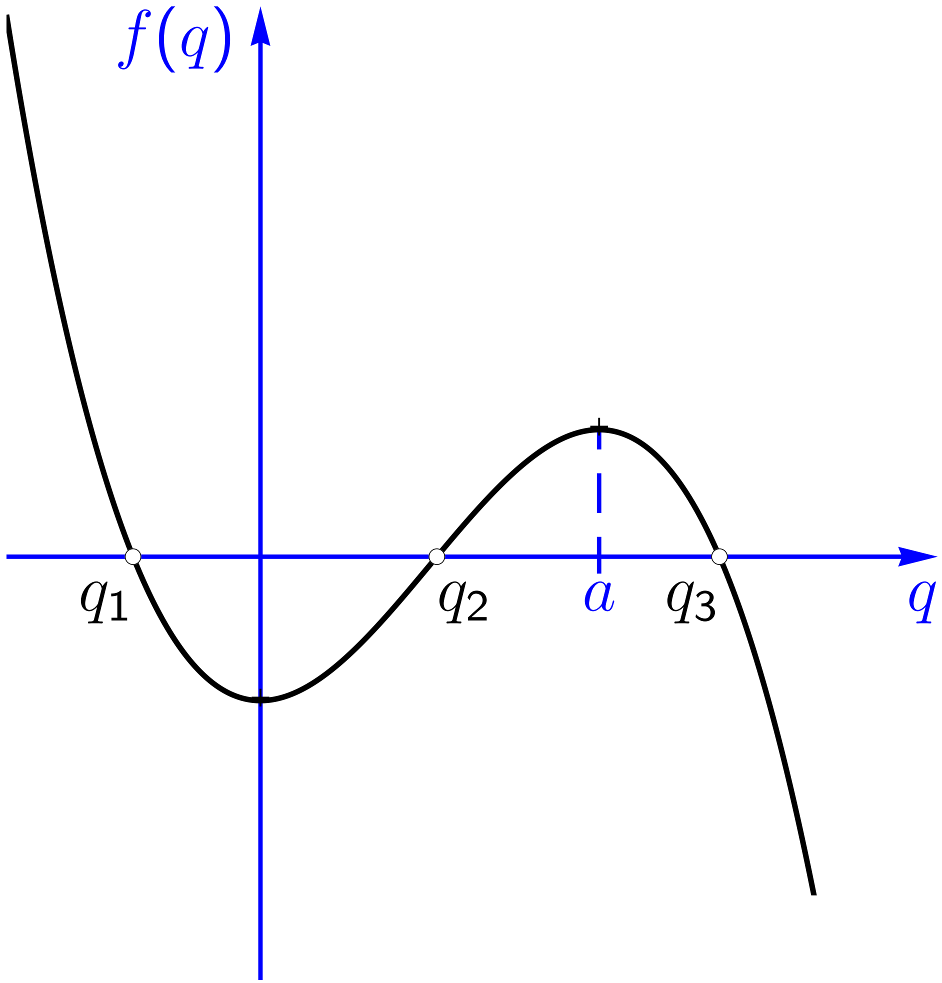

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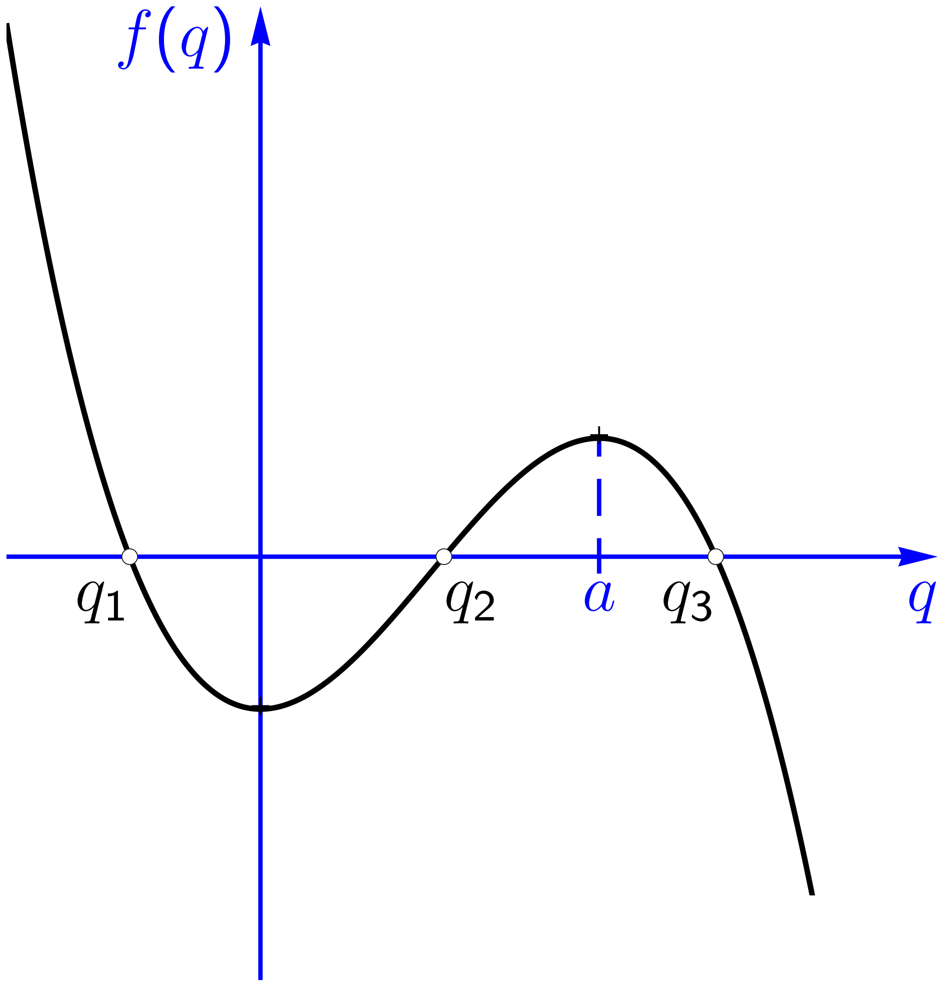

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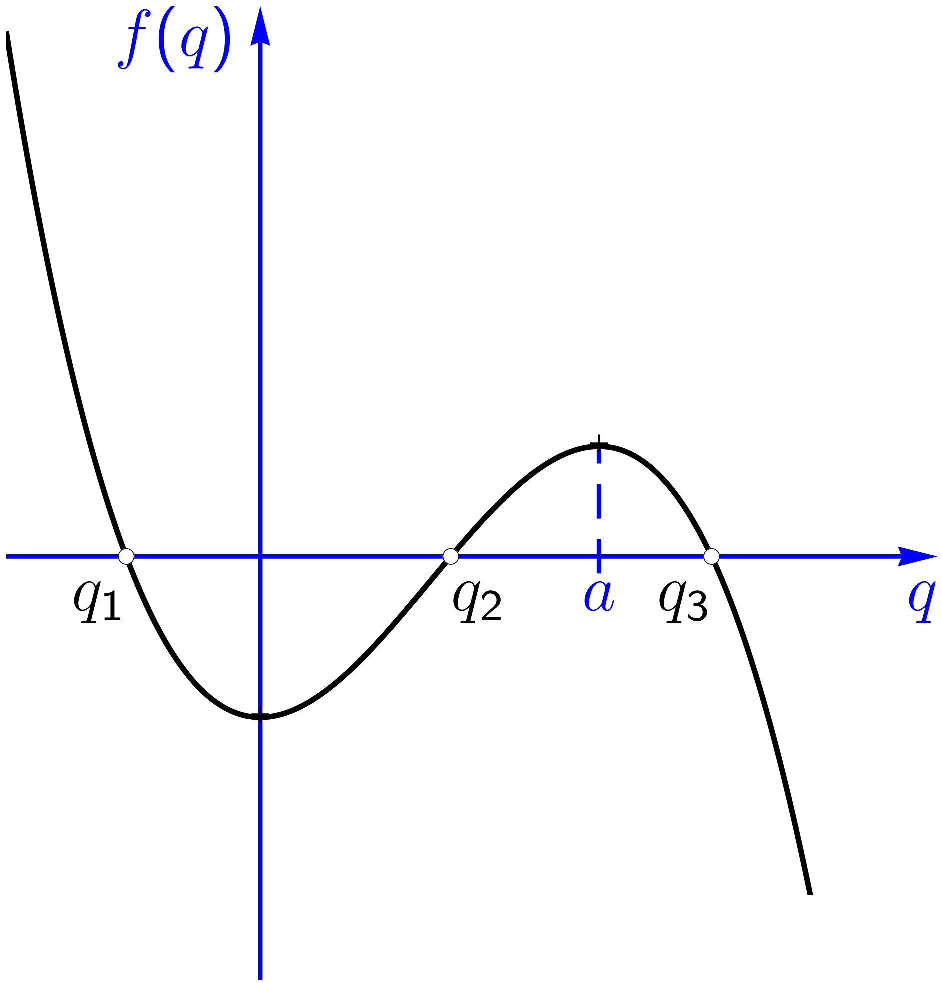

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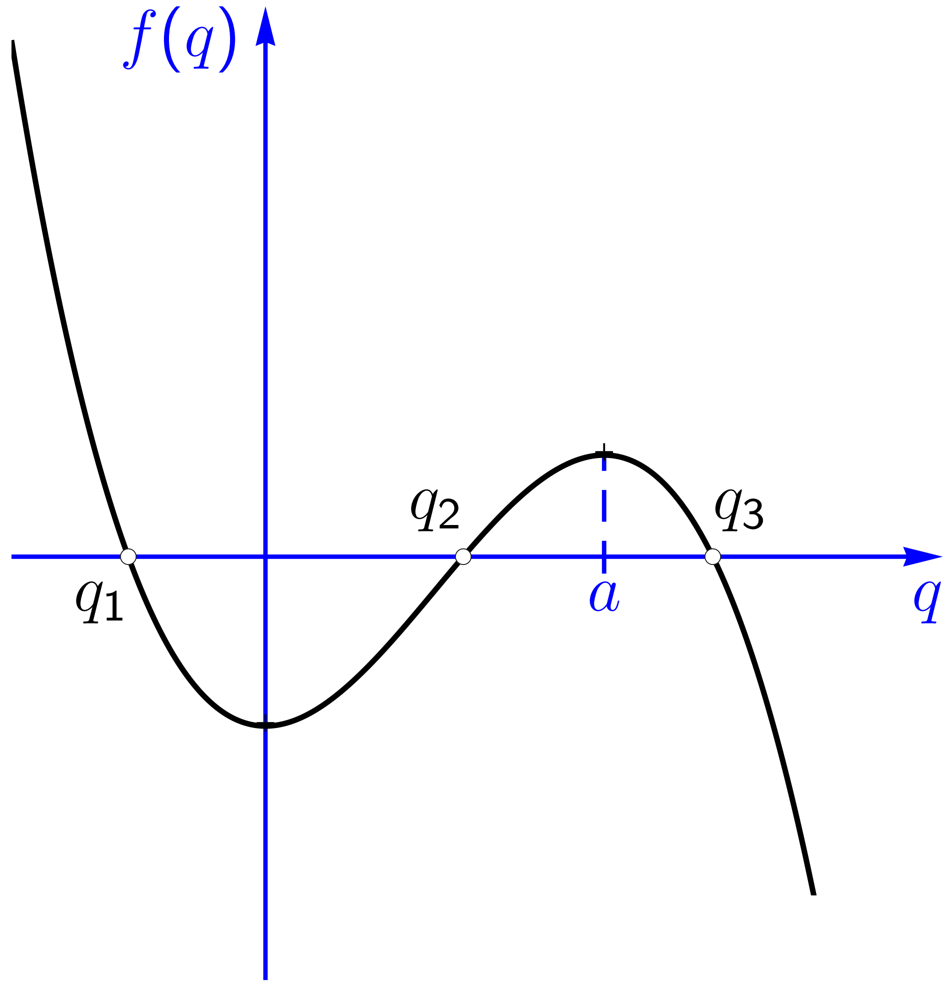

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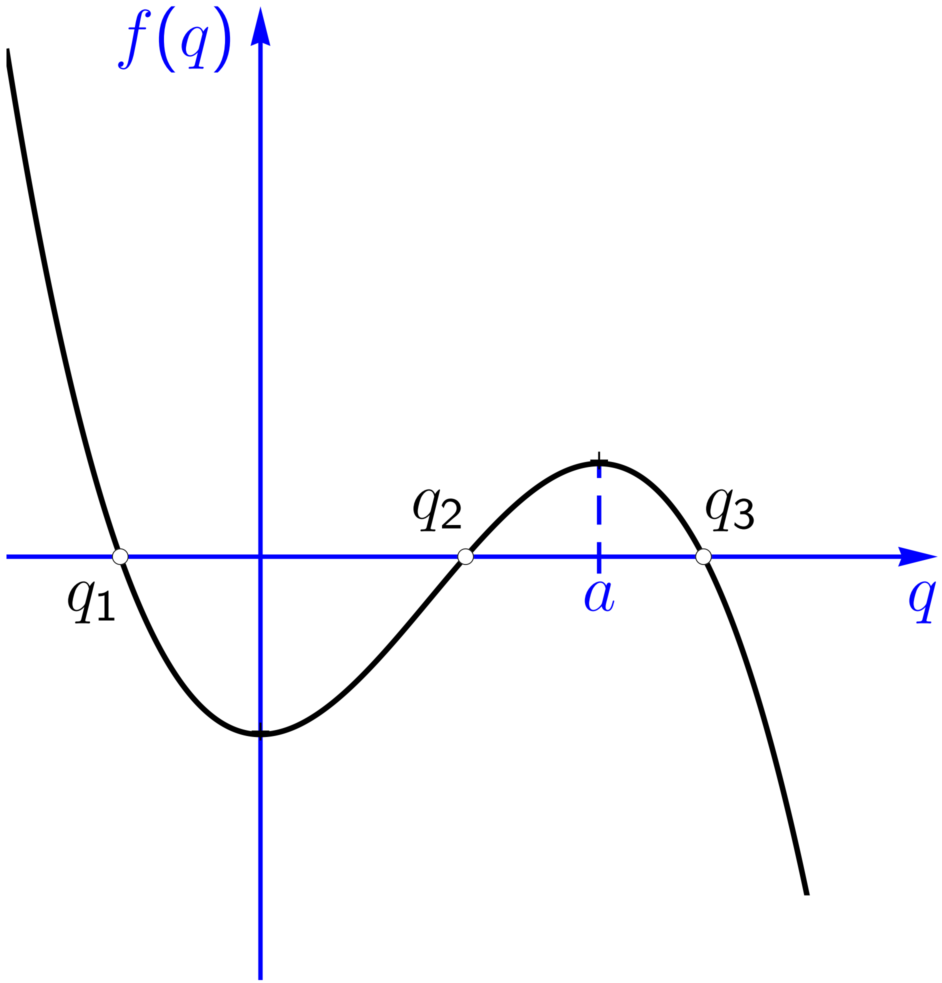

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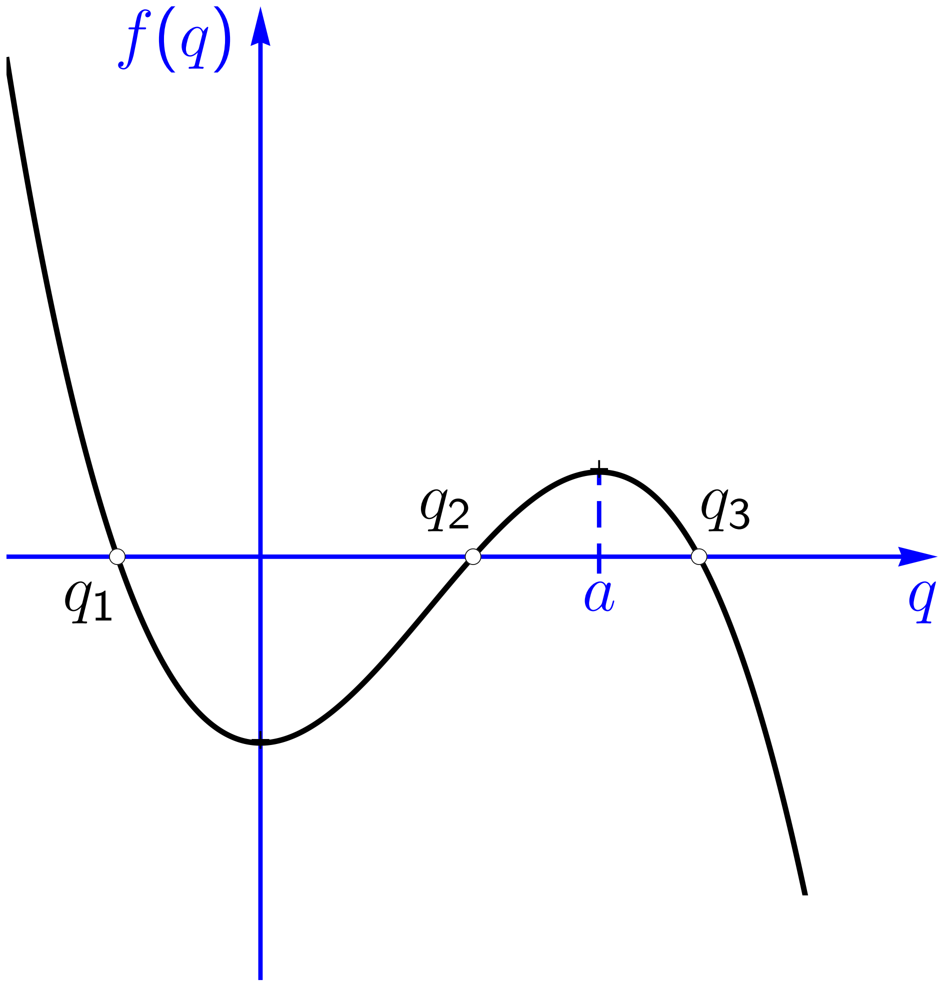

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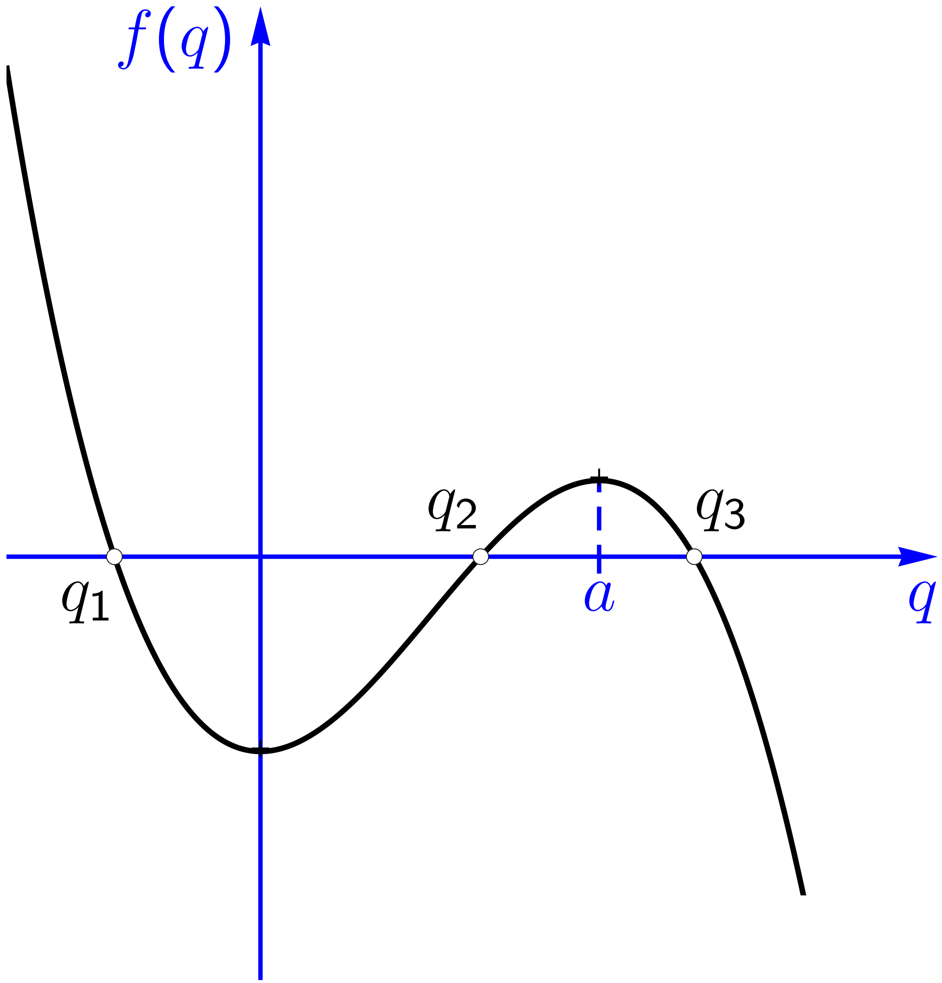

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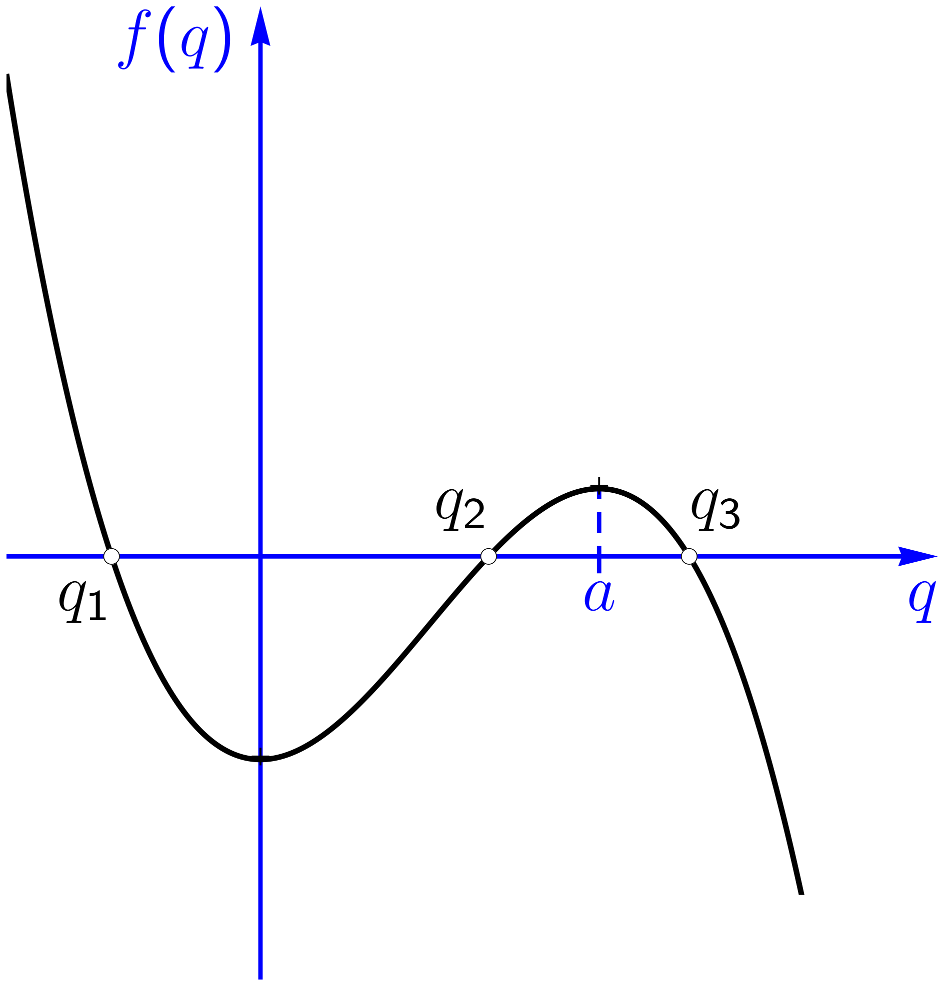

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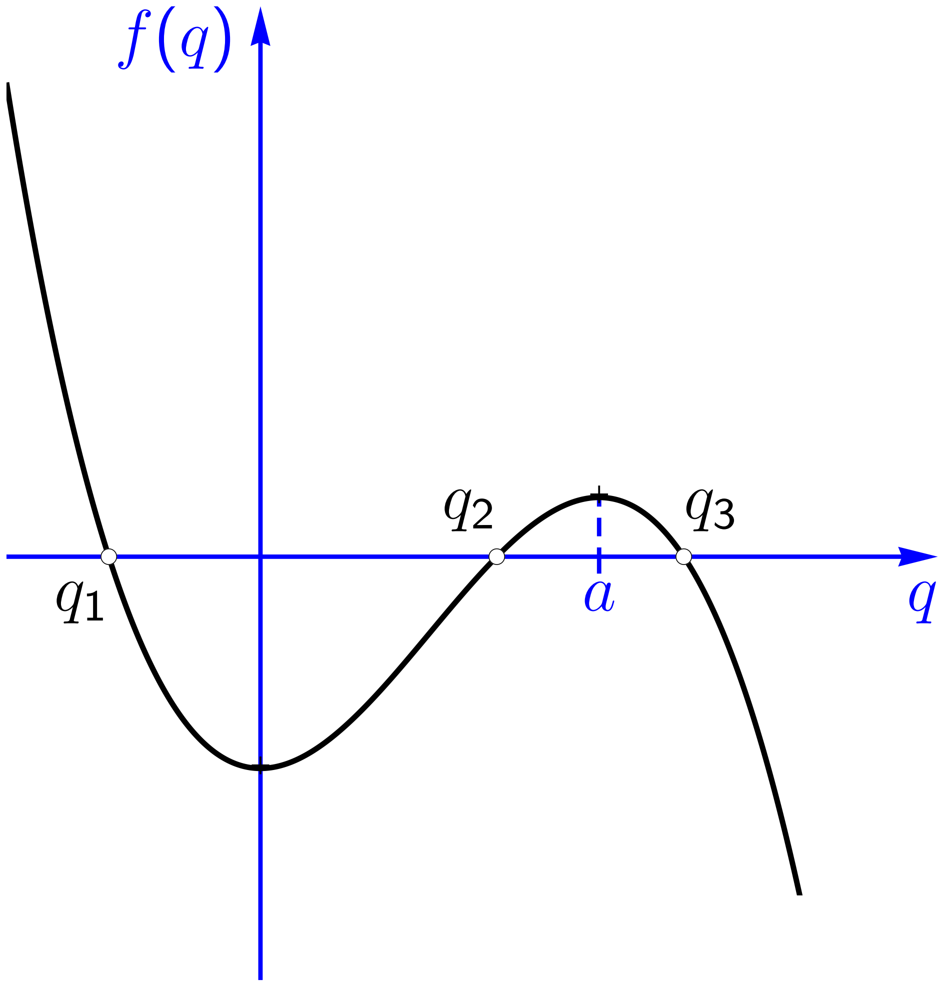

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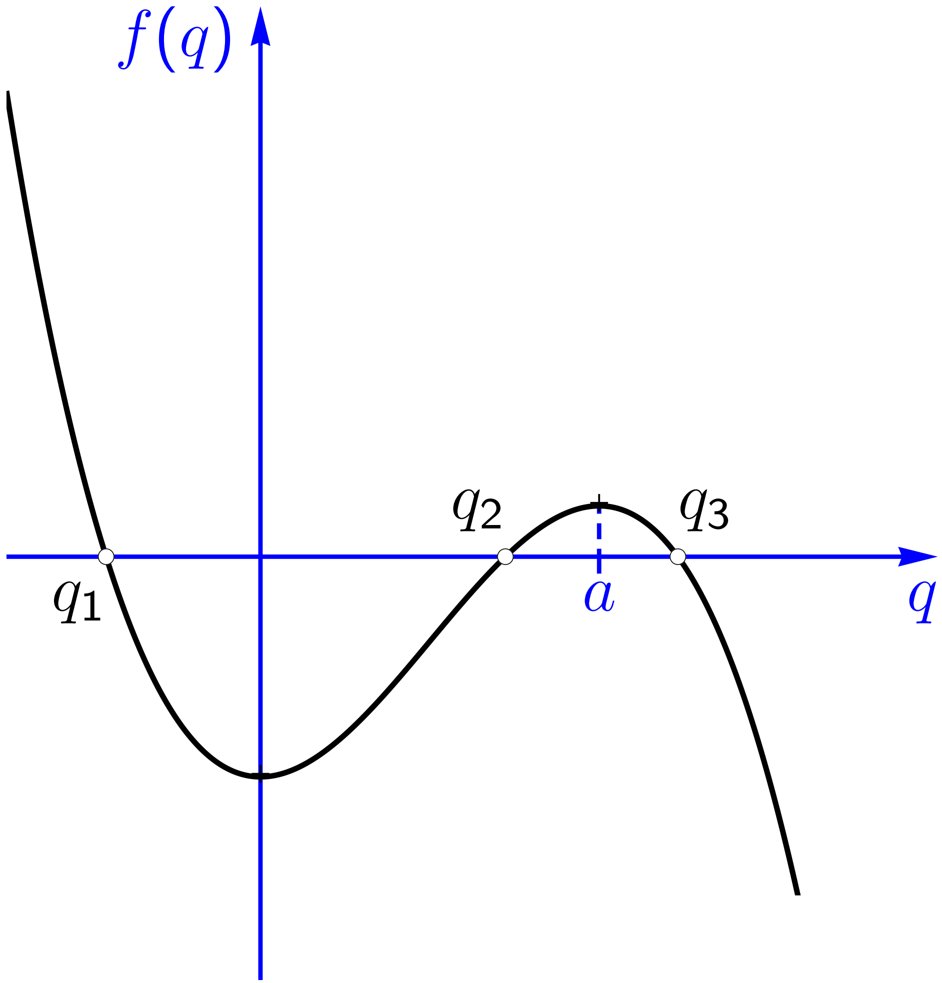

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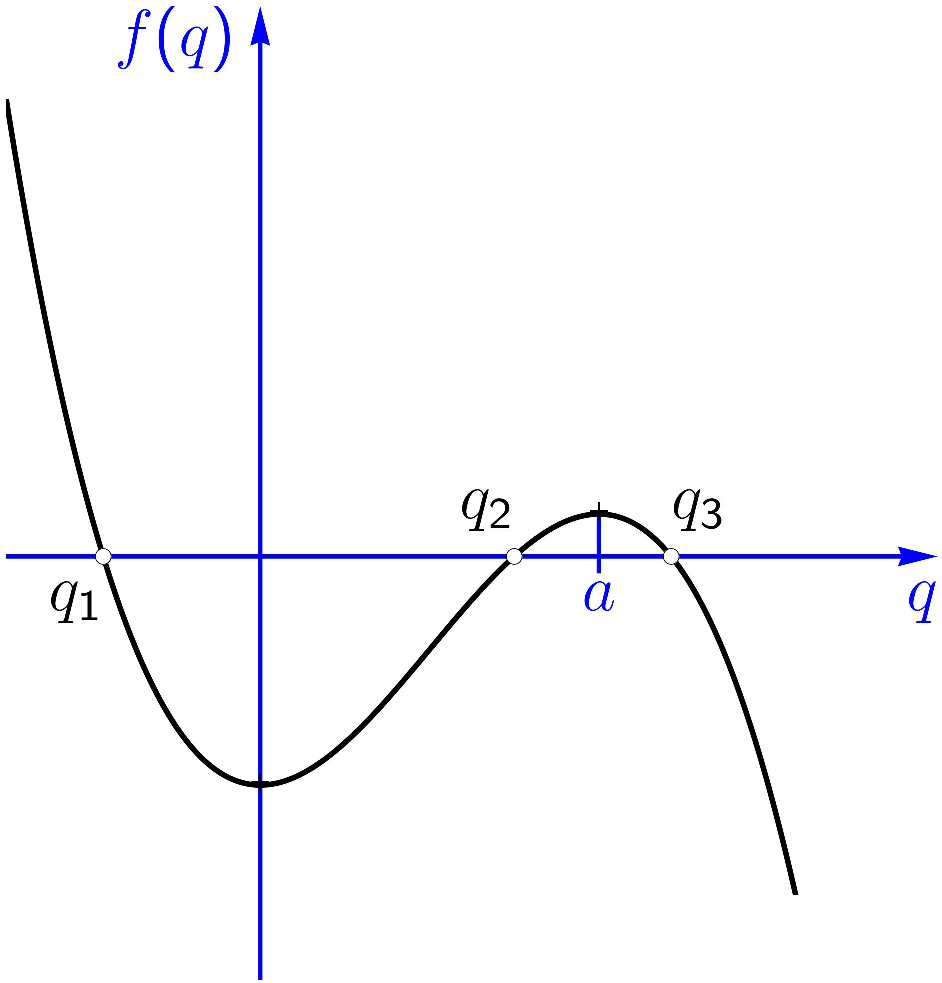

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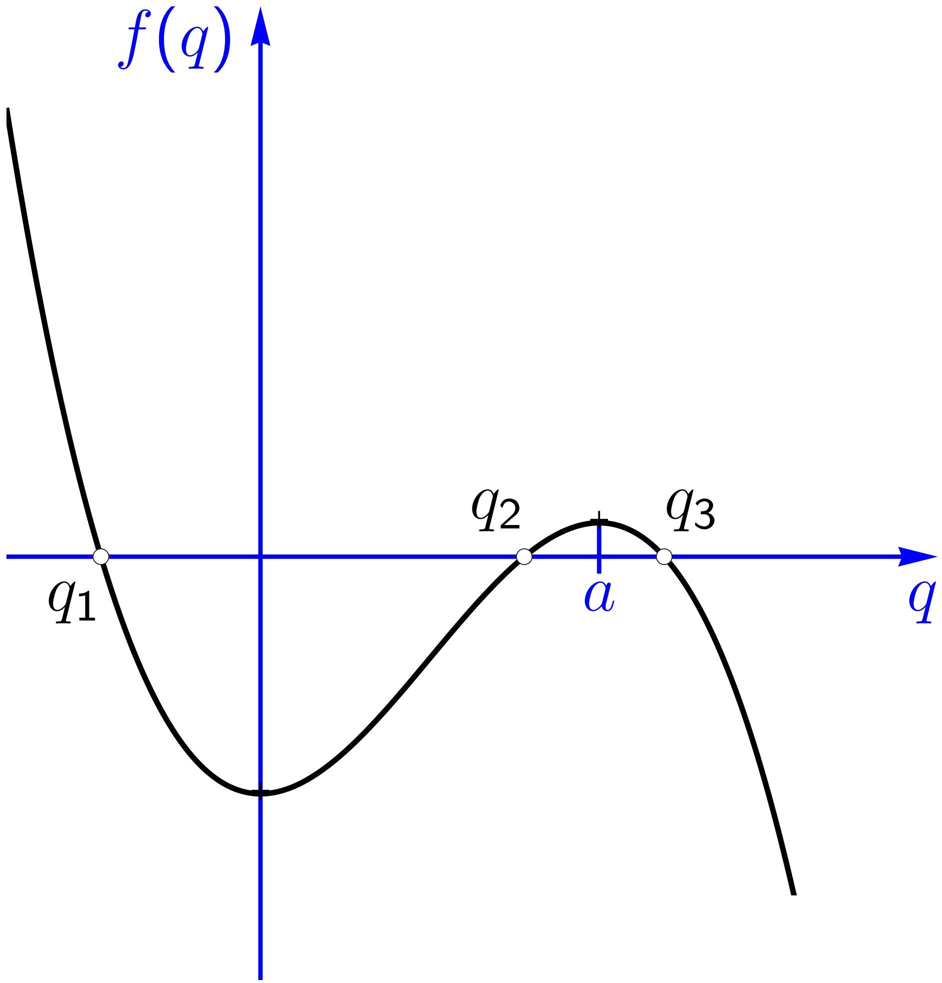

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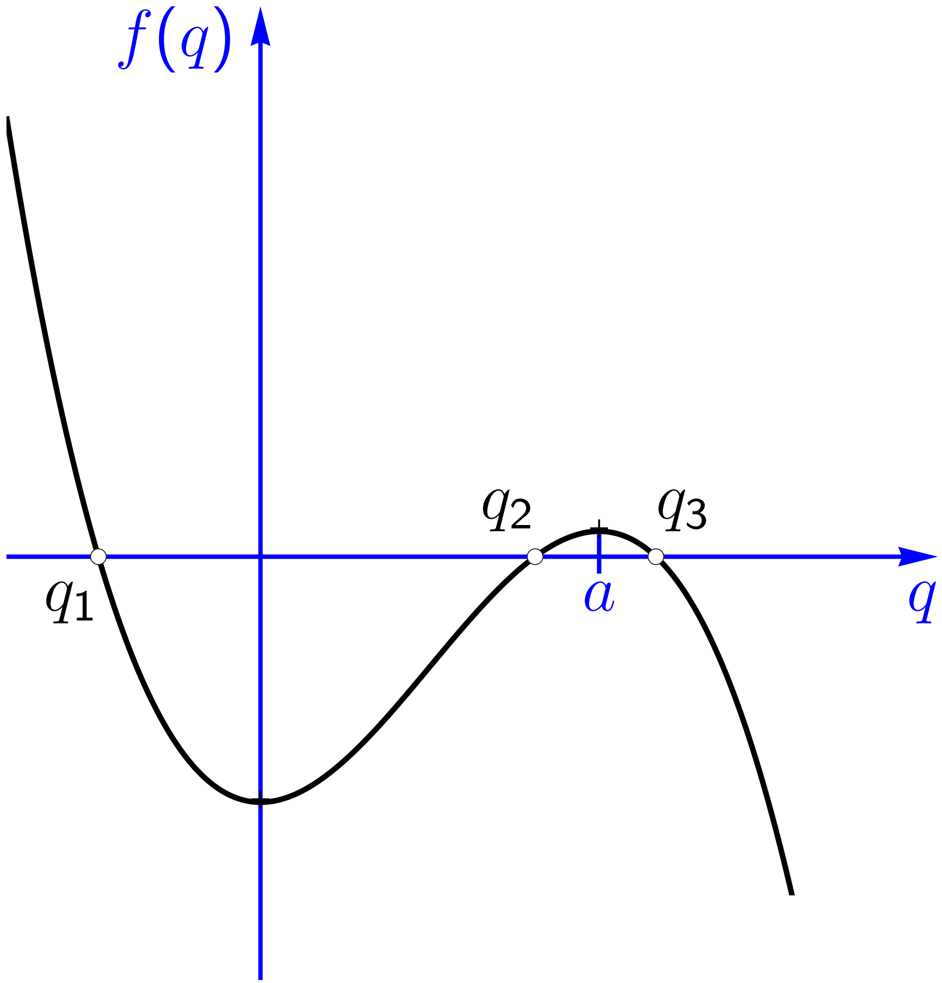

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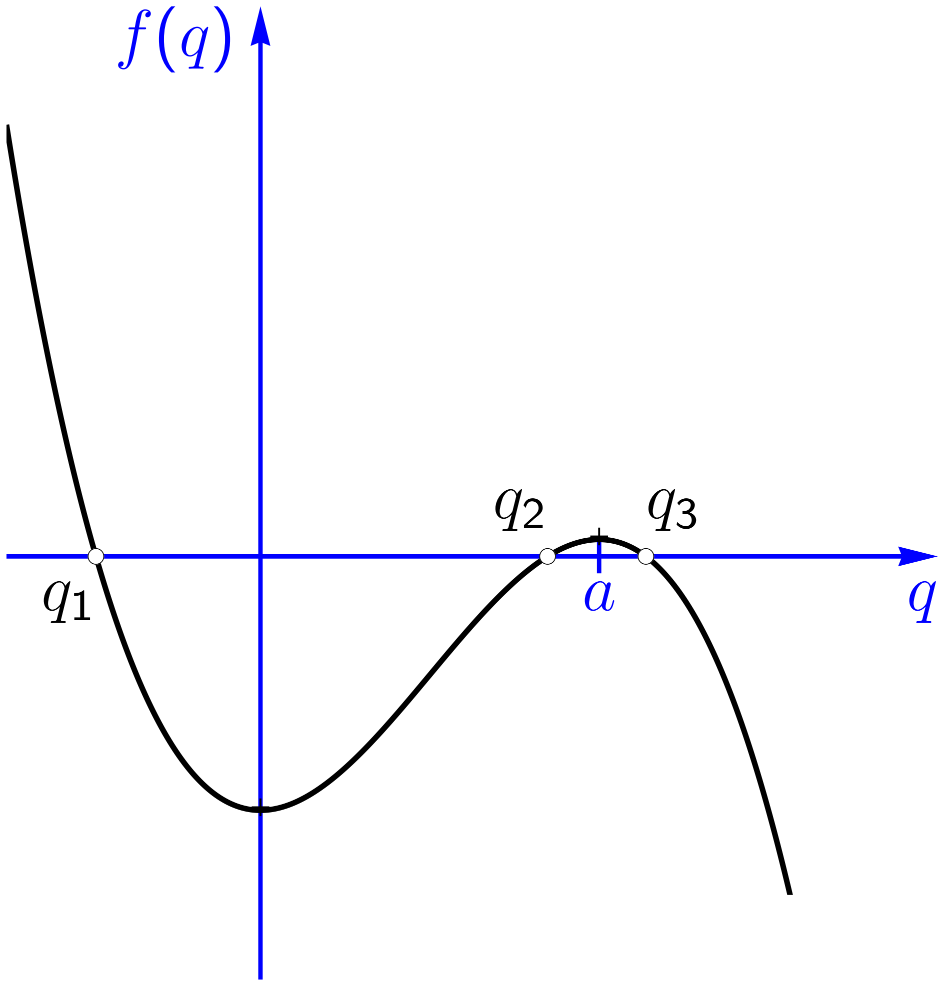

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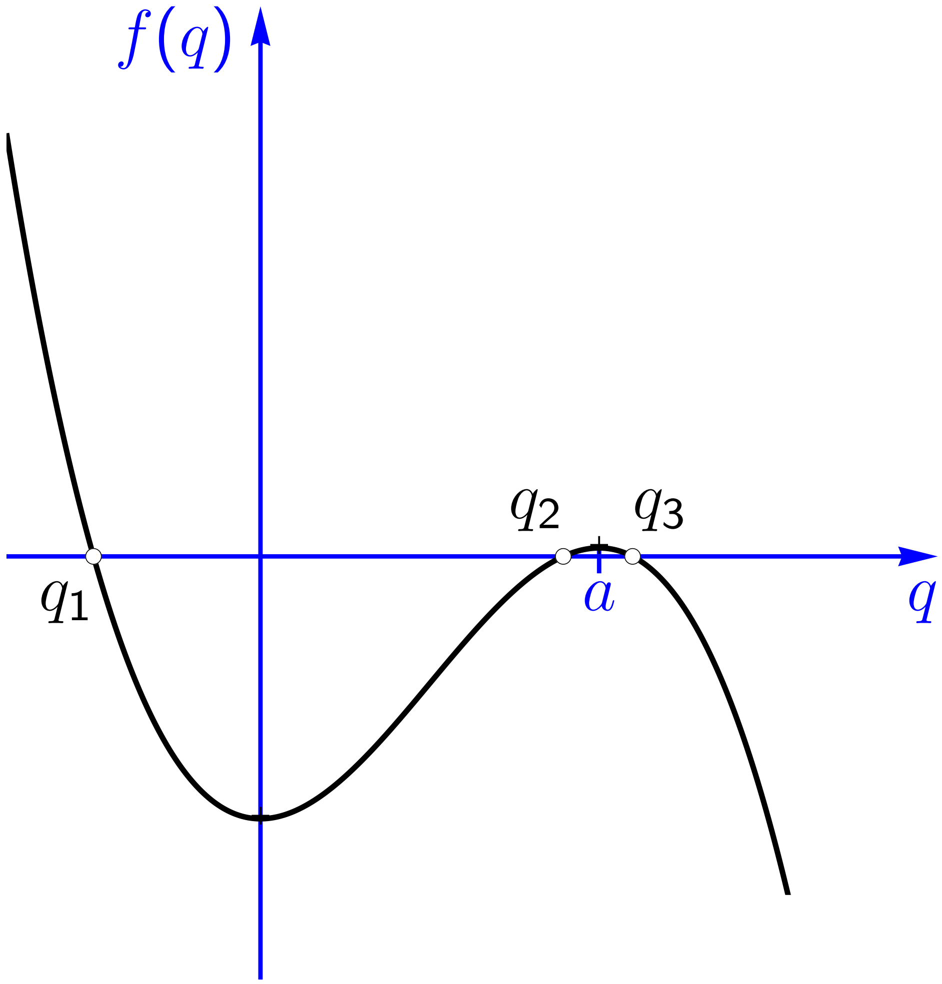

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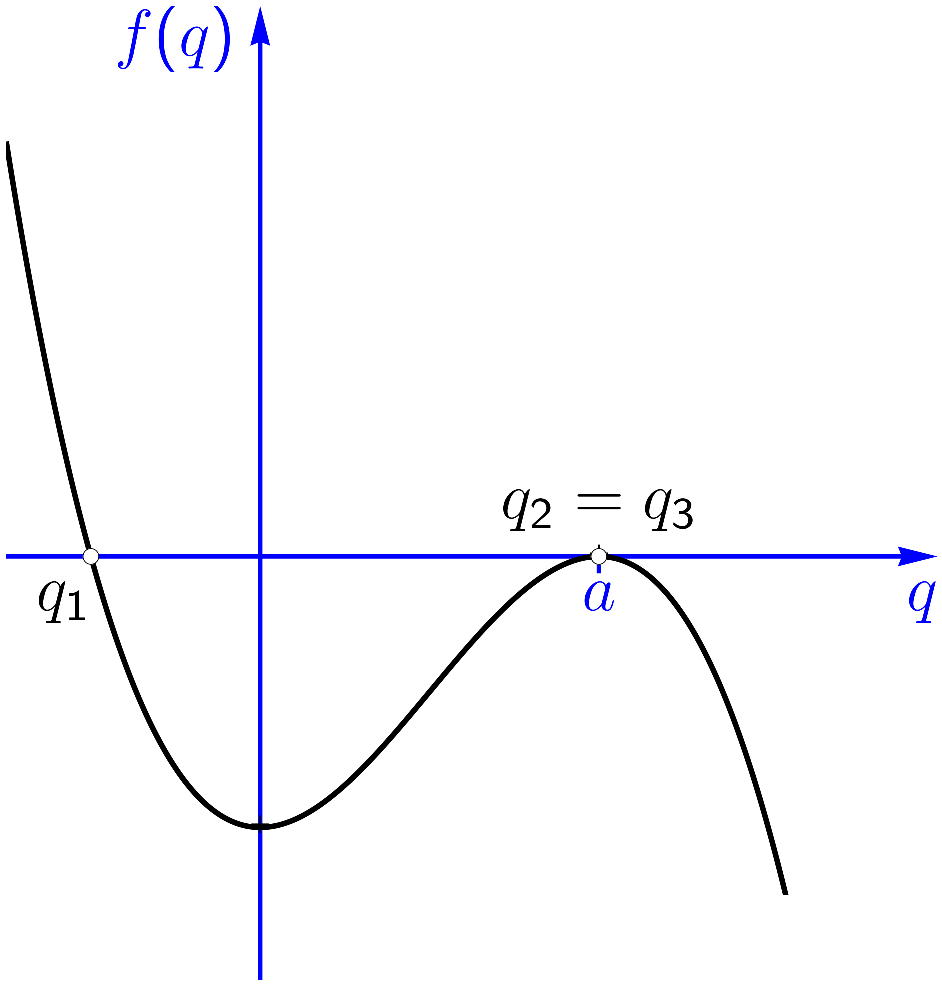

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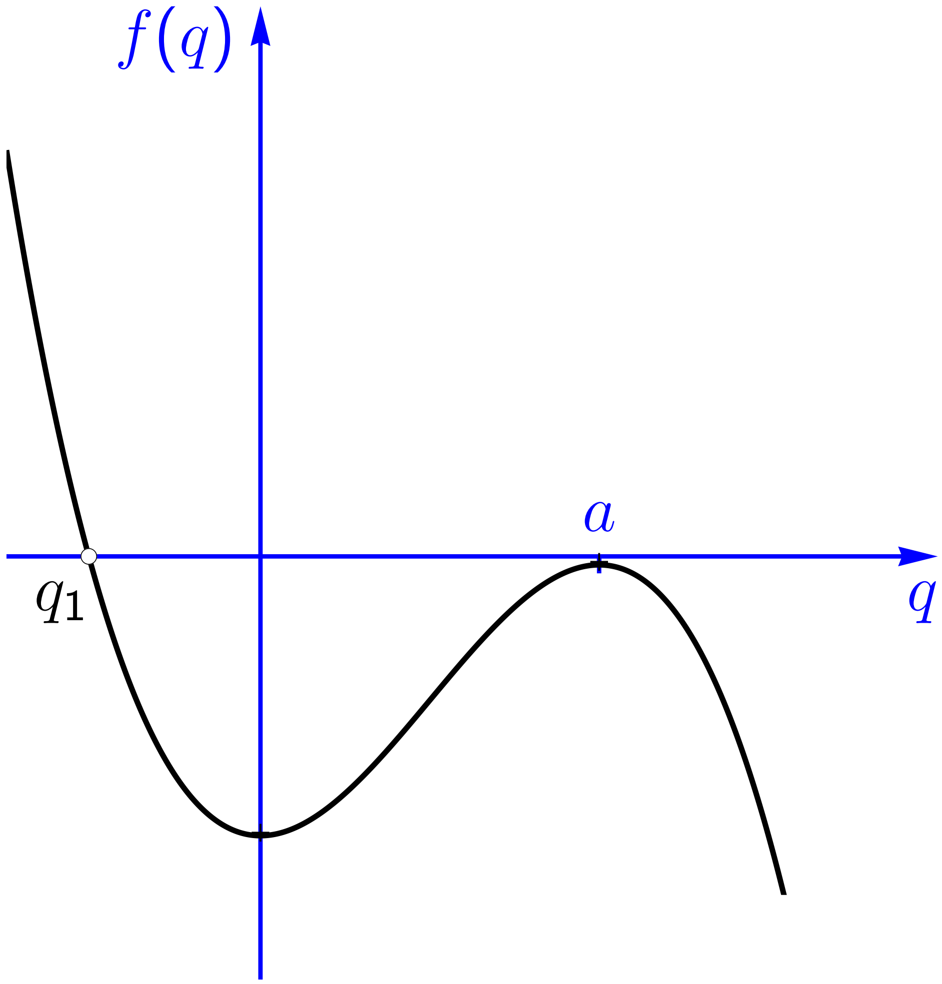

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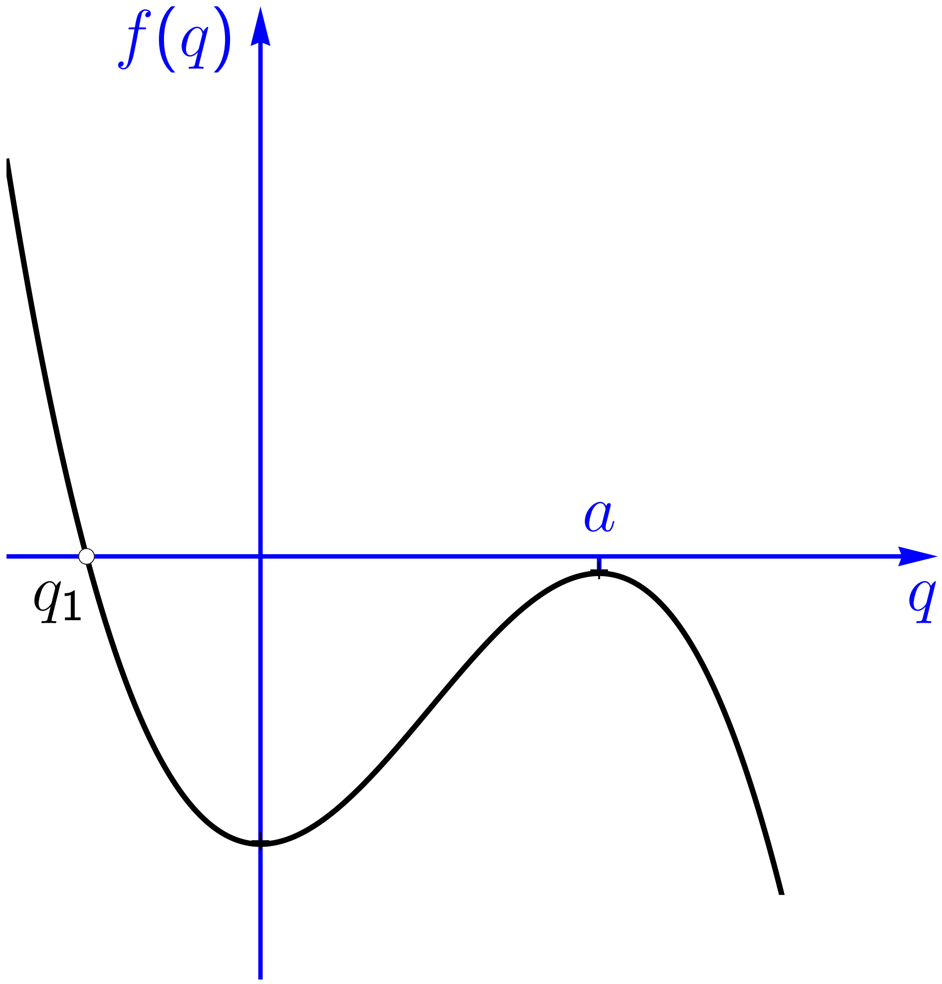

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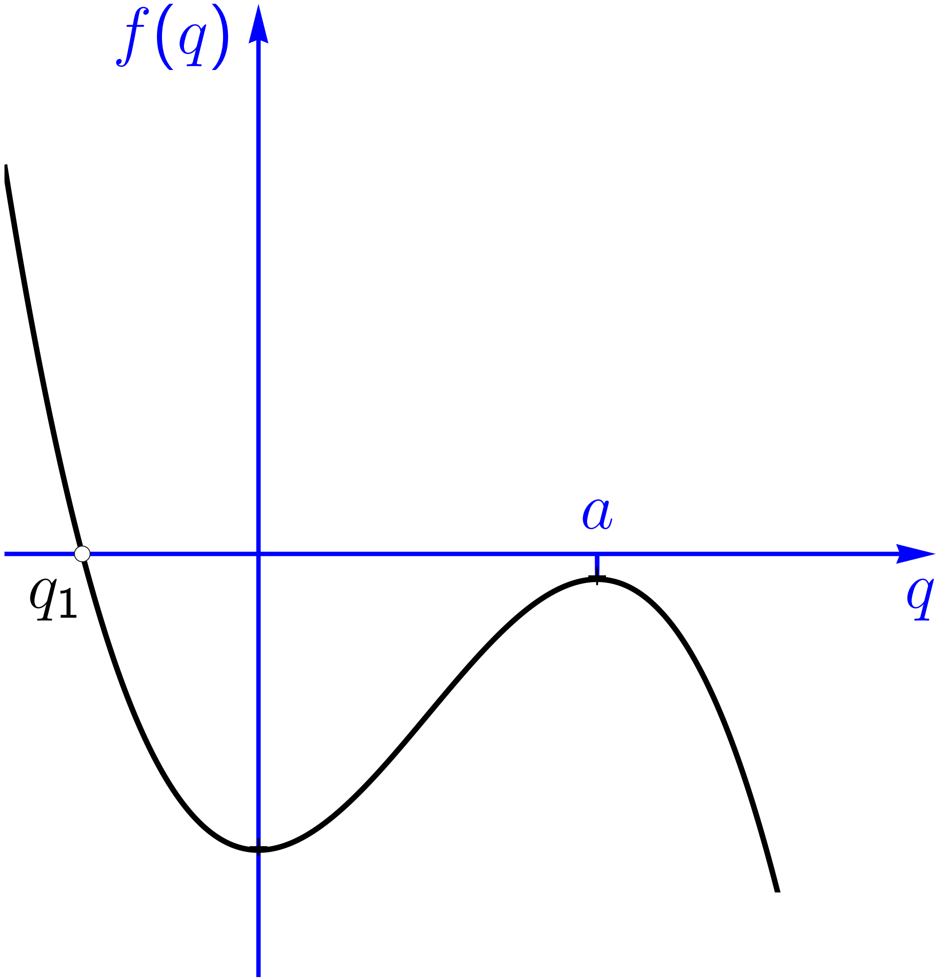

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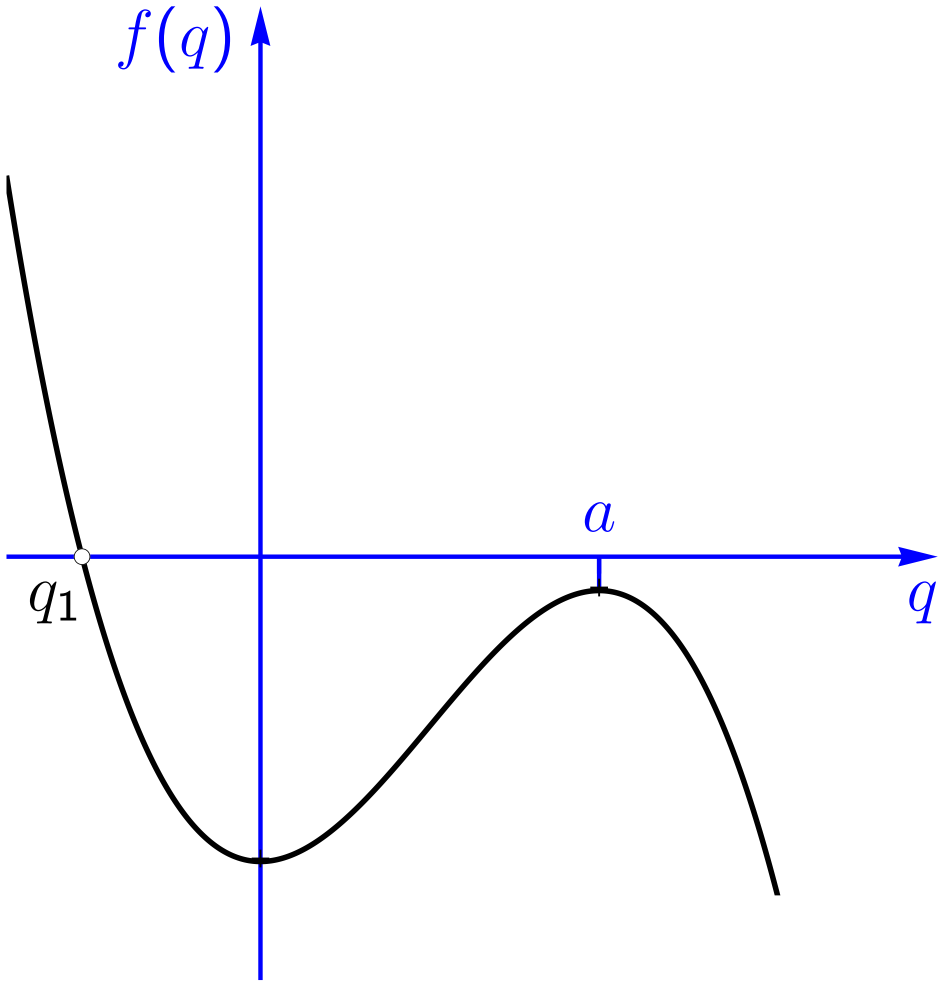

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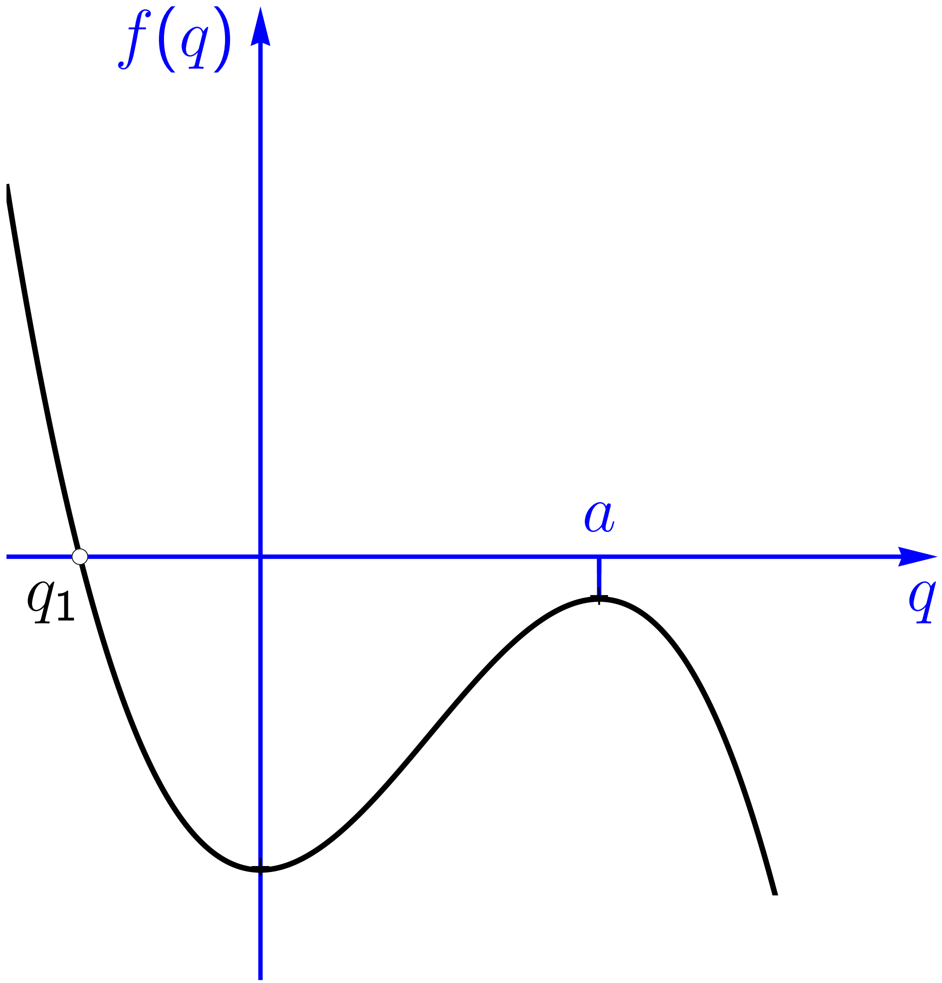

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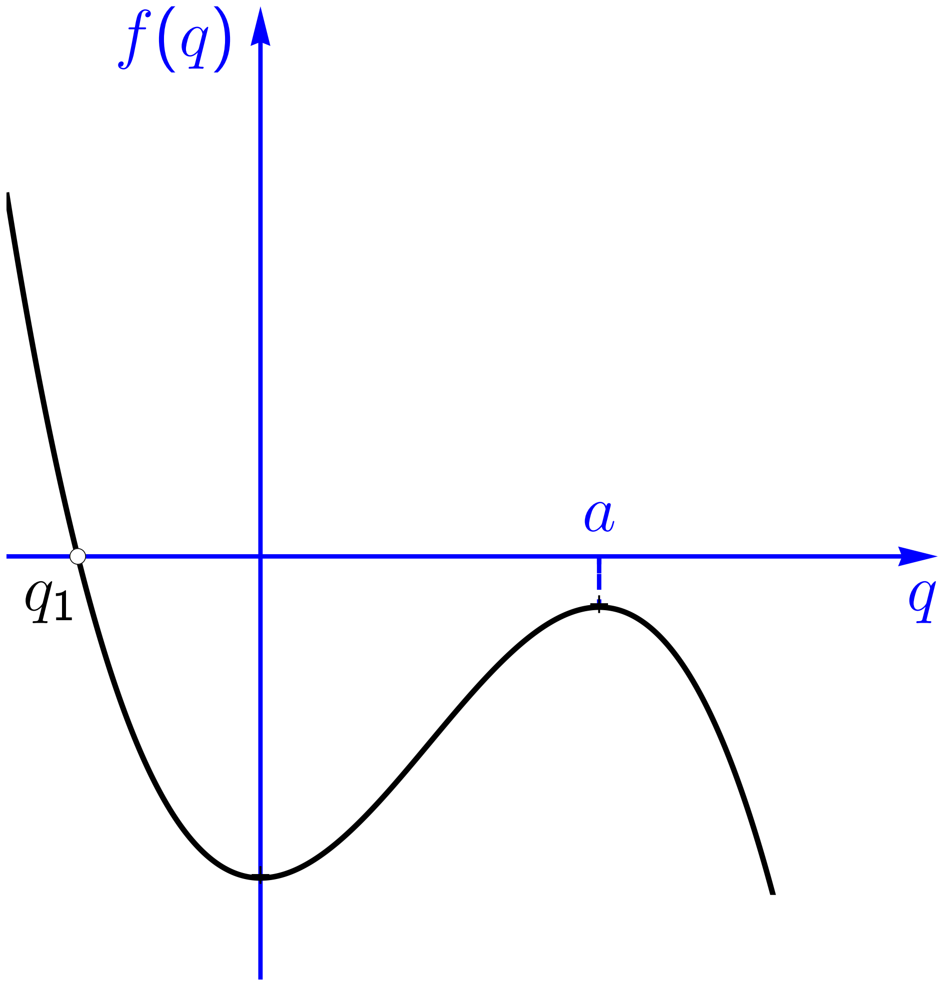

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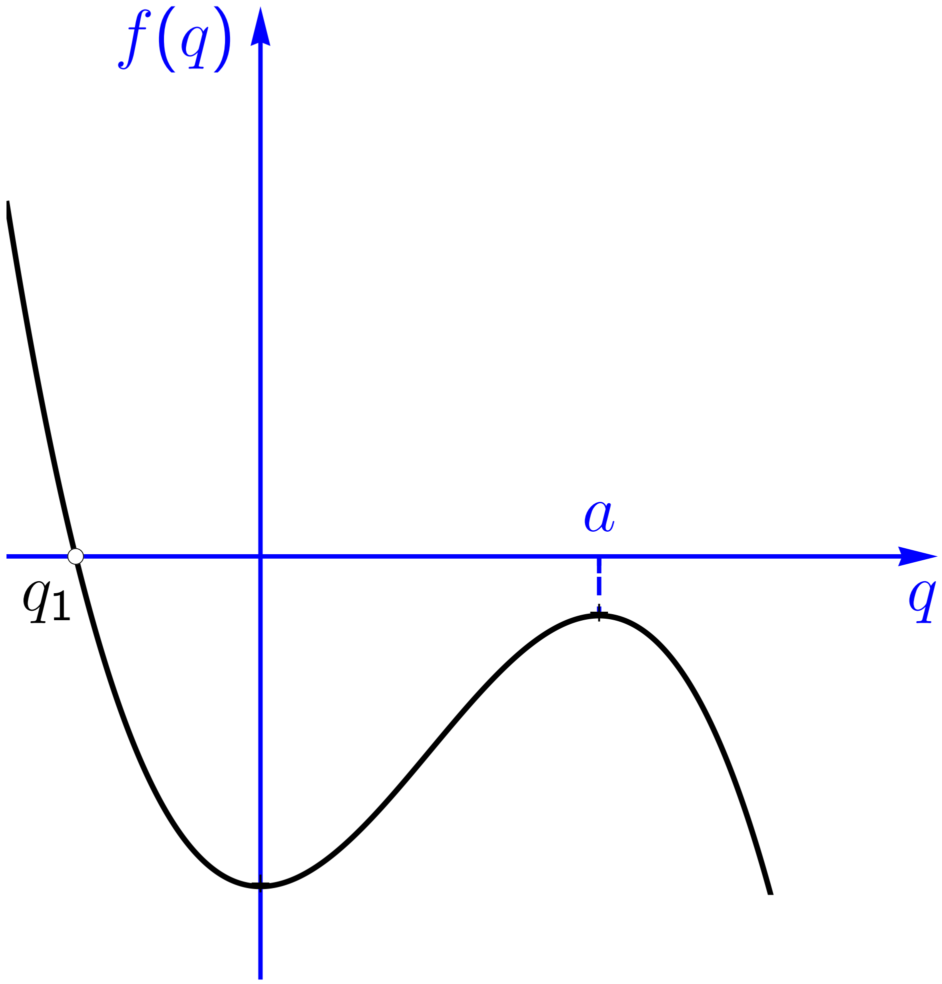

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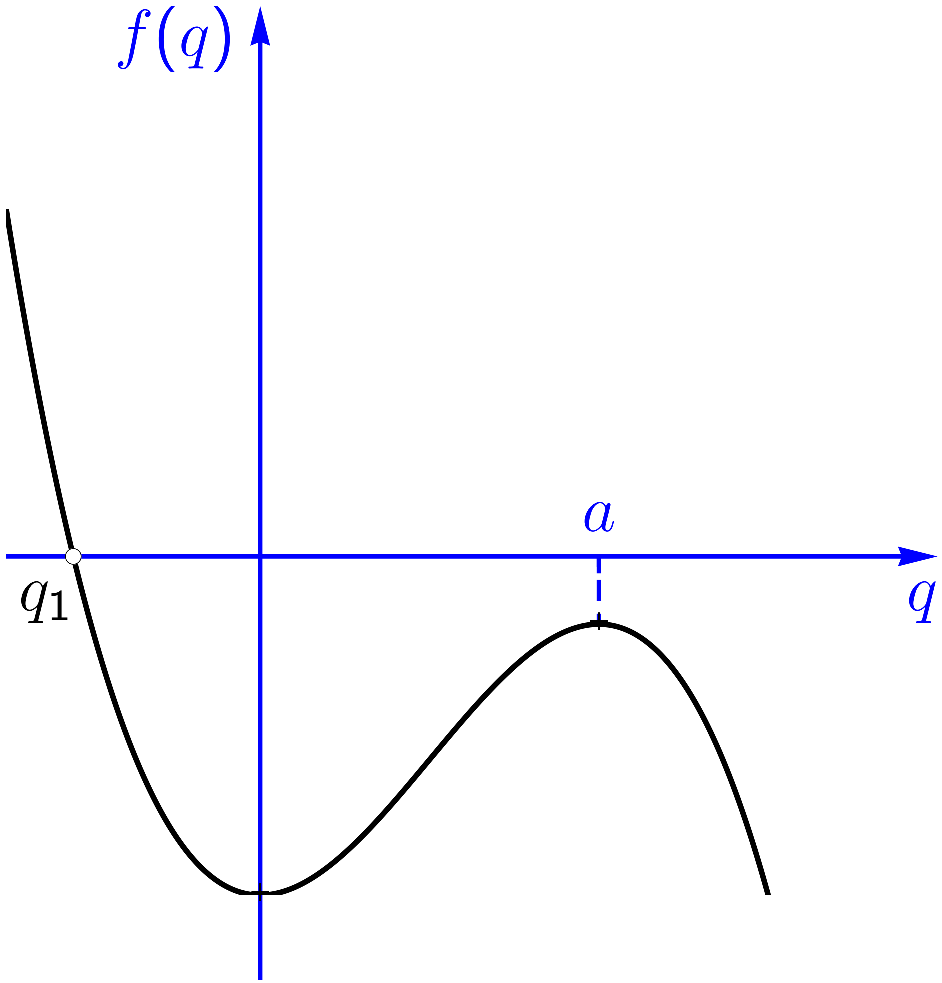

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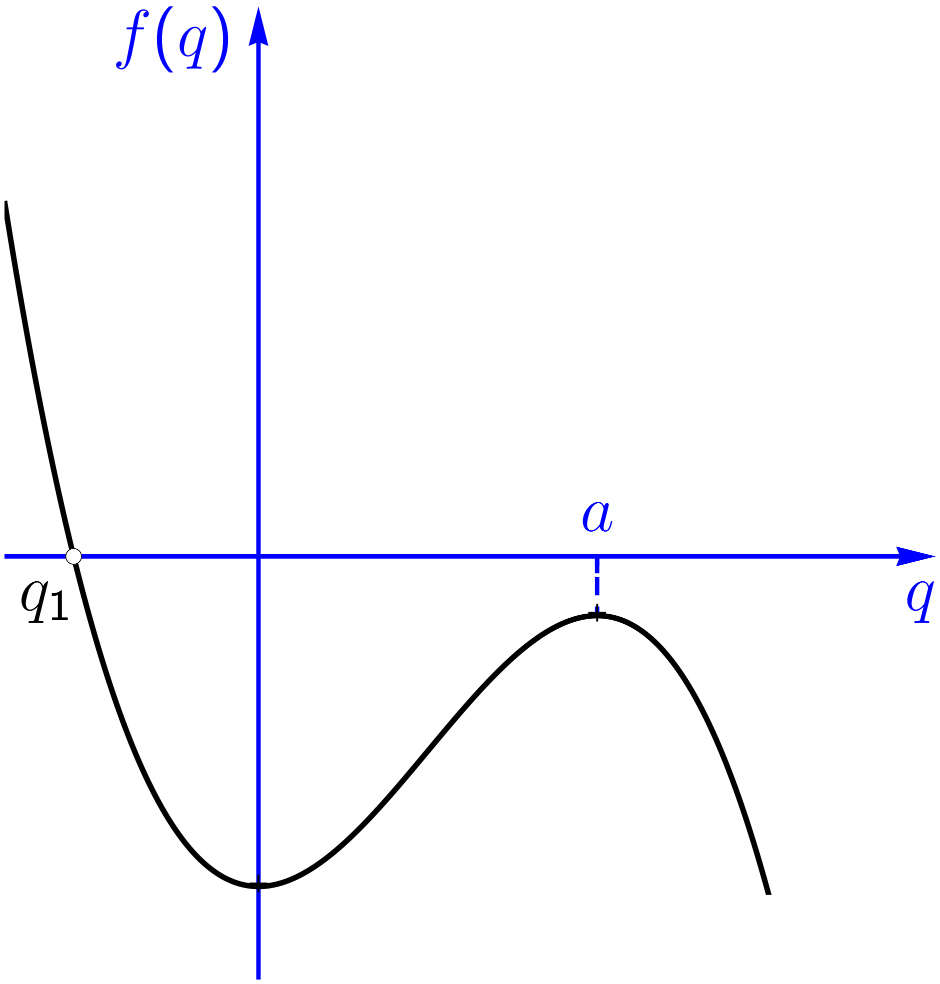

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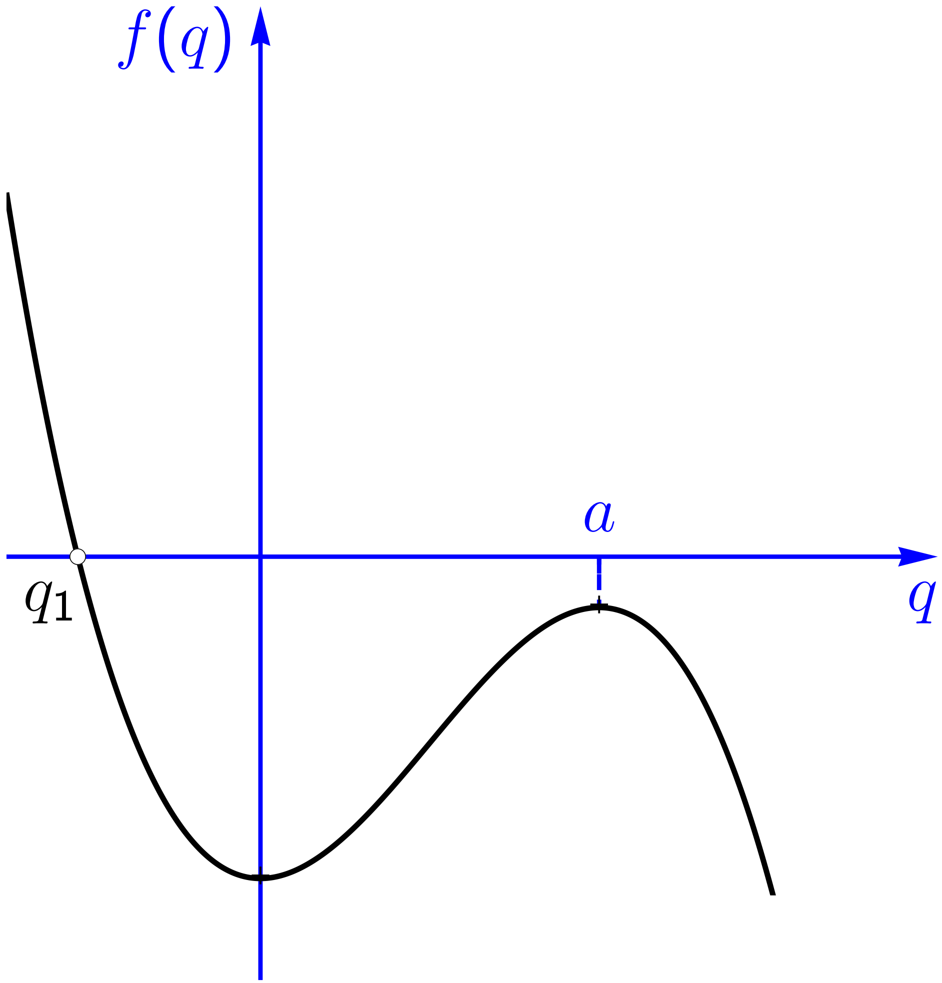

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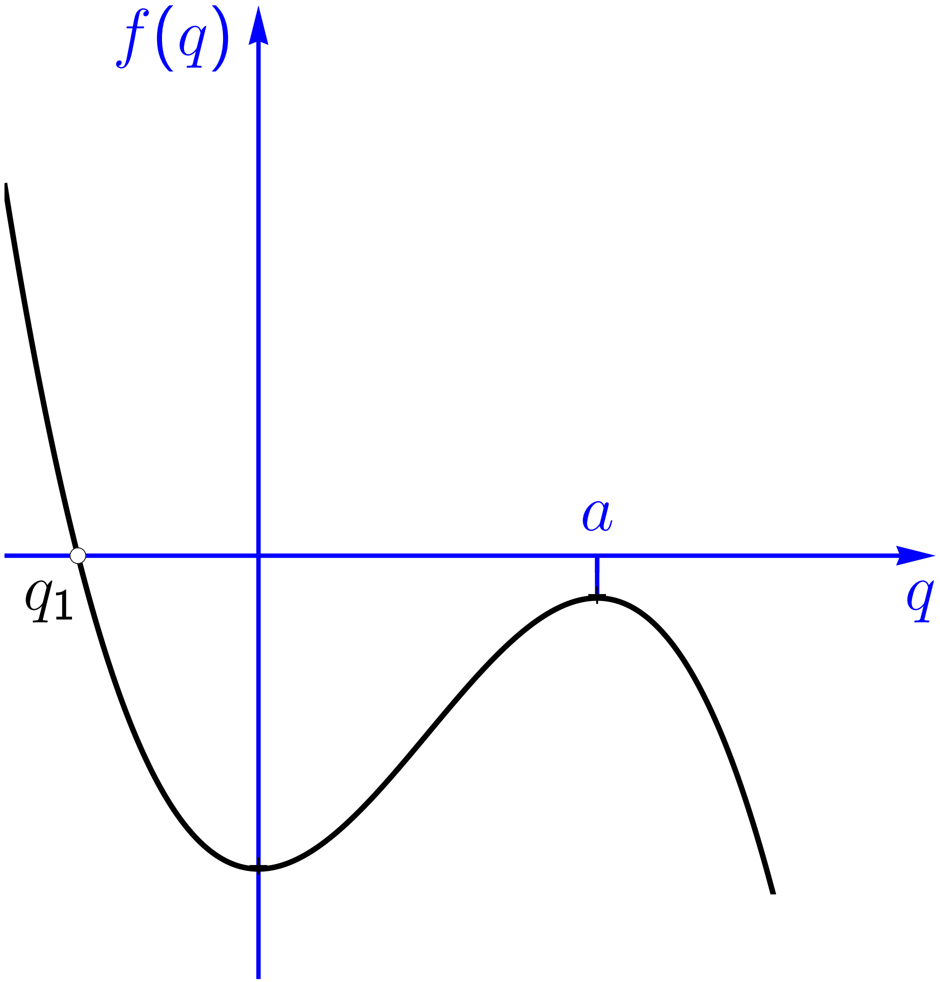

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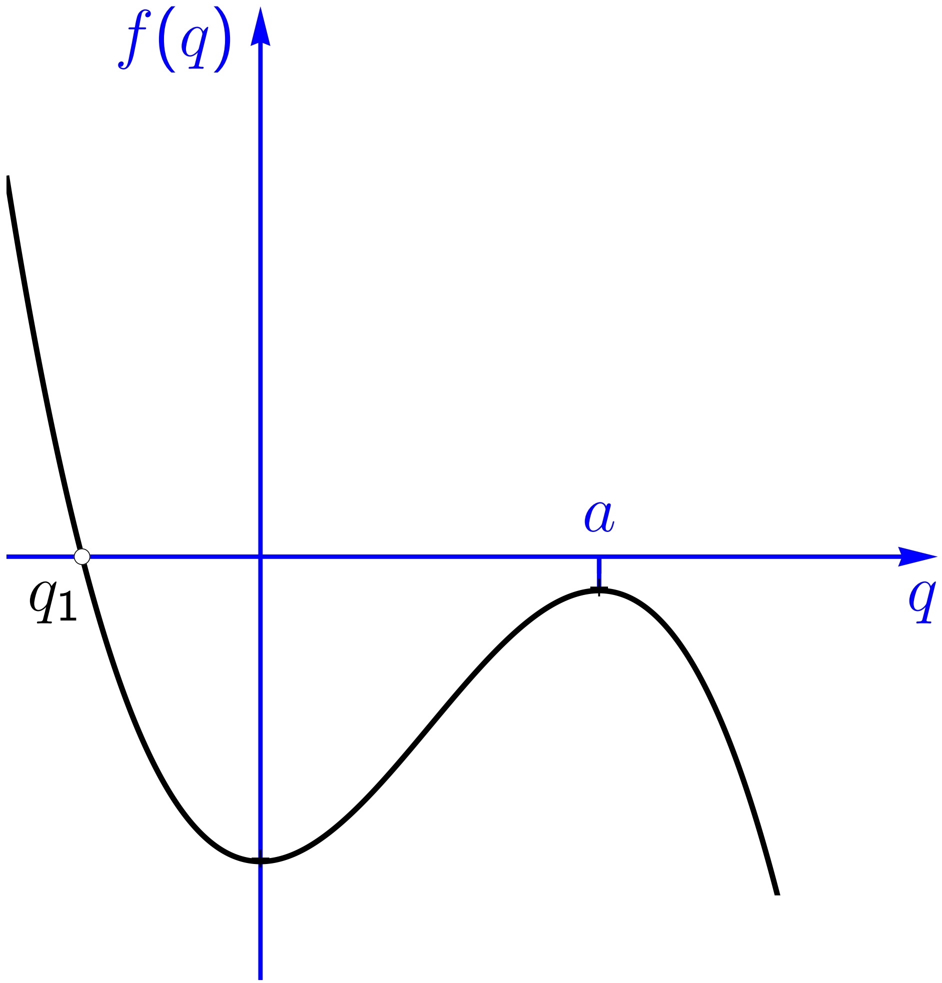

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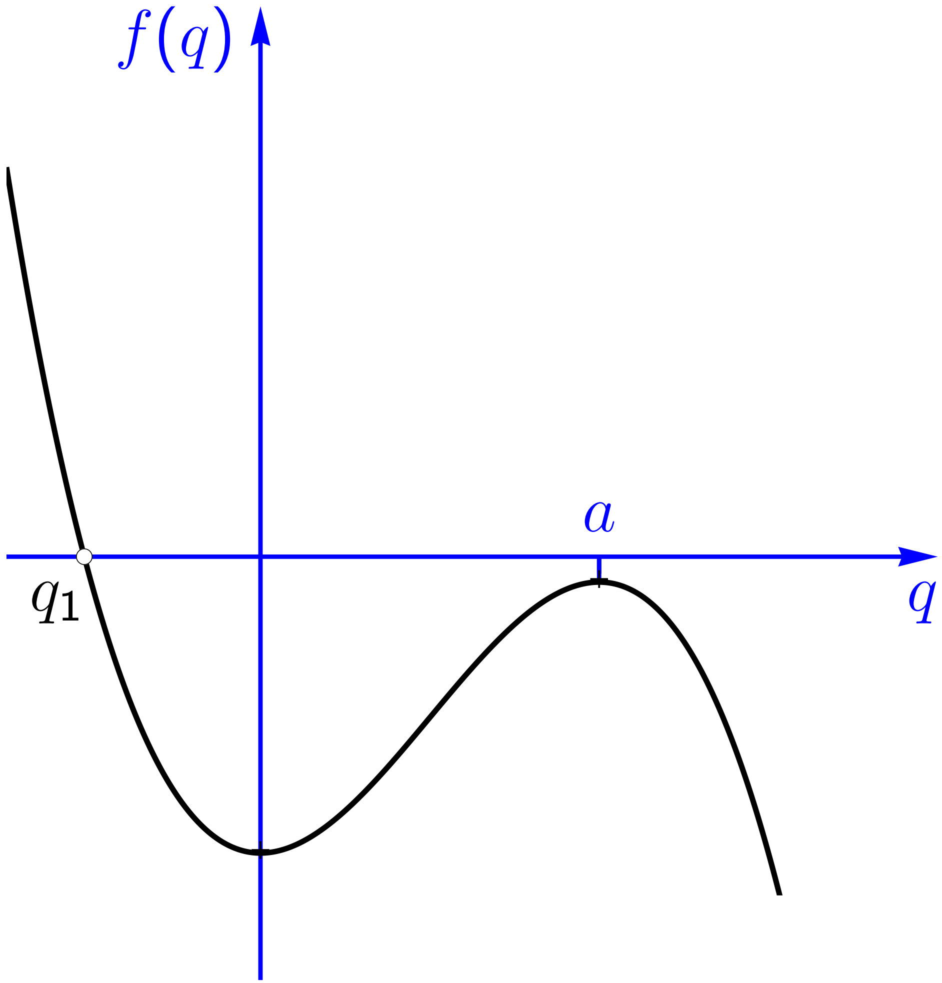

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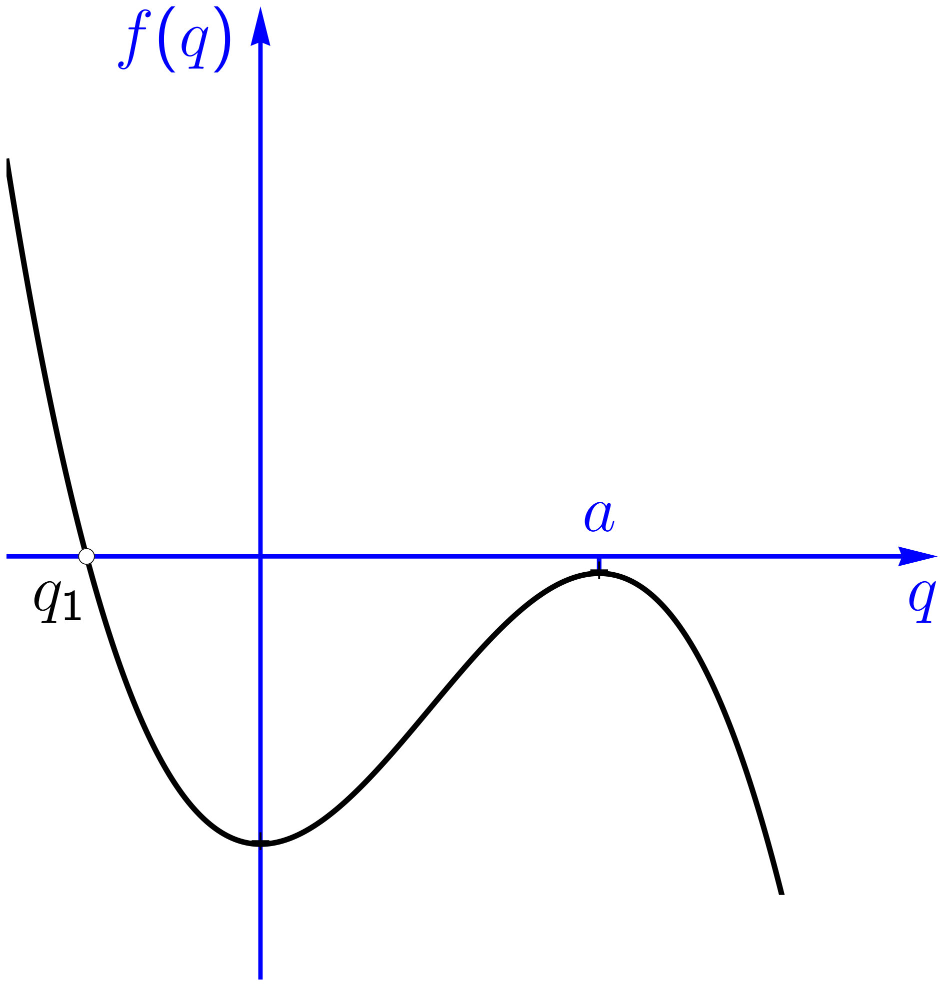

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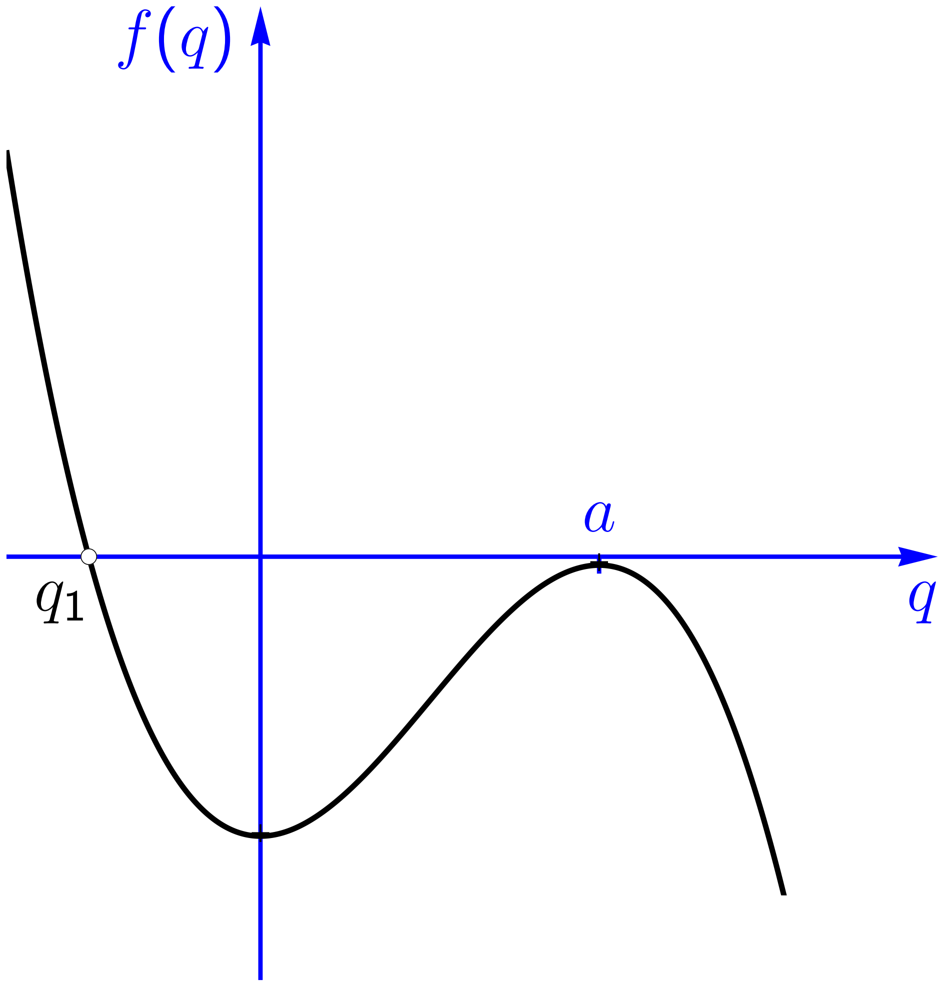

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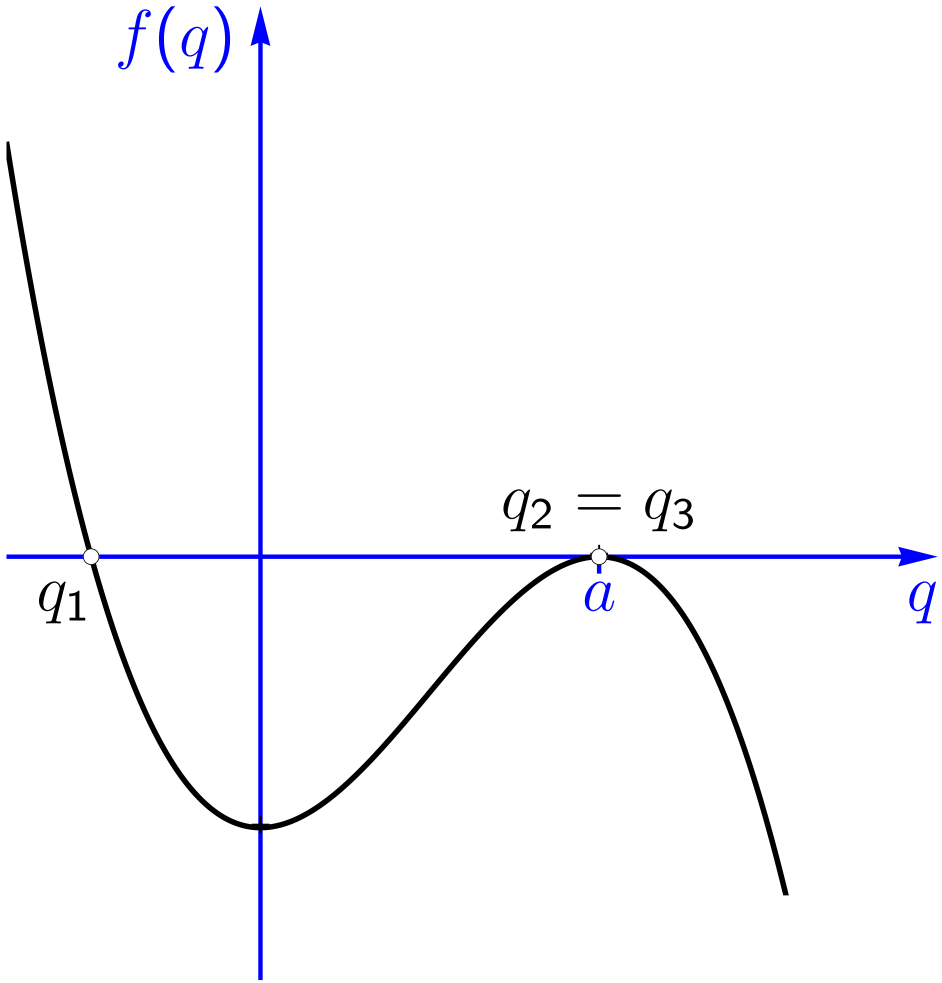

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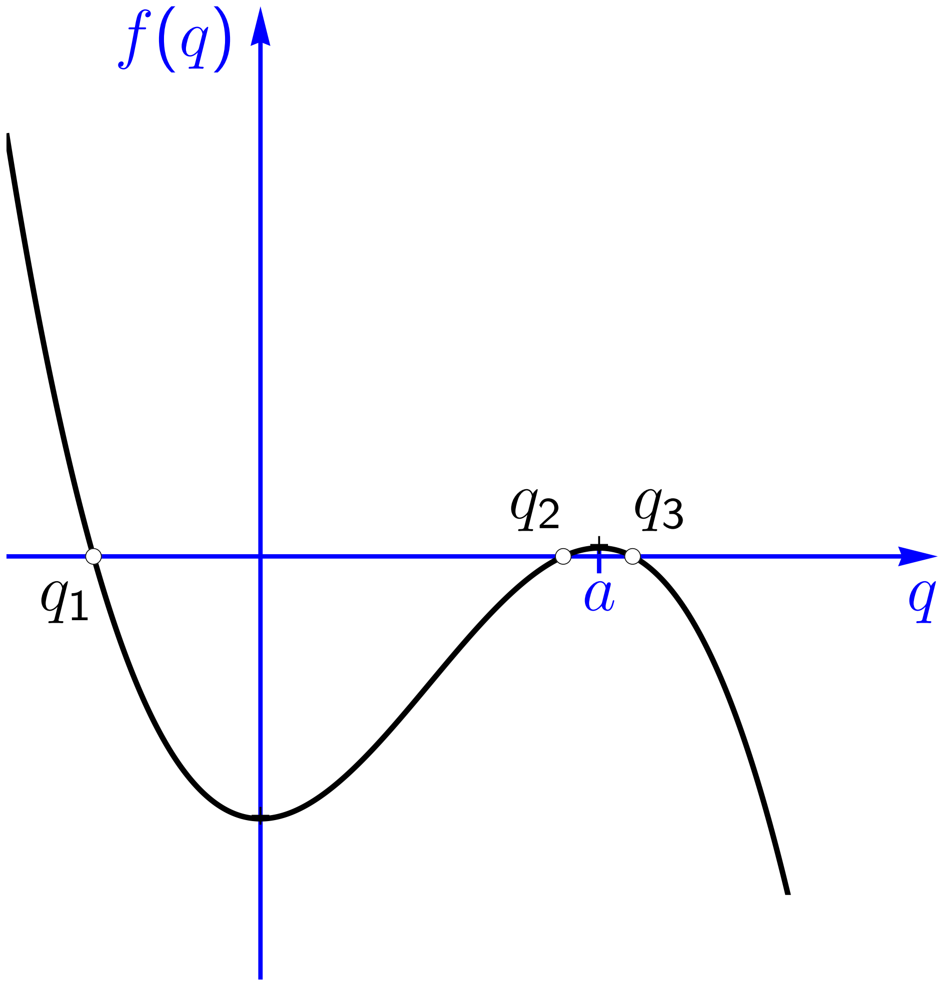

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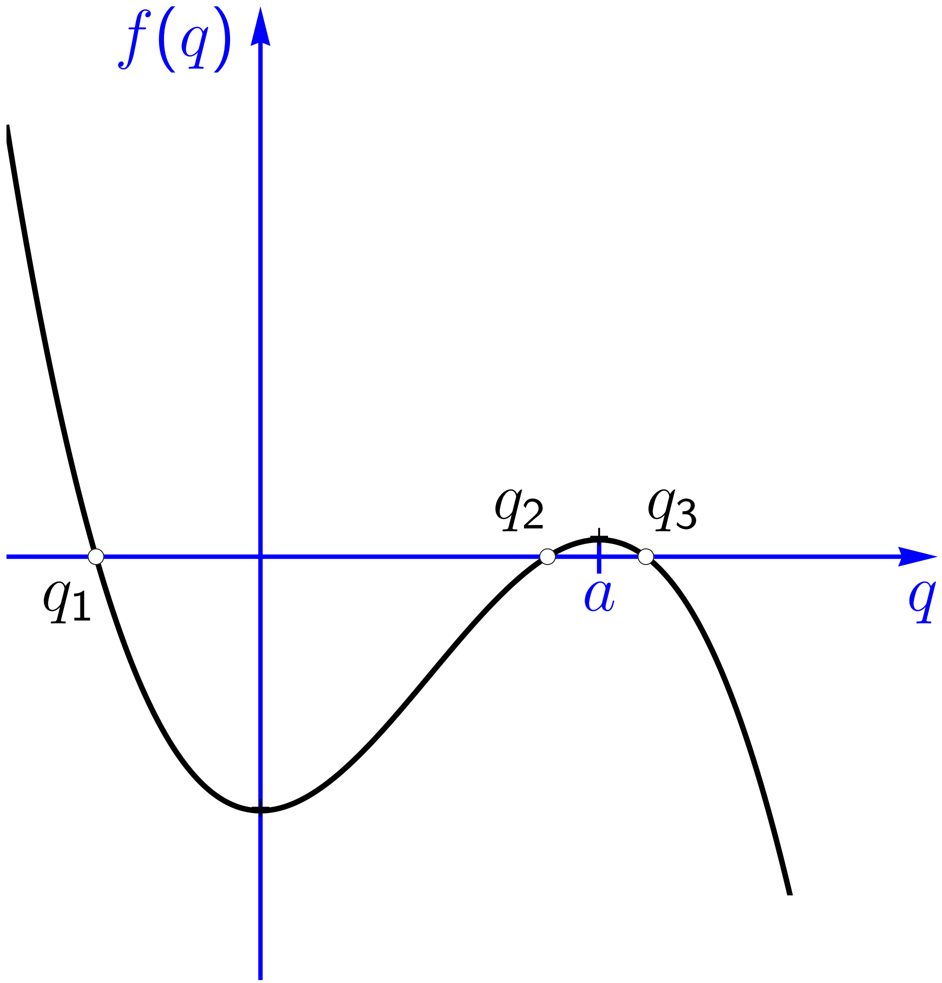

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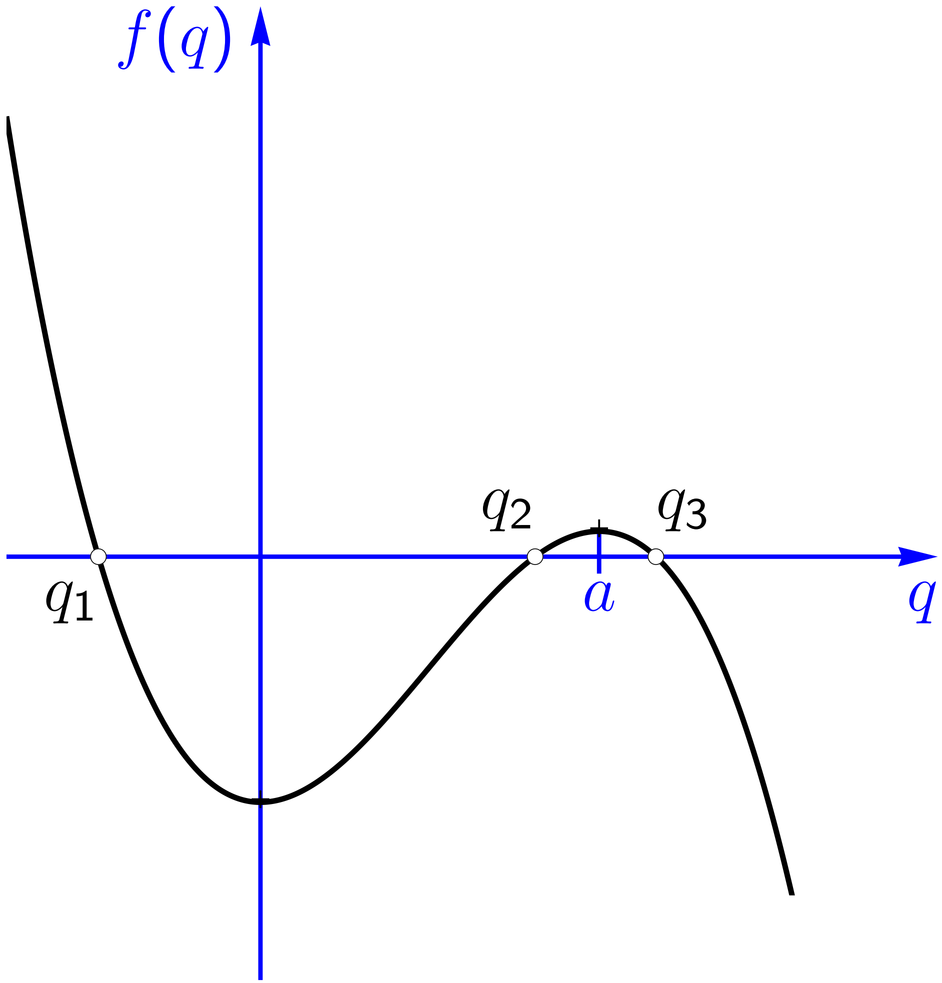

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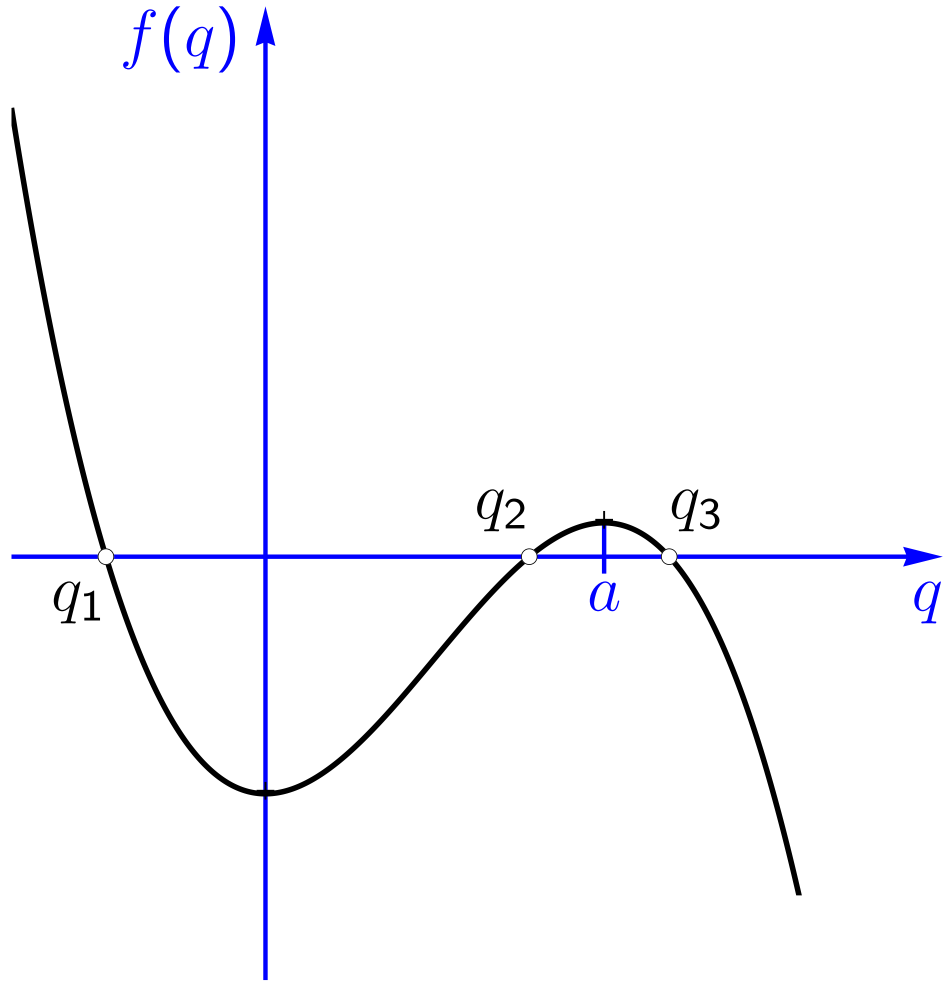

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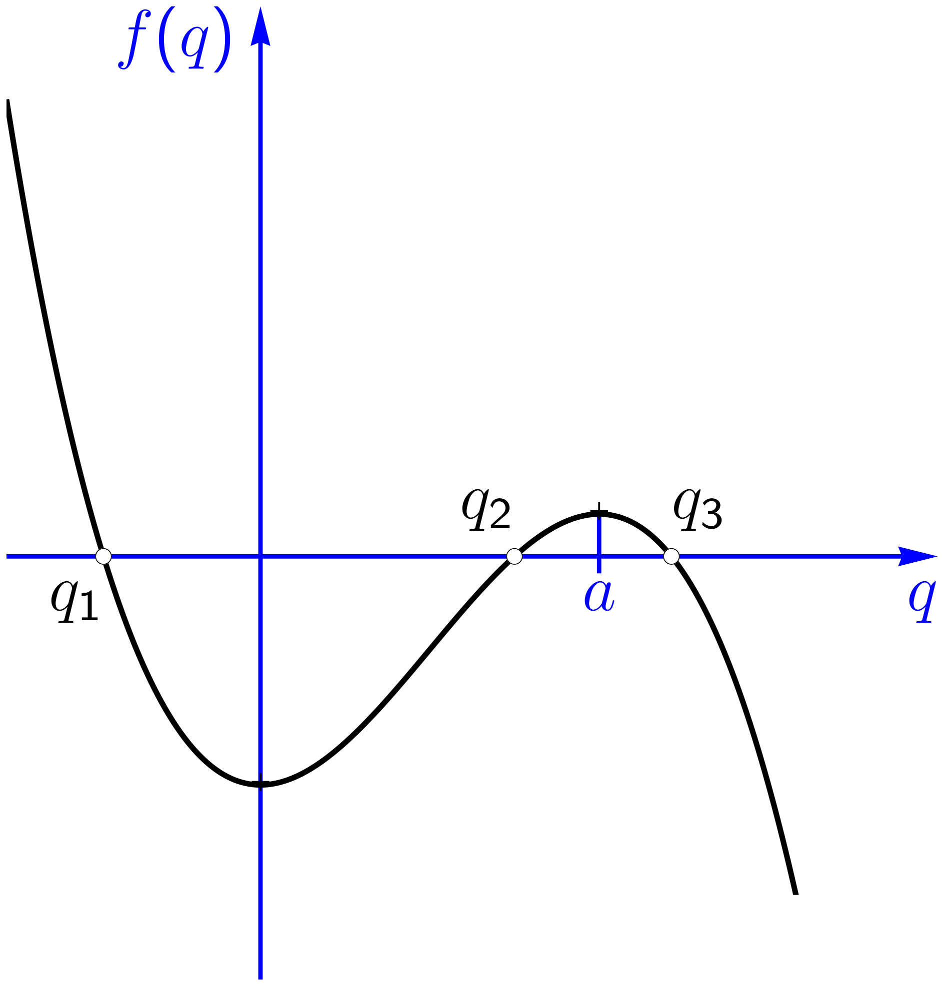

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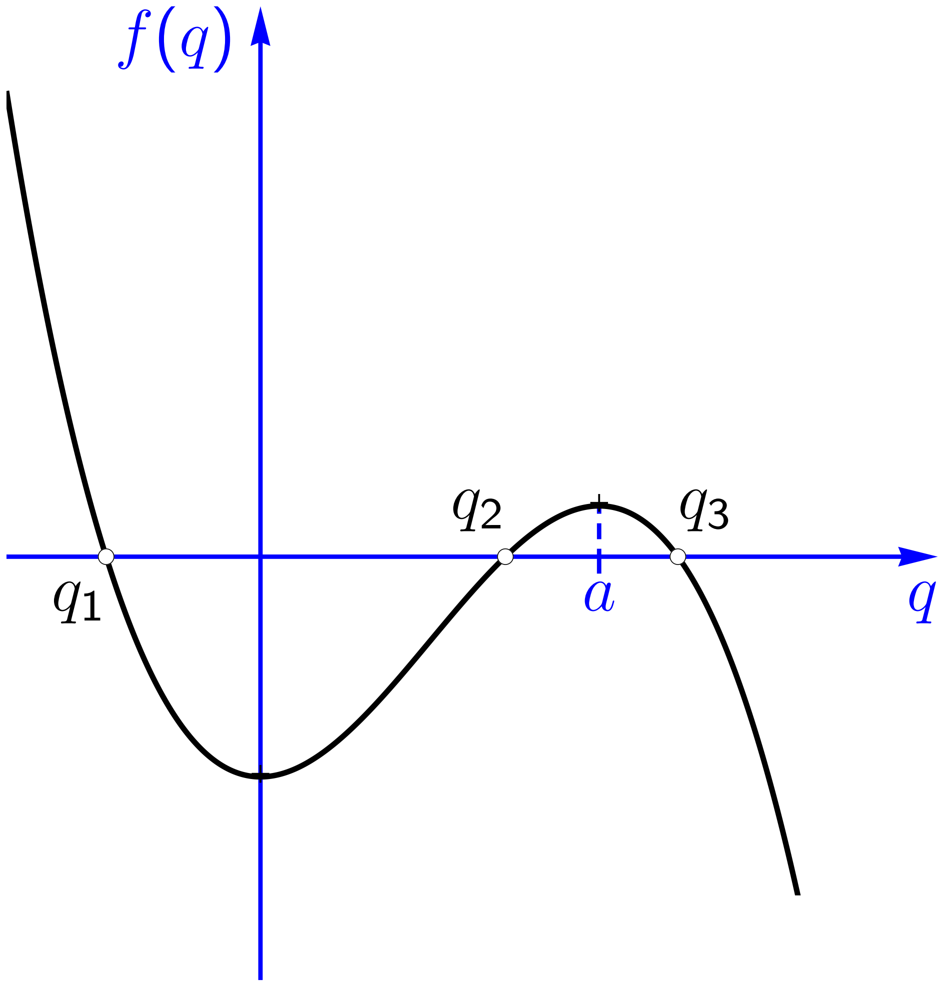

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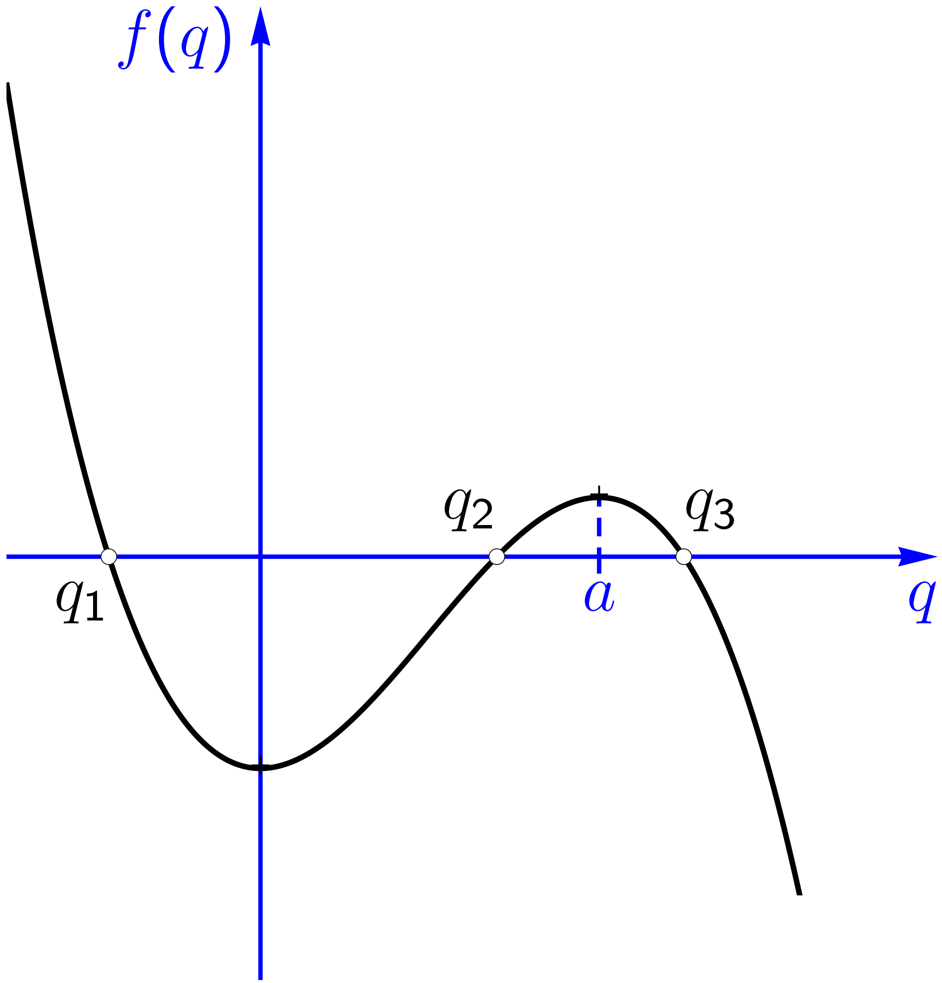

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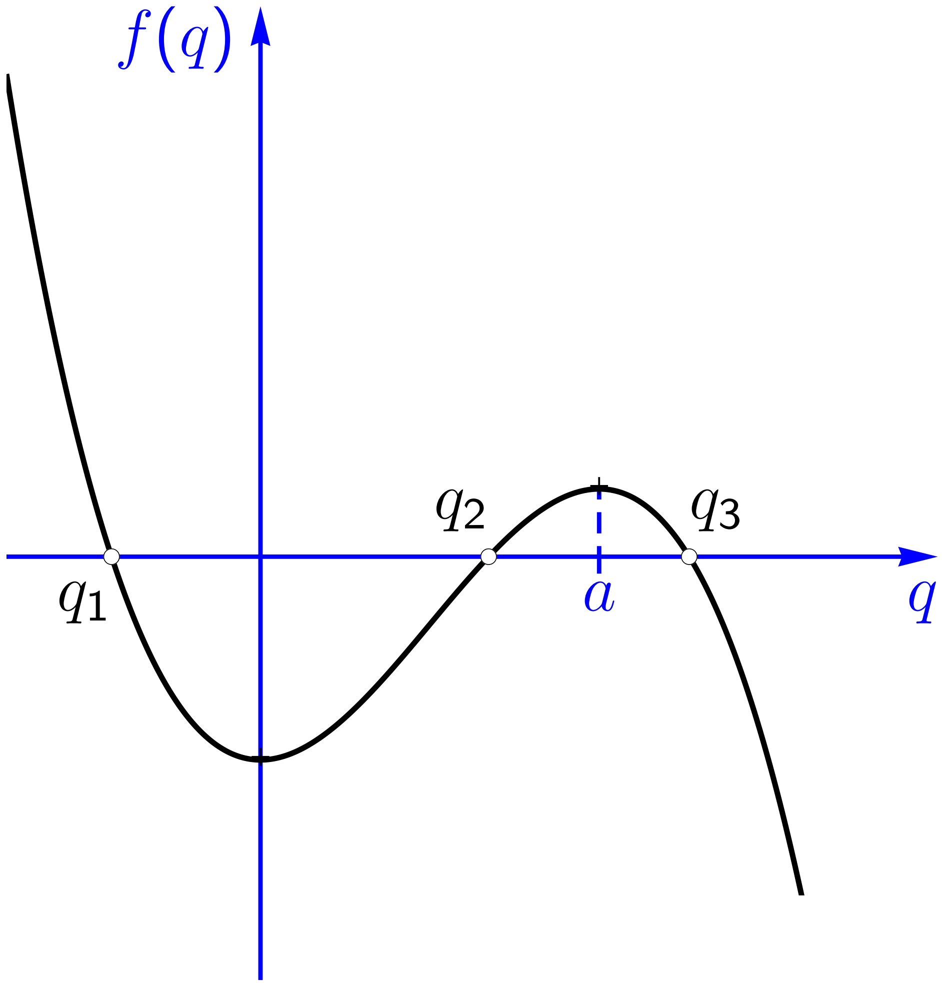

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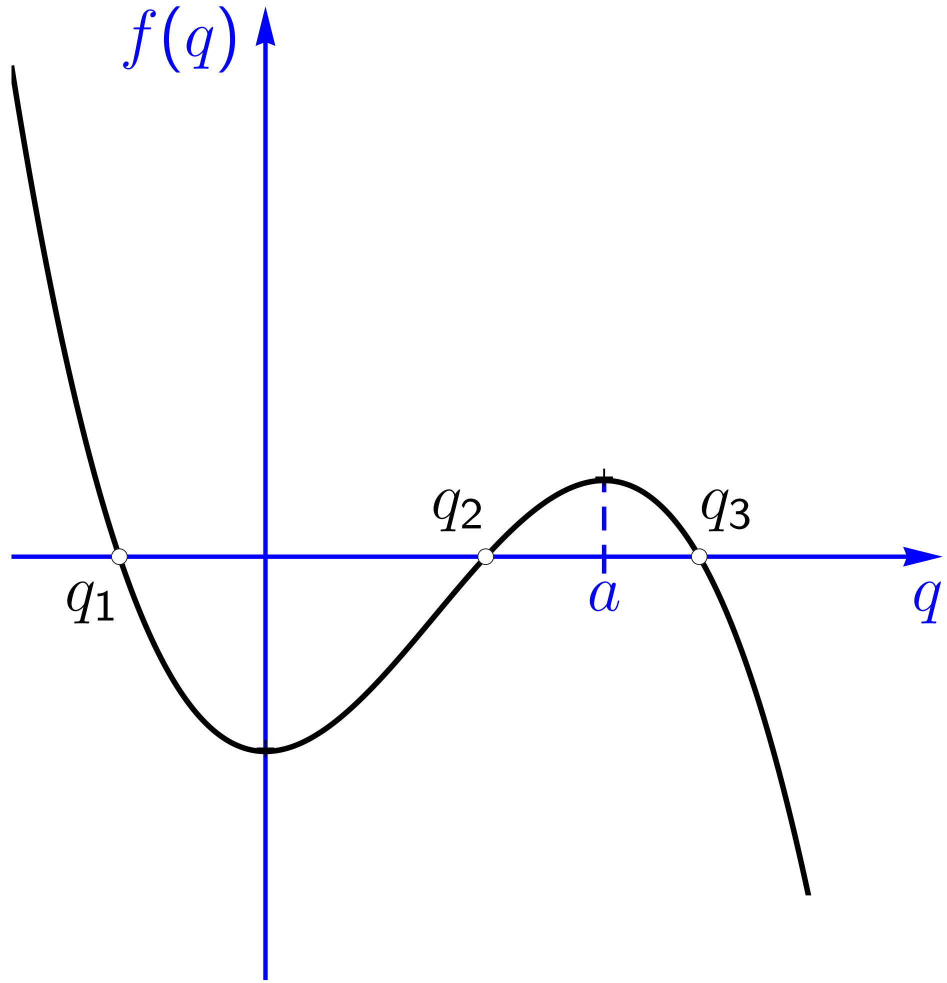

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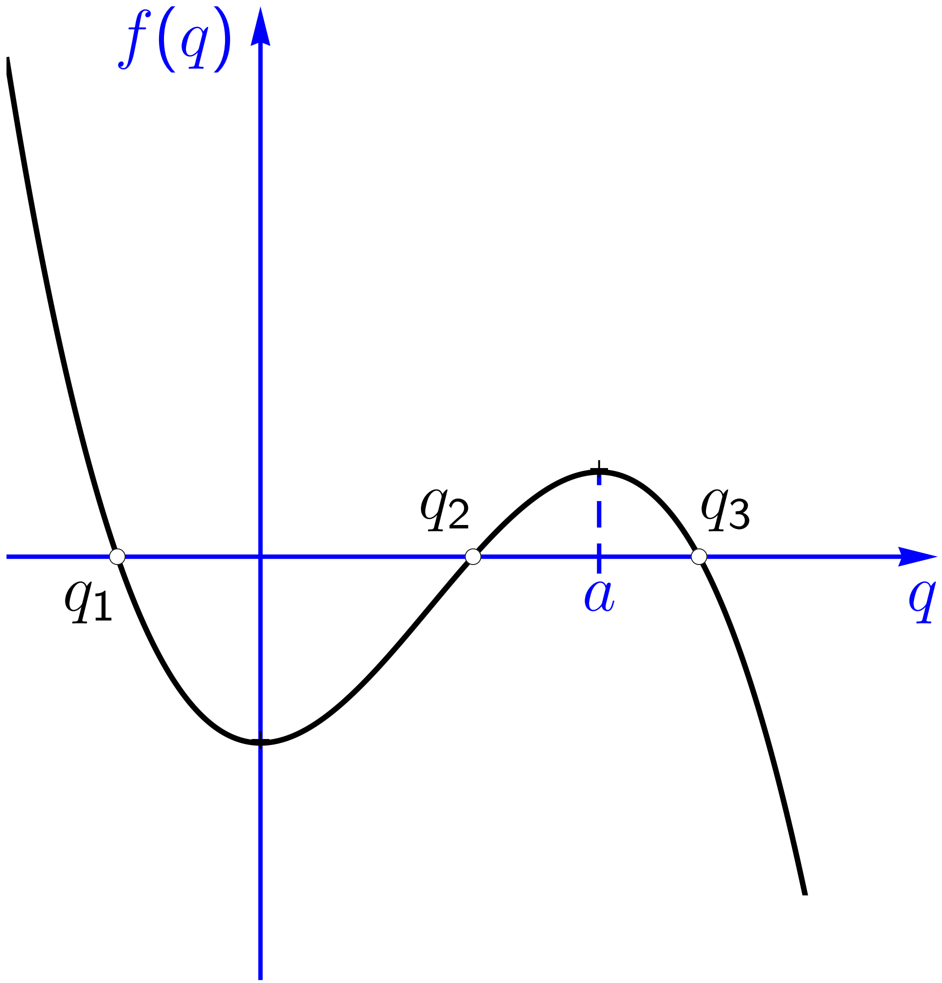

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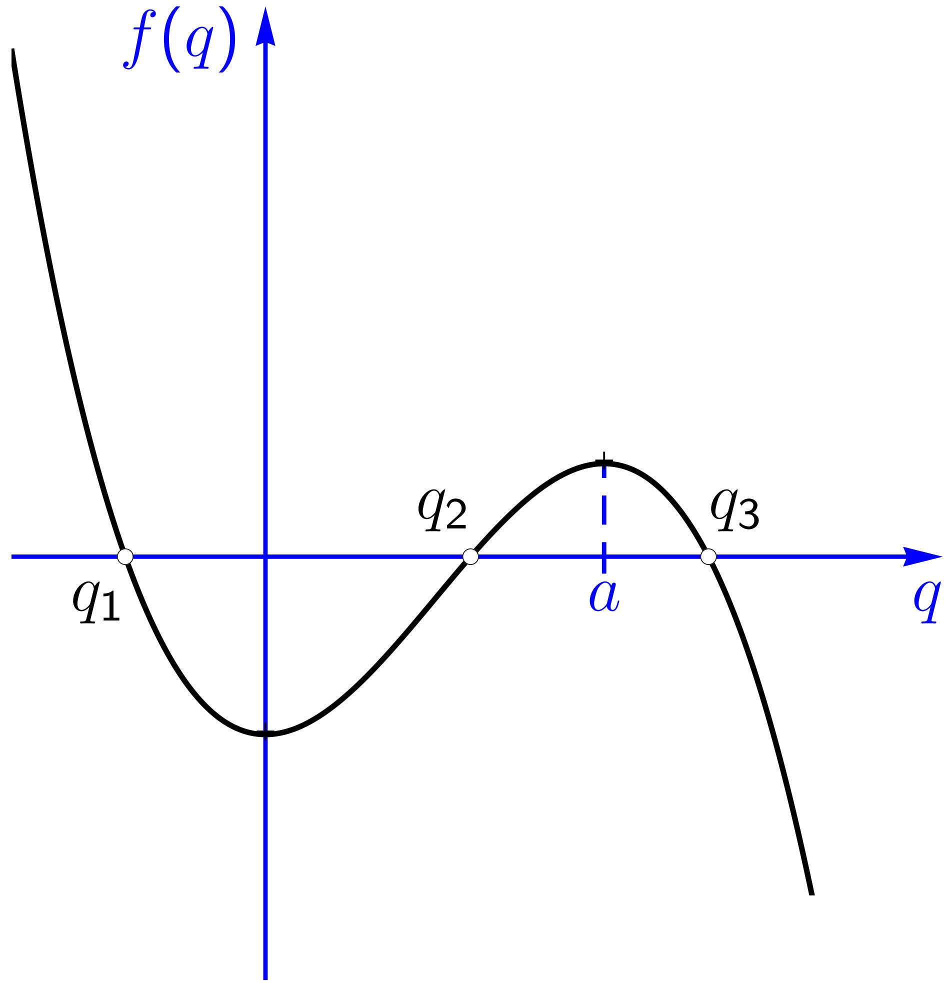

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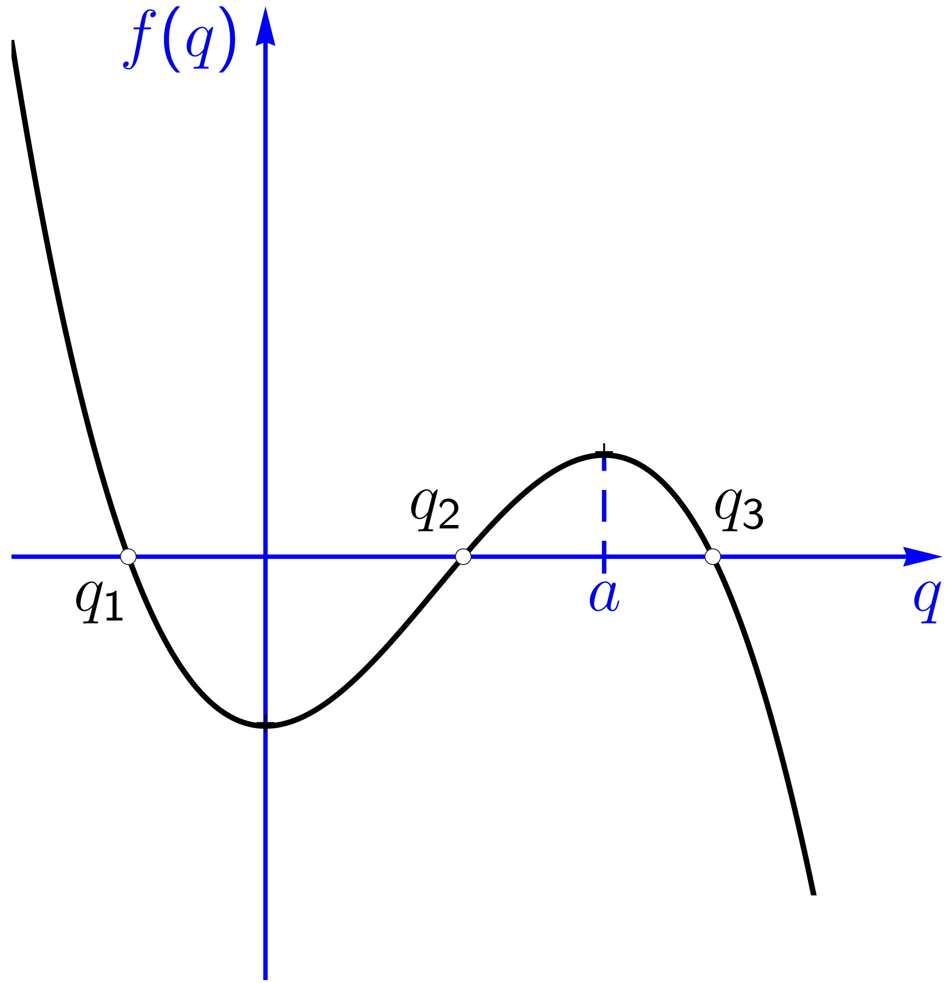

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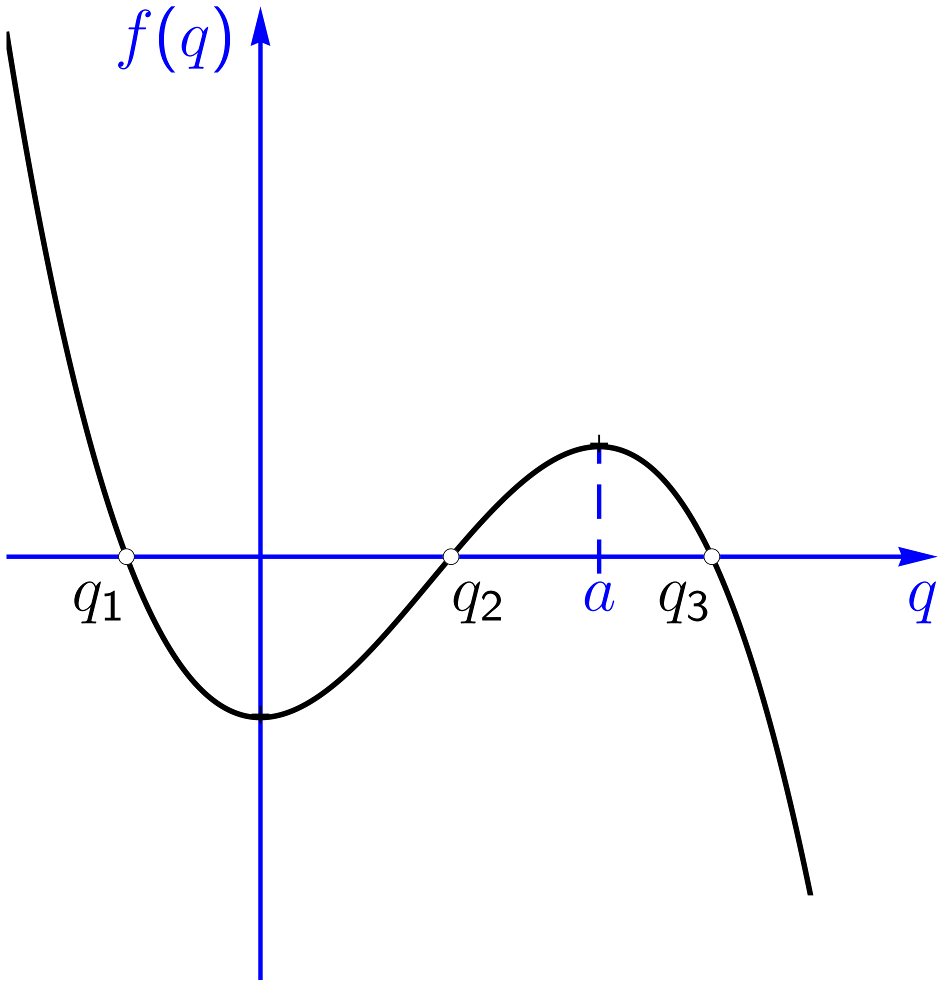

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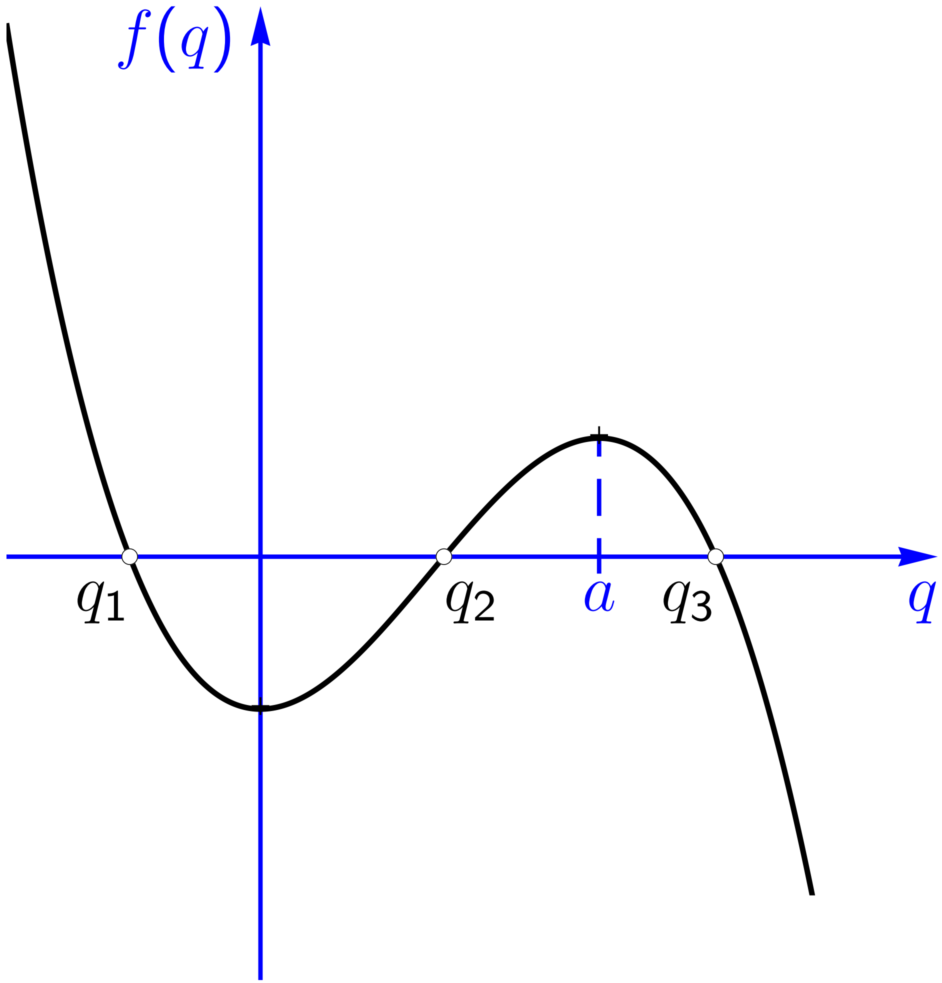

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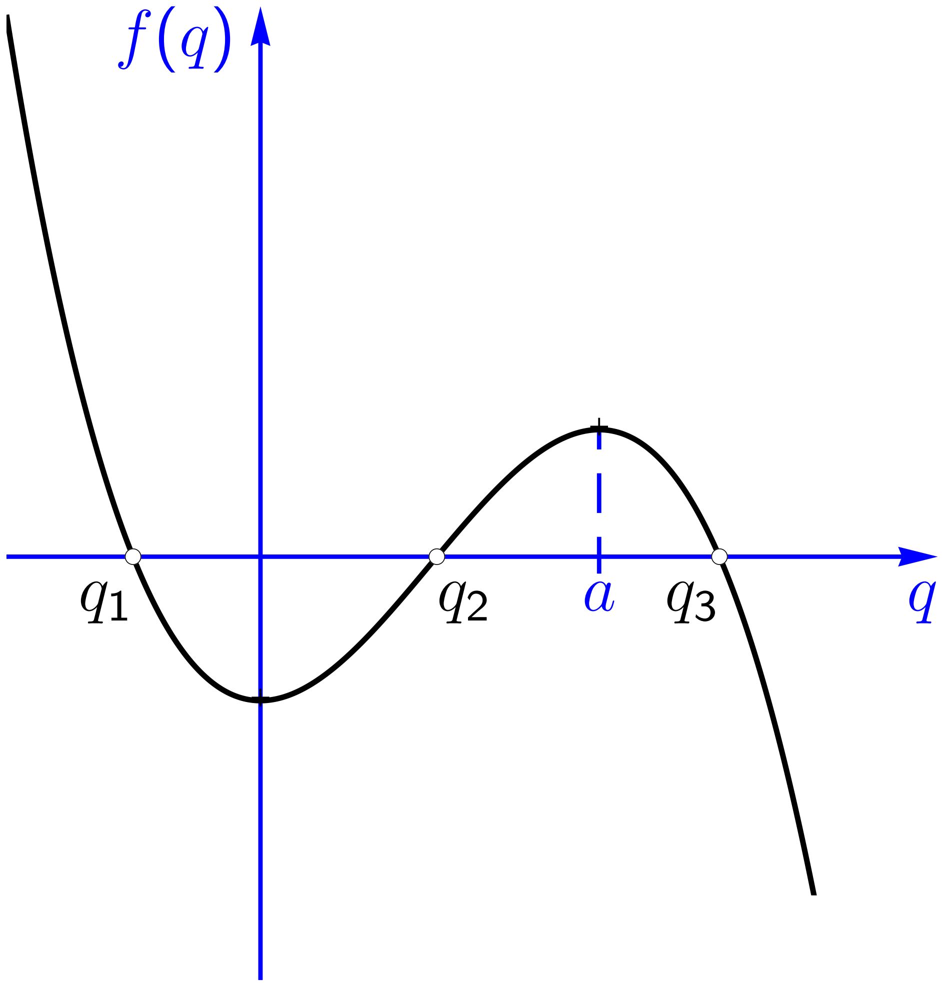

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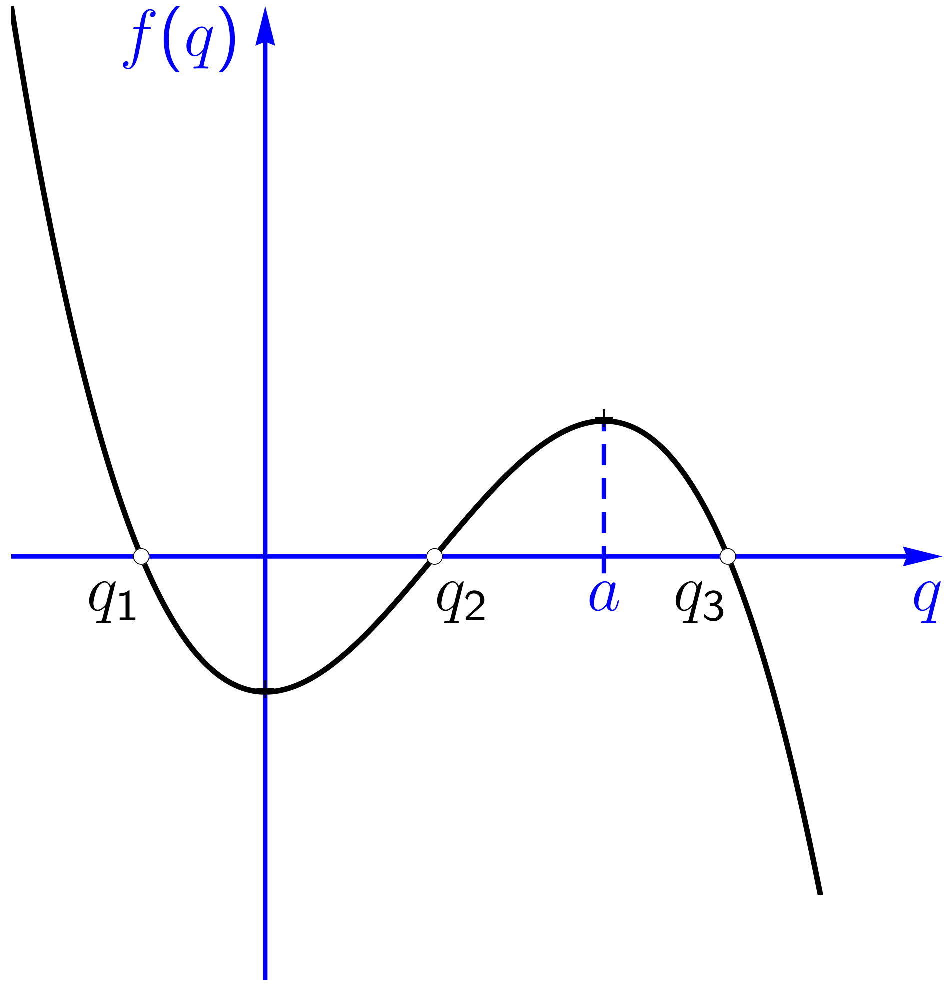

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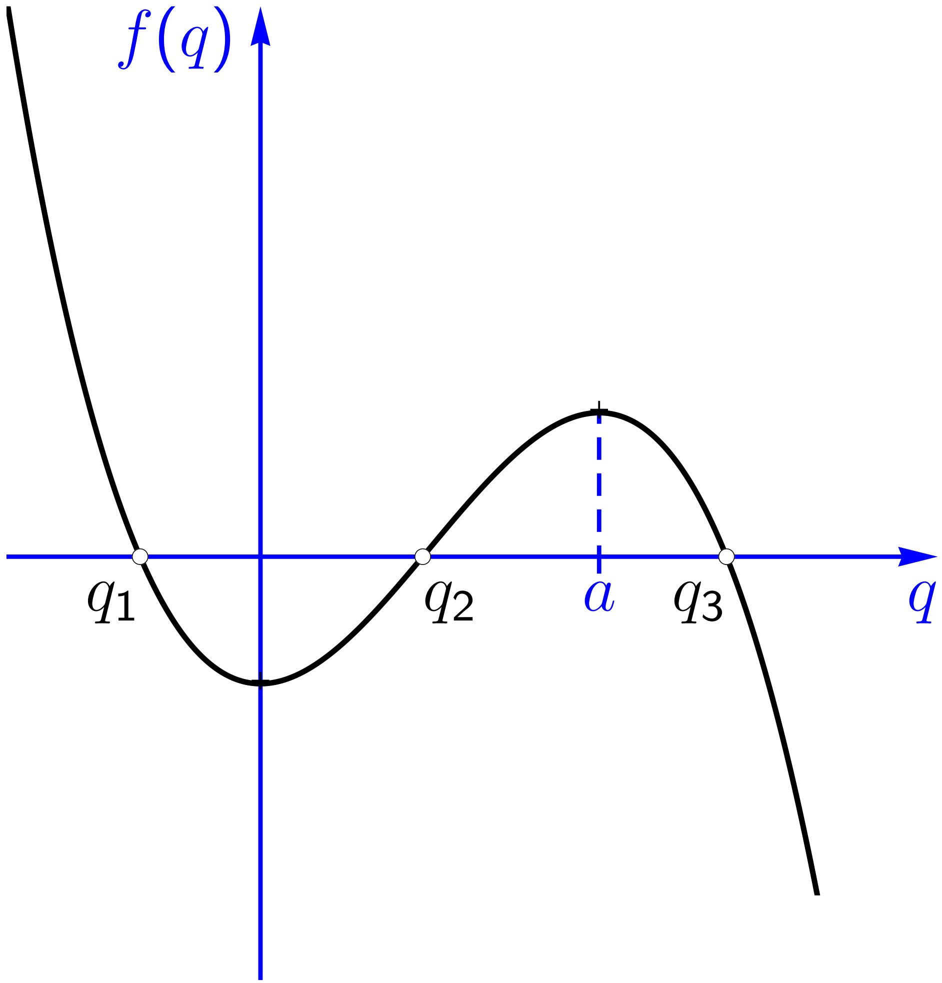

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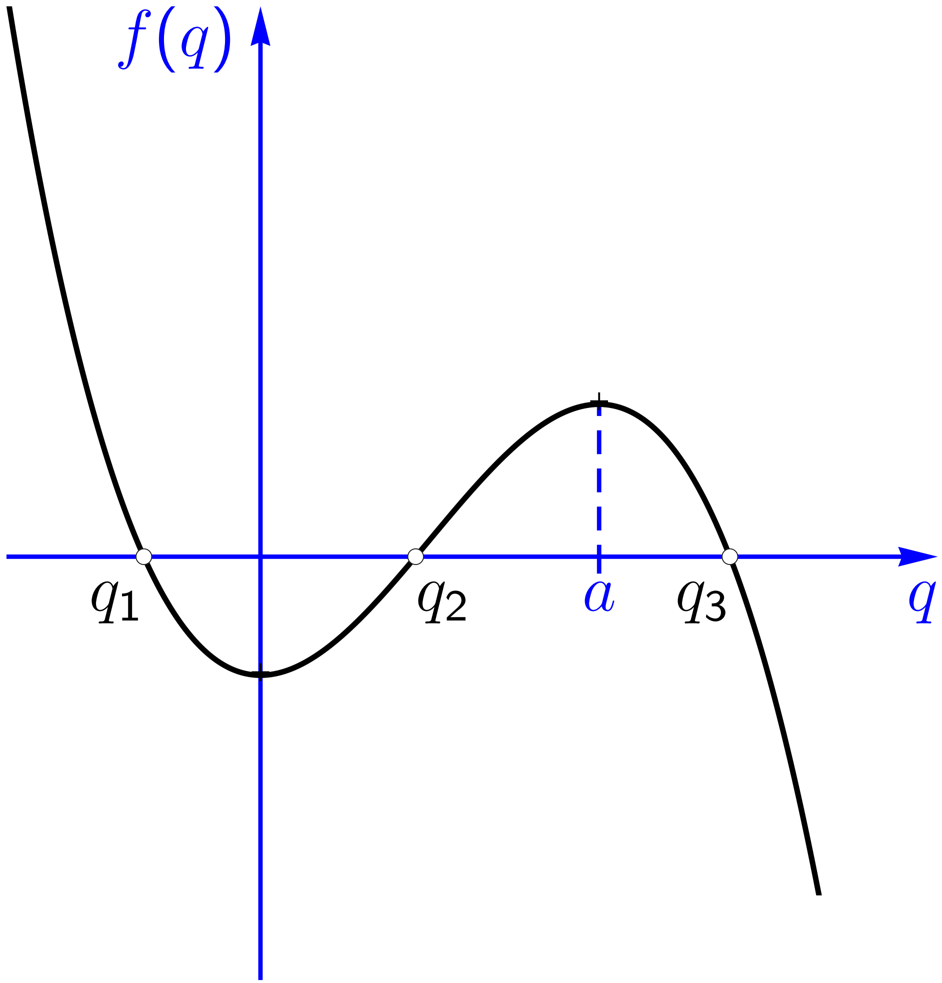

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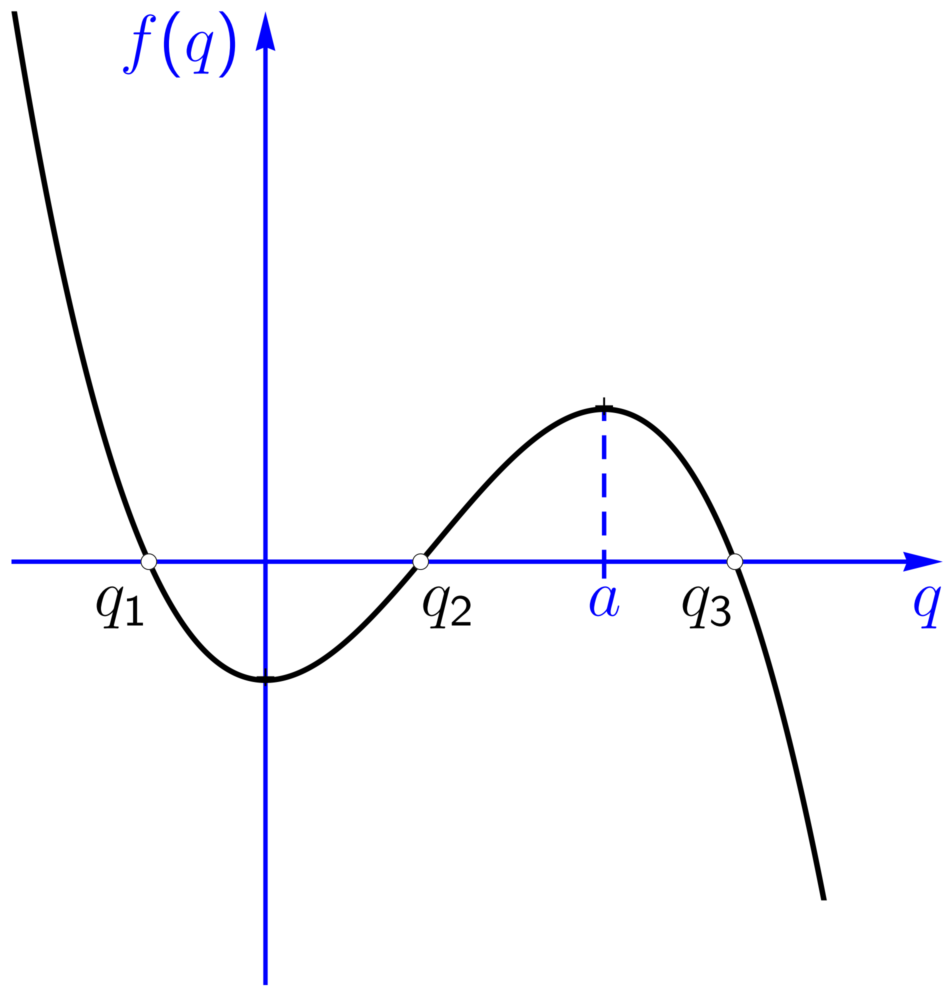
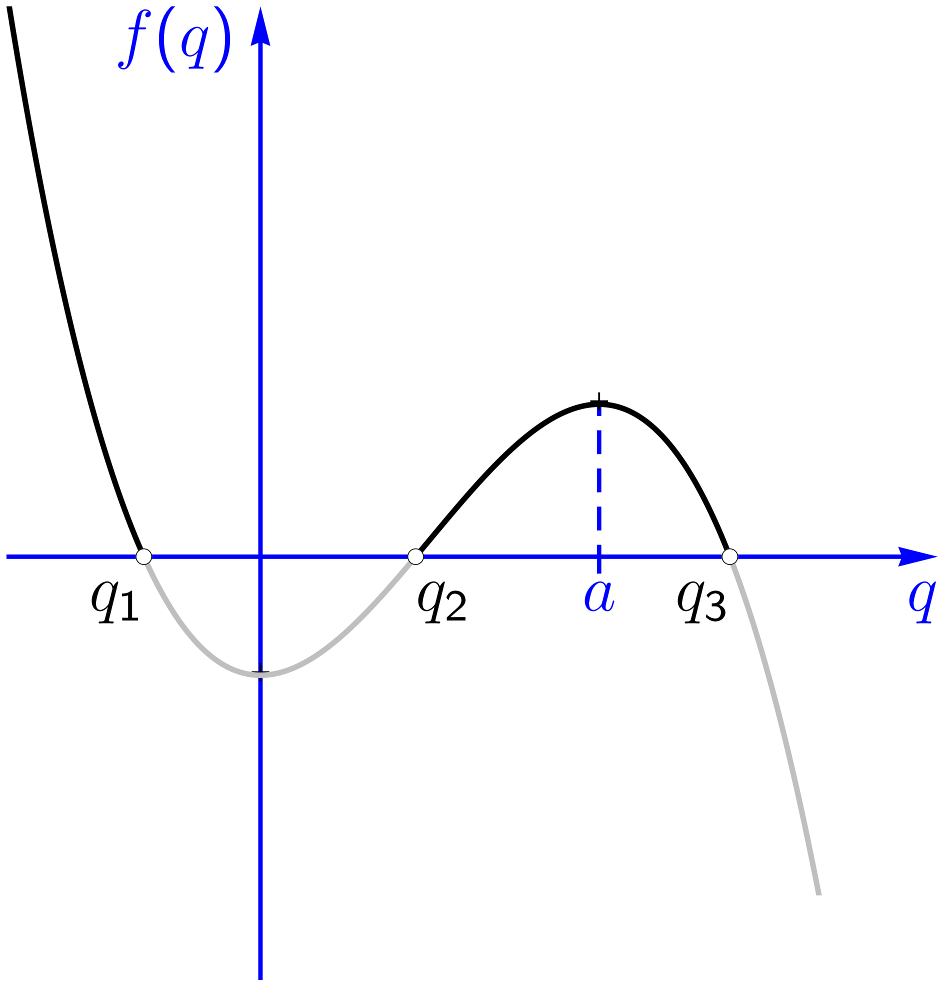


$$\frac{\mathrm{d}q}{\mathrm{d}\theta} = \pm\sqrt{f(q)}$$

Function f and Phase Curves



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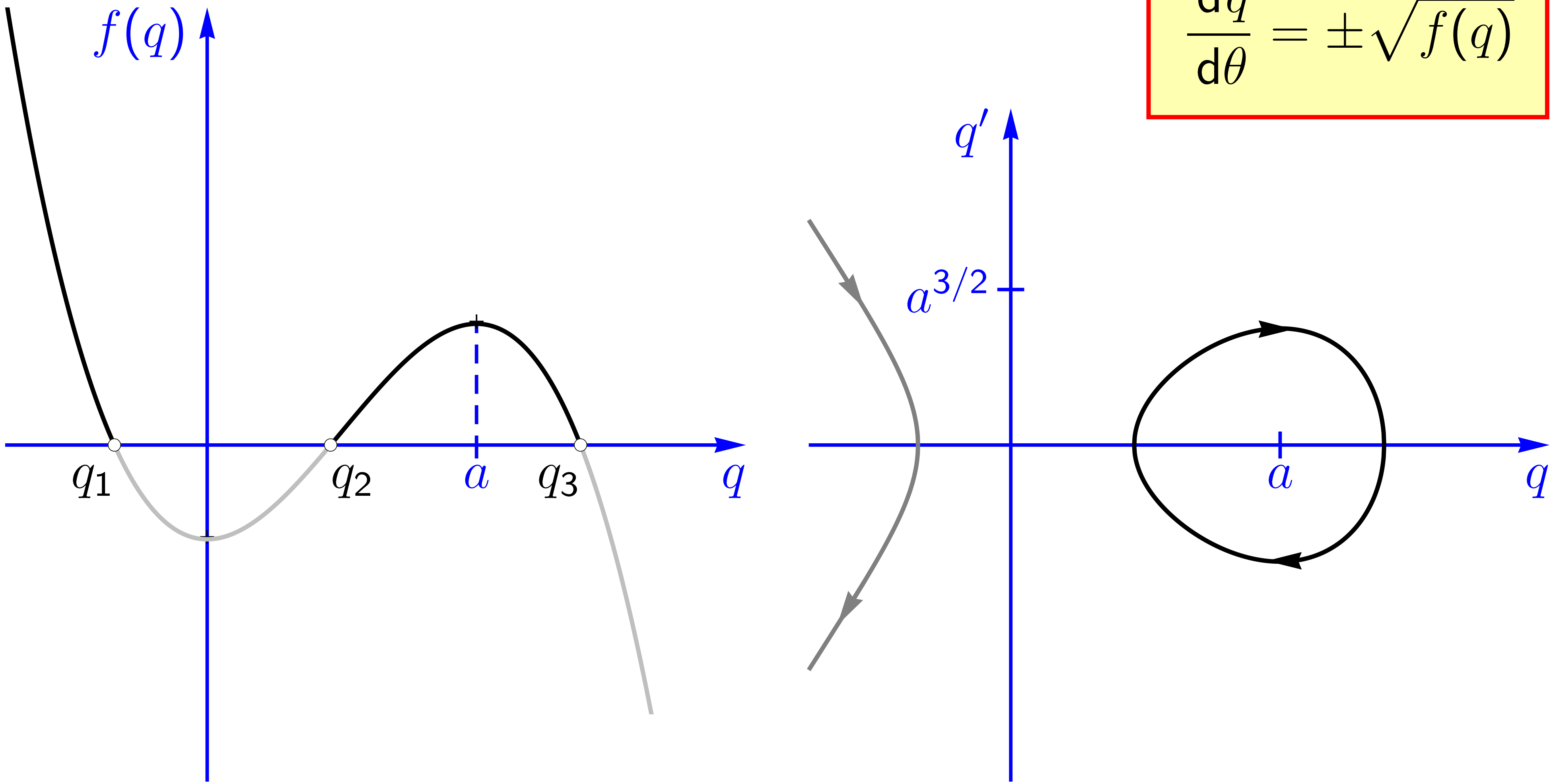

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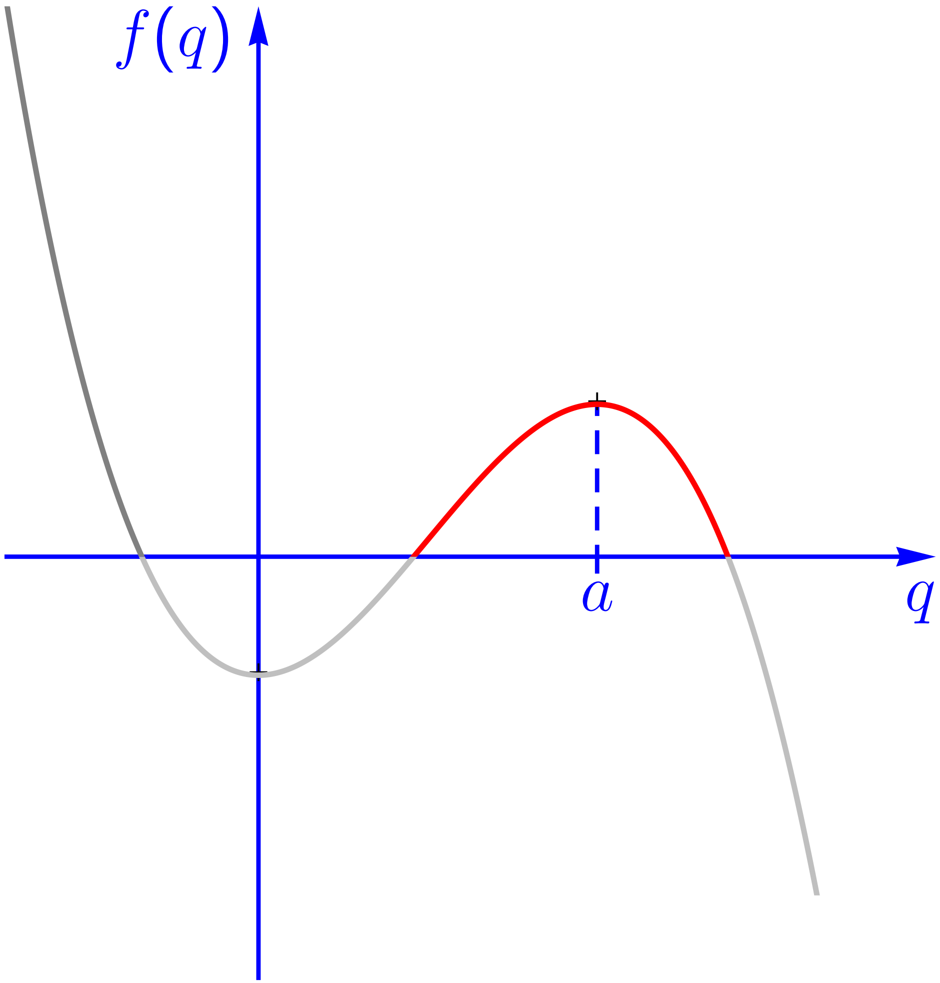

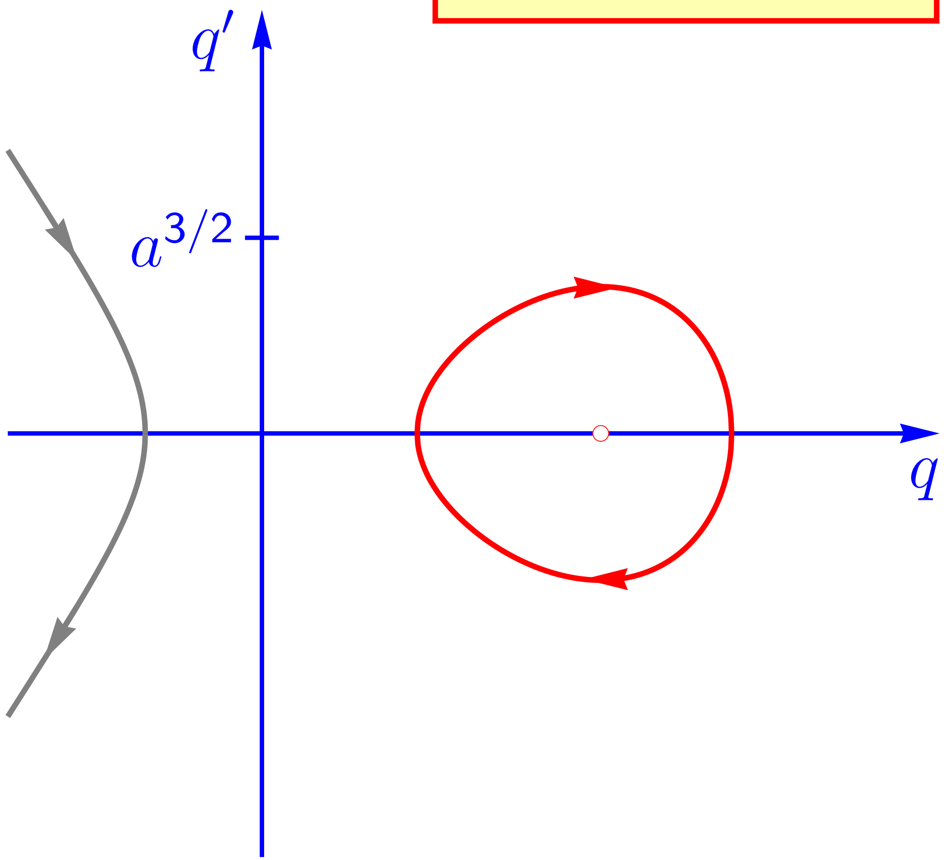

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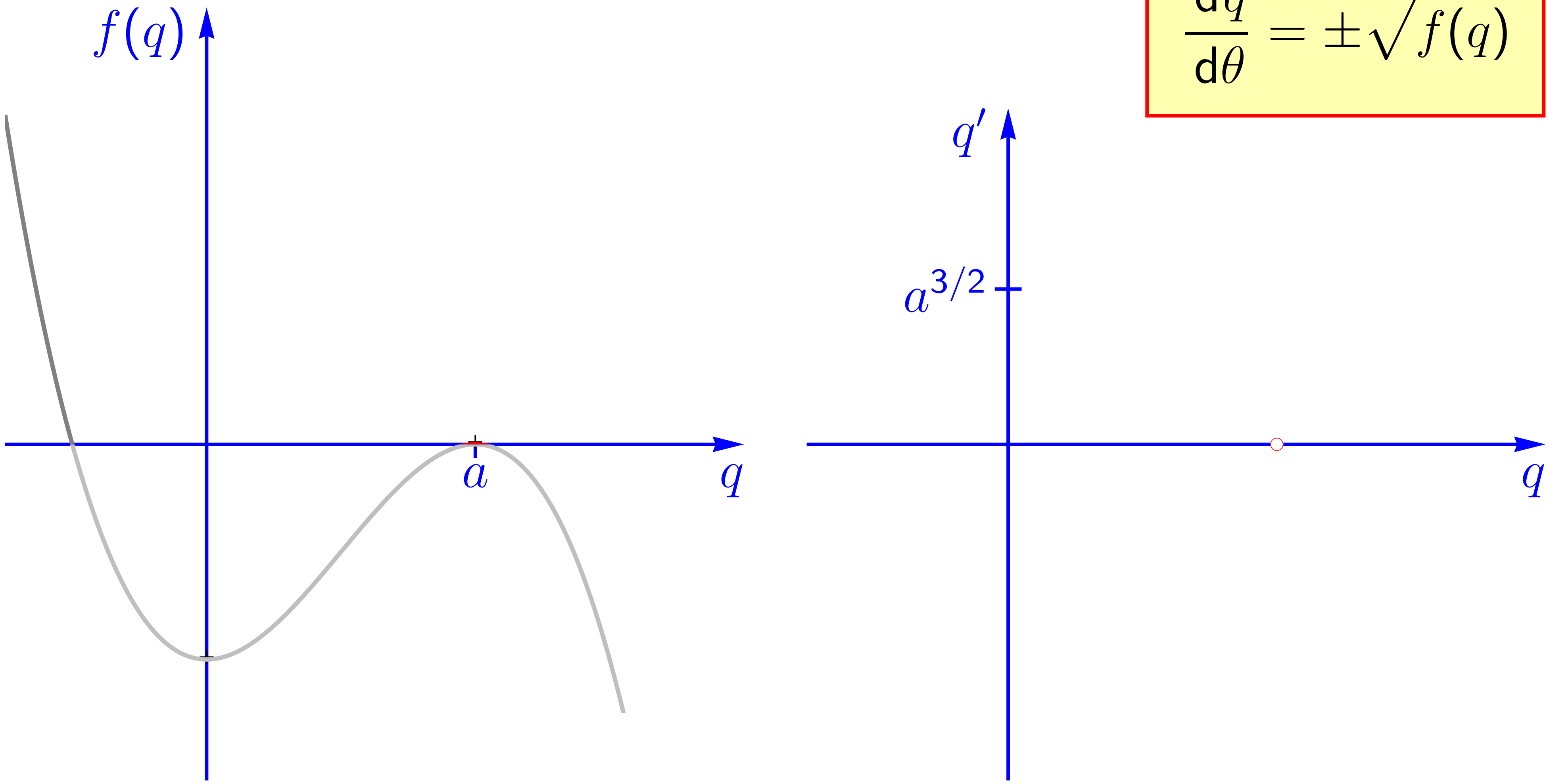

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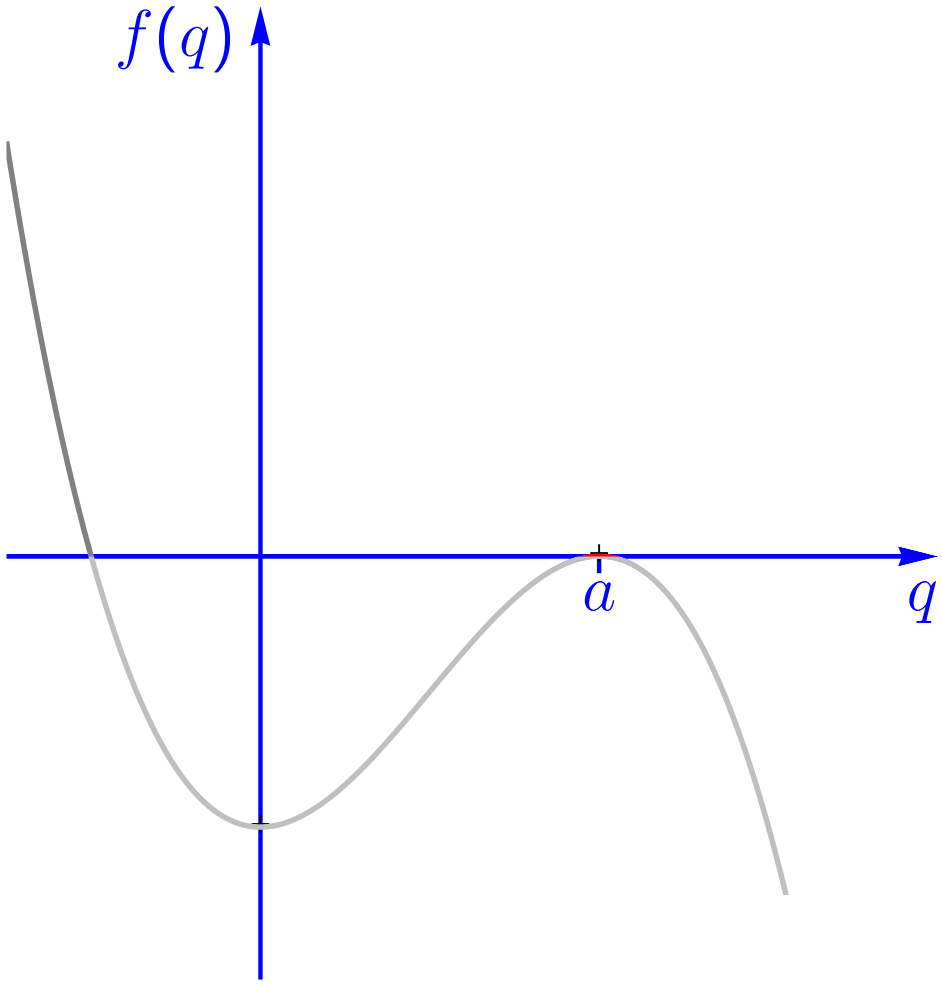

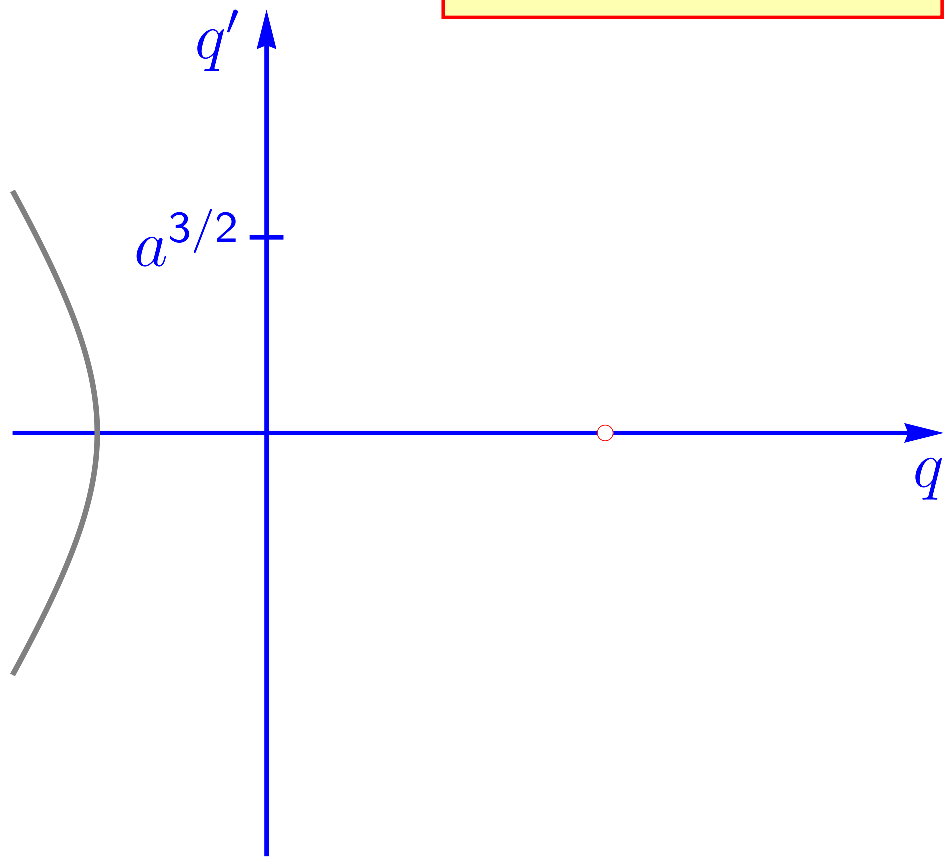

Function f and Phase Curves

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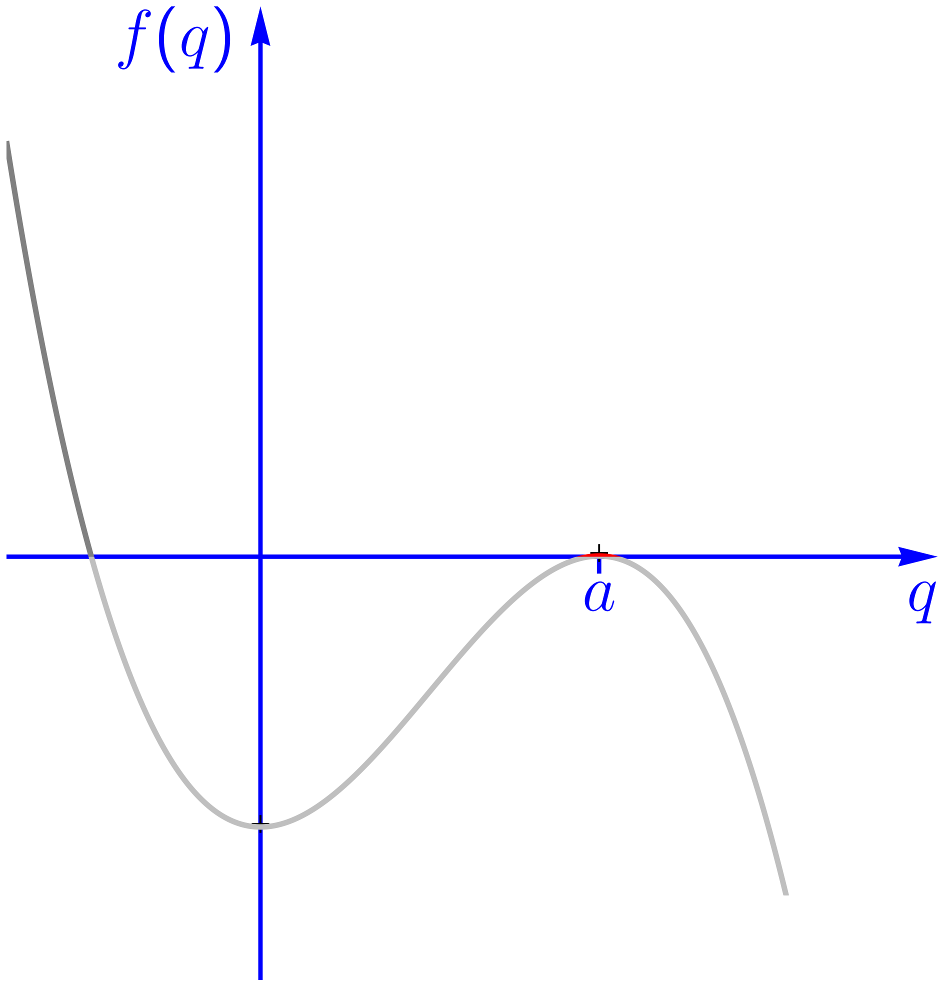

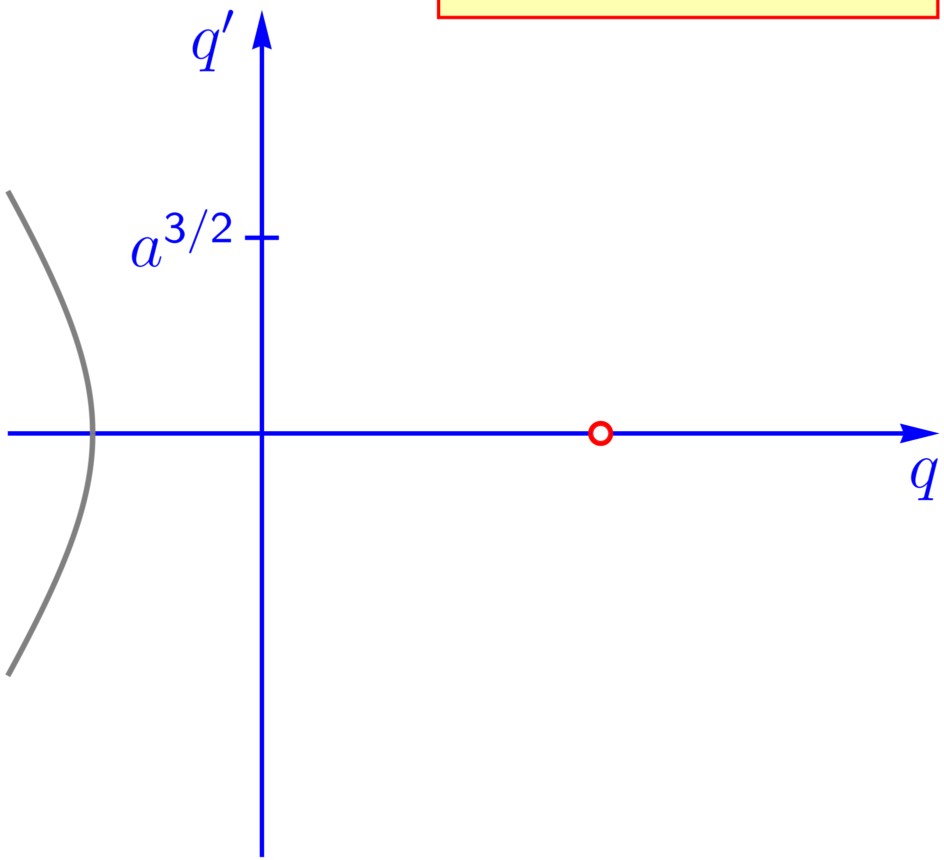

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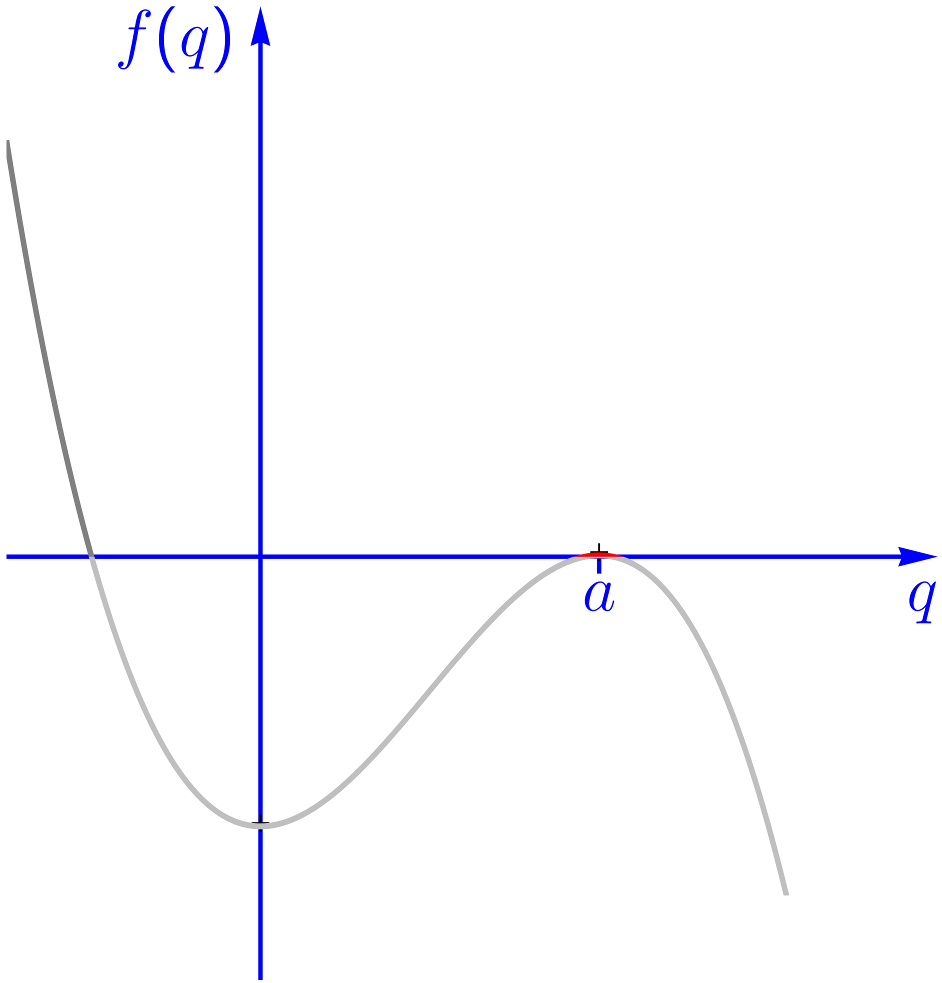

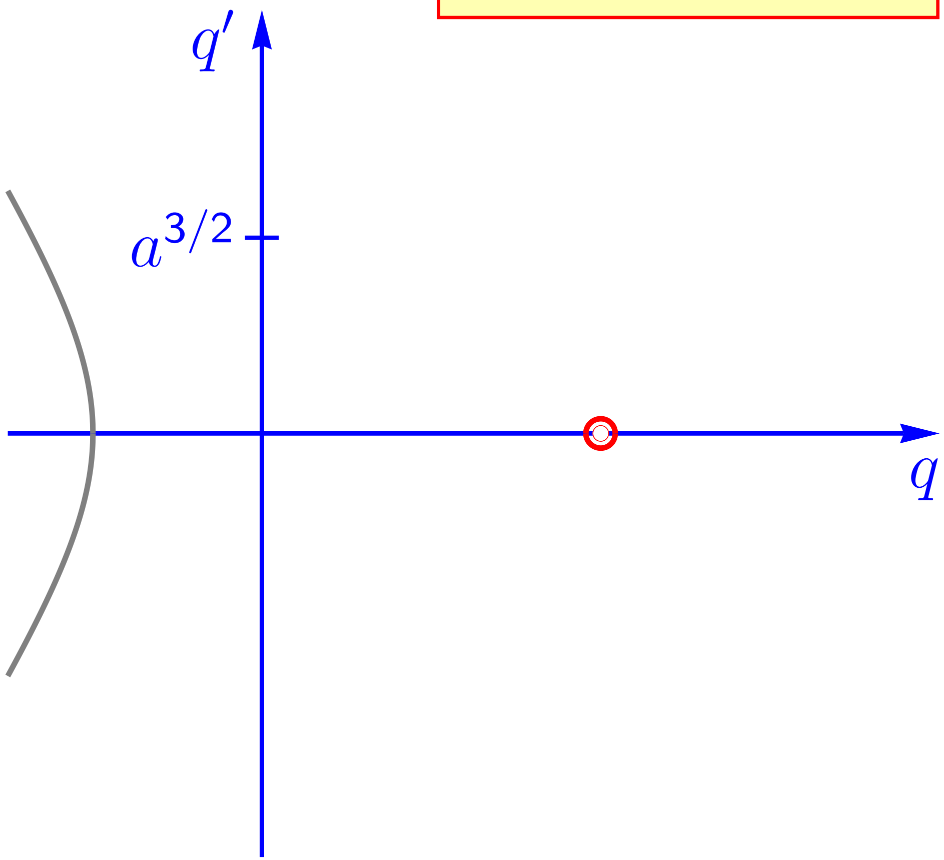

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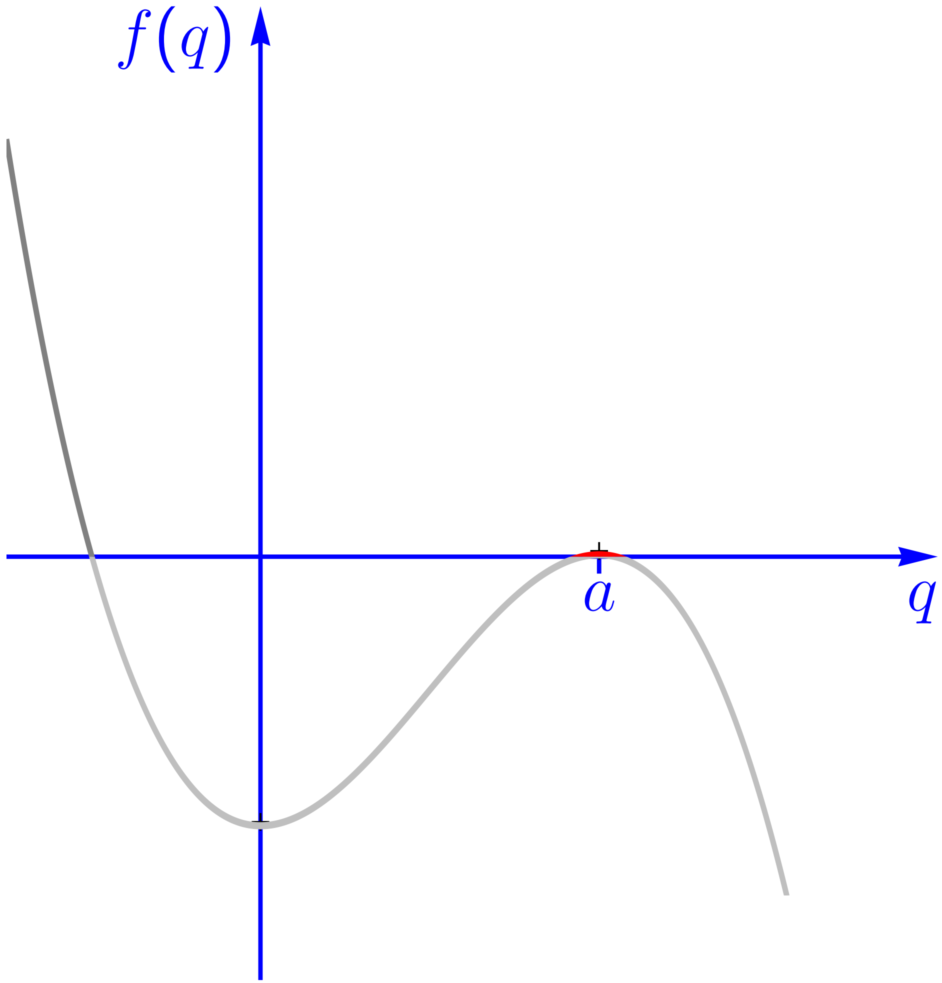

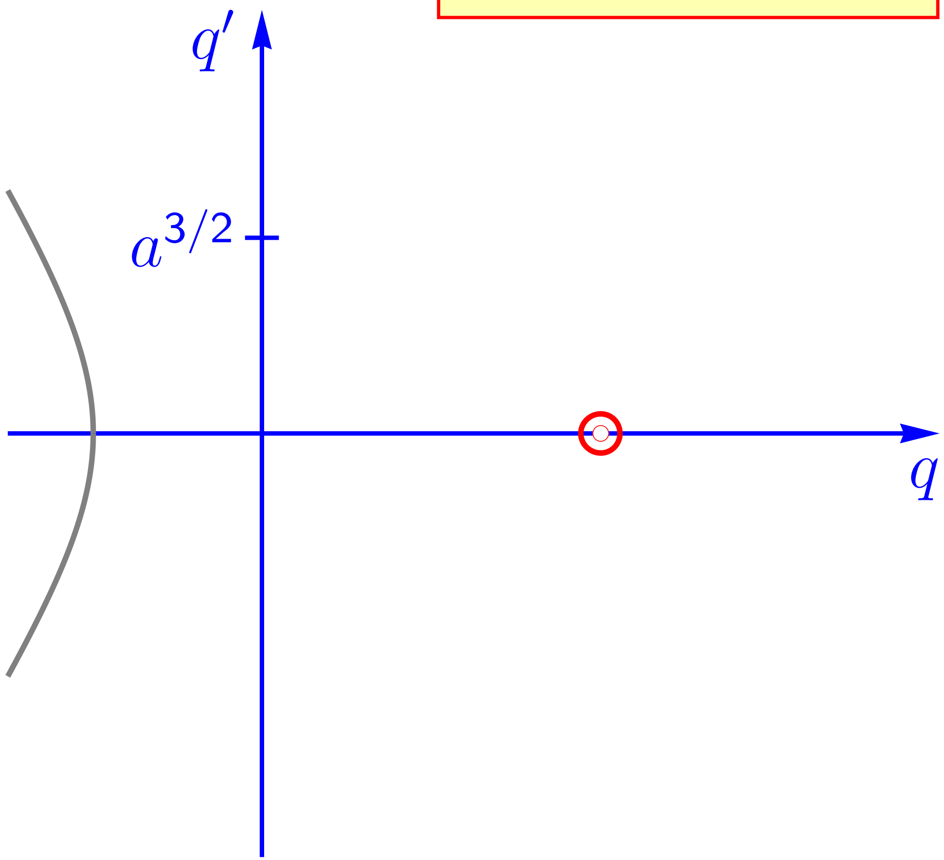

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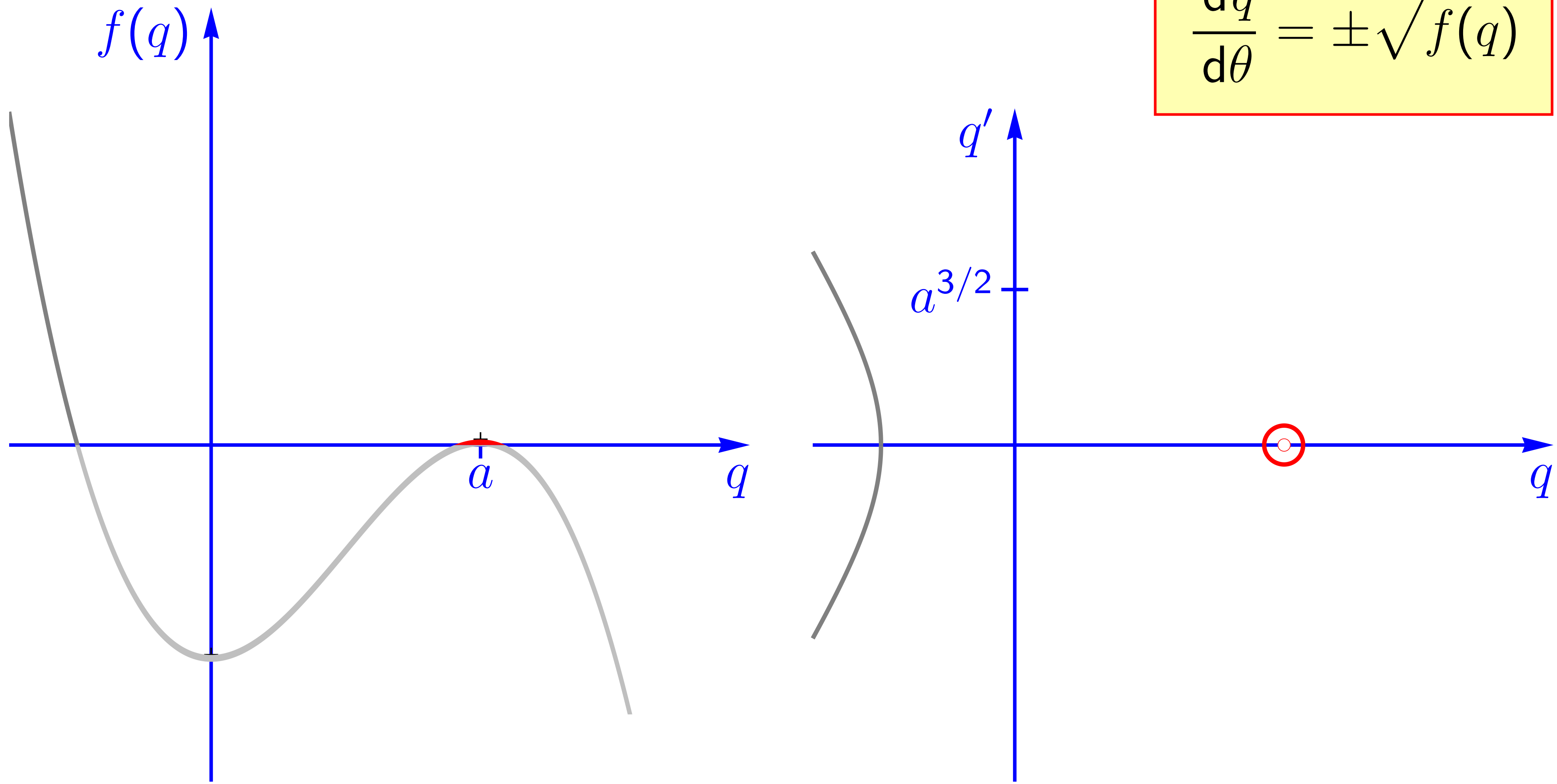

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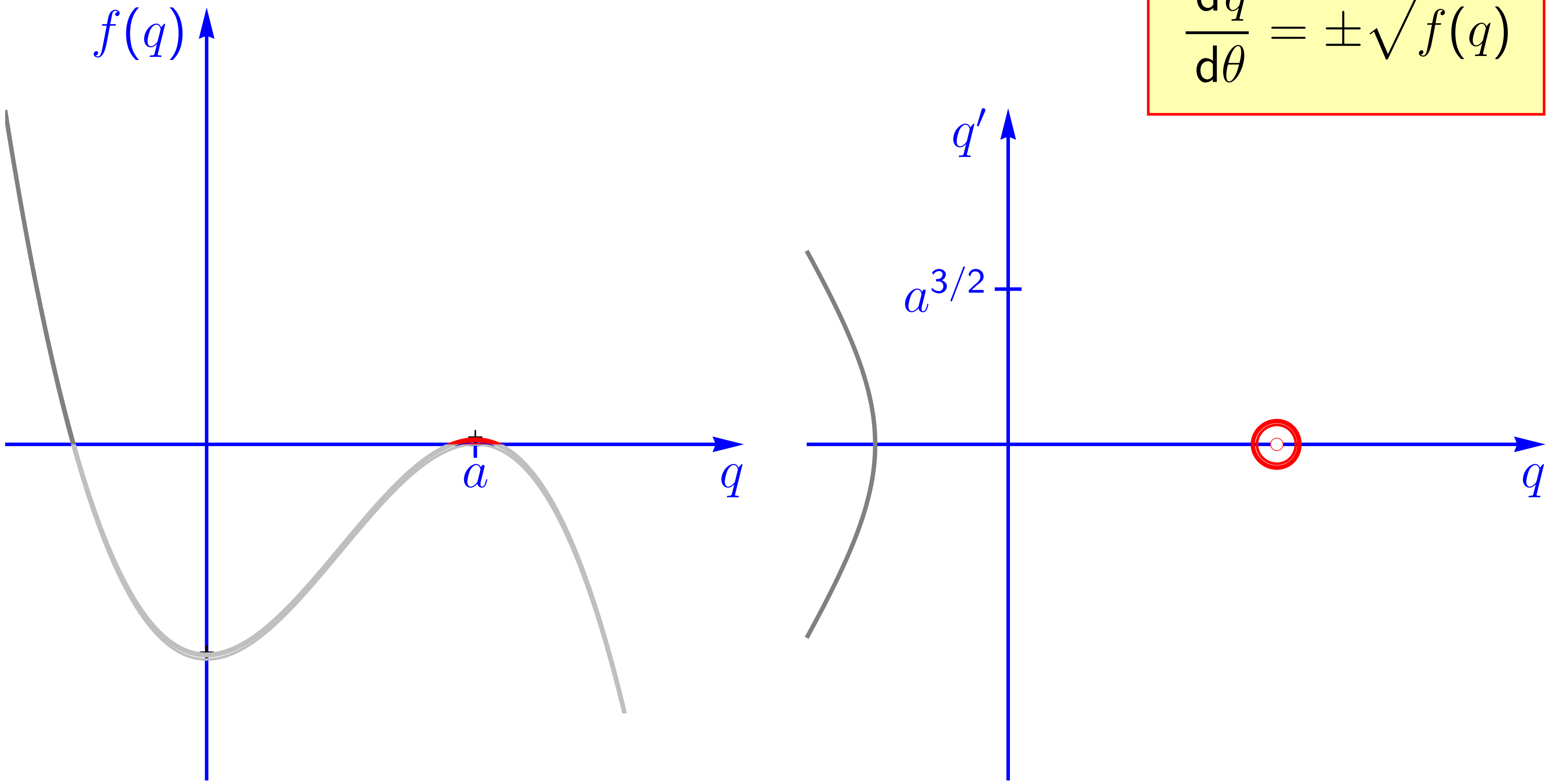

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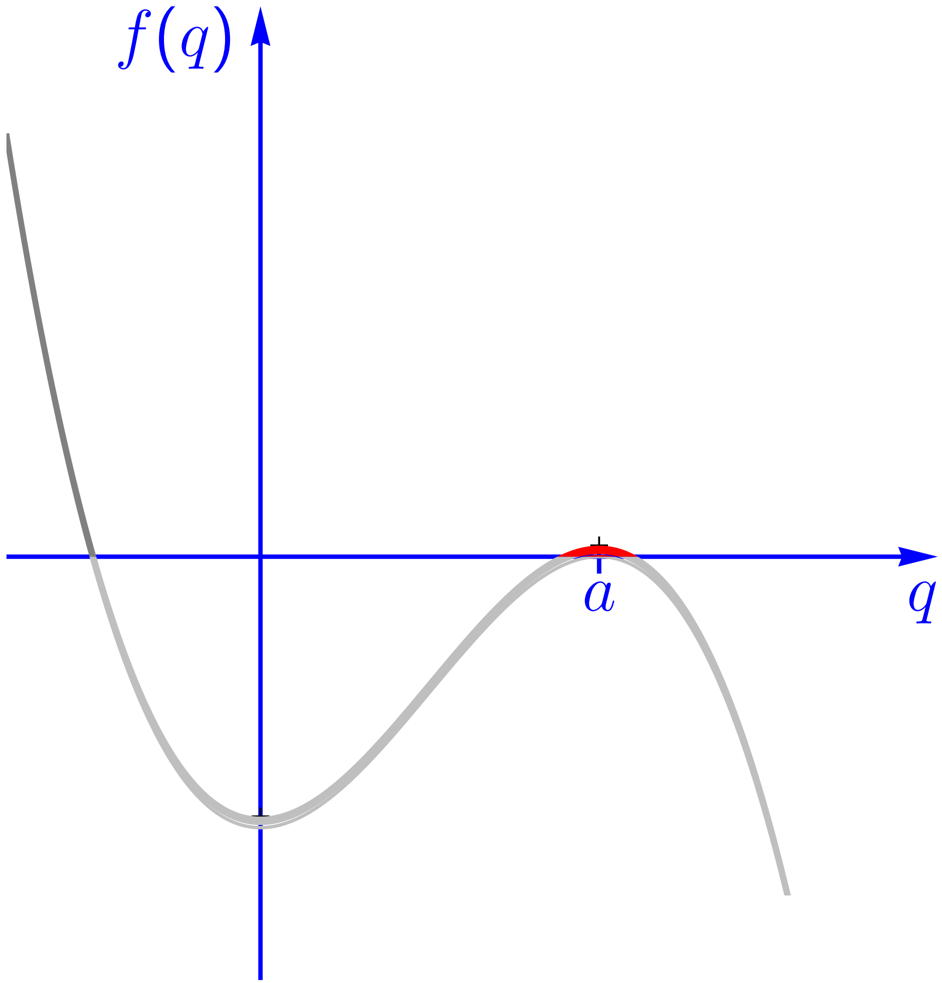

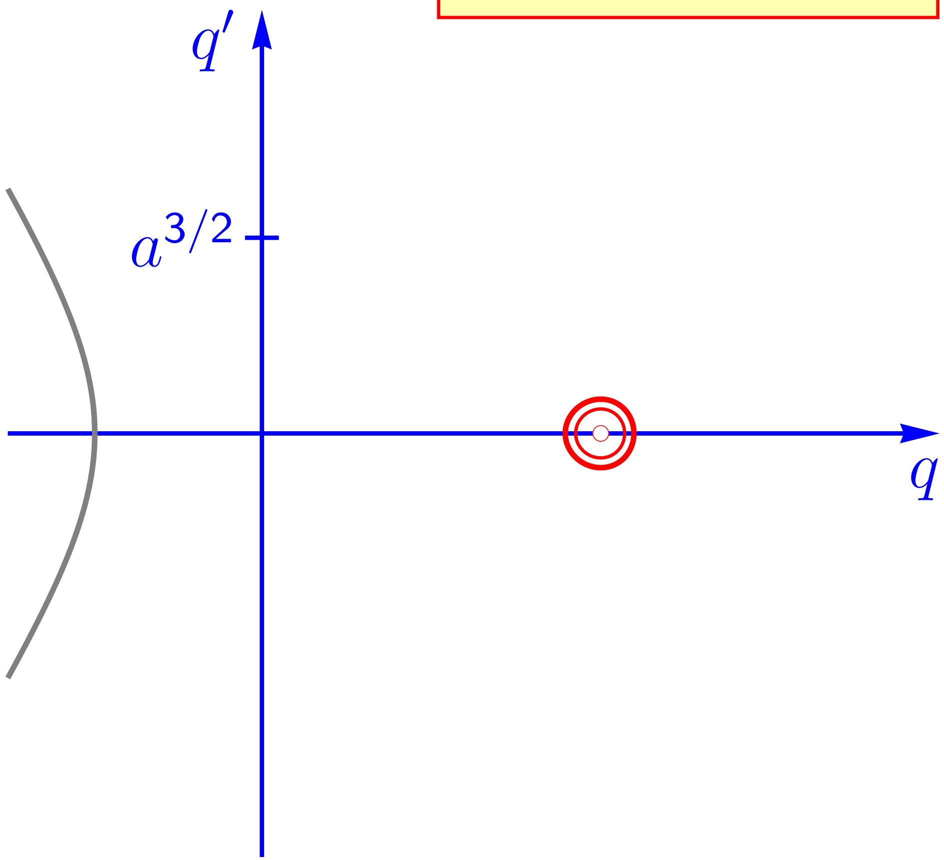

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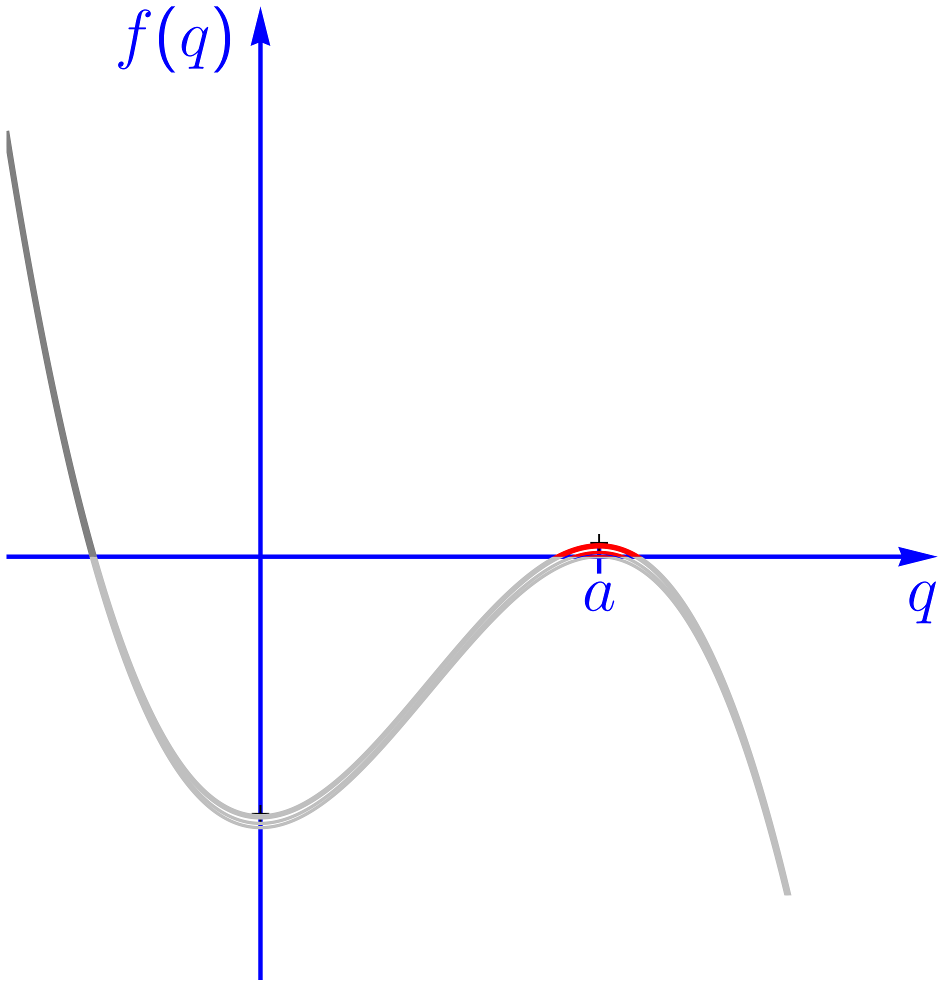

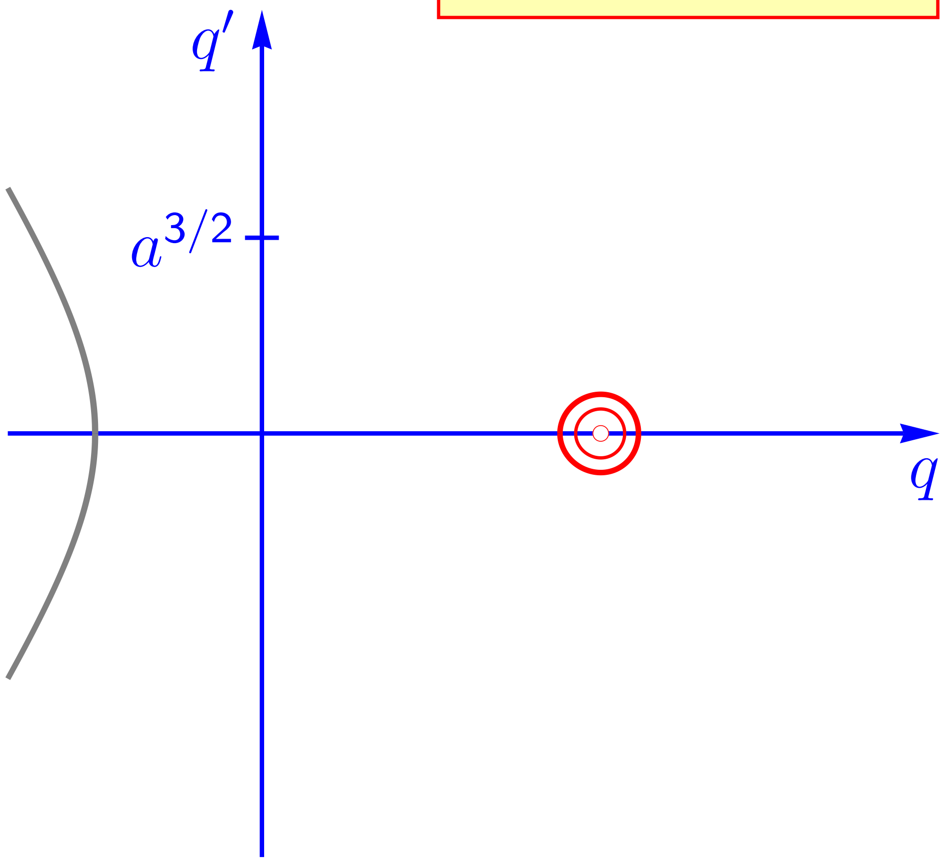

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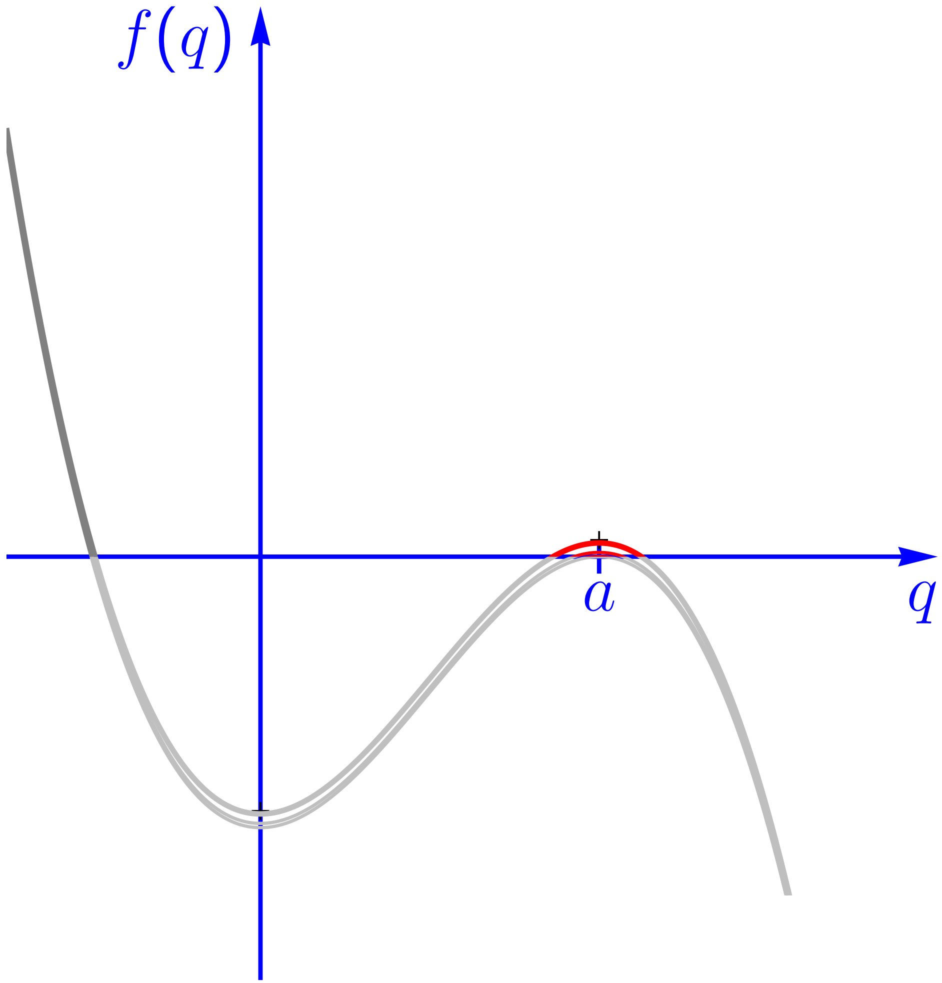

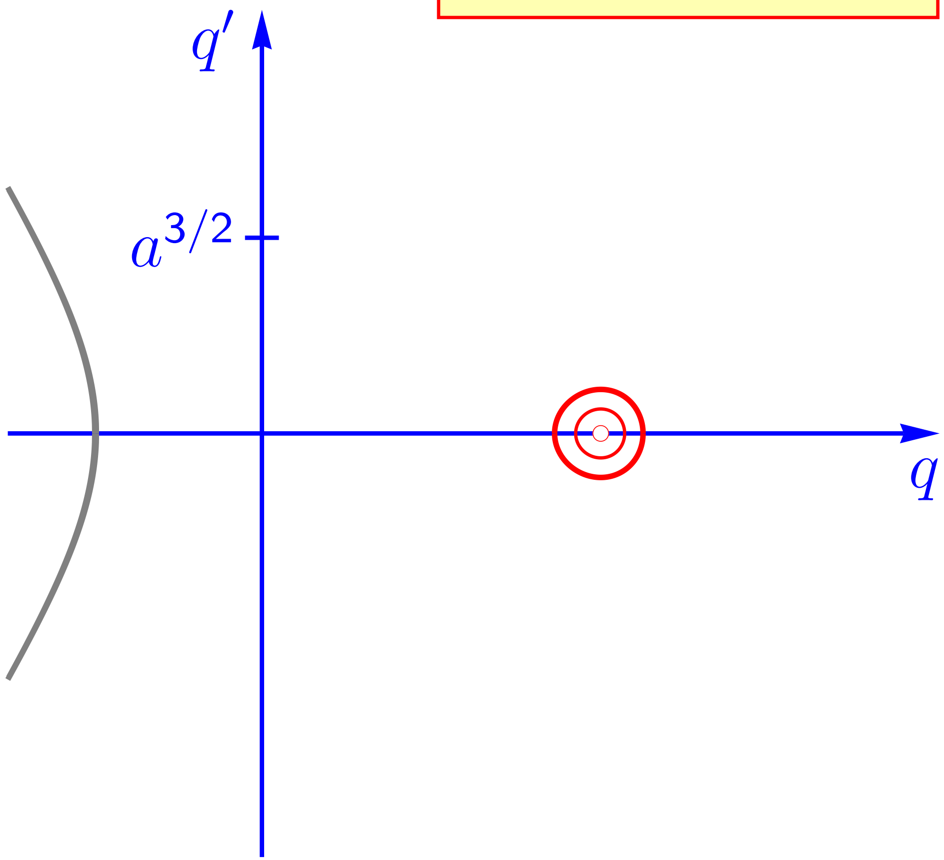

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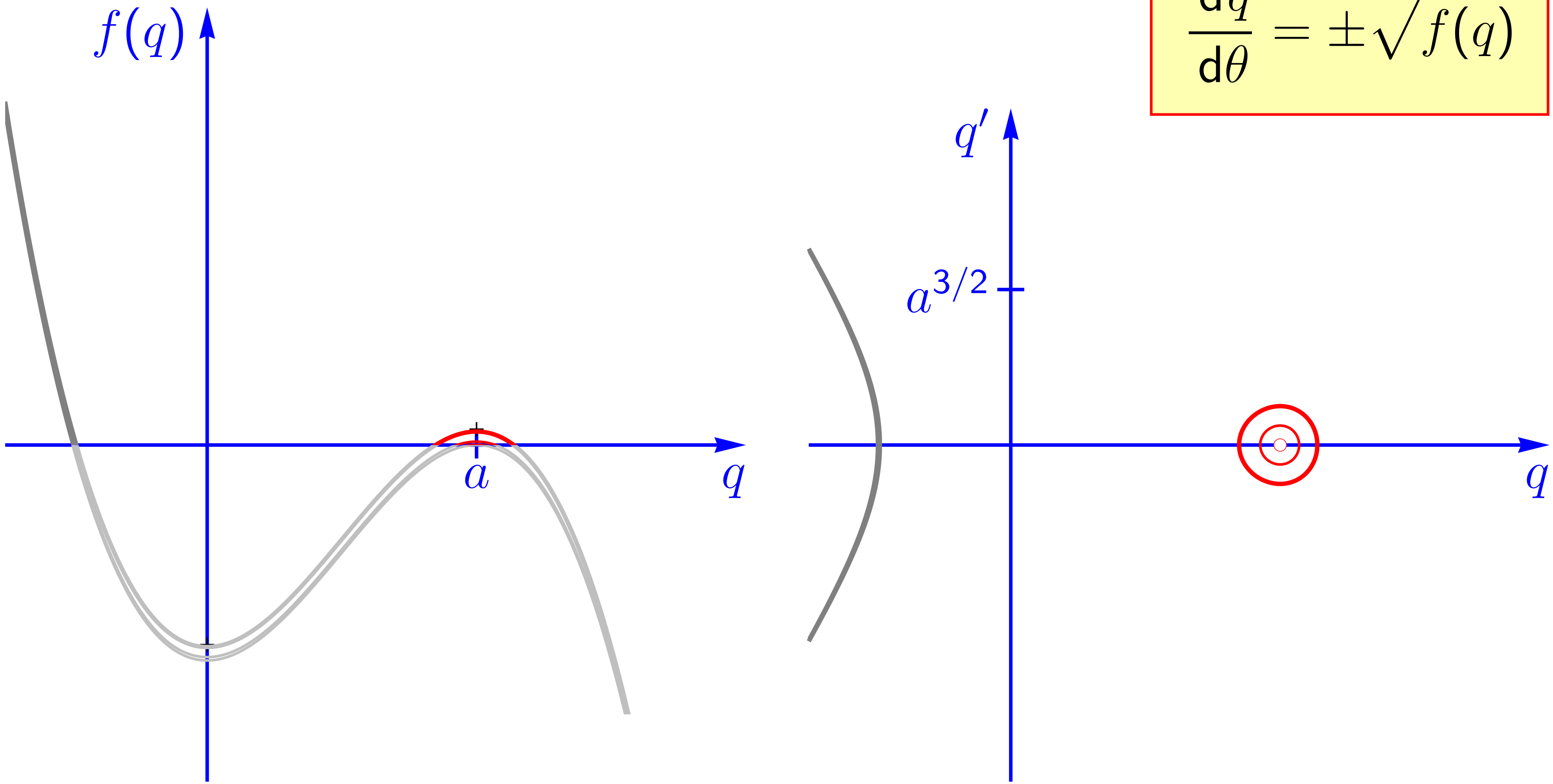

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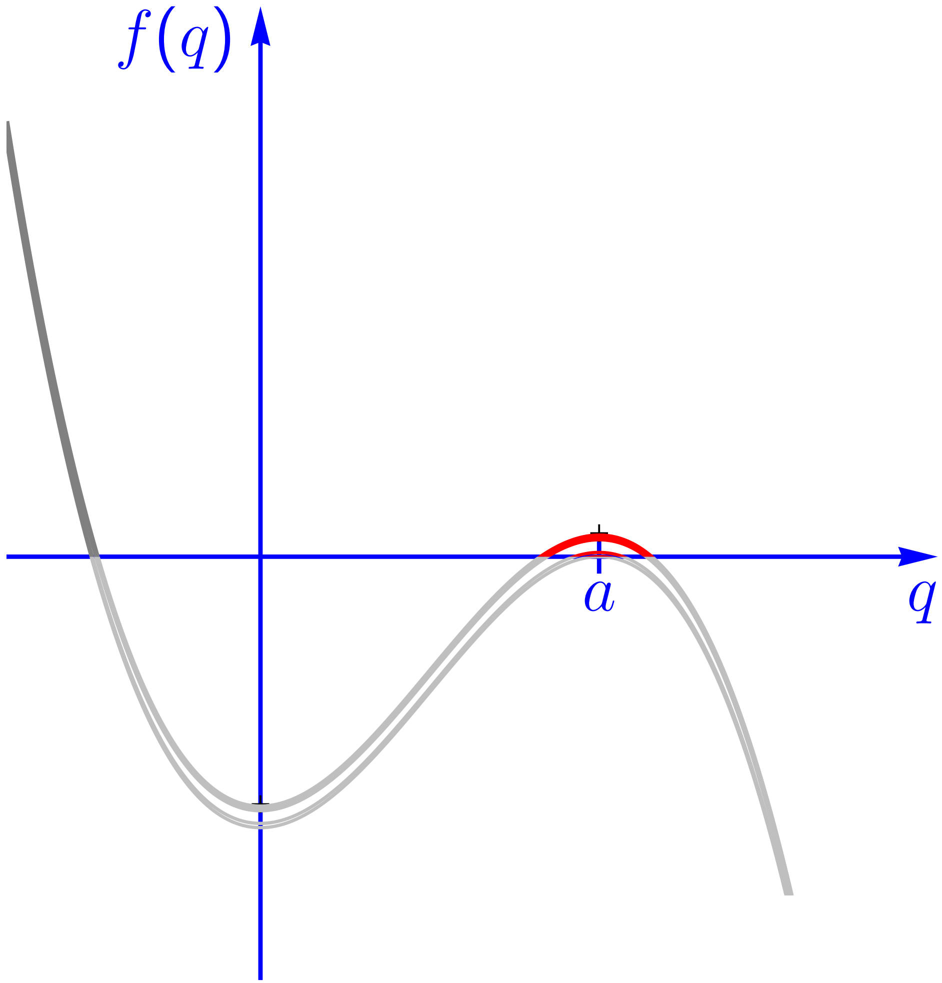

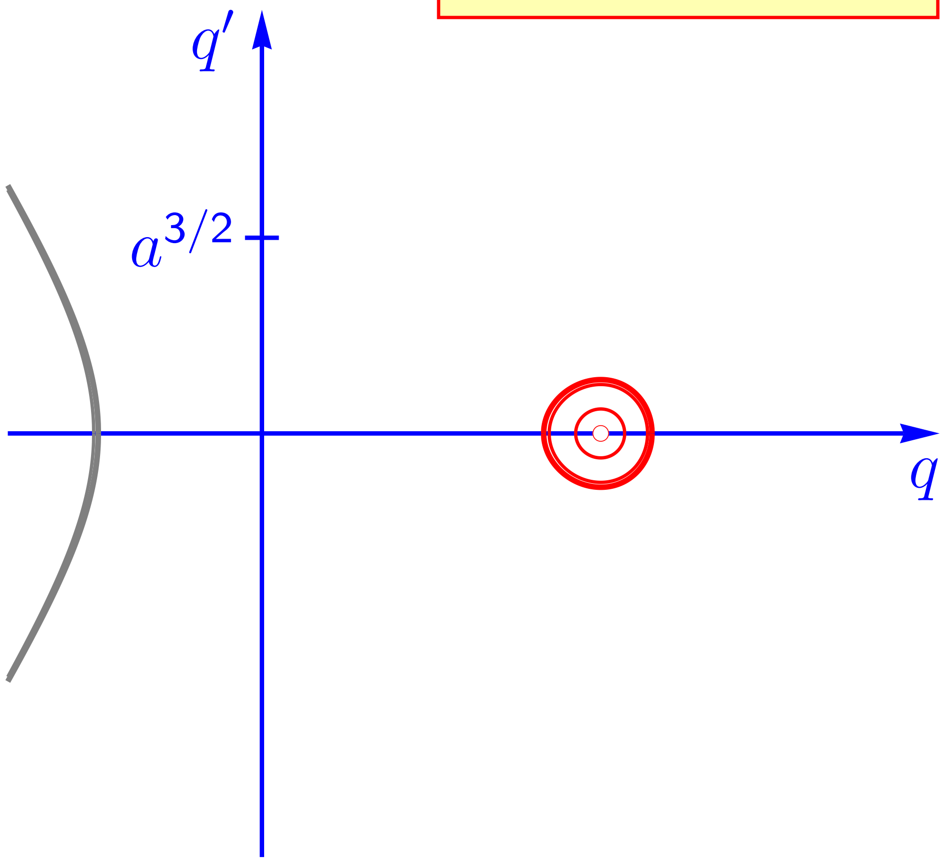

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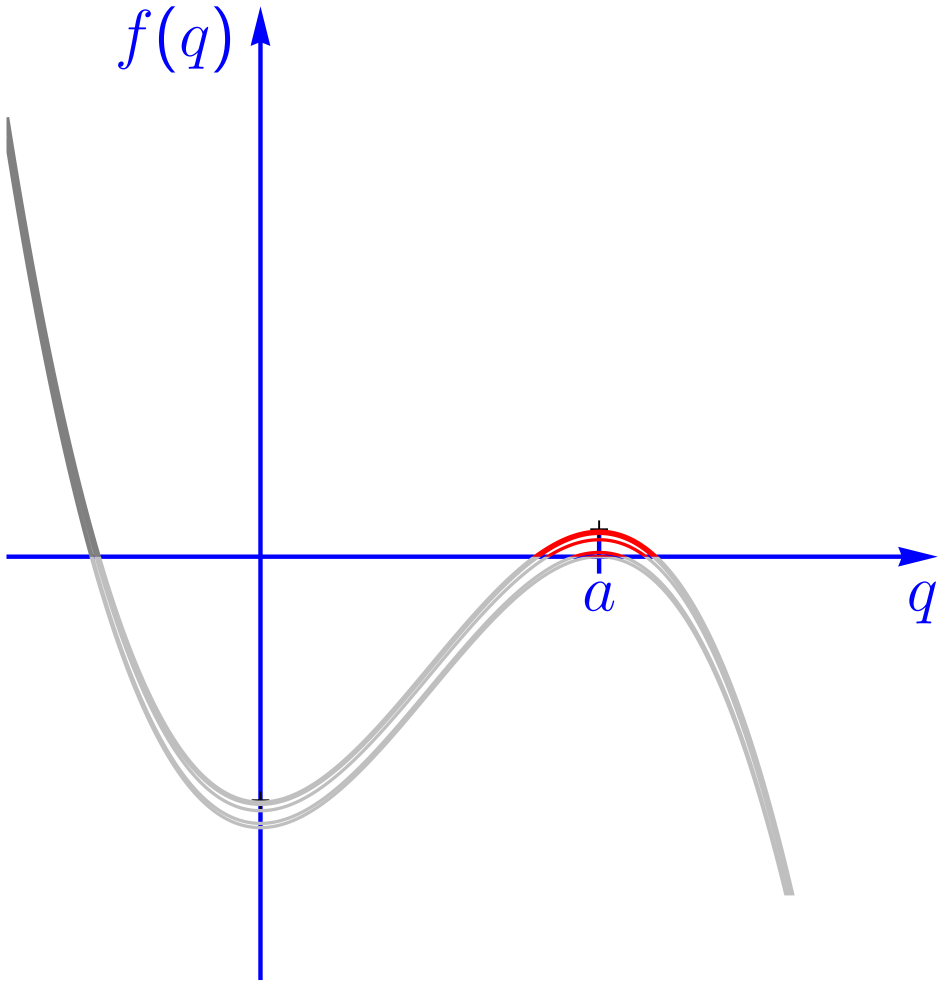

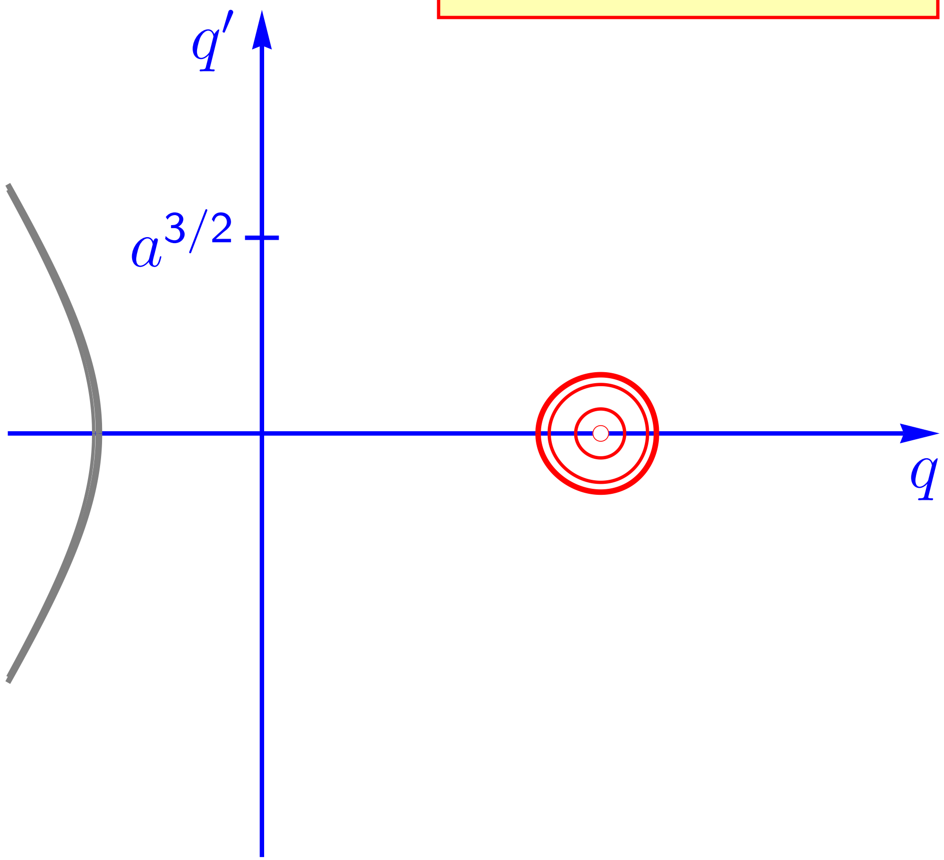

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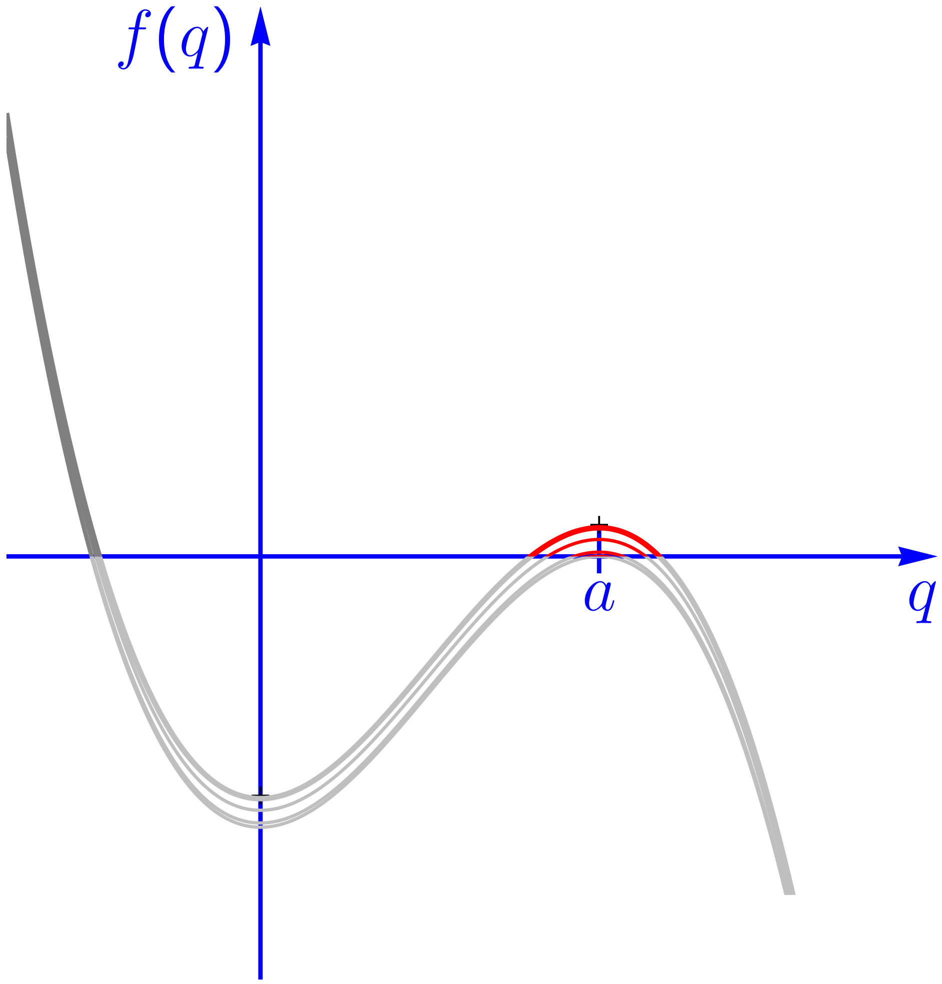

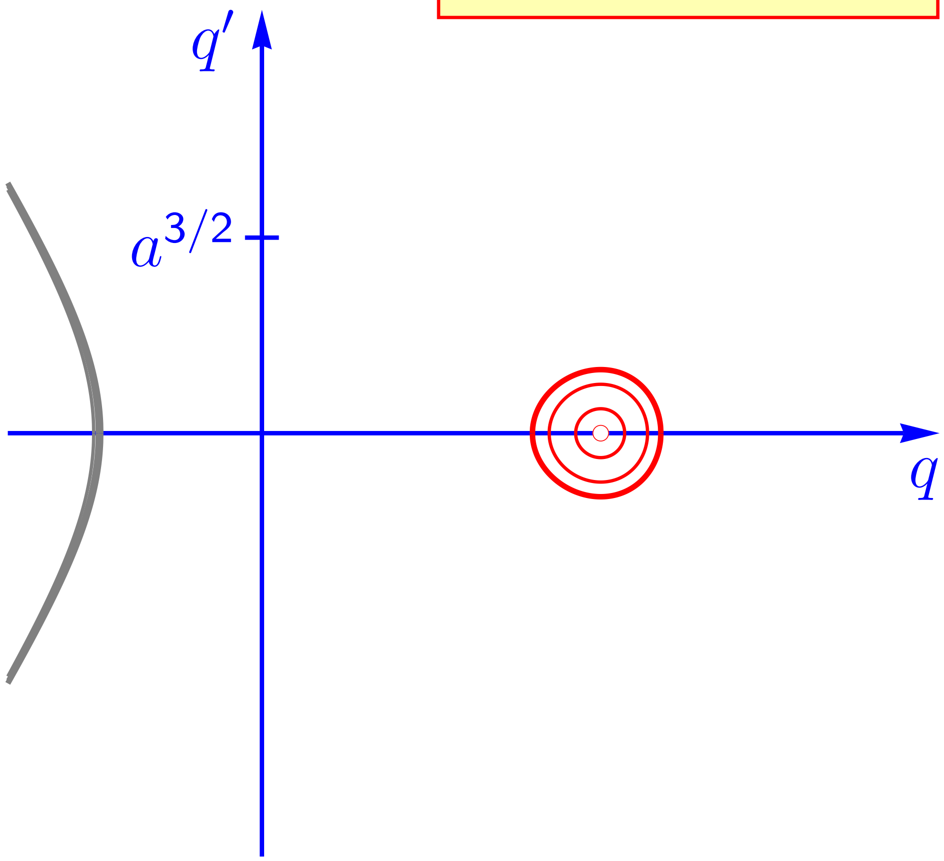

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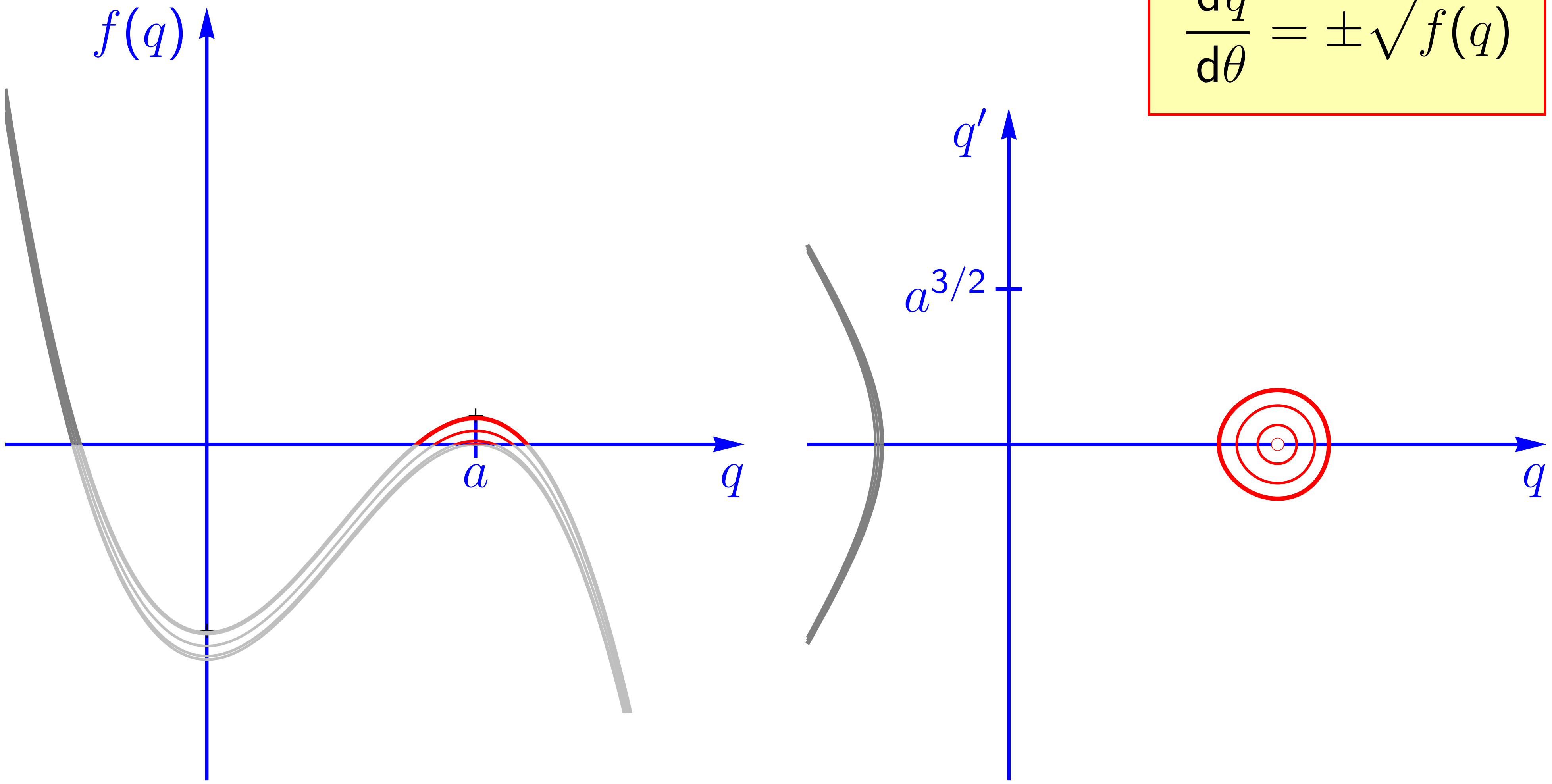

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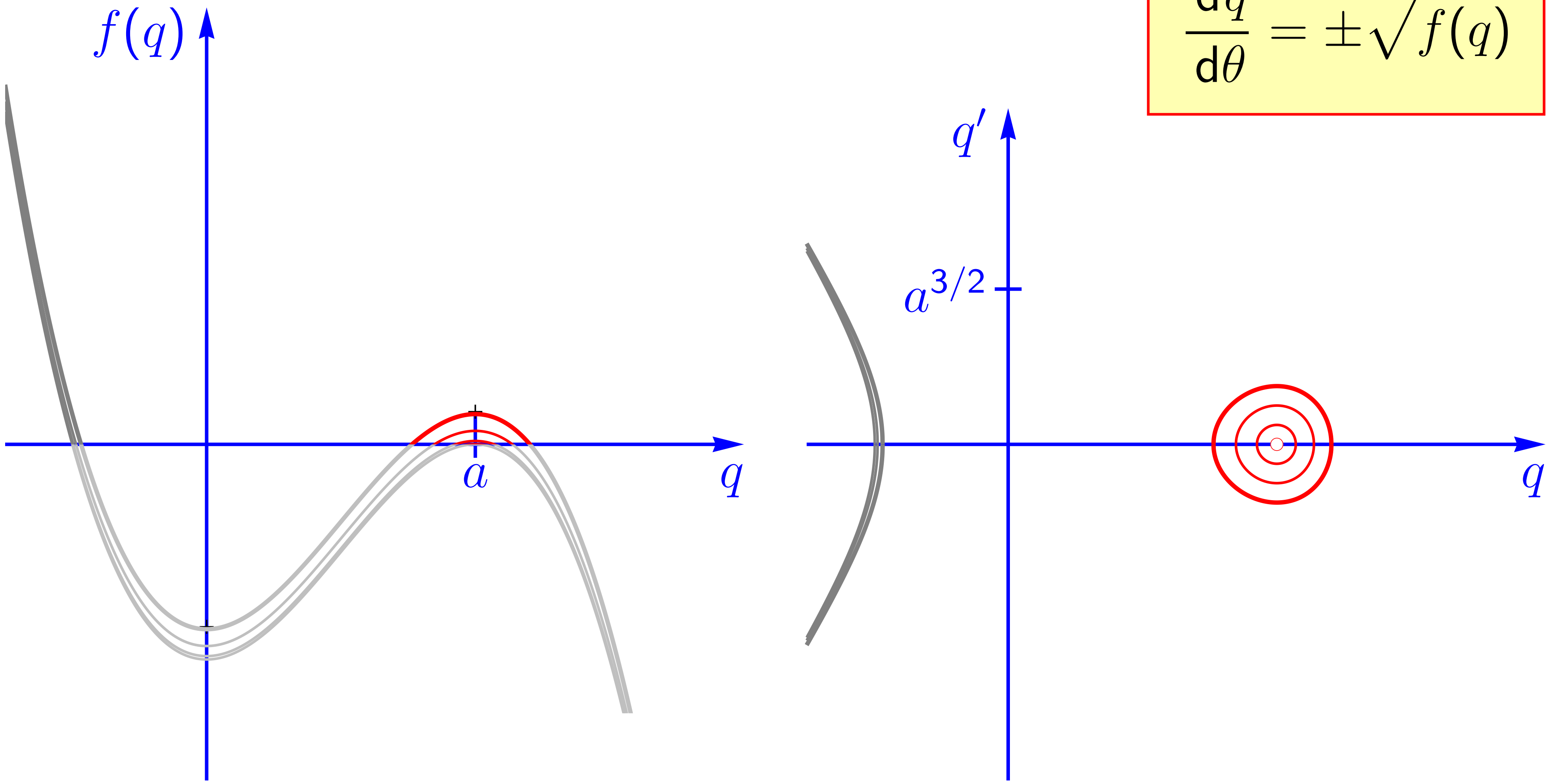

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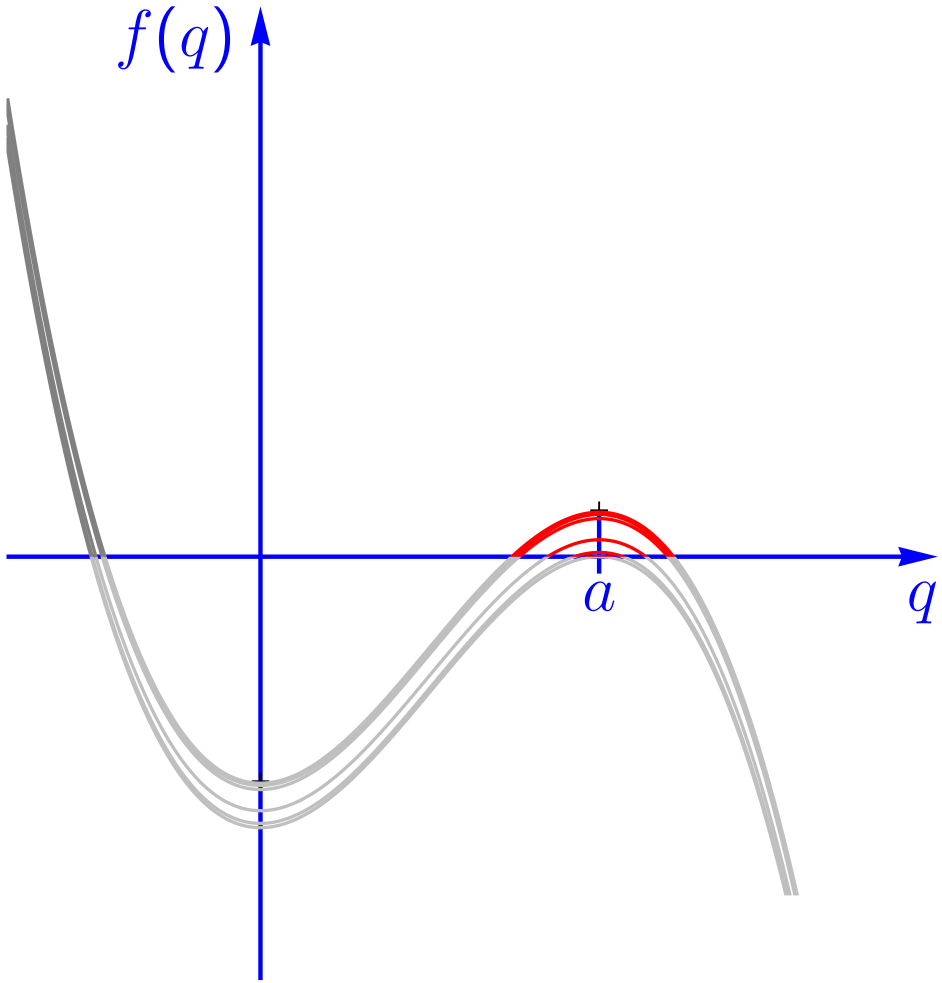

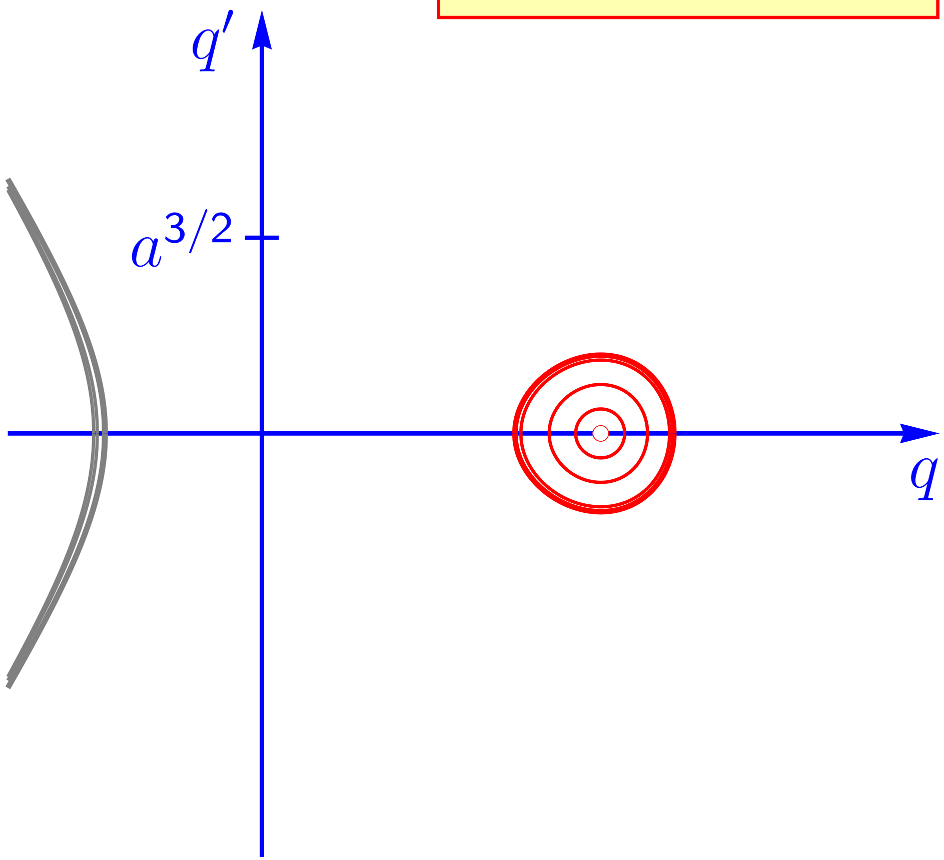

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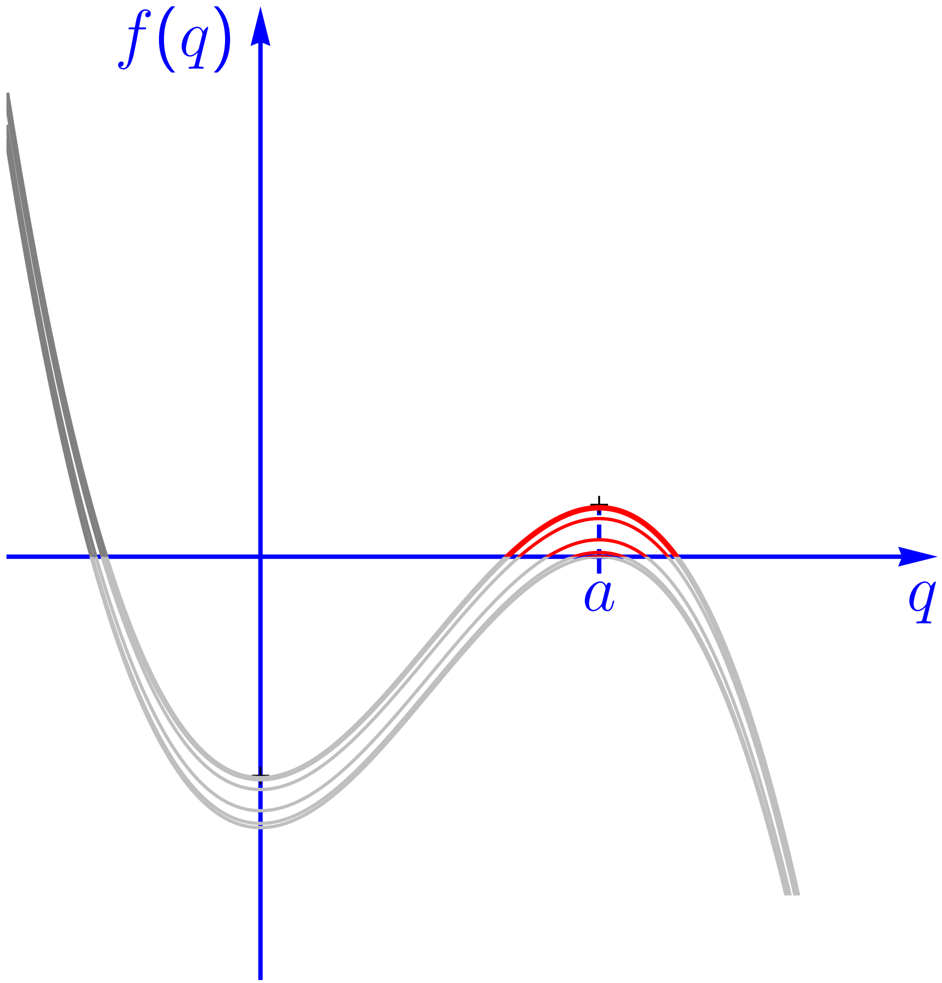

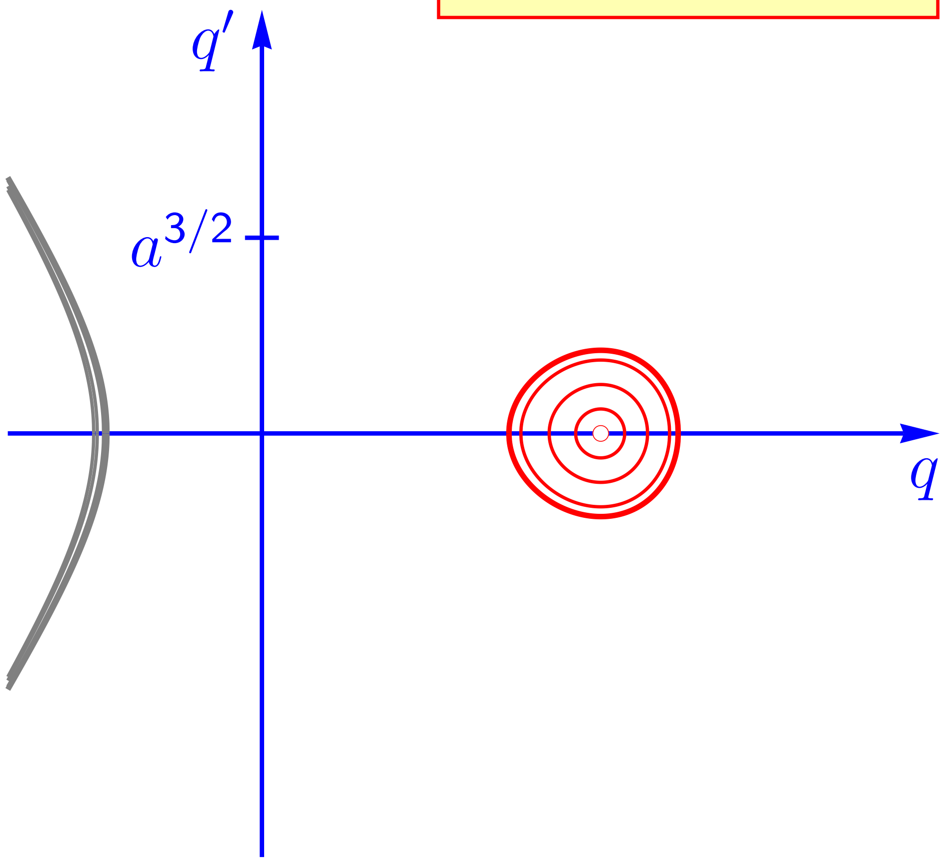

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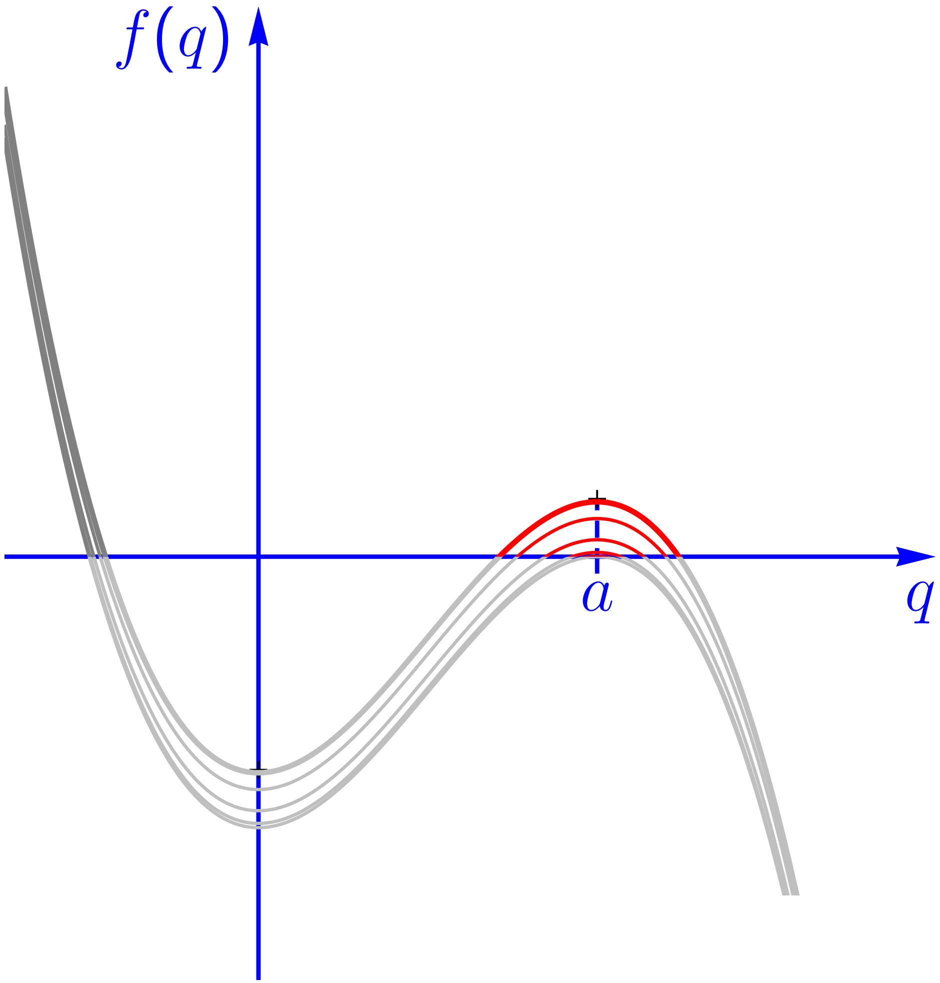

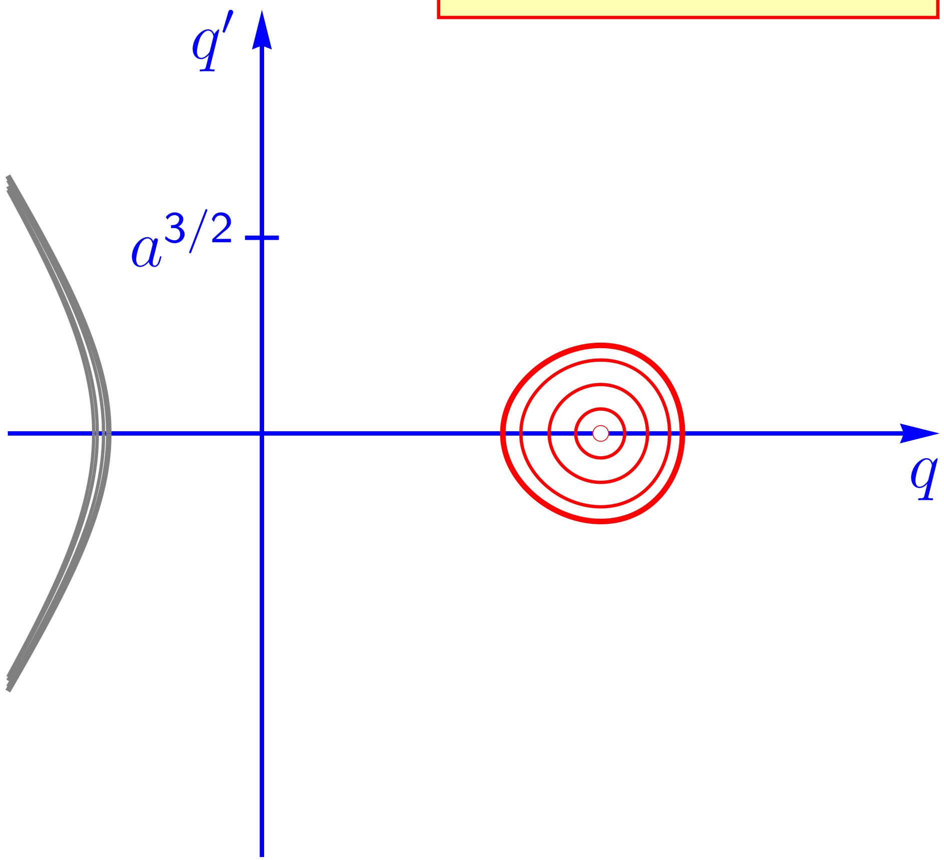

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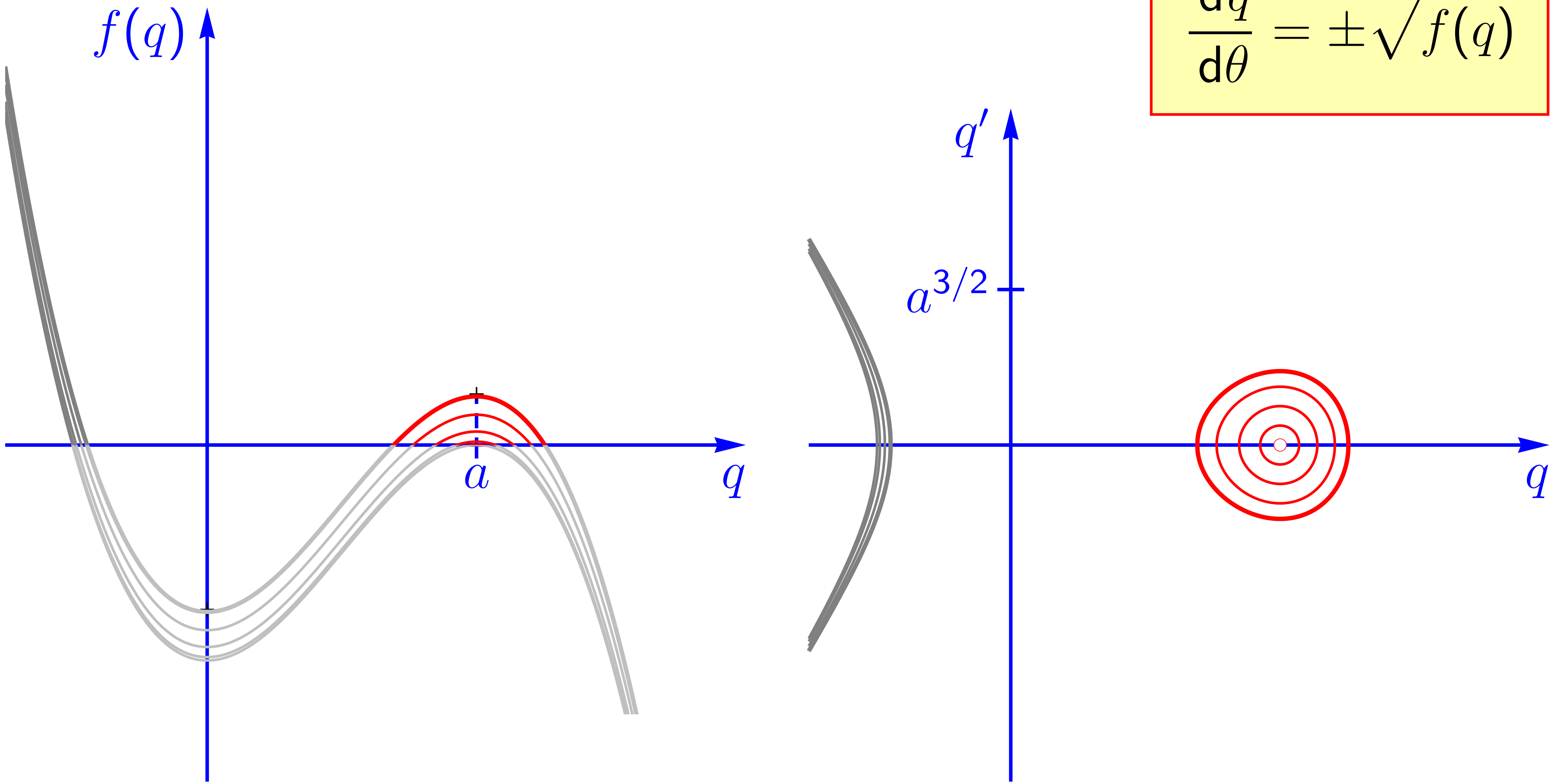

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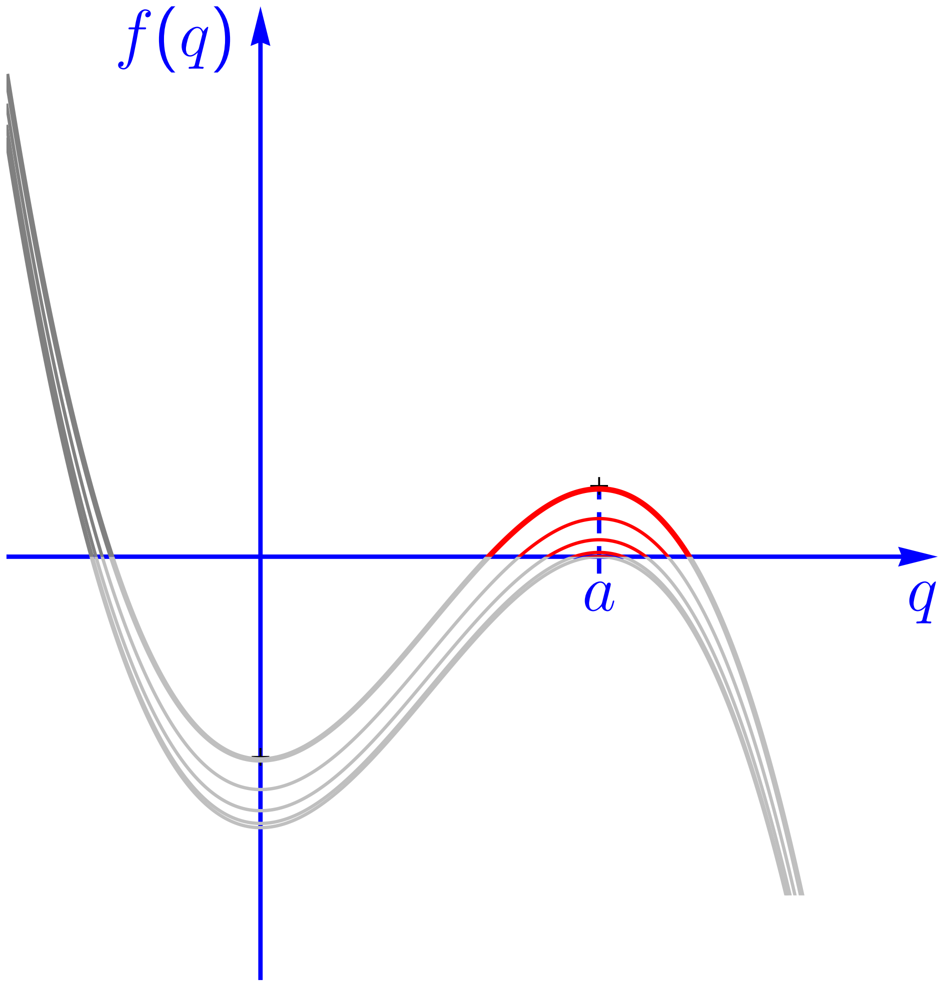

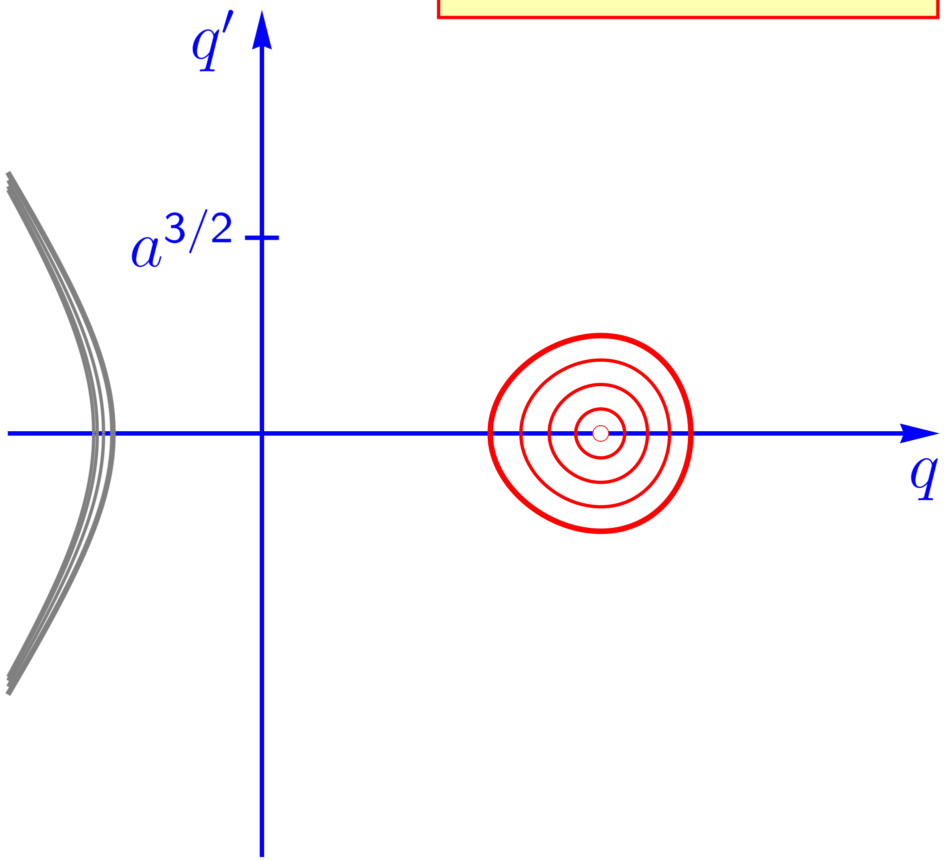

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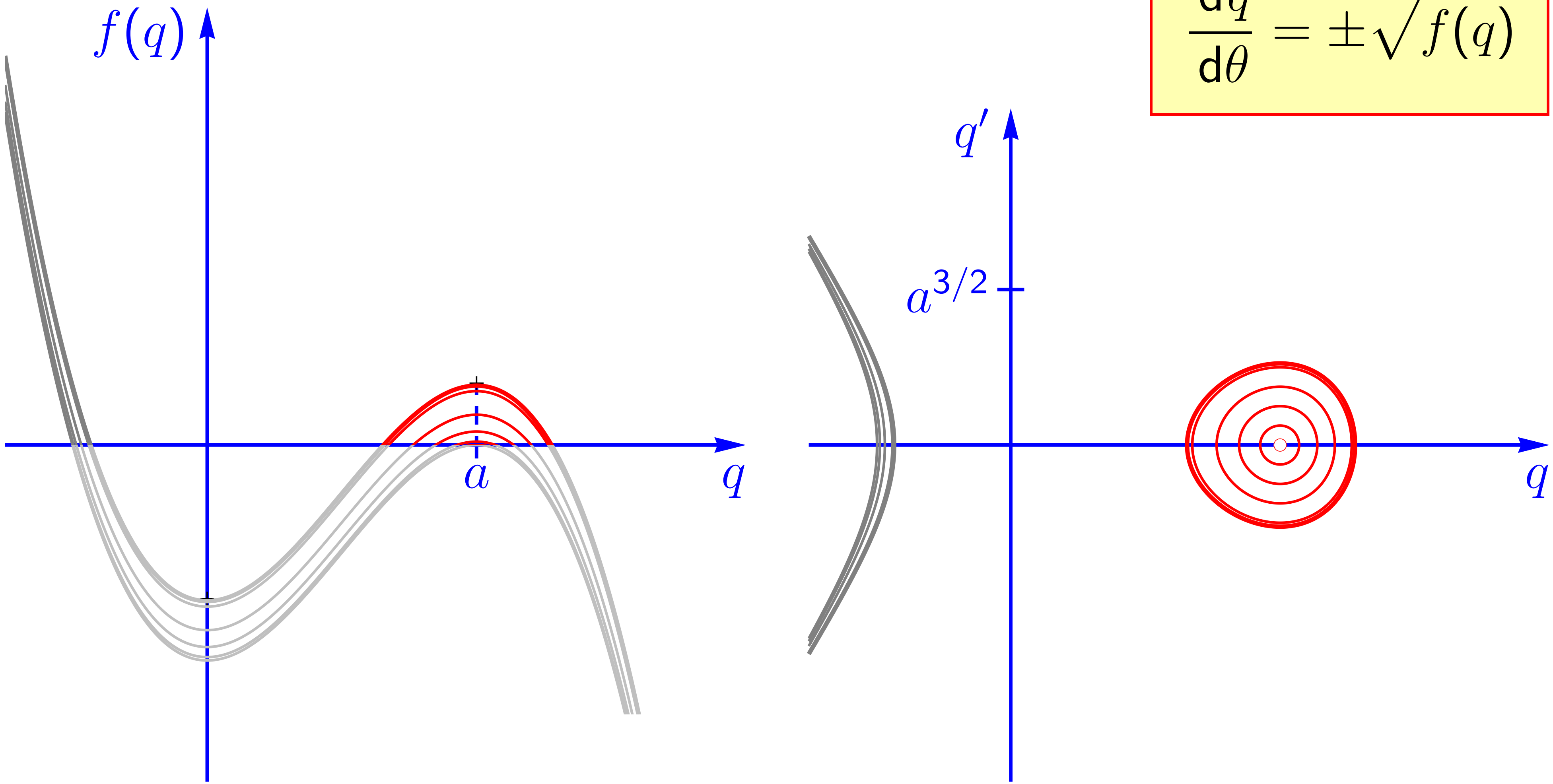

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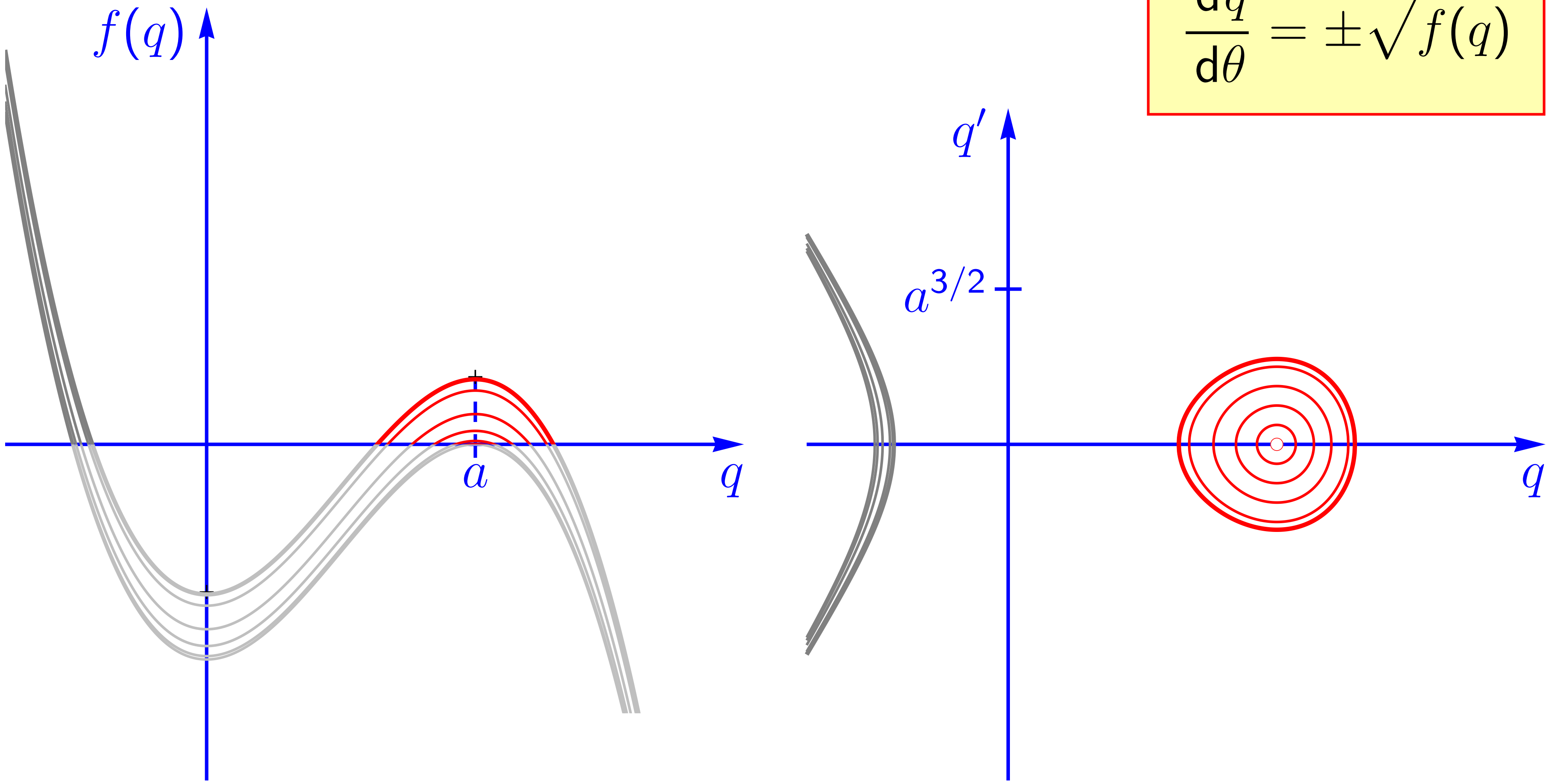

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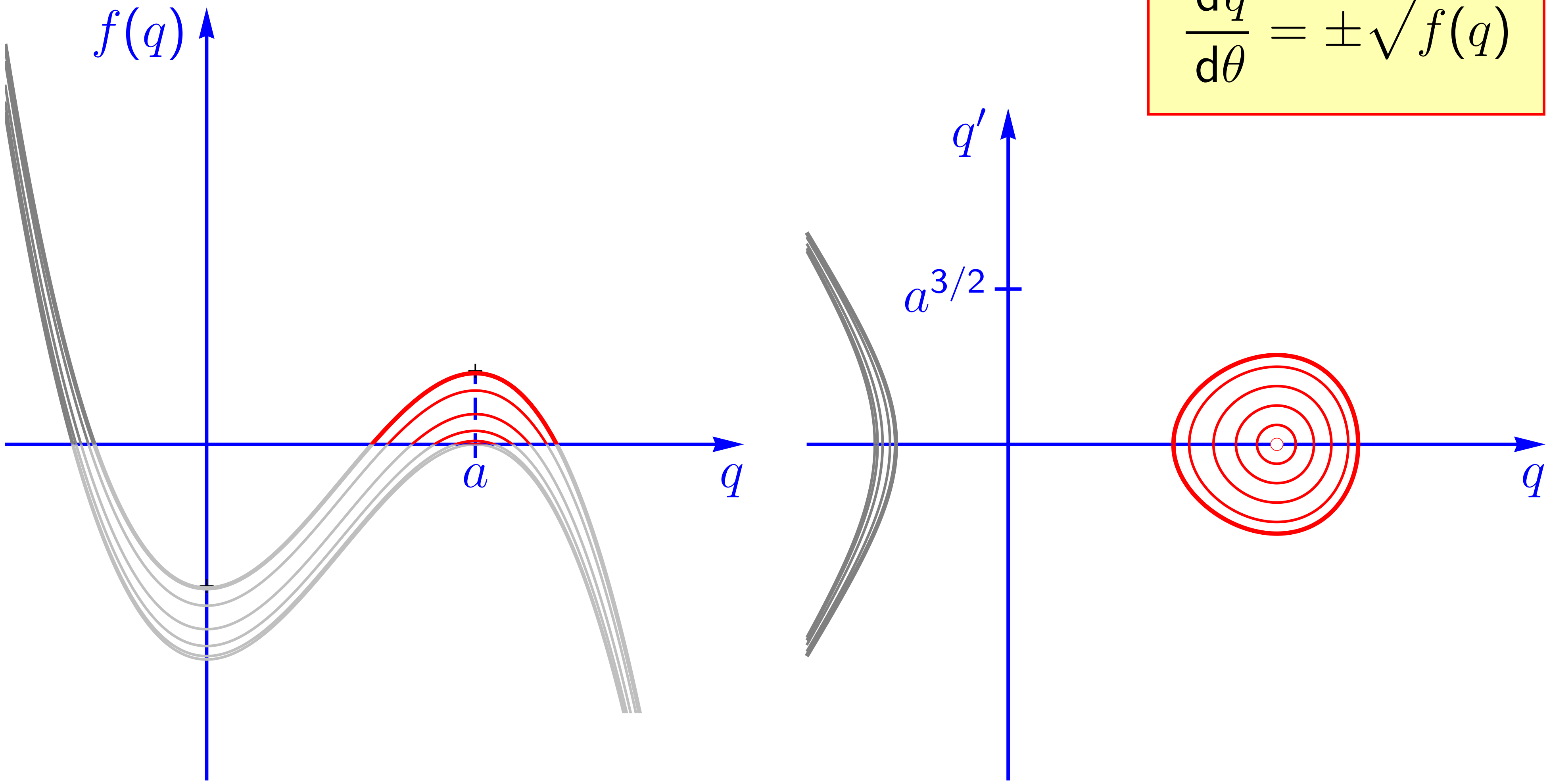

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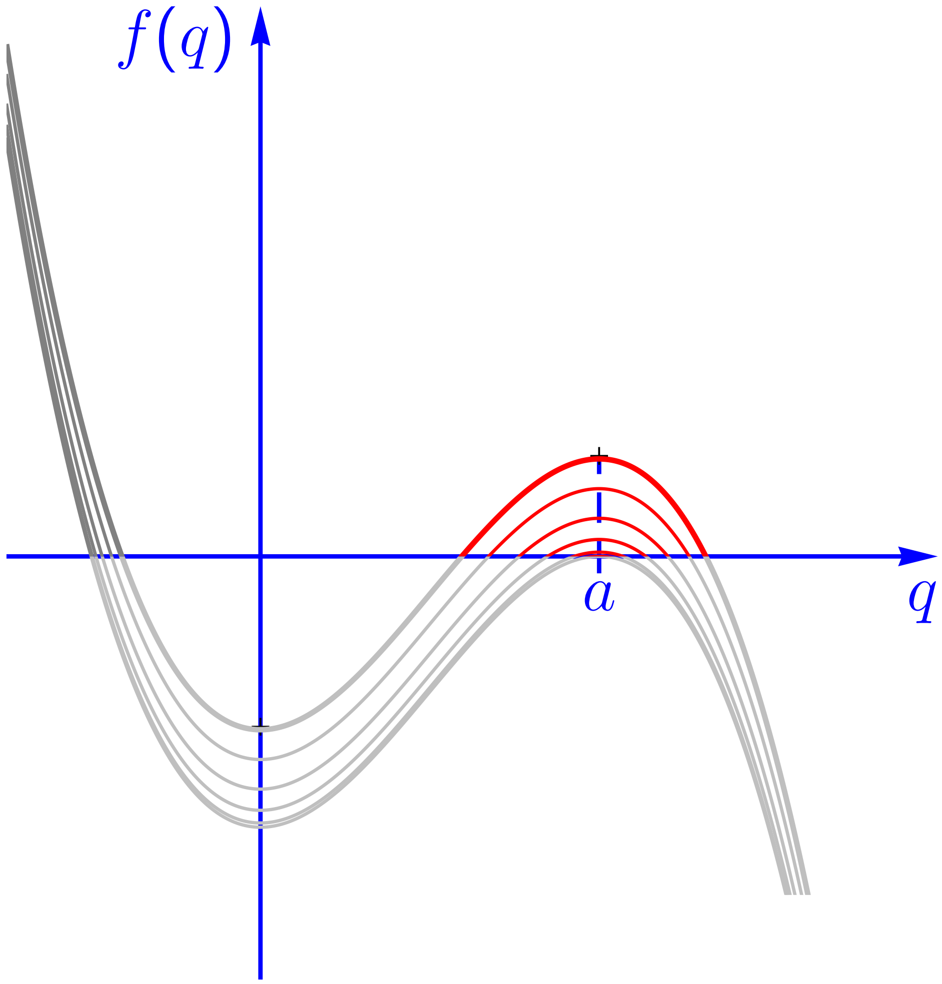

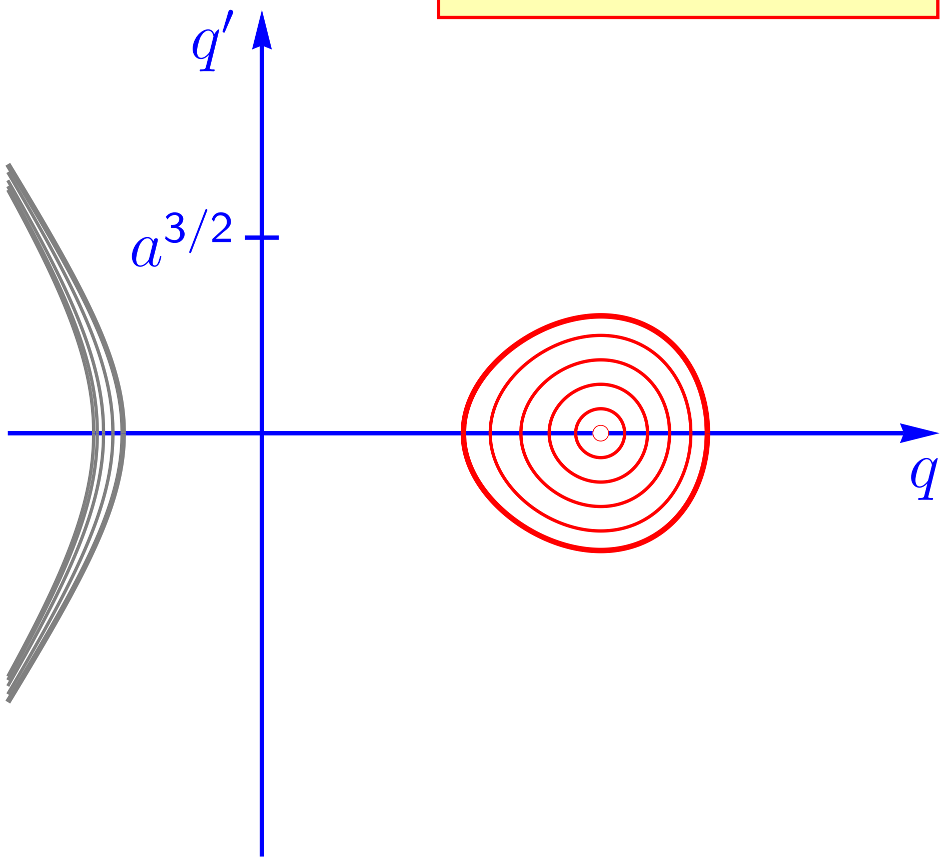

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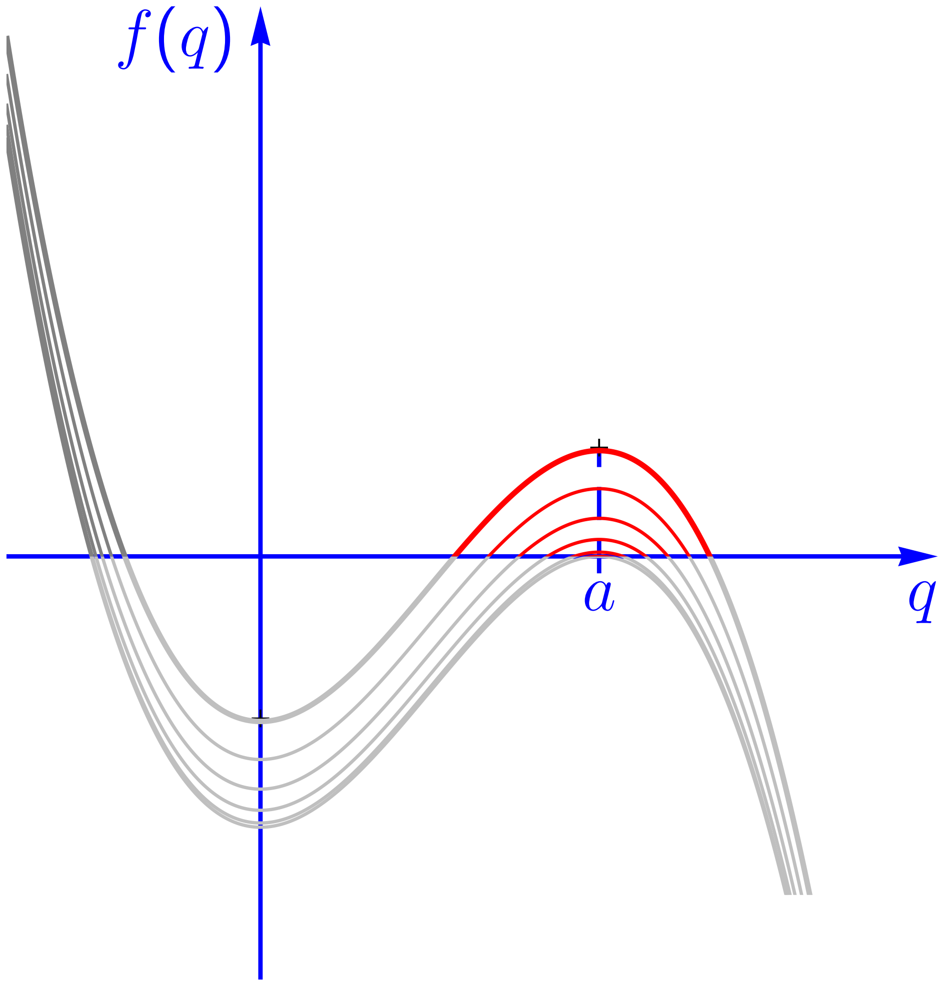

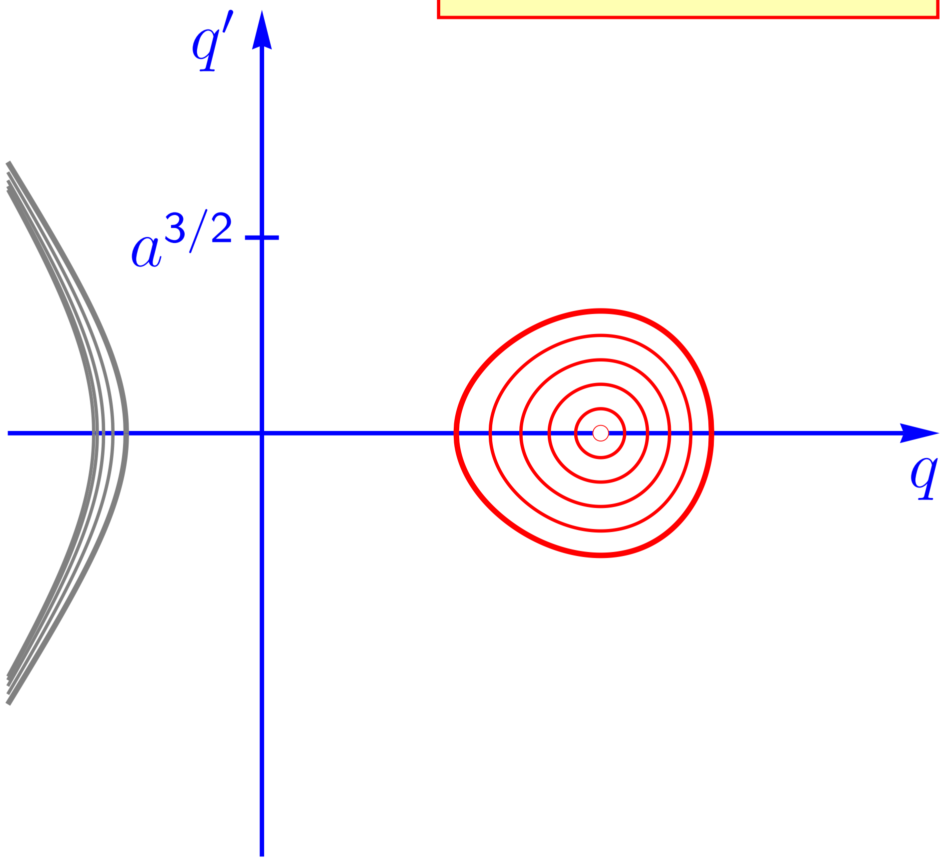

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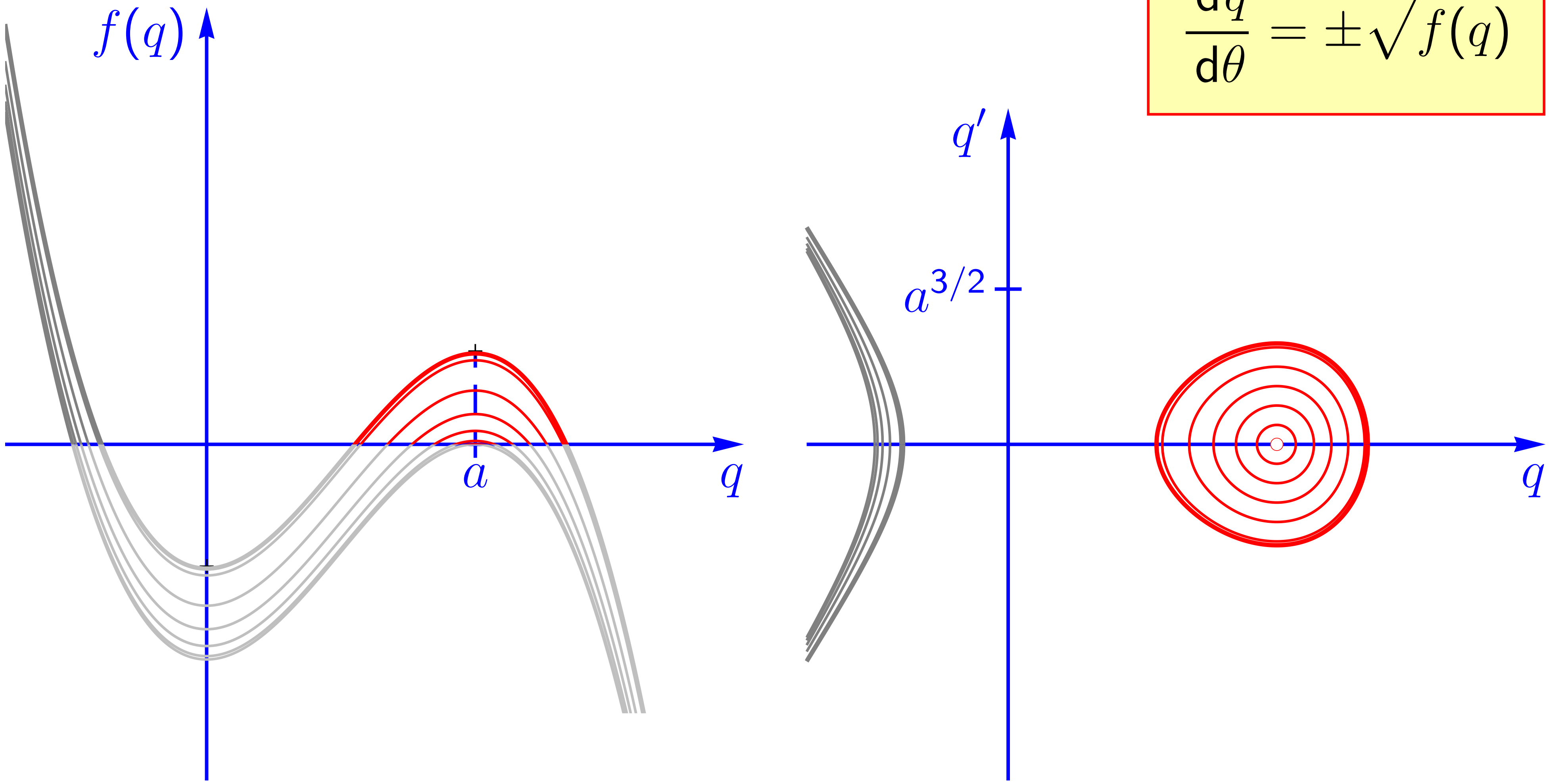

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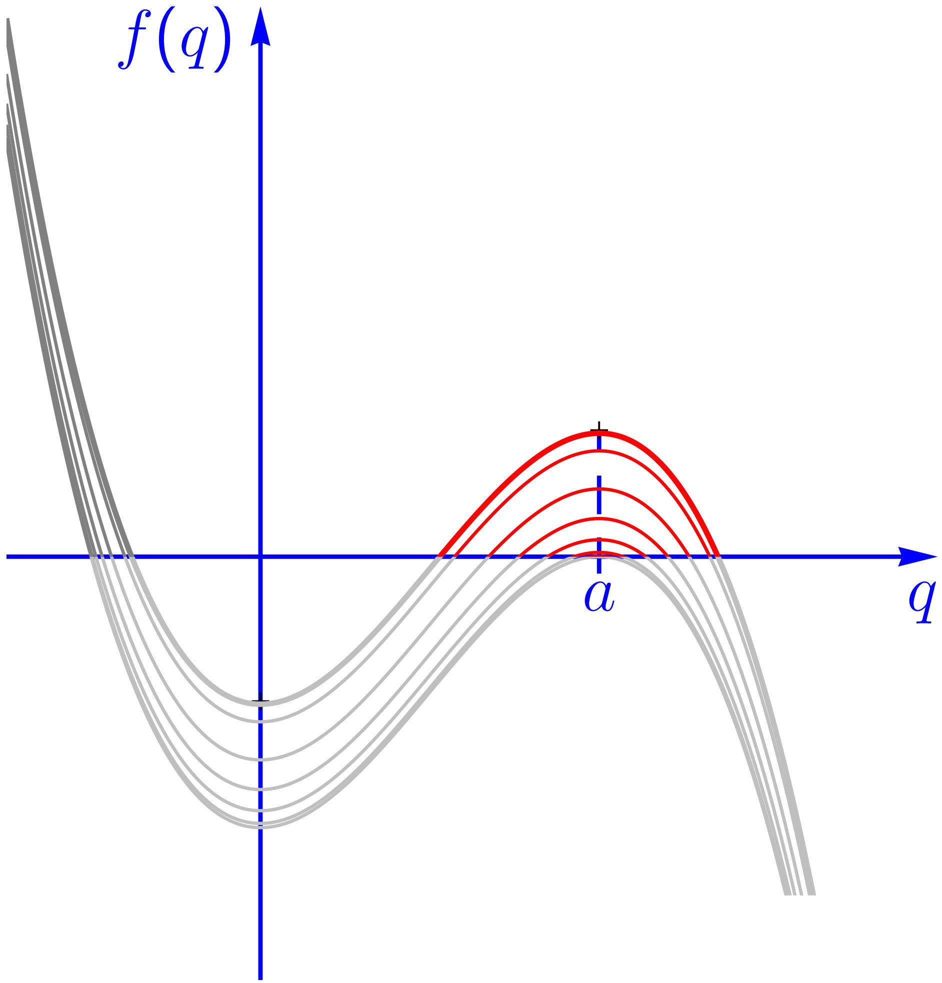

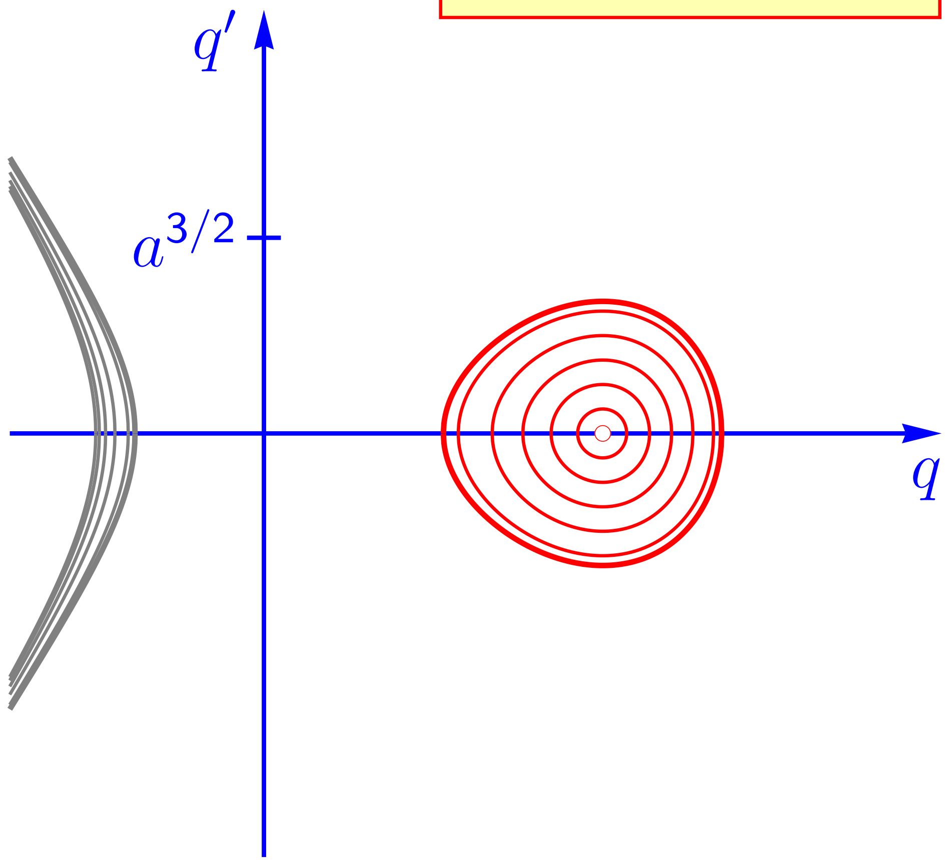

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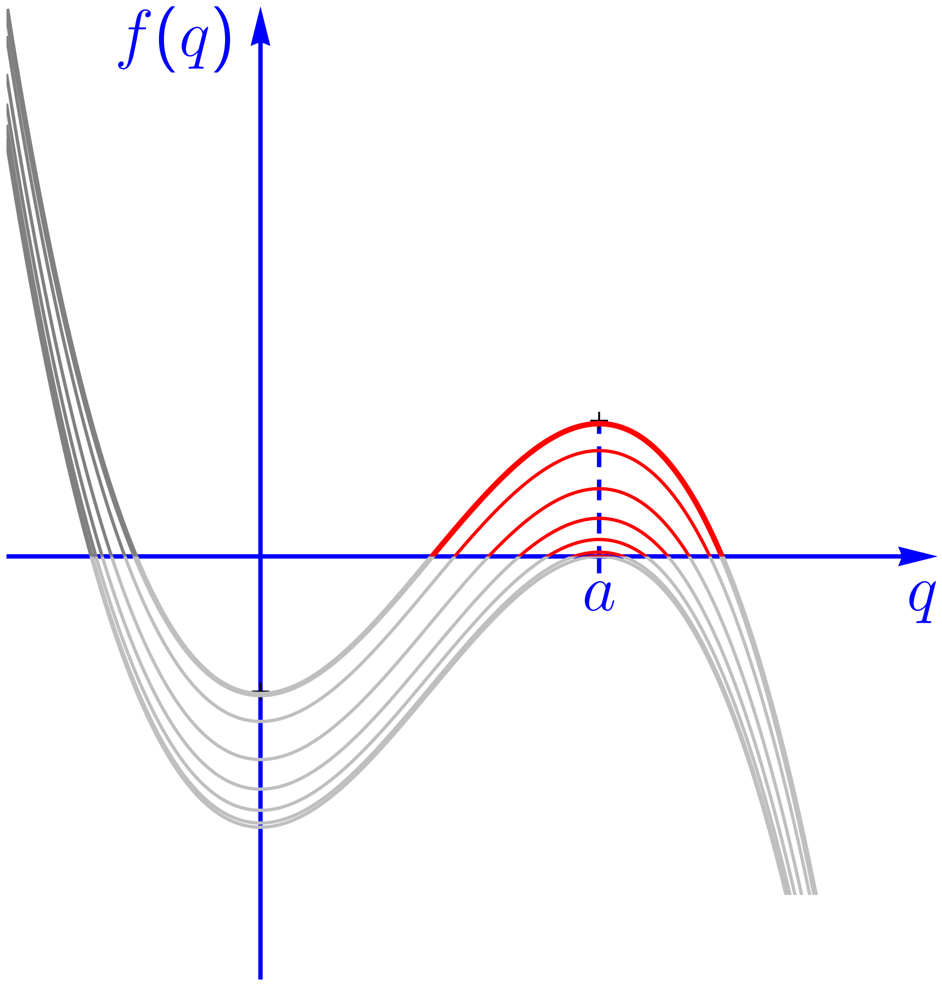

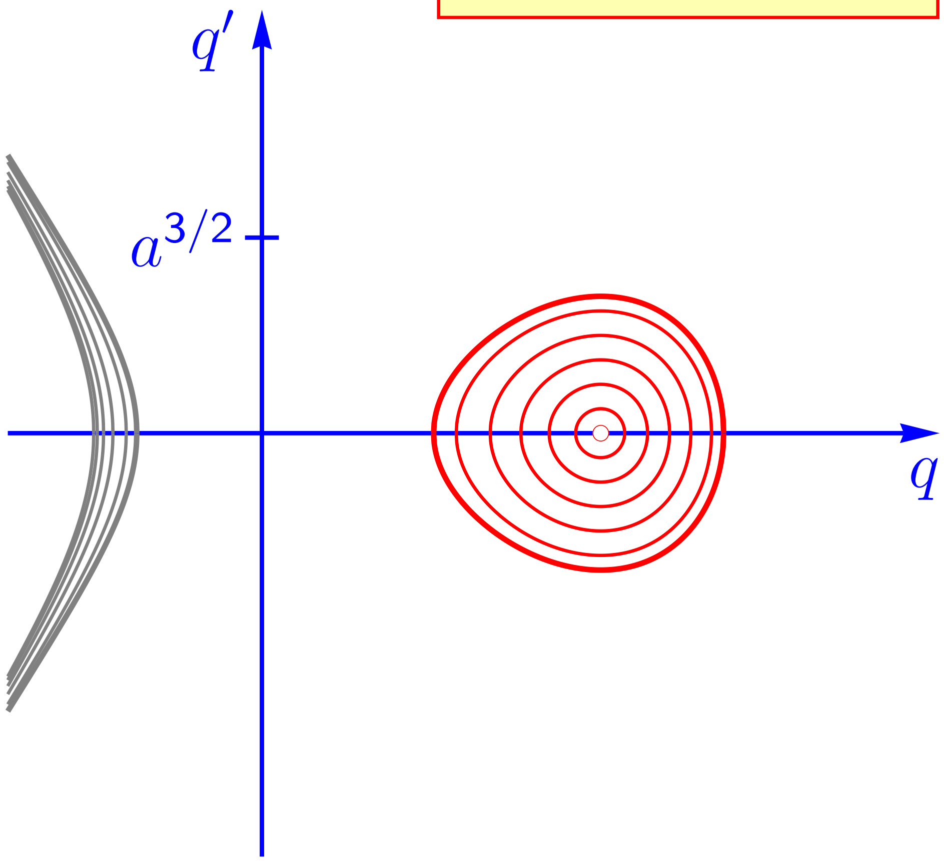

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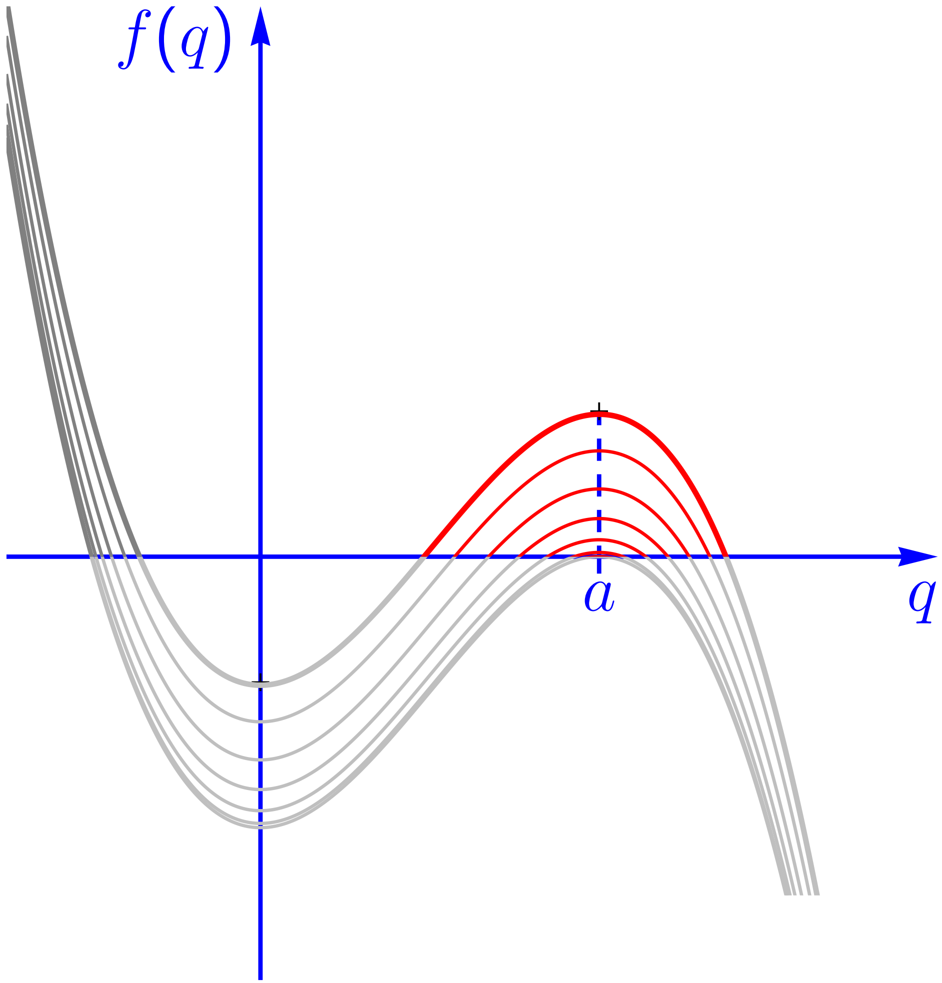

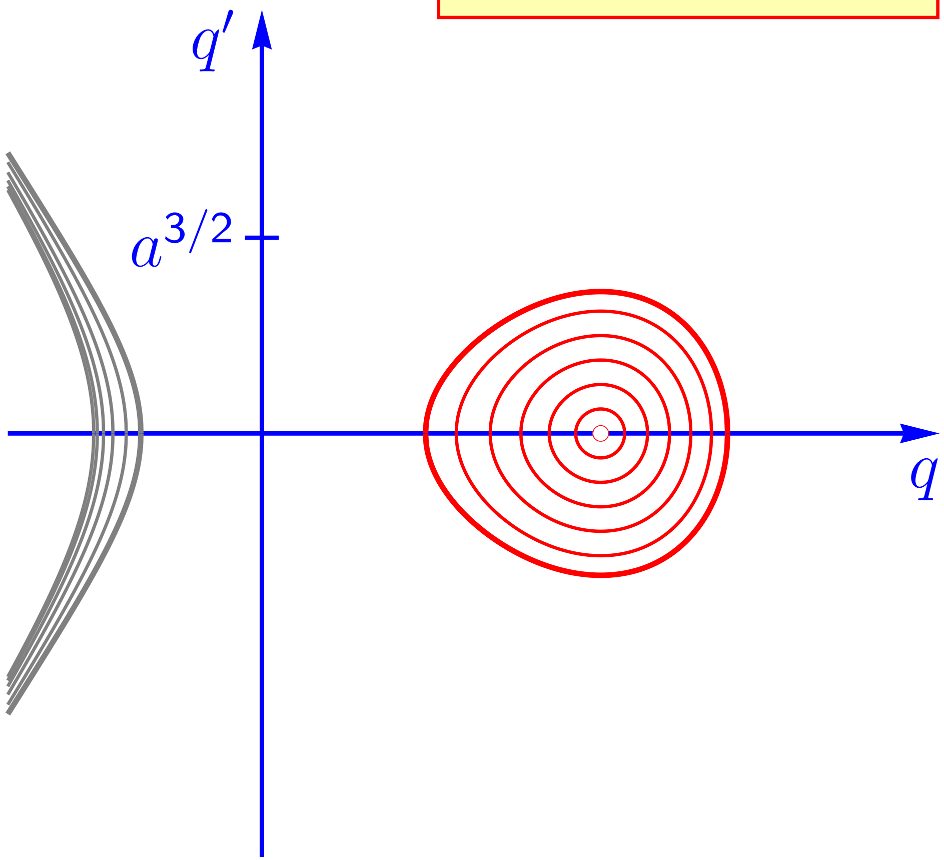

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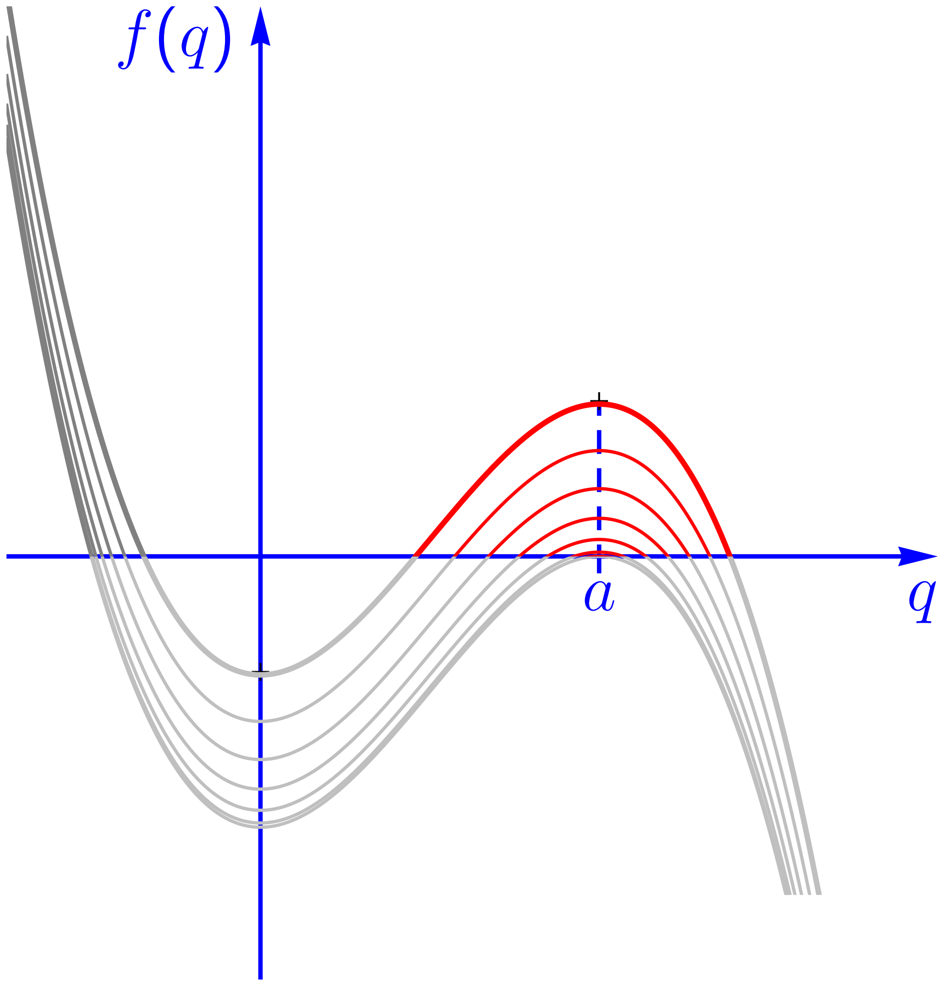

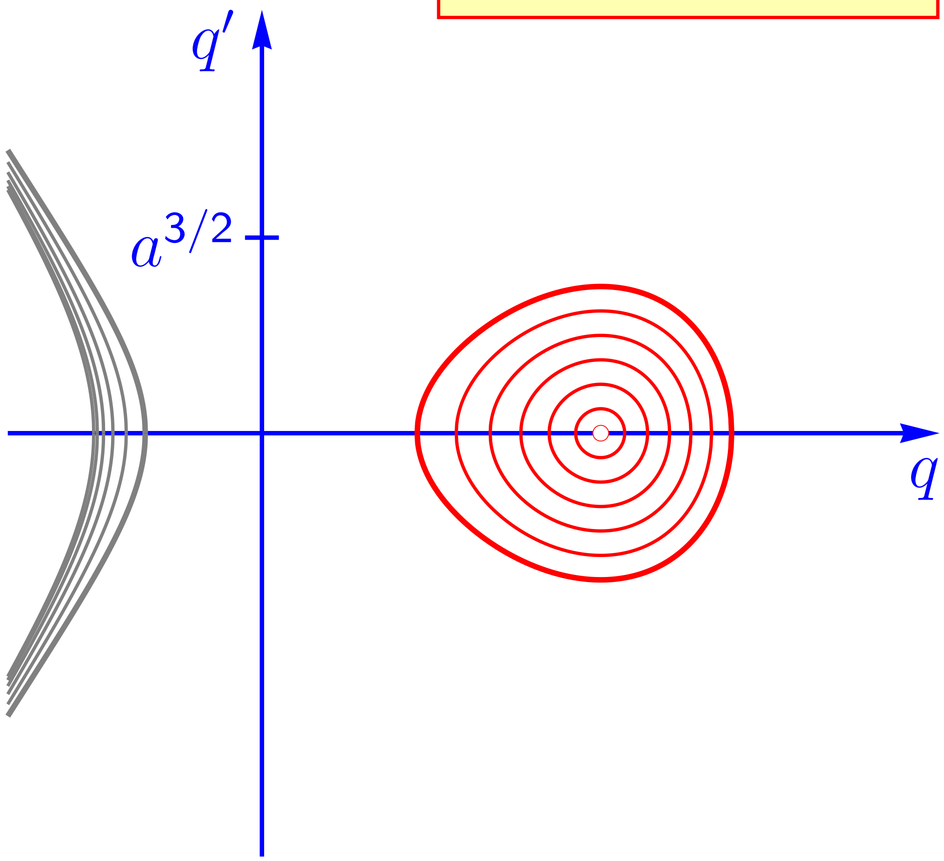

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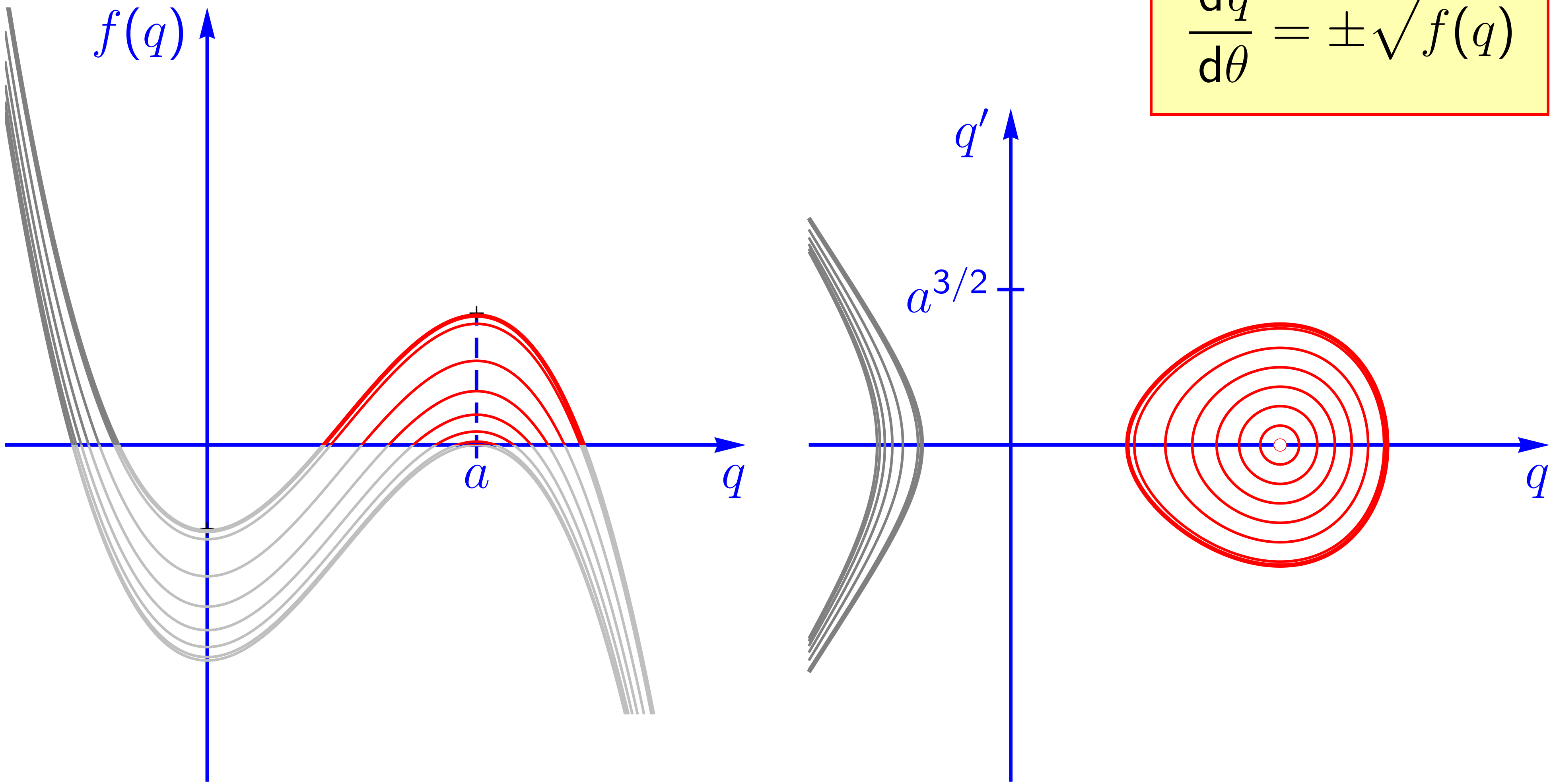

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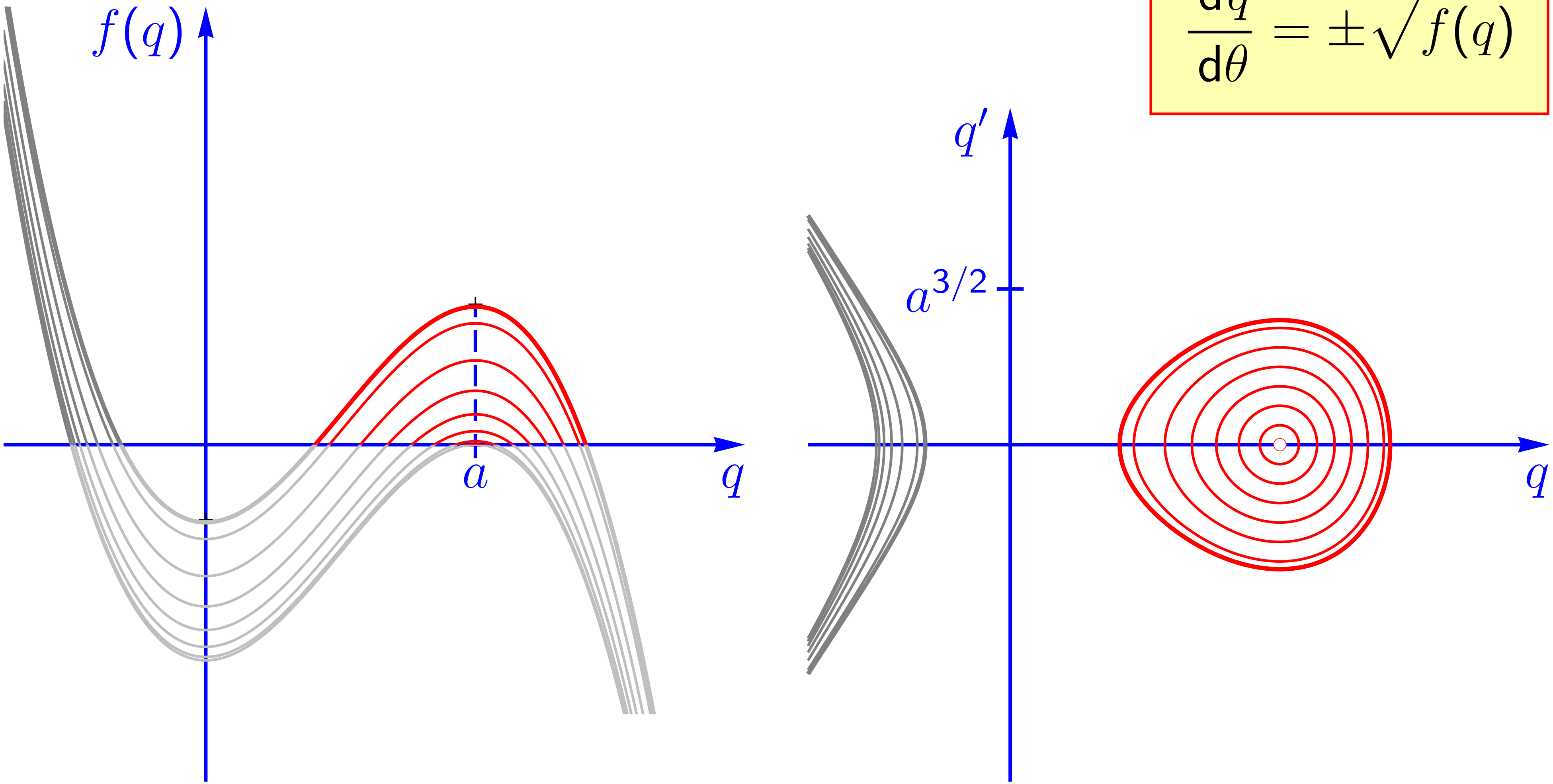

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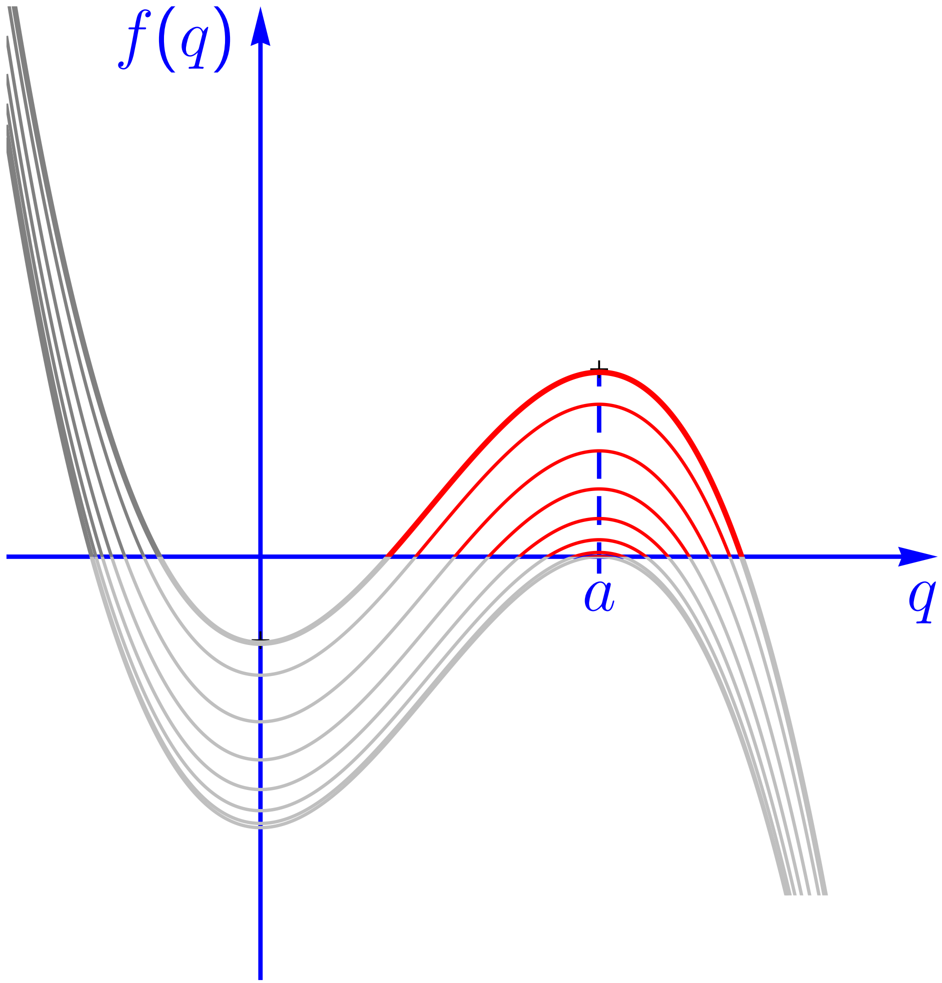

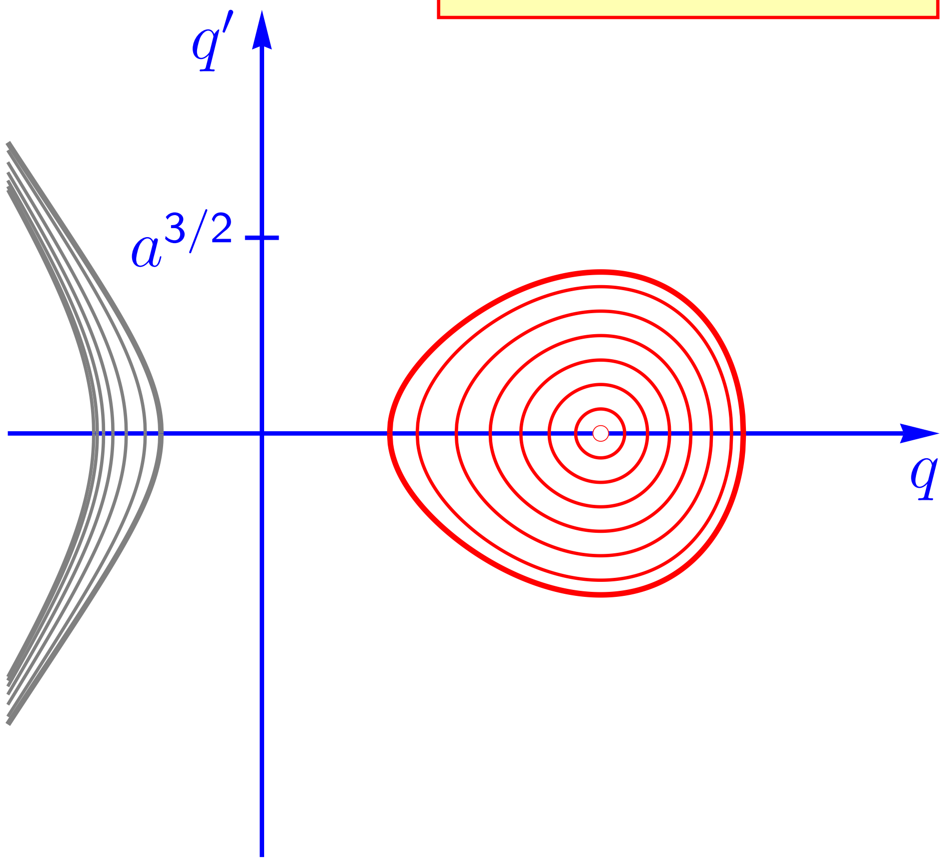

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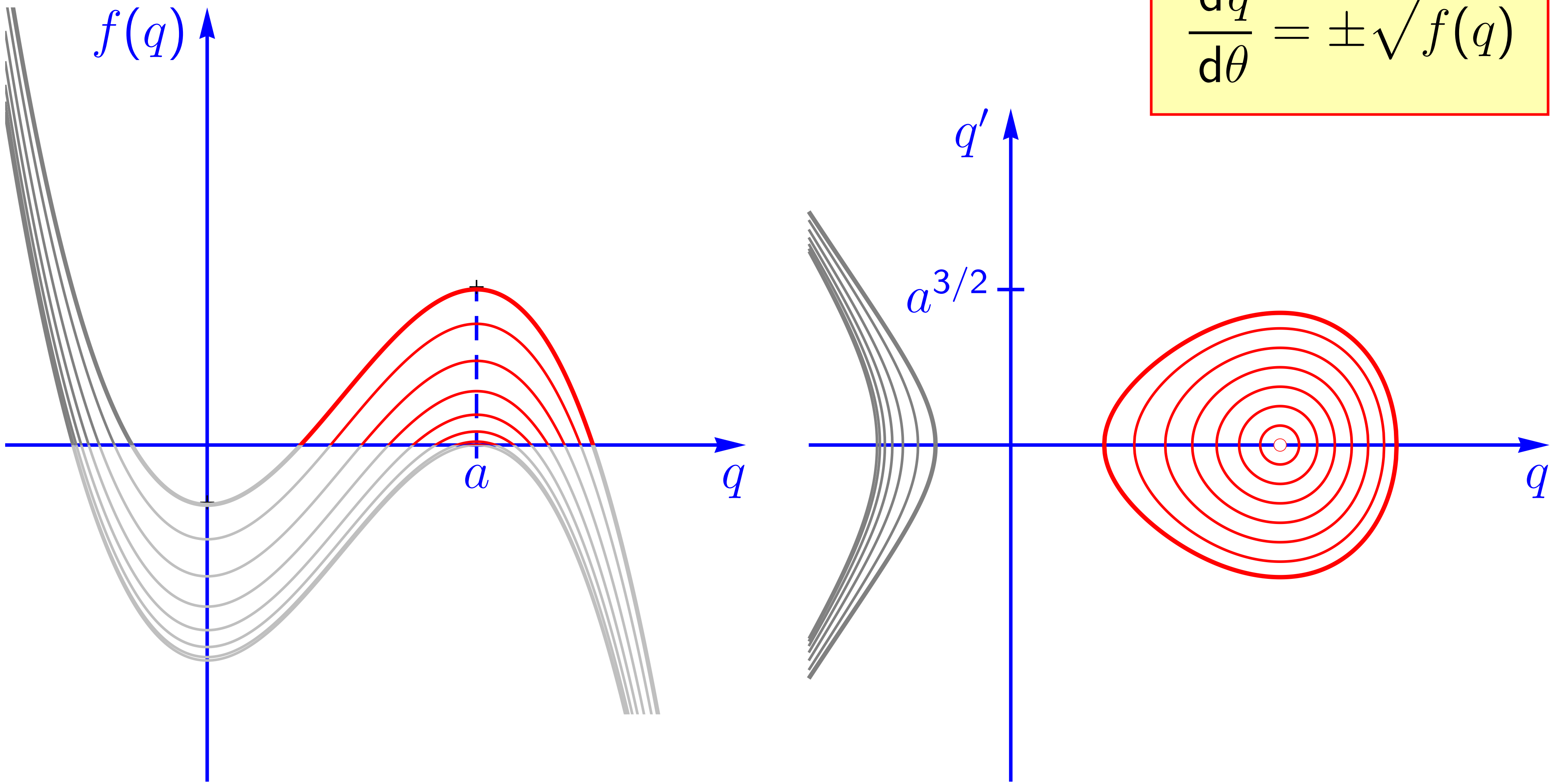

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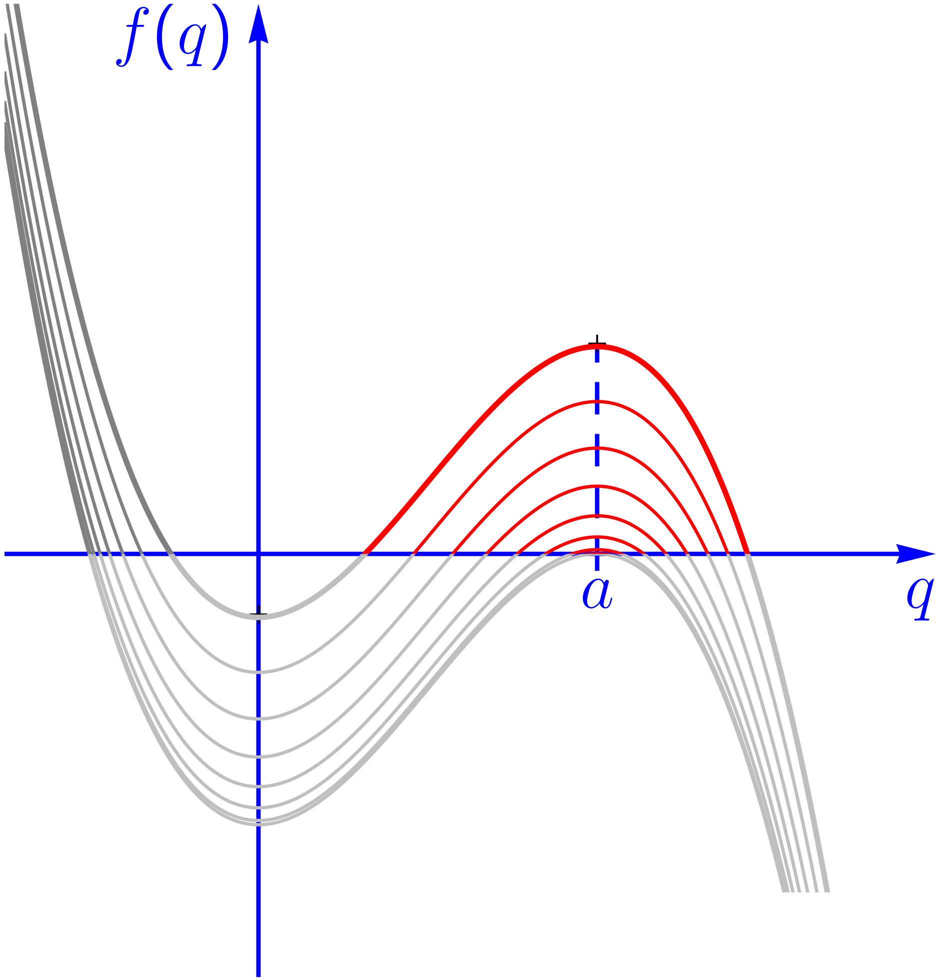

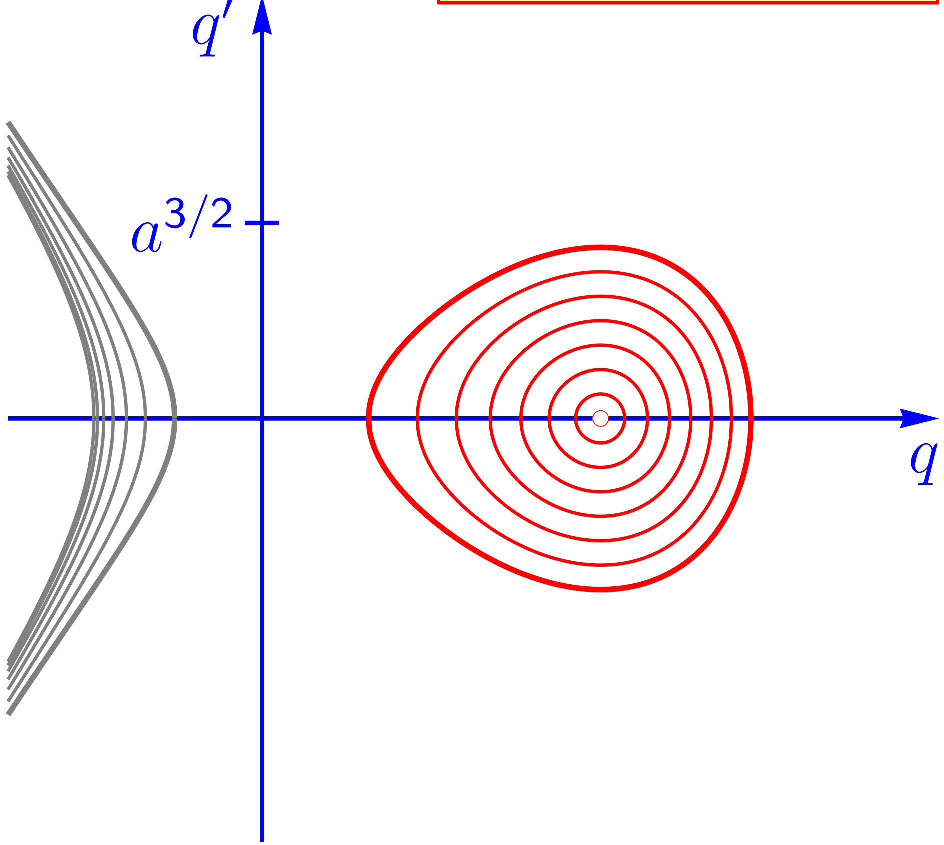

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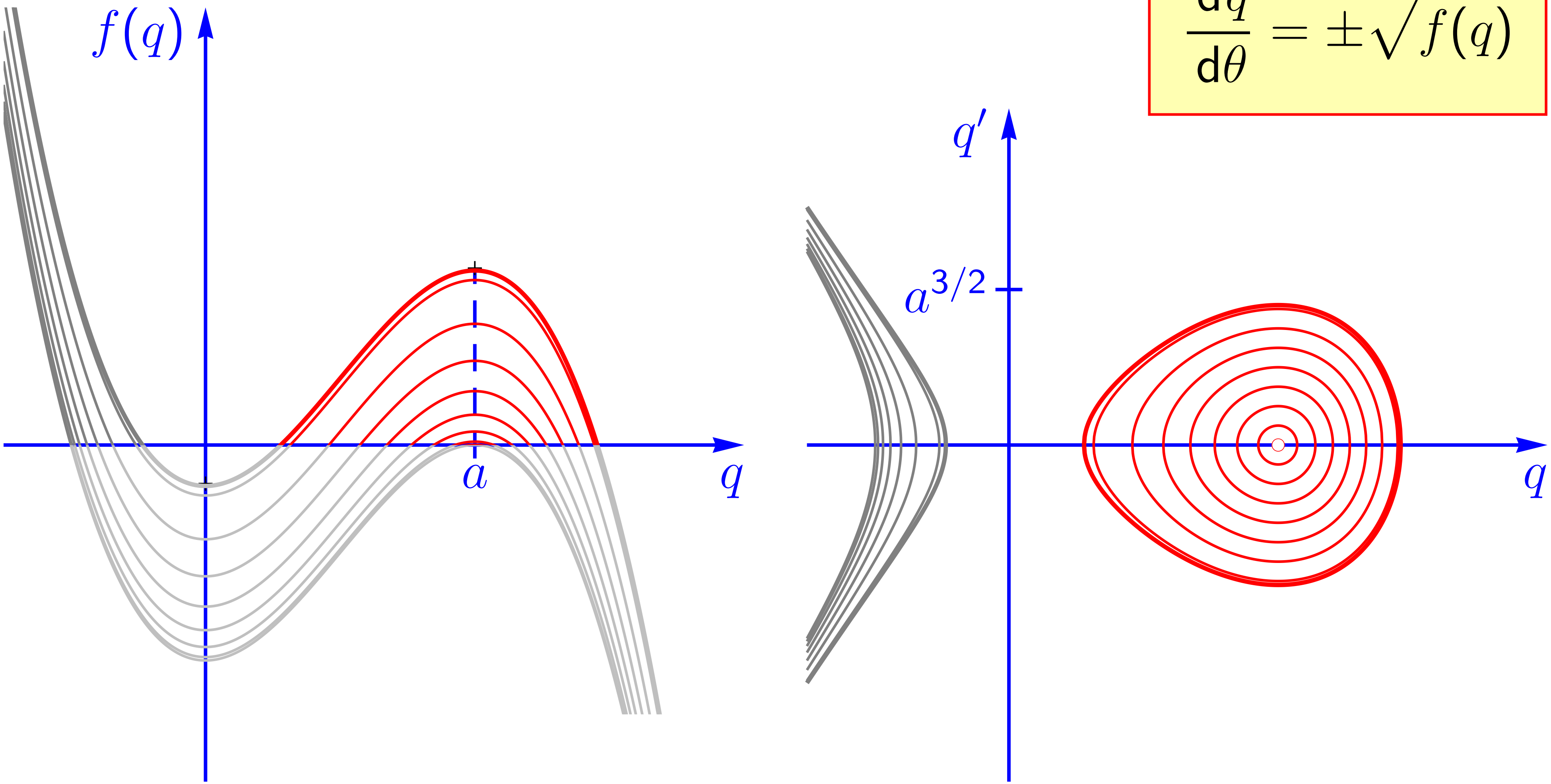

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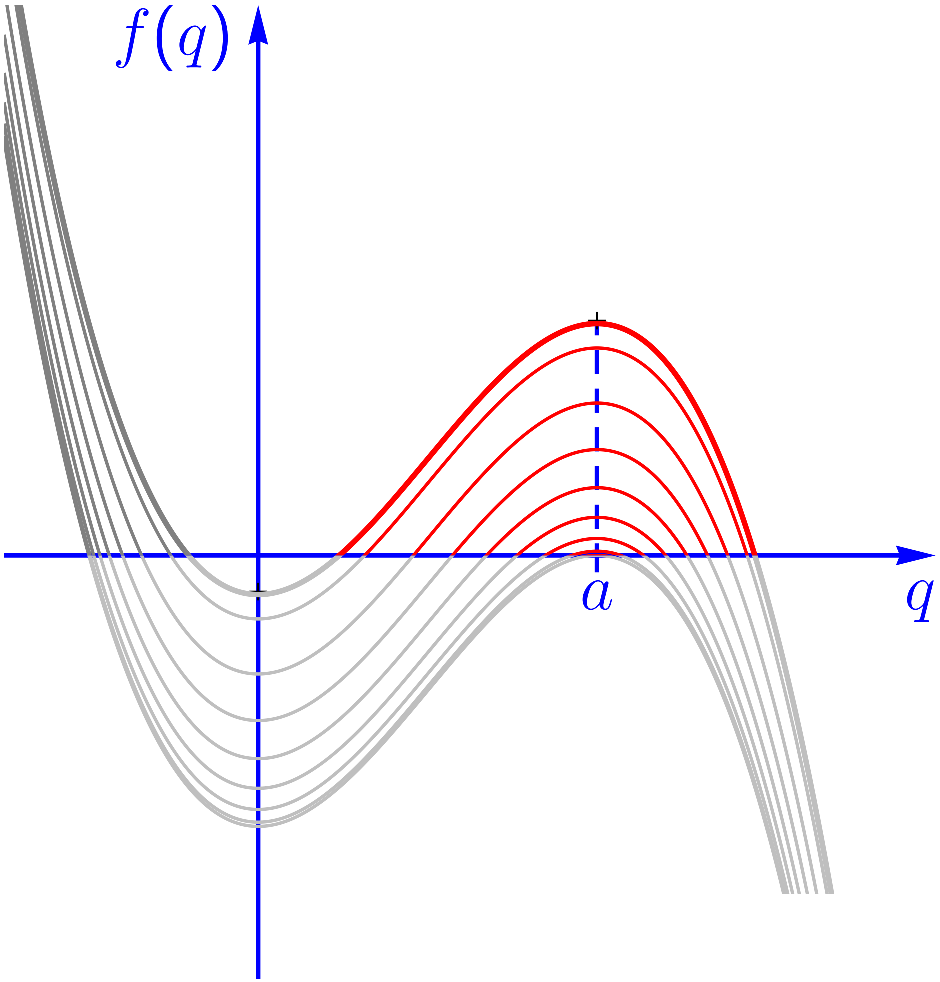

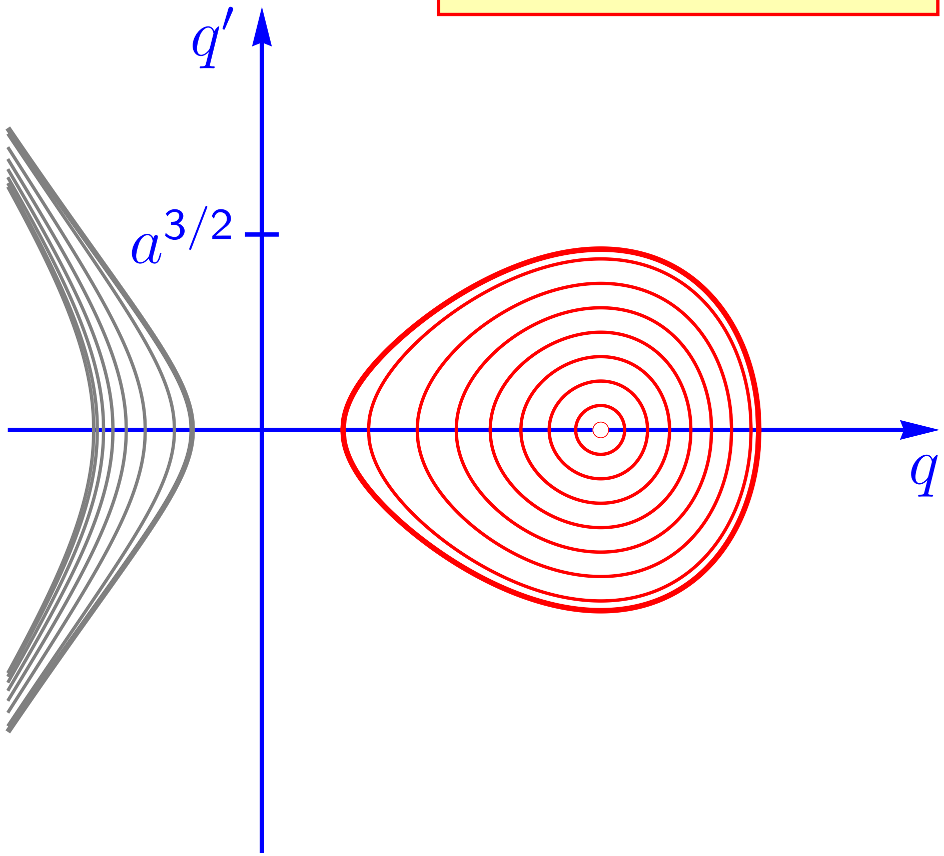

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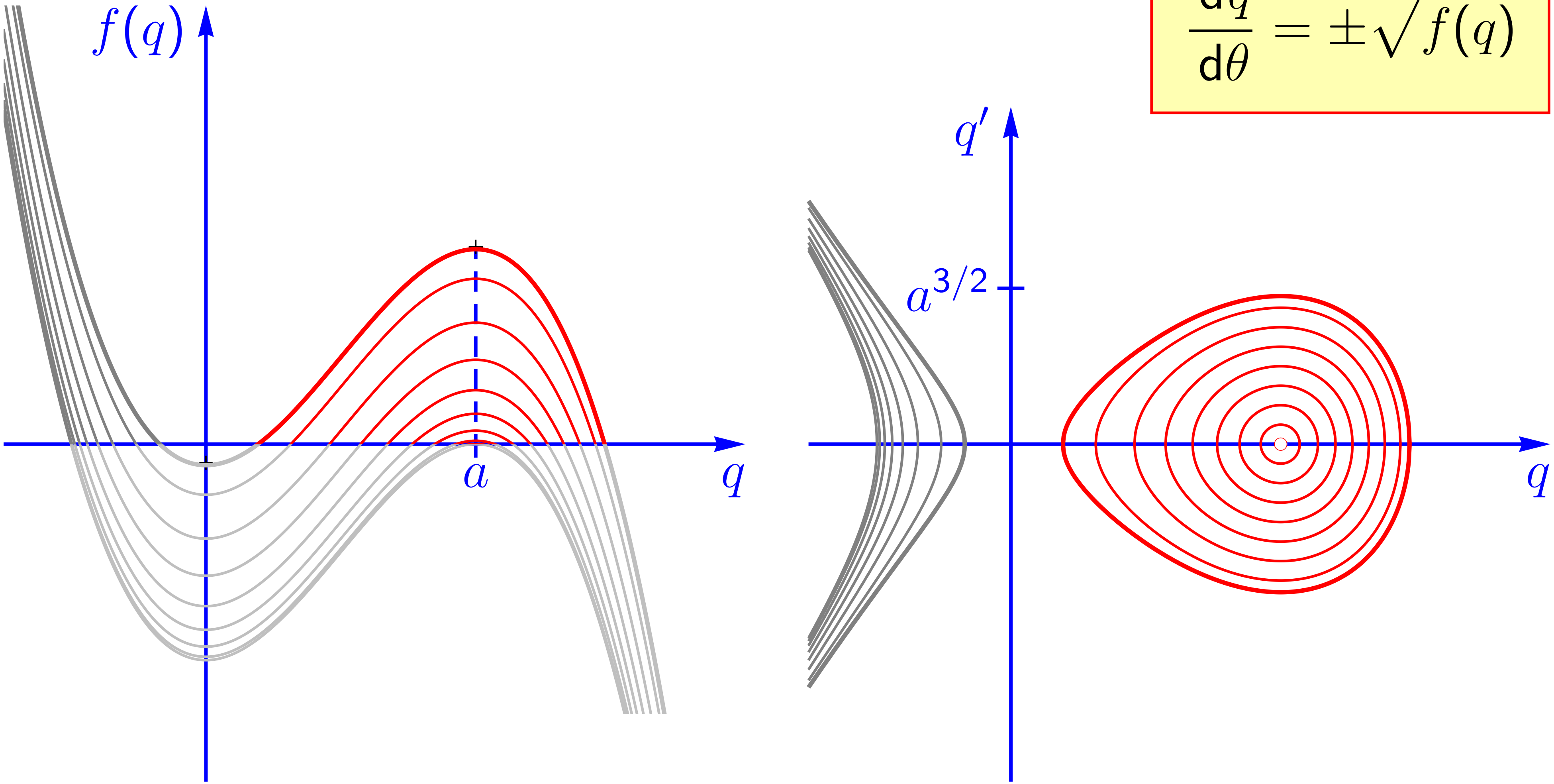

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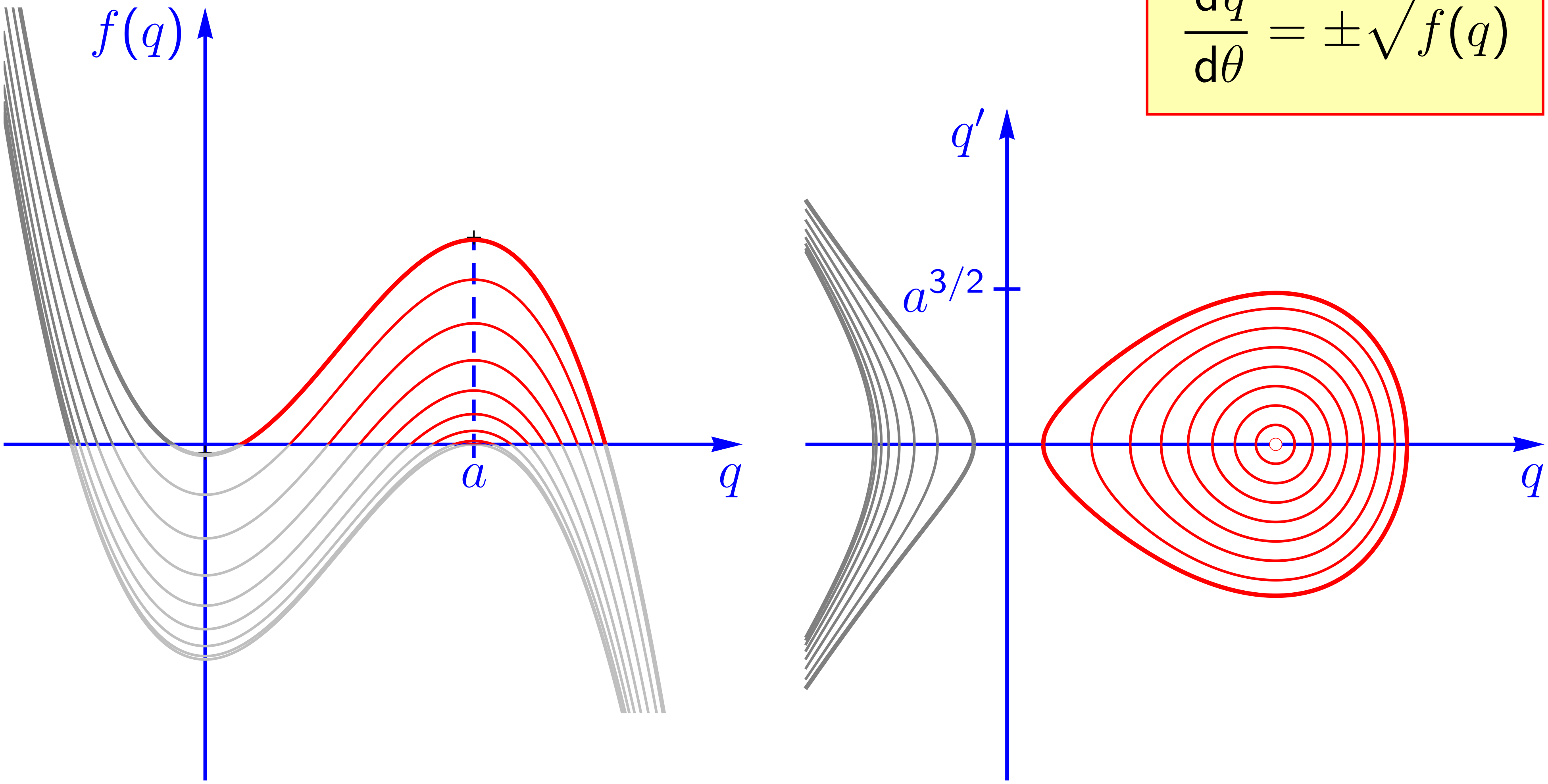

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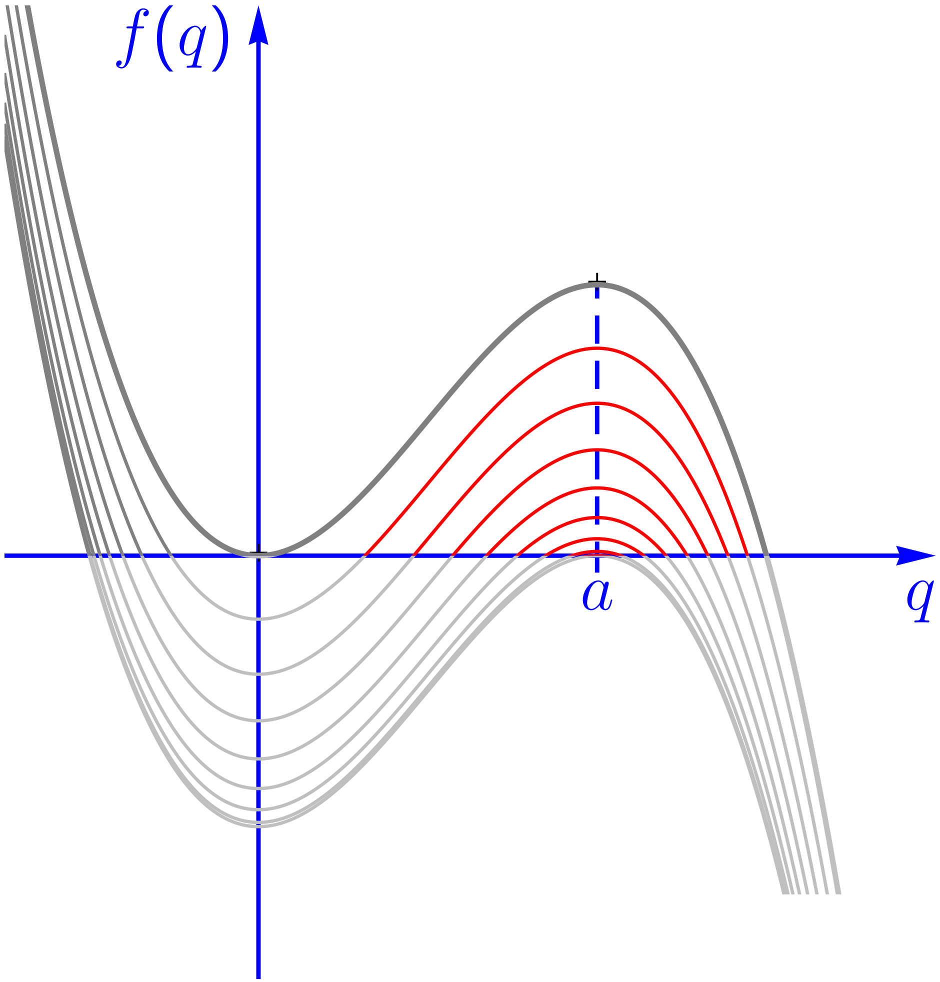

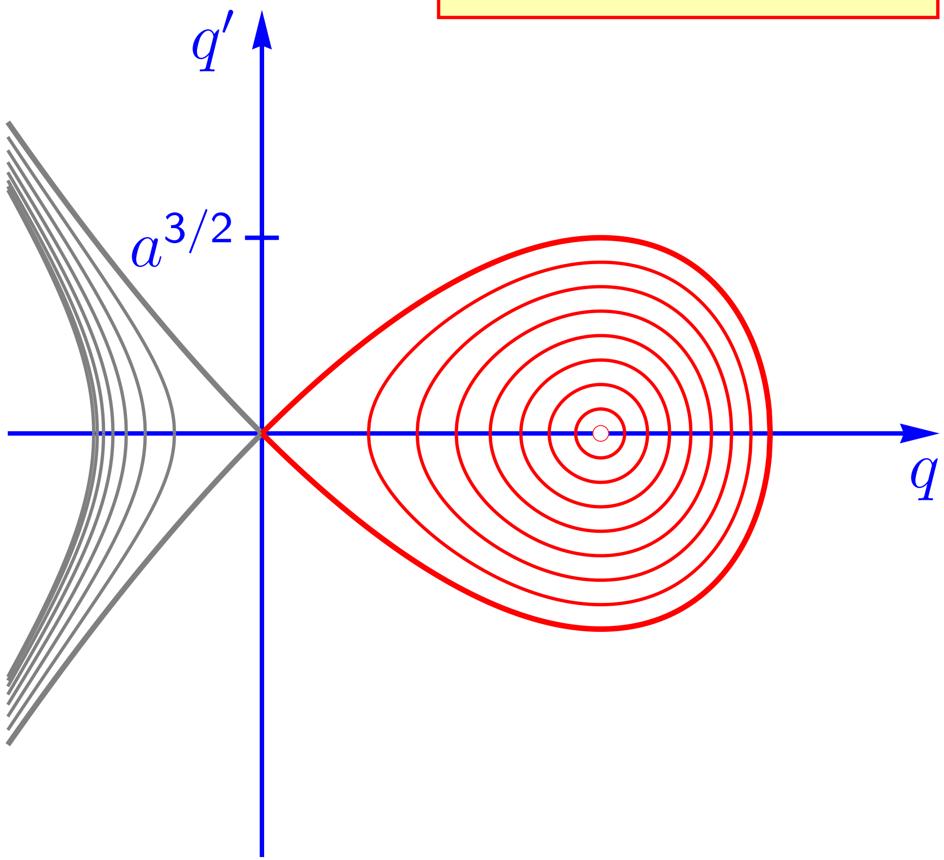

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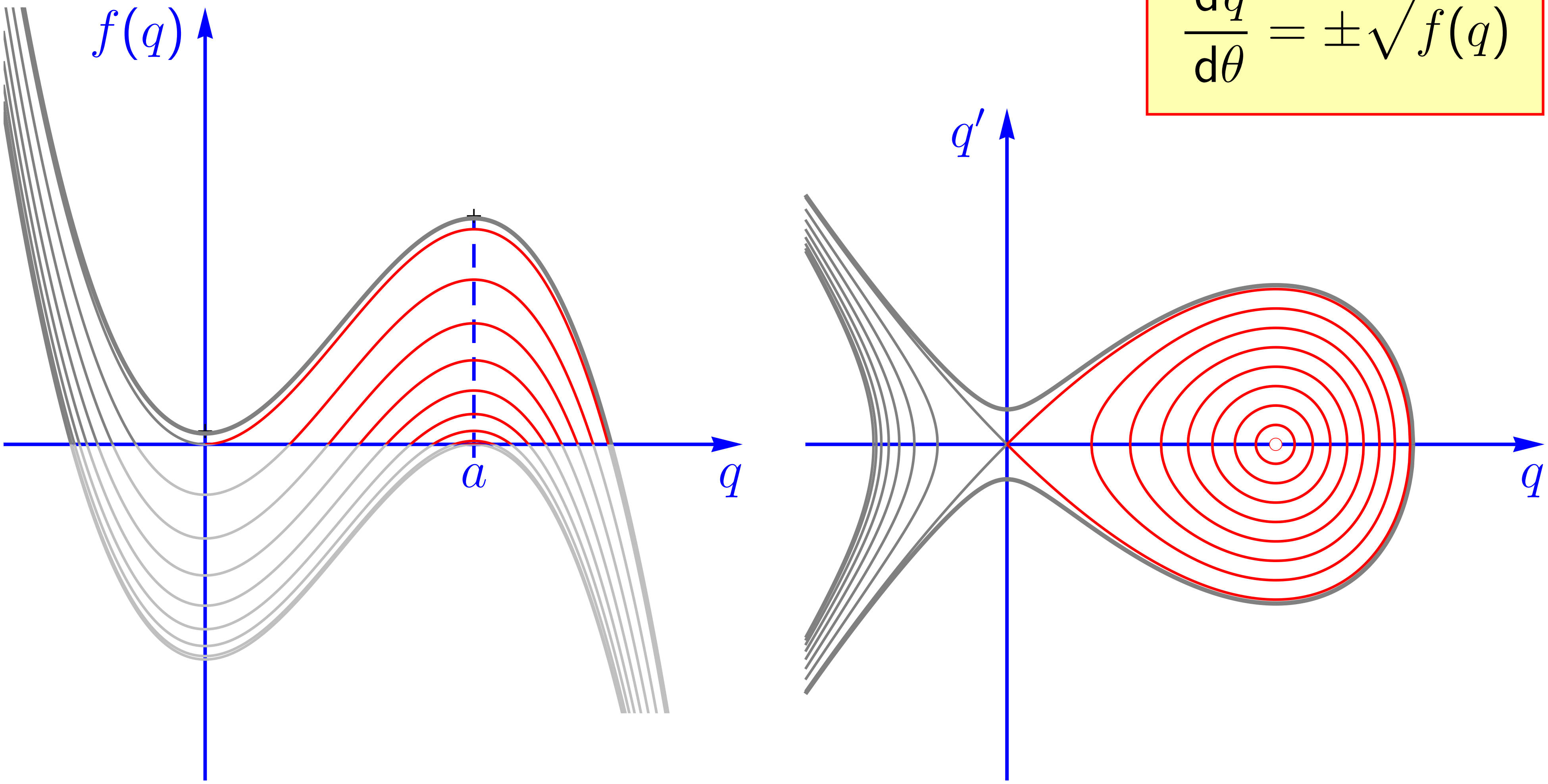
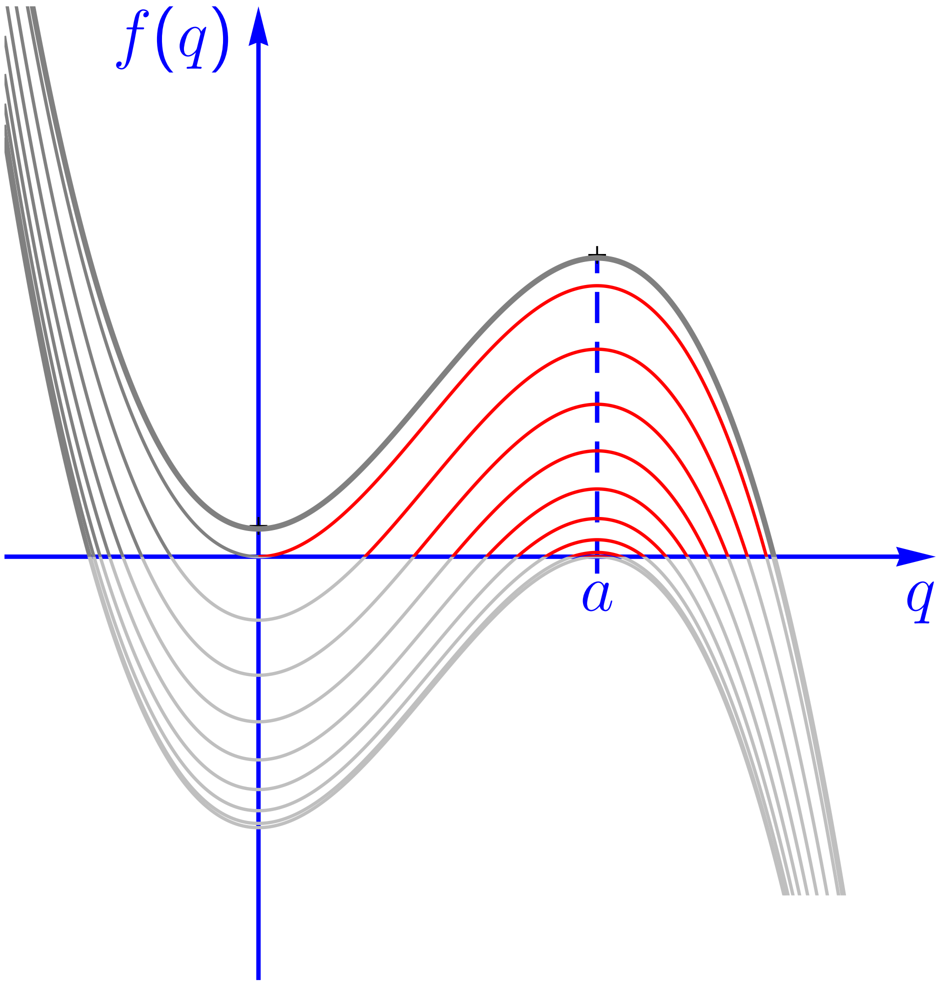
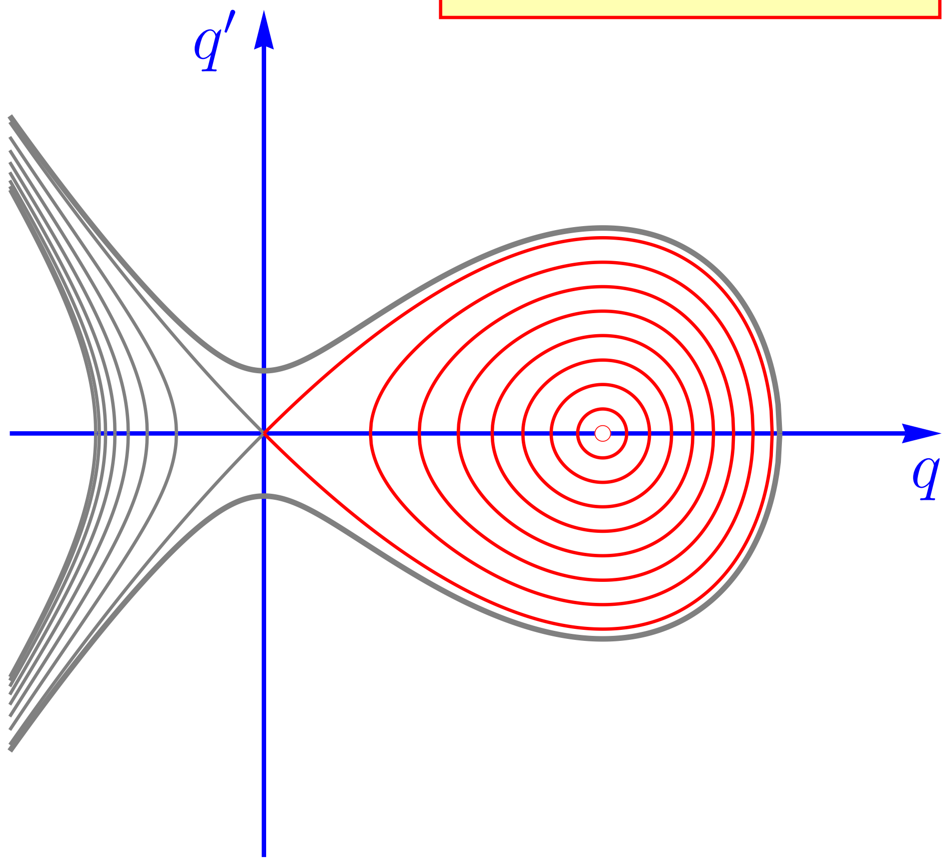


Function f and Phase Curves

$$\frac{\mathrm{d}q}{\mathrm{d}\theta} = \pm\sqrt{f(q)}$$





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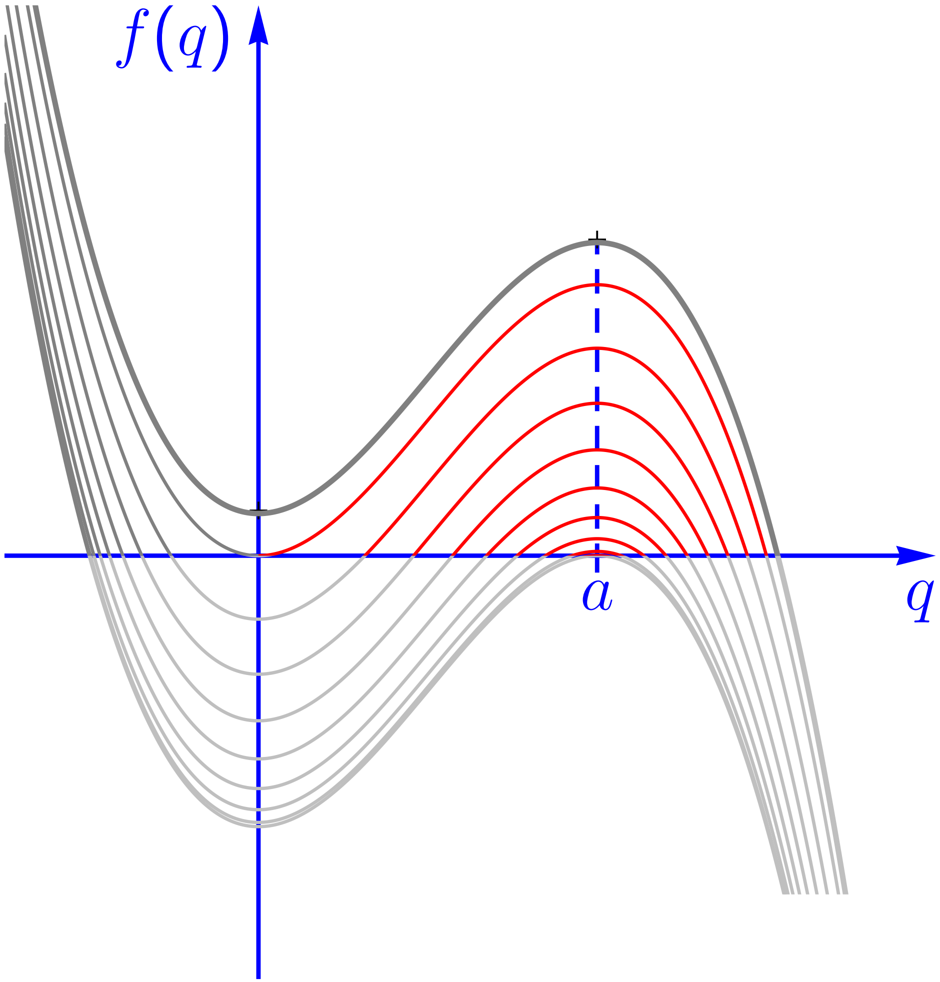

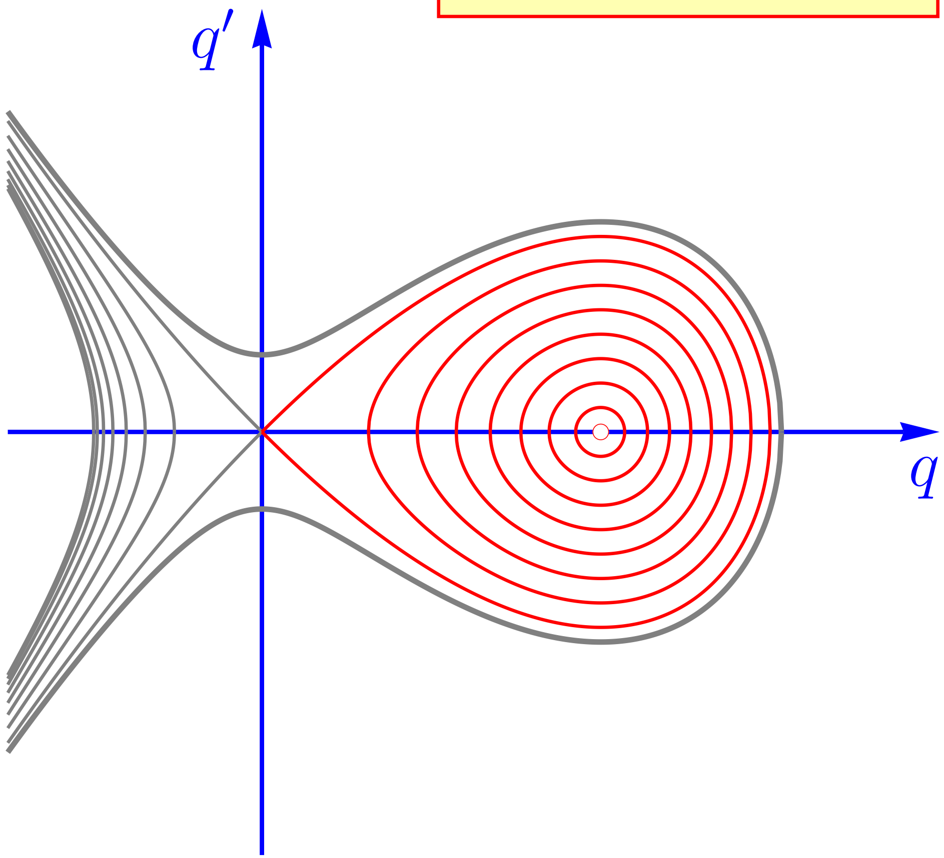

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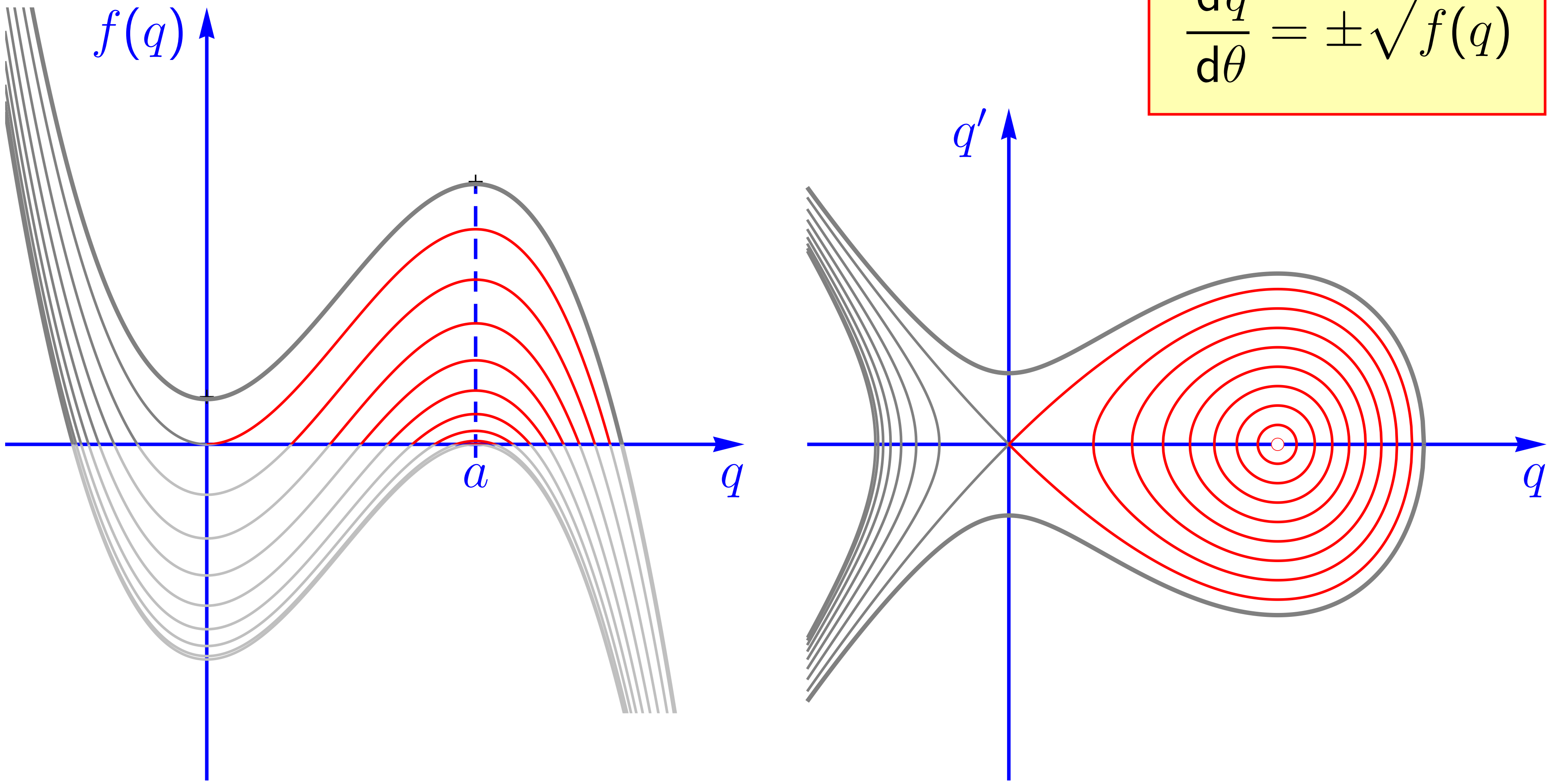

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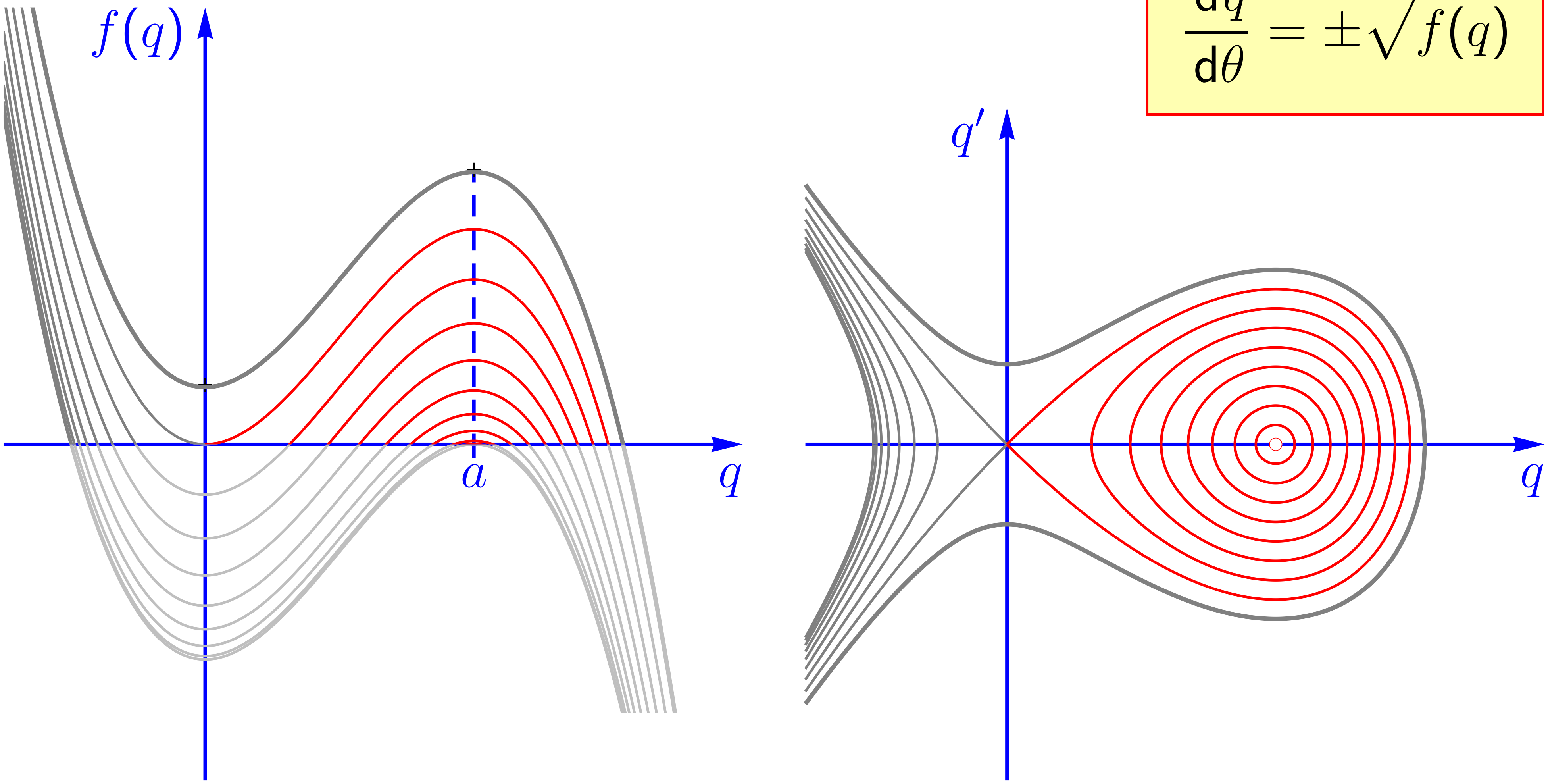

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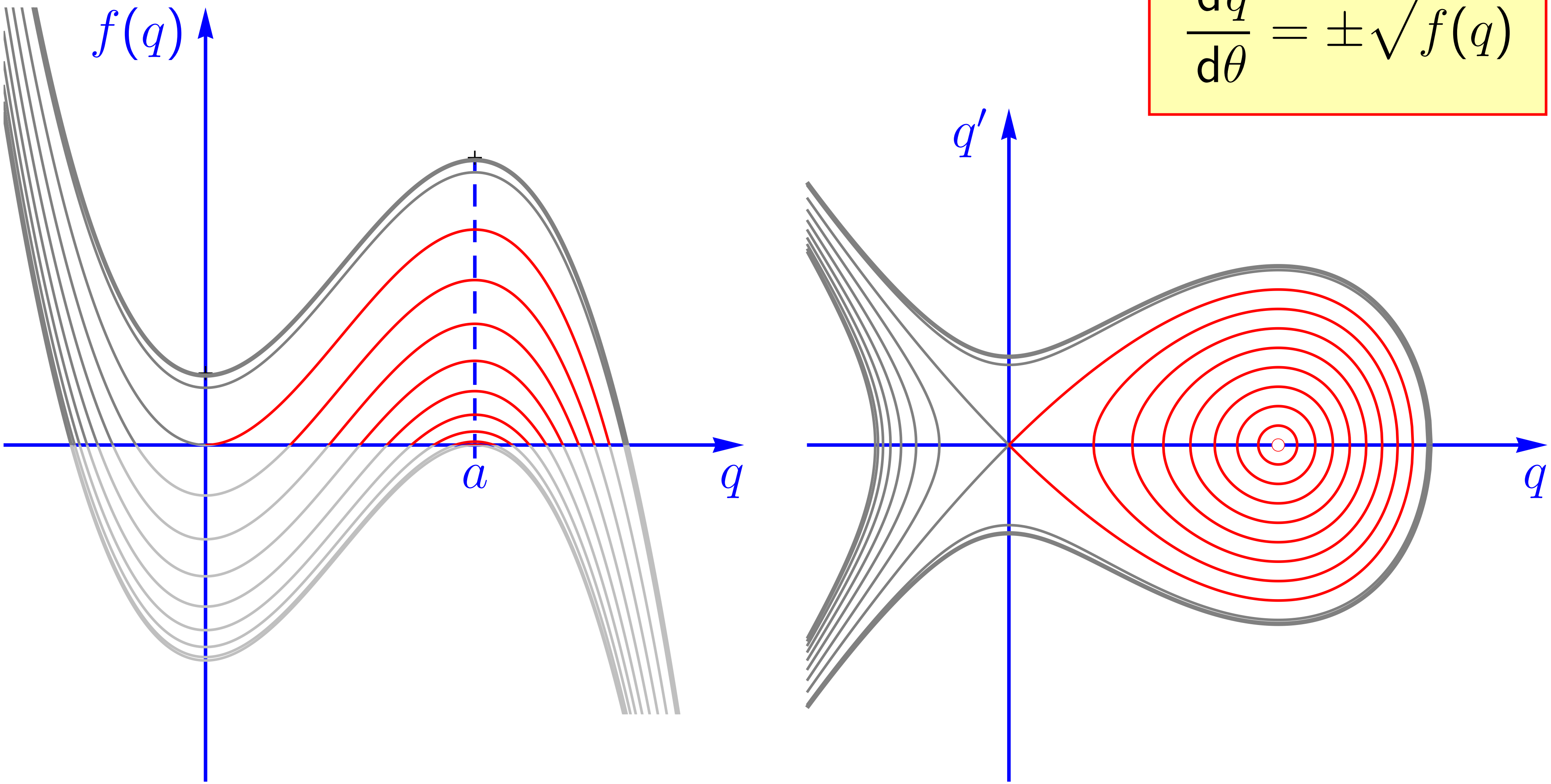

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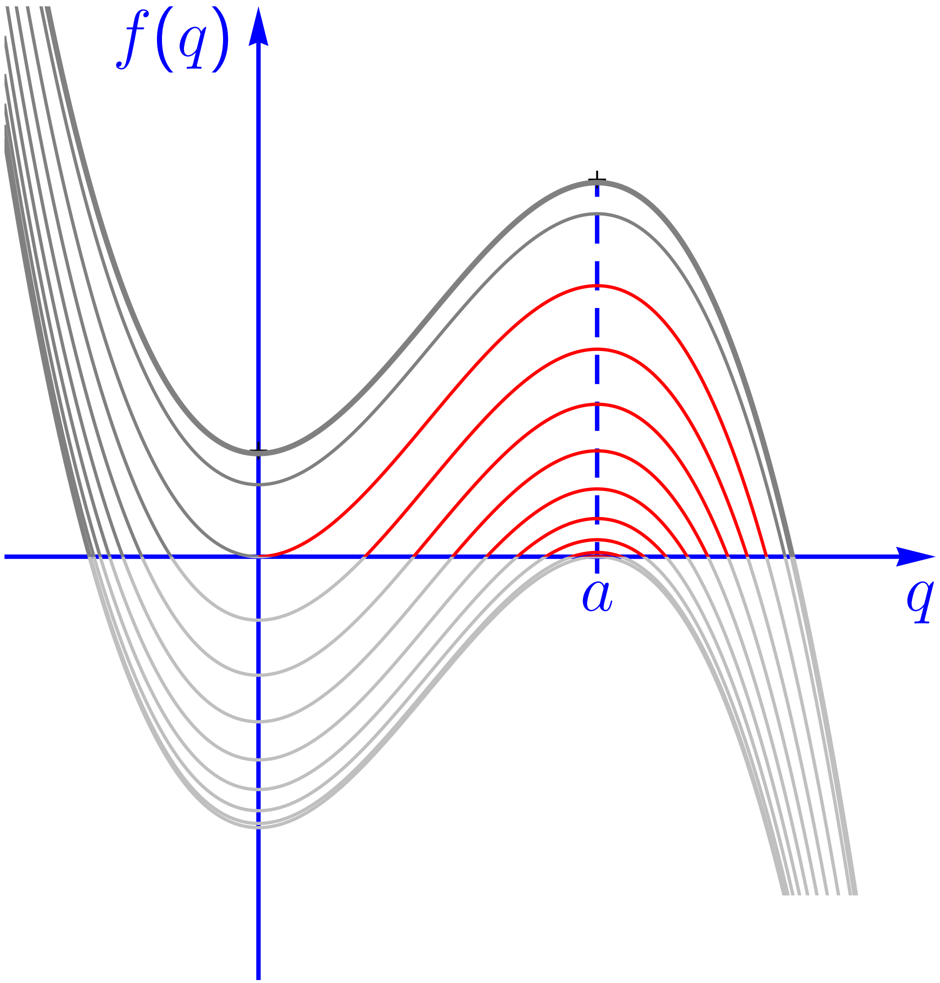

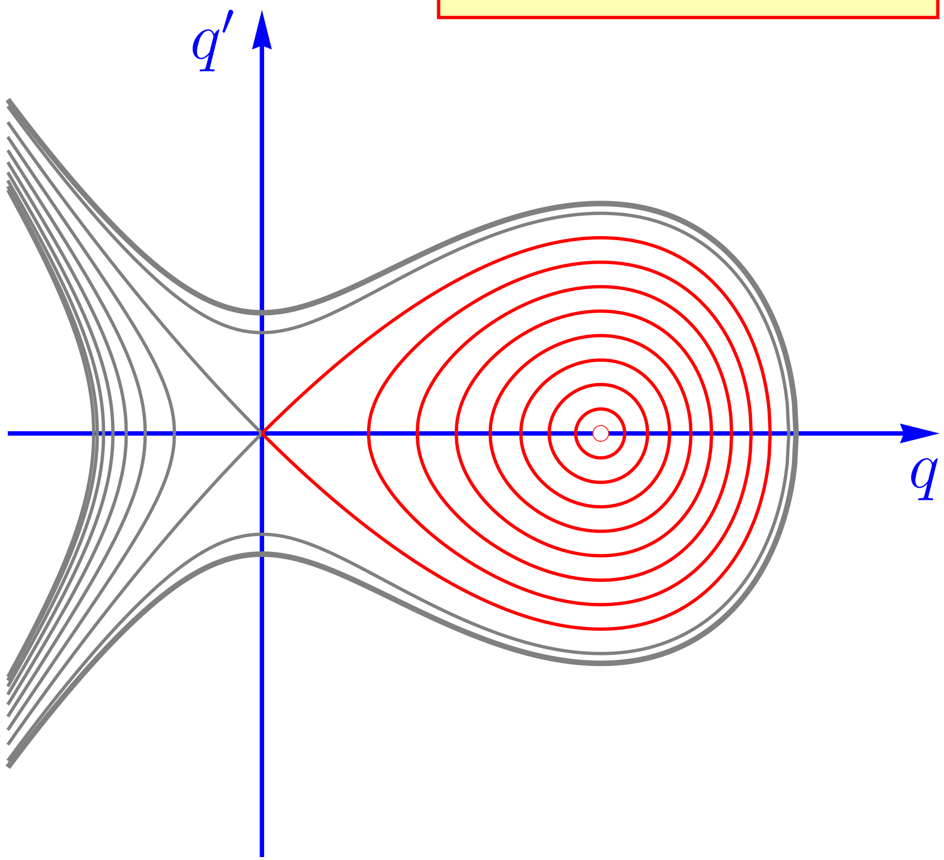

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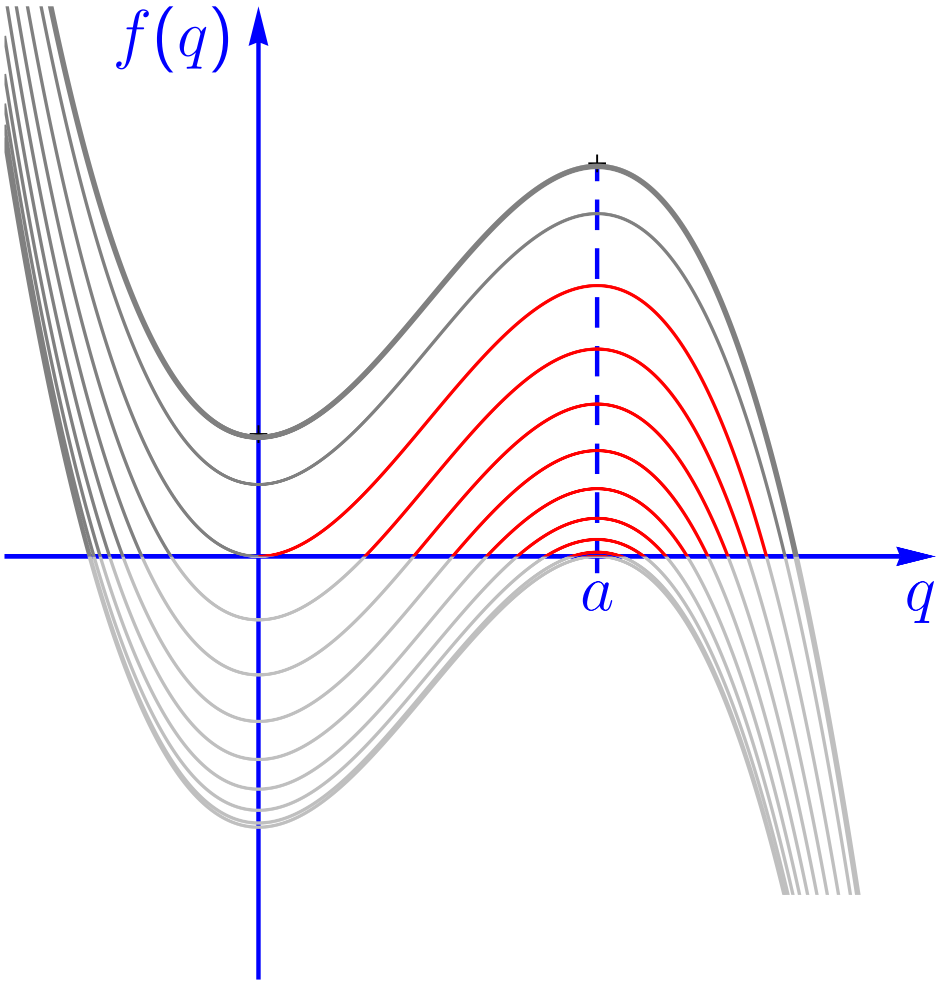

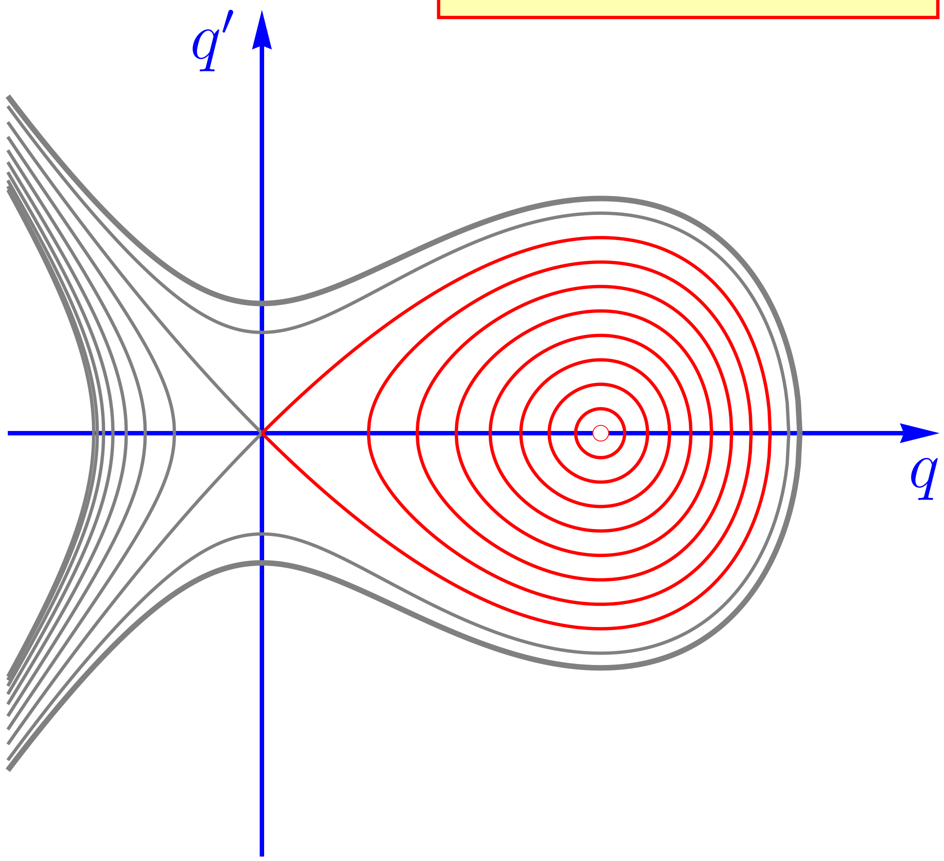

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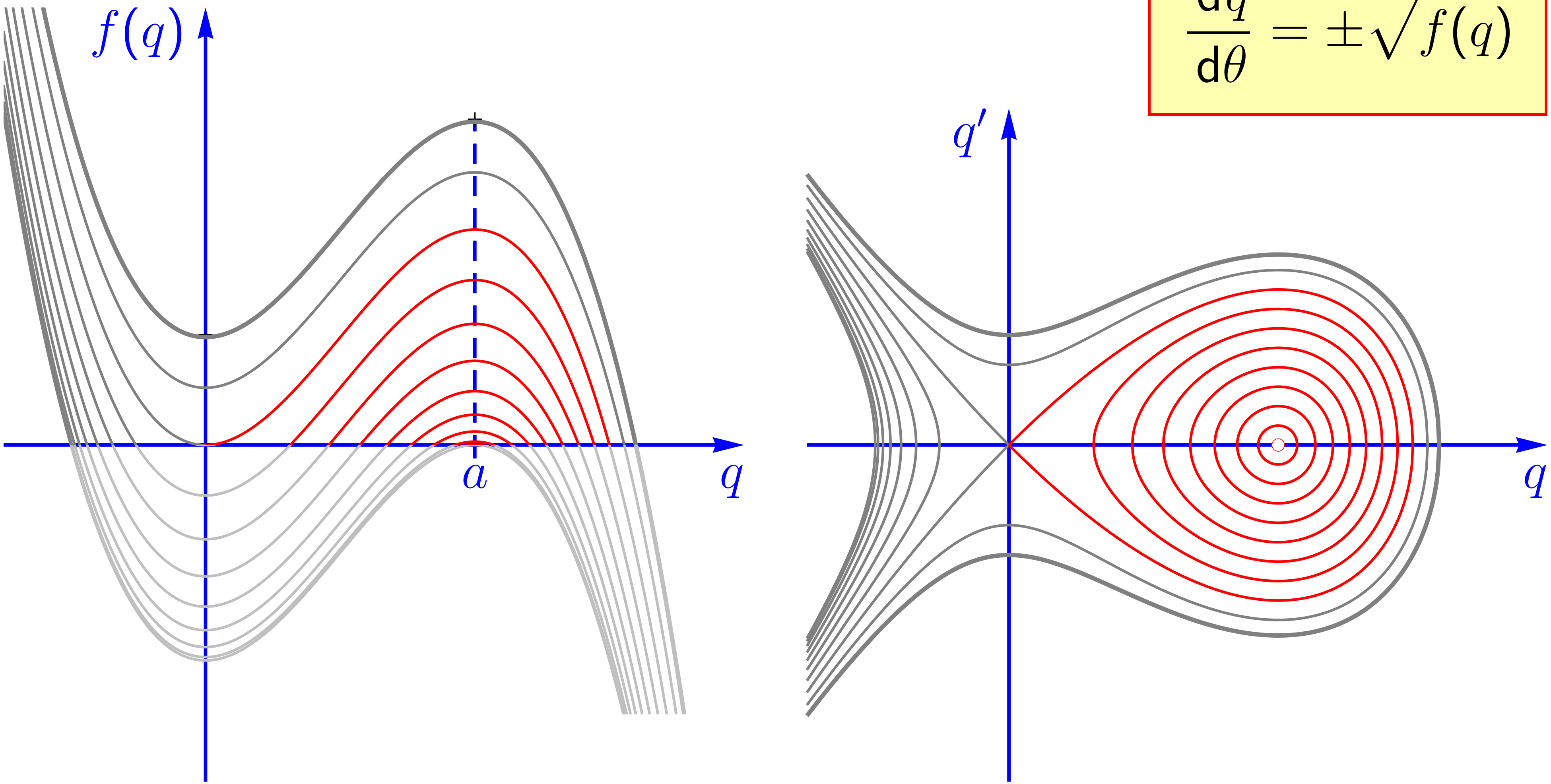

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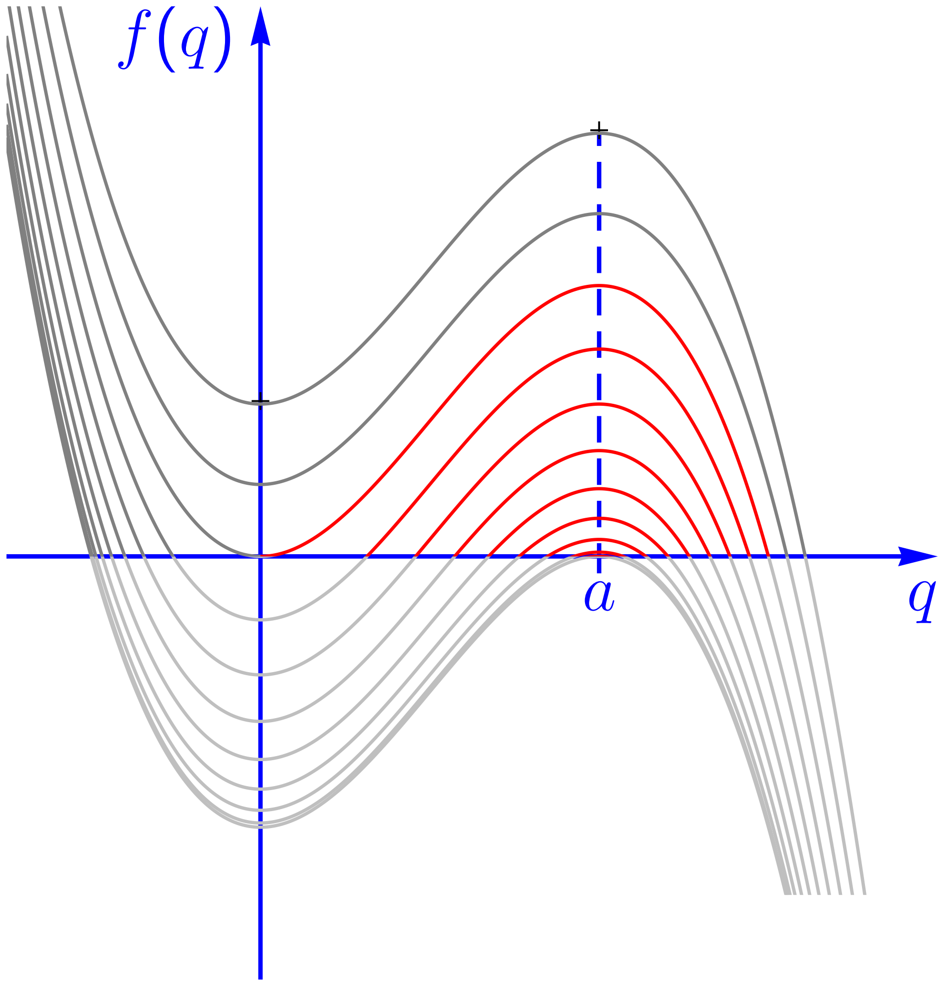

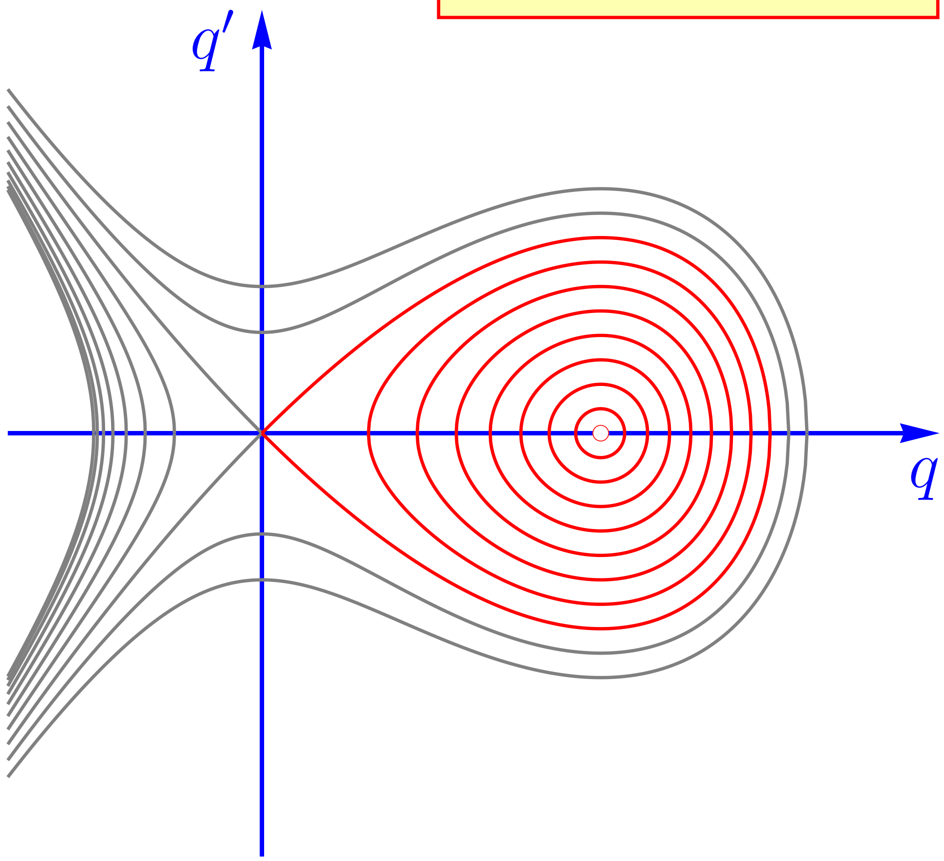

Final Integration

Differential equation

$$\frac{\mathsf{d}q}{\mathsf{d}\theta} = \pm\sqrt{f(q)}$$

Substitution

$$q = q_2 + (q_3 - q_2)\cos^2\varphi$$

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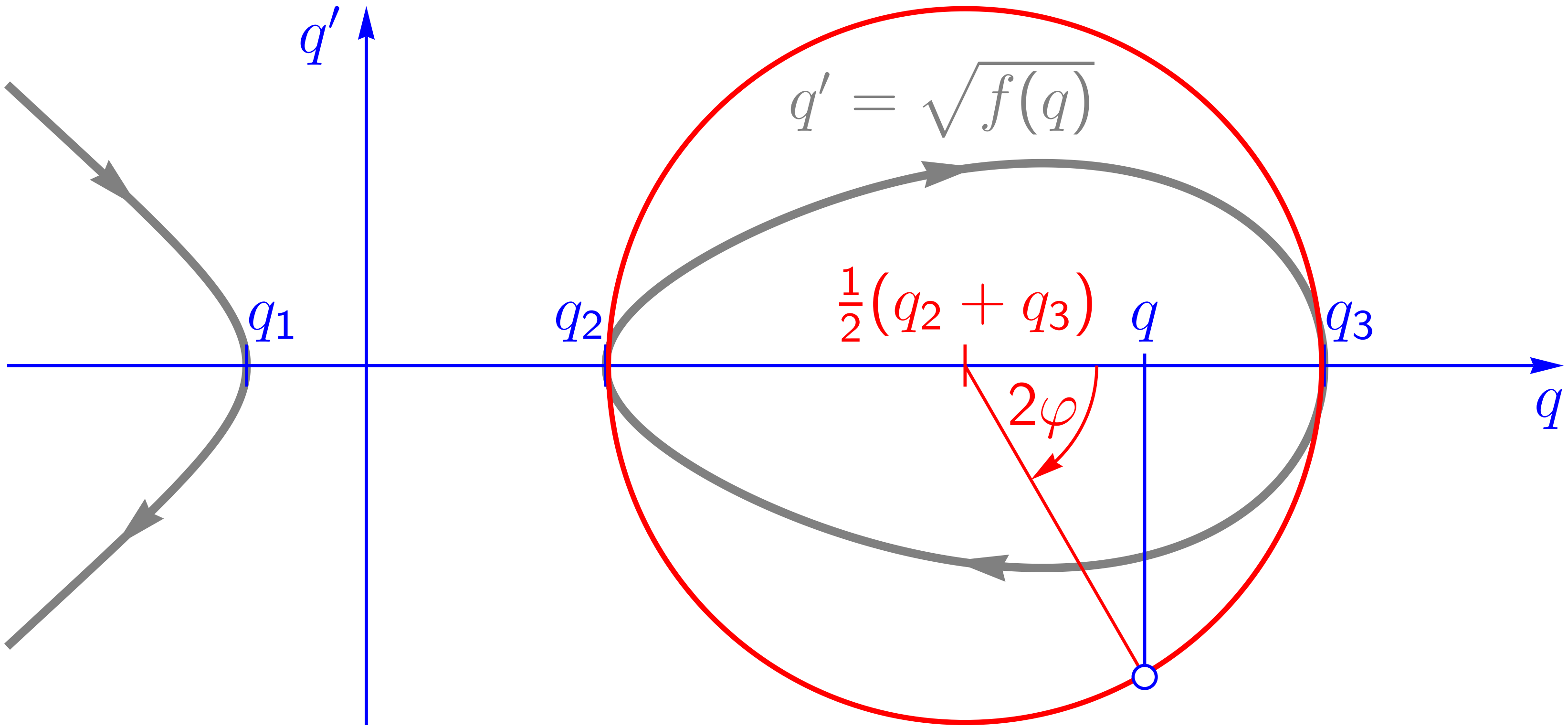

Definite integration

$$\theta = \int_{q_3}^{q} \frac{-\,\mathrm{d}q}{\sqrt{f(q)}} = \frac{1}{\eta}\int_0^{\varphi} \frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}}$$

Parameters

$$\eta^2 = \frac{1}{2}(q_3 - q_1) \qquad k^2 = \frac{q_3 - q_2}{q_3 - q_1}$$

Incomplete elliptic integral of the first kind

$$F(\varphi;k) = \int_0^{\varphi} \frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}}$$

Result of integration

$$\eta\theta = F(\varphi; k) \quad \Longleftrightarrow \quad \varphi = \mathsf{am}(\eta\theta; k) \qquad \text{“amplitudo”}$$

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cosinus amplitudinis $\qquad \mathrm{cn}(x; k) = \cos \mathrm{am}(x; k)$

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cosinus amplitudinis
(amplituudi koosinus)

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---

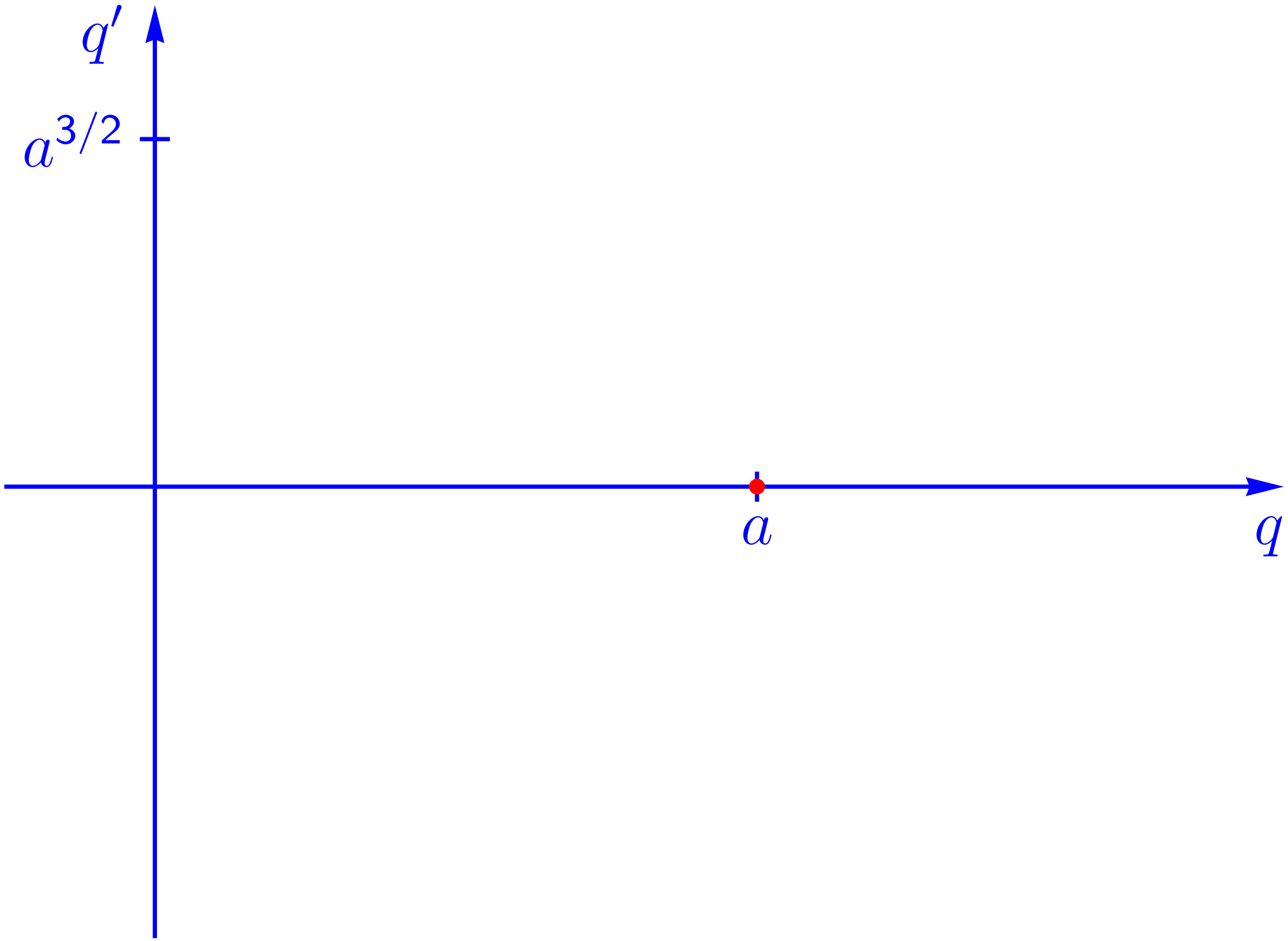
cosinus amplitudinis
(amplituudi koosinus)

$$\mathrm{cn}(x; k) = \cos\mathrm{am}(x; k)$$

Cnoidal waves

$$q = q_2 + (q_3 - q_2)\,\mathrm{cn}^2(\eta\theta; k)$$

Phase Portrait



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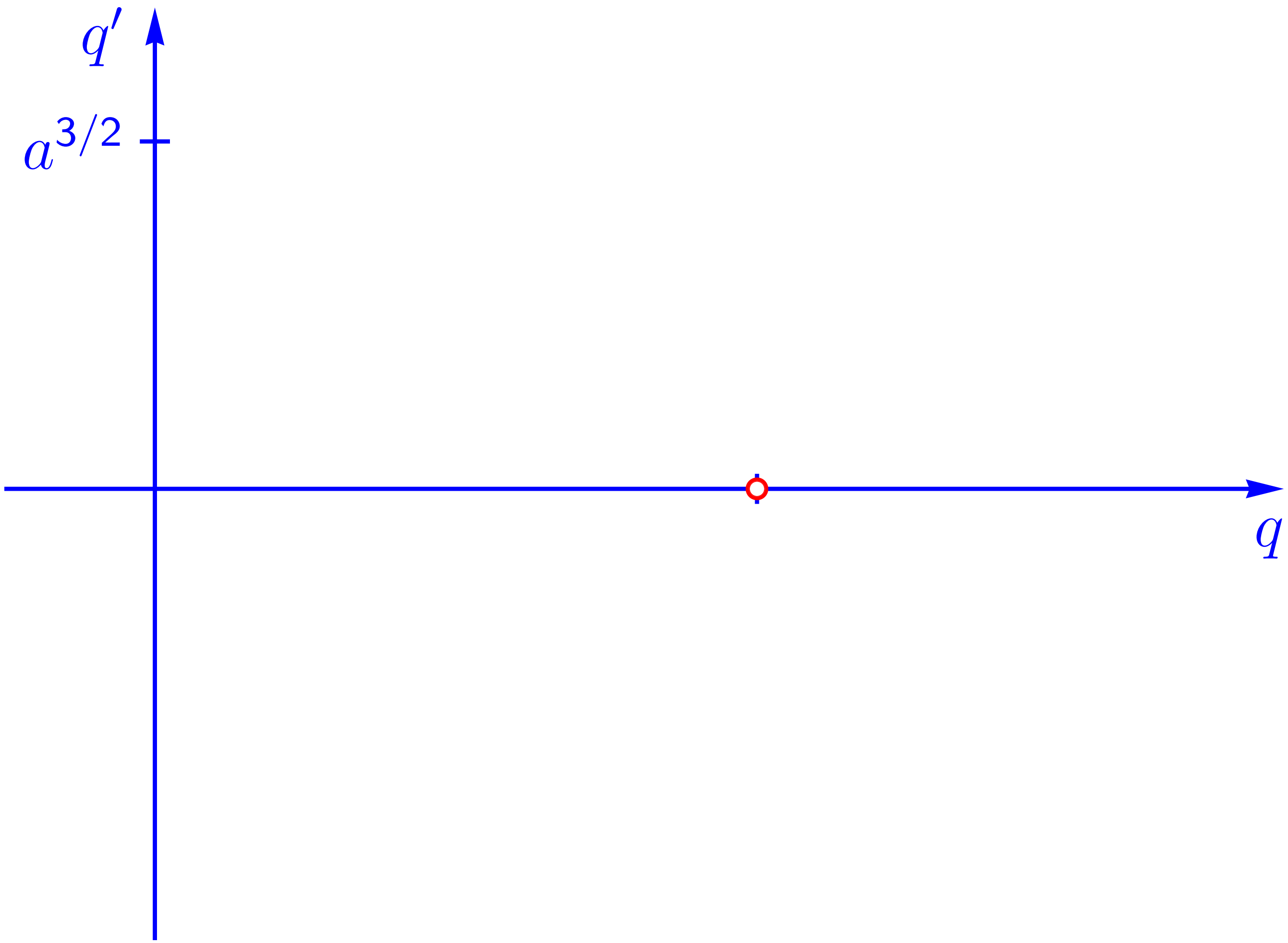

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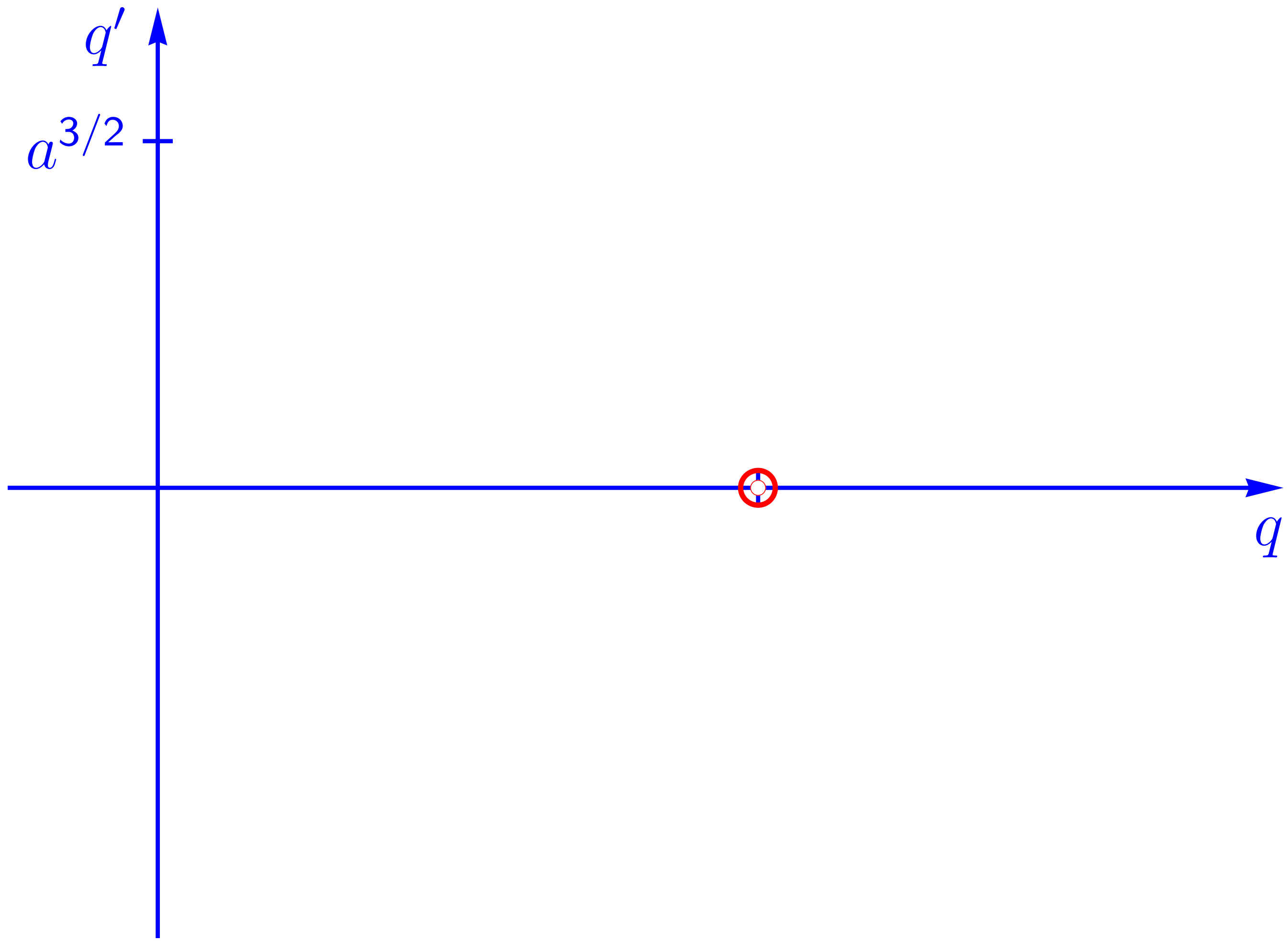

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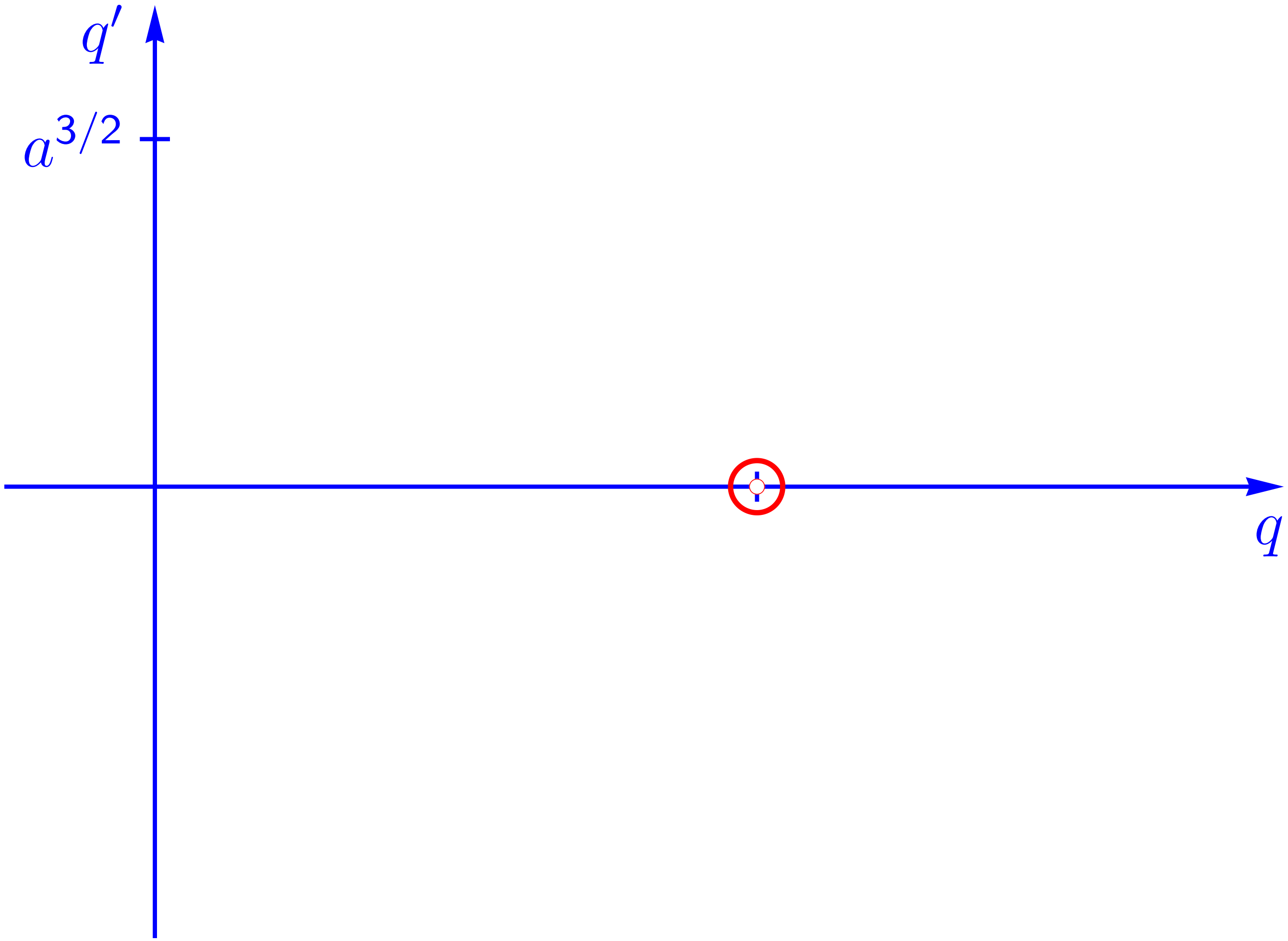

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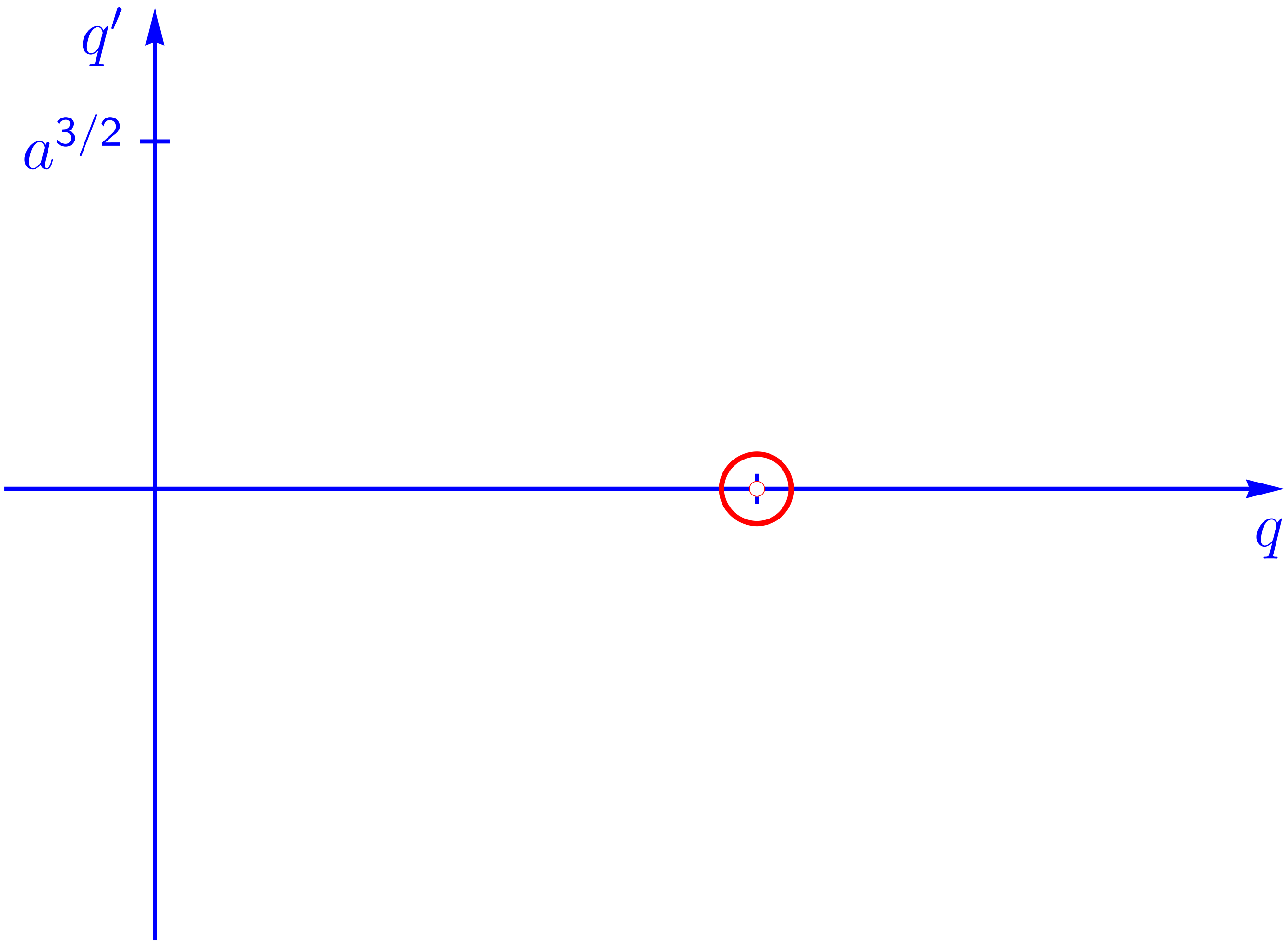

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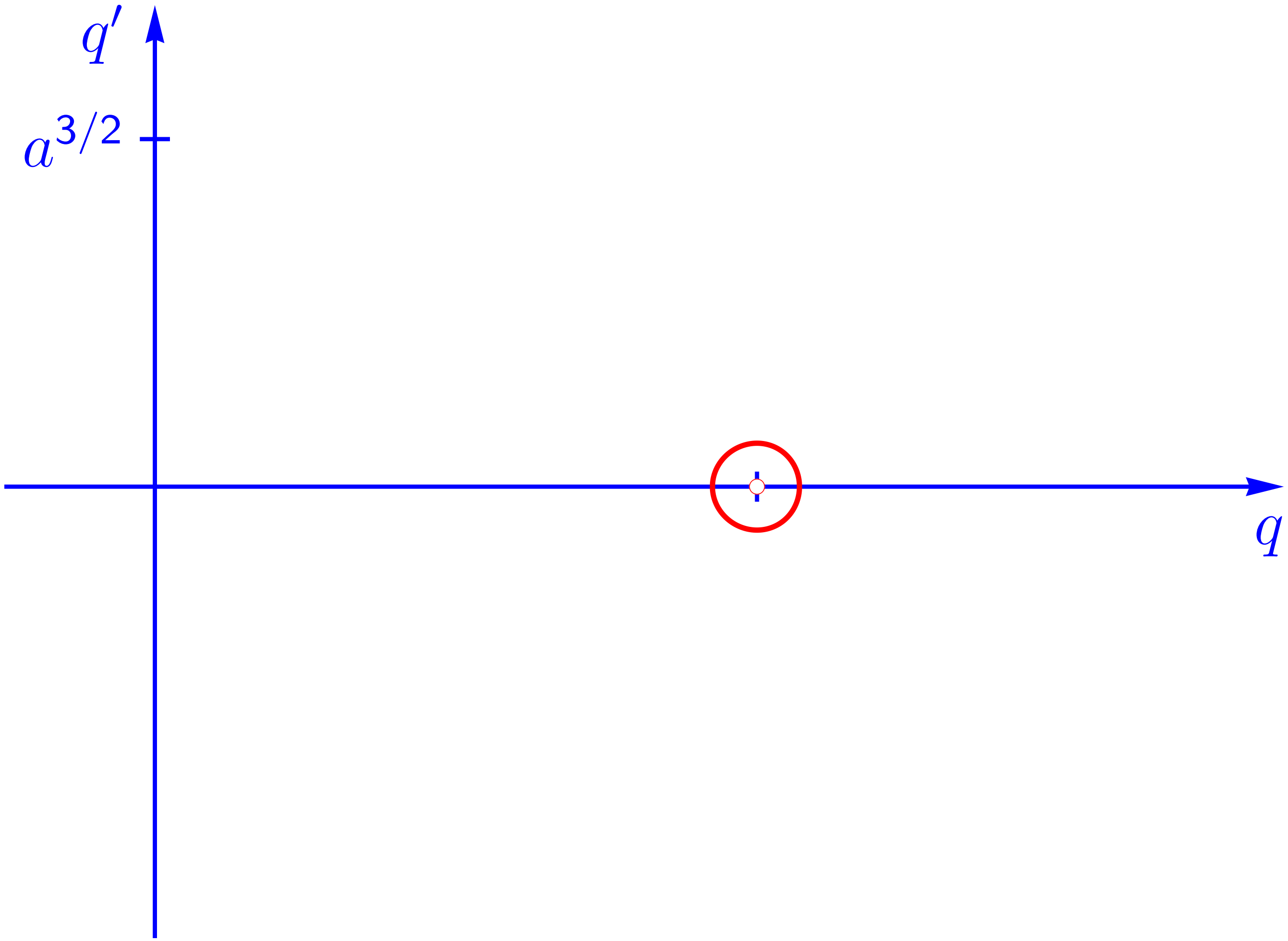

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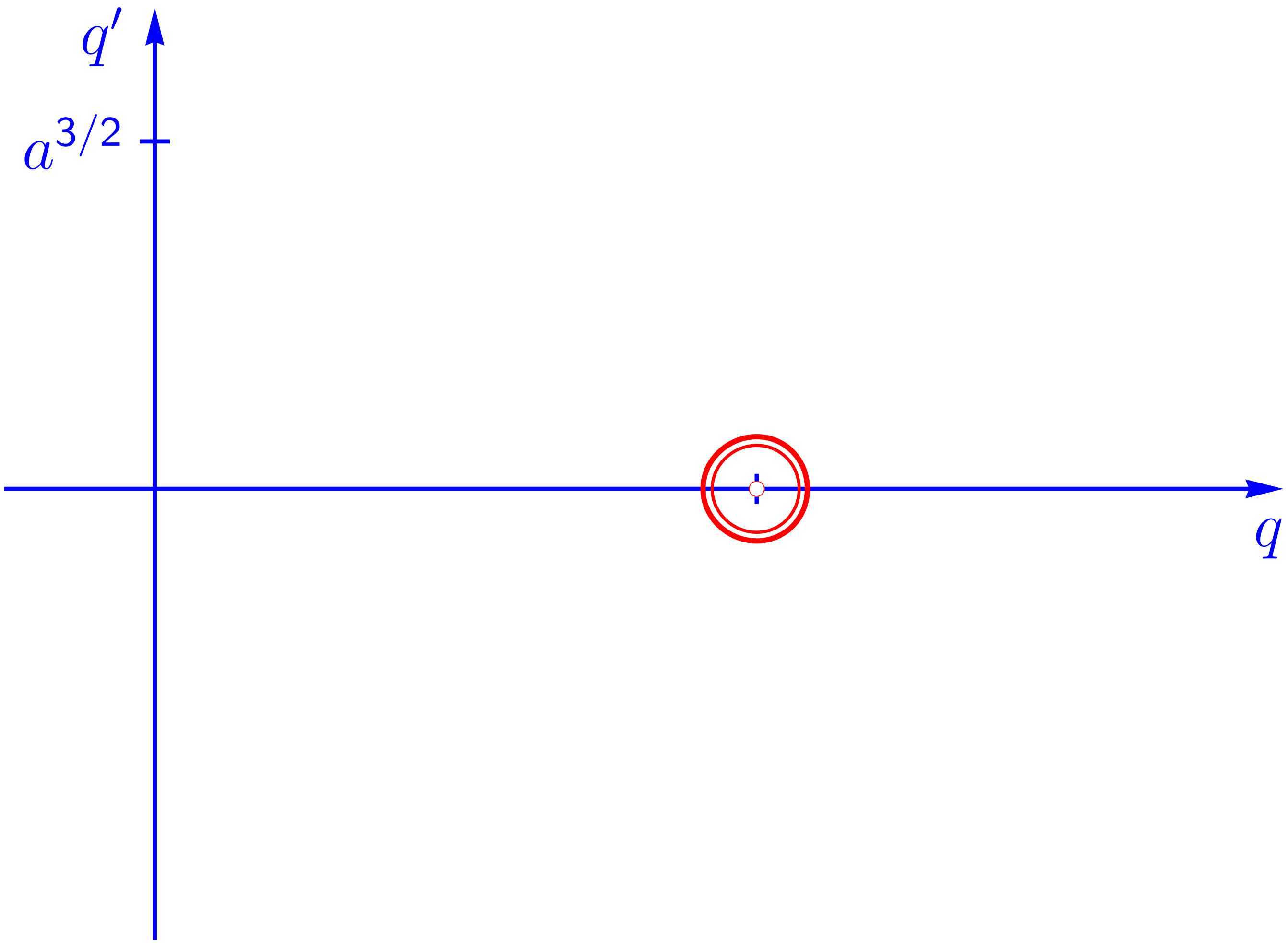

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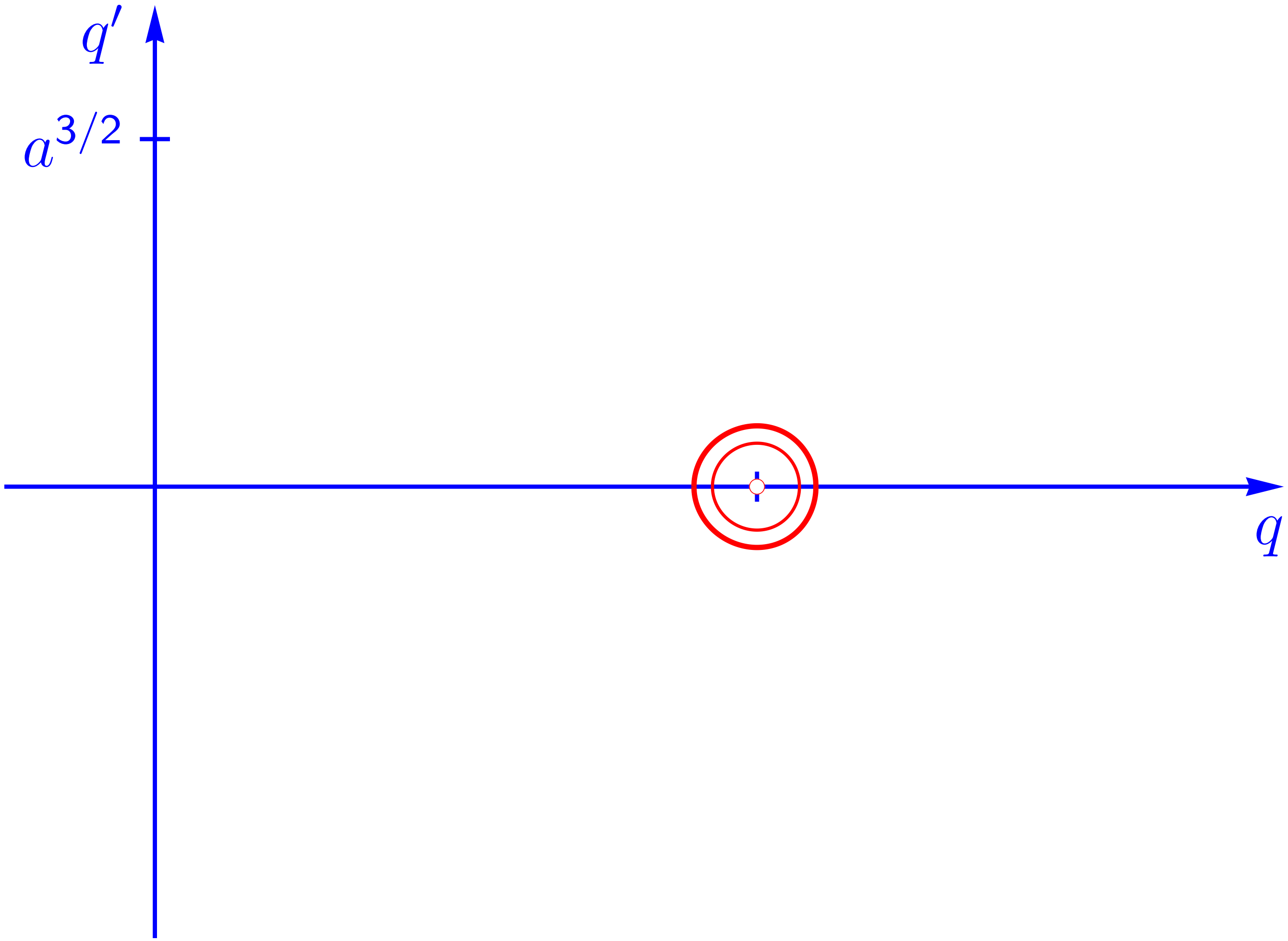

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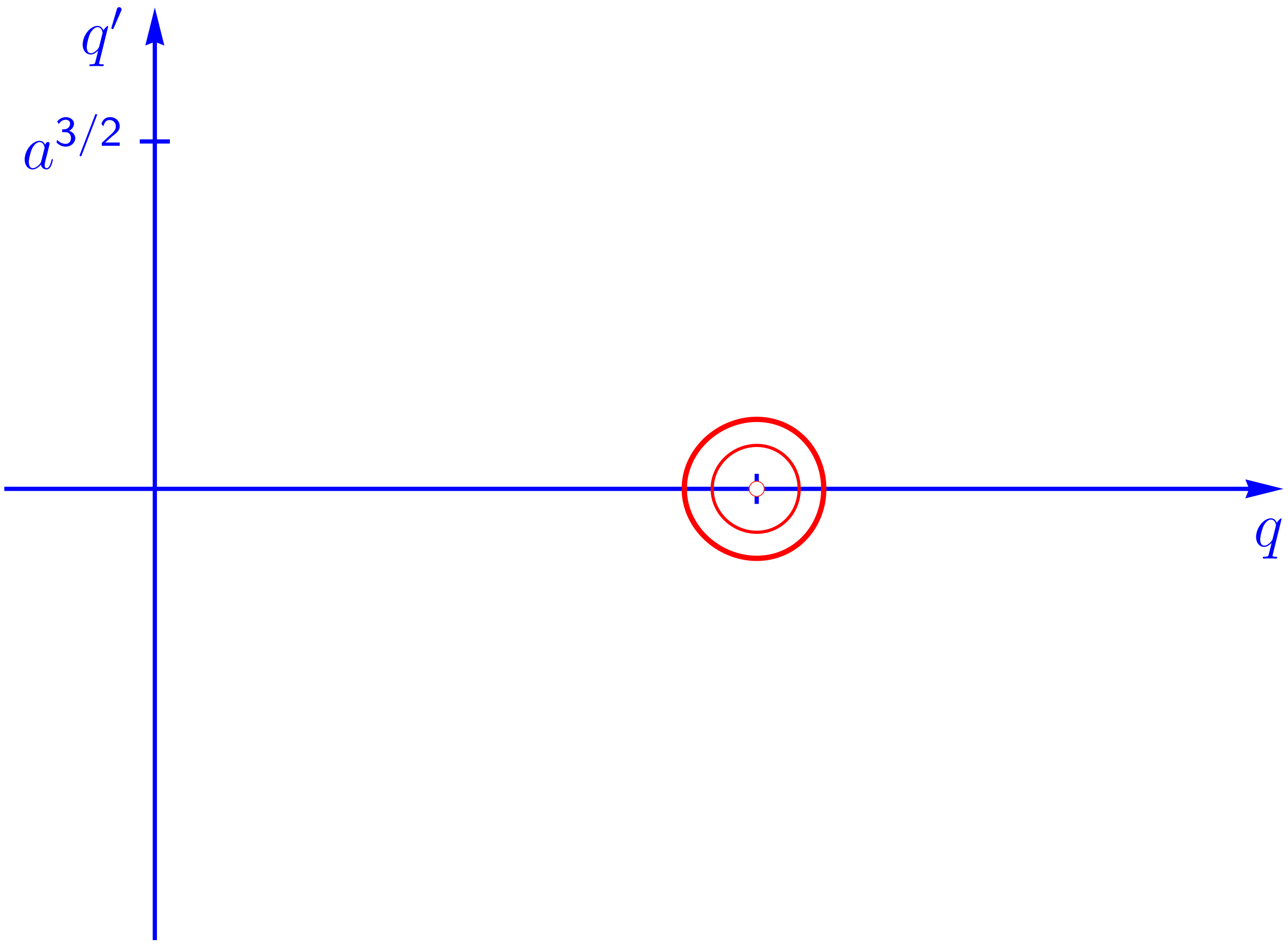

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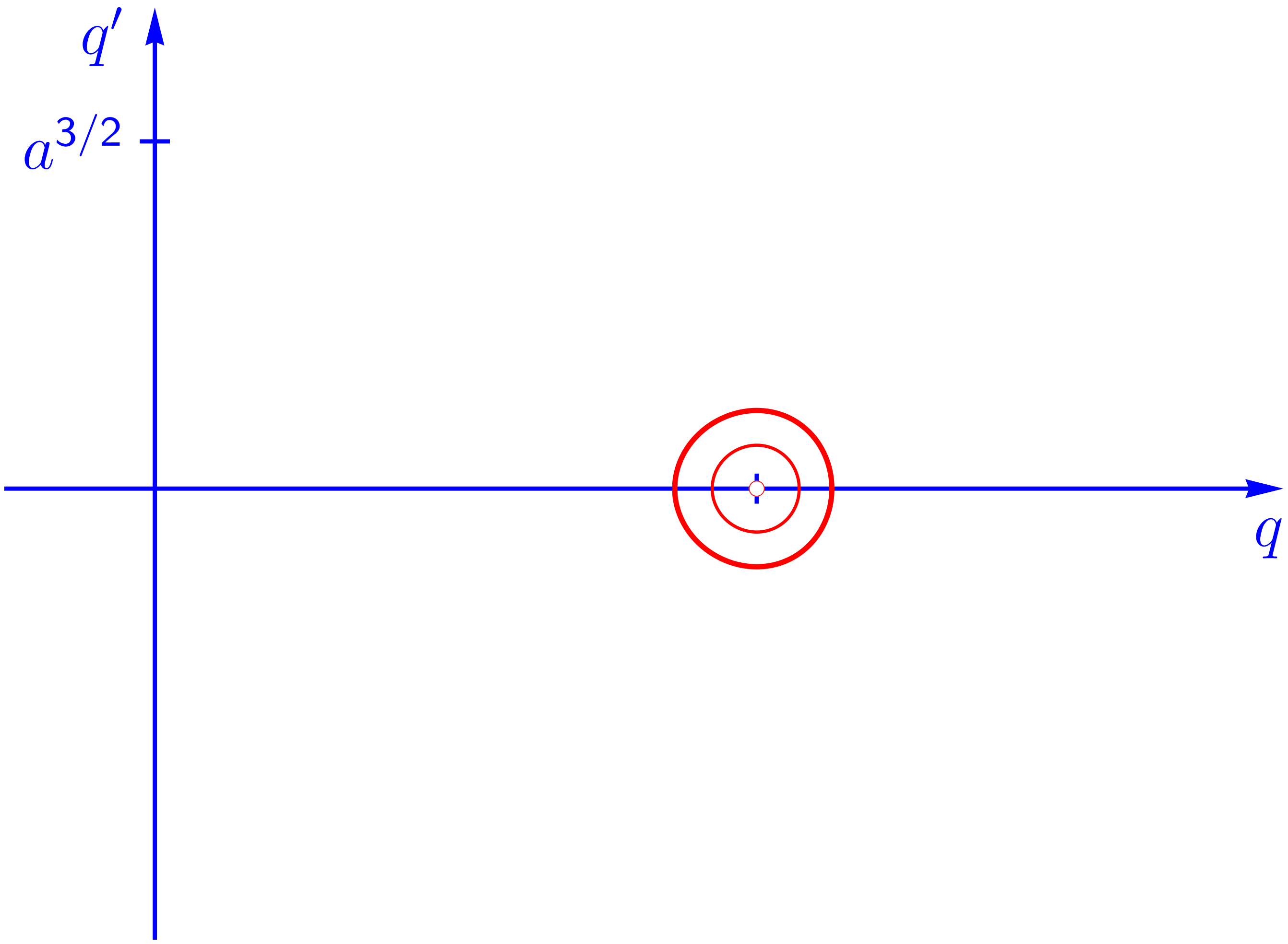

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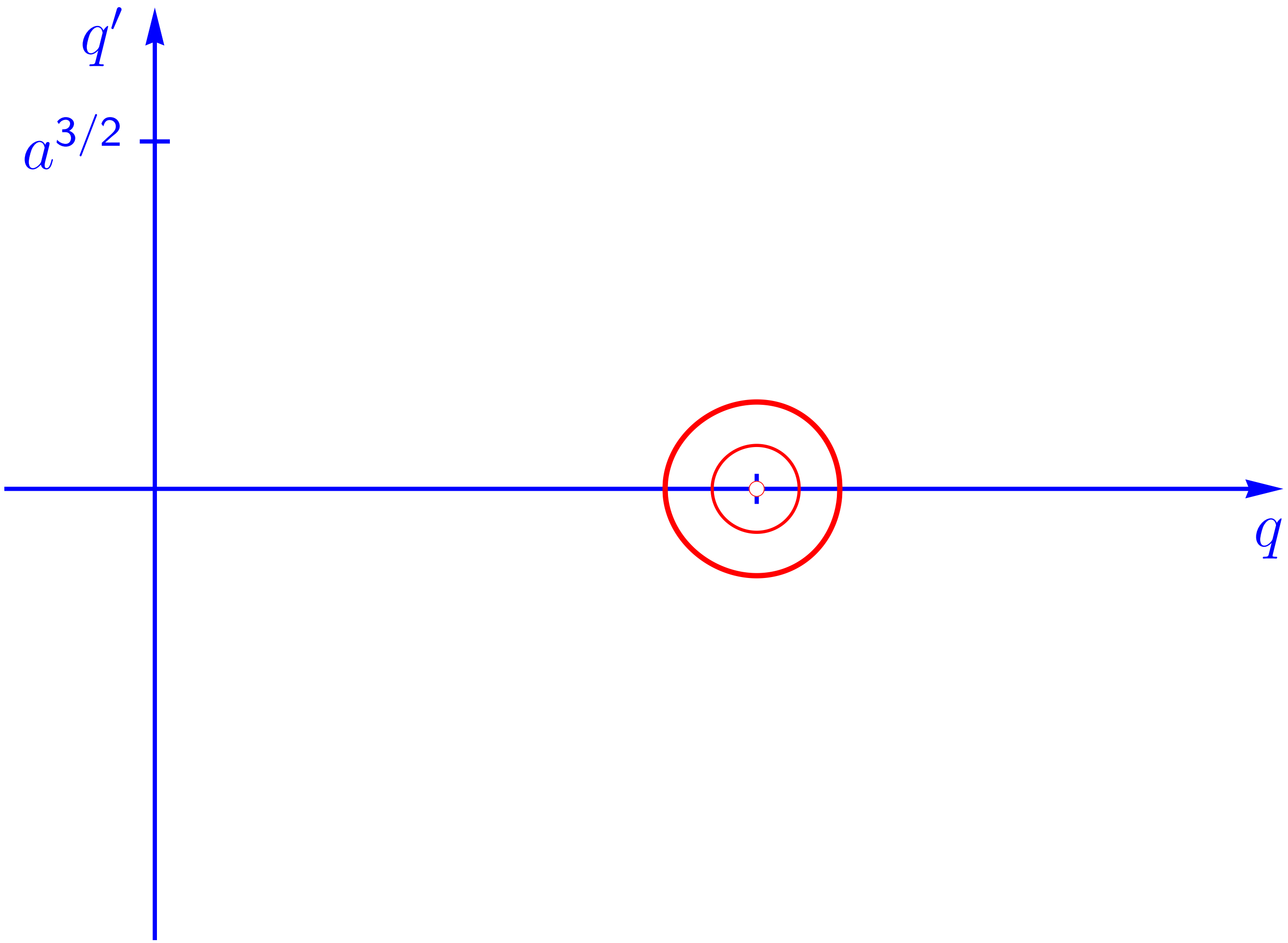

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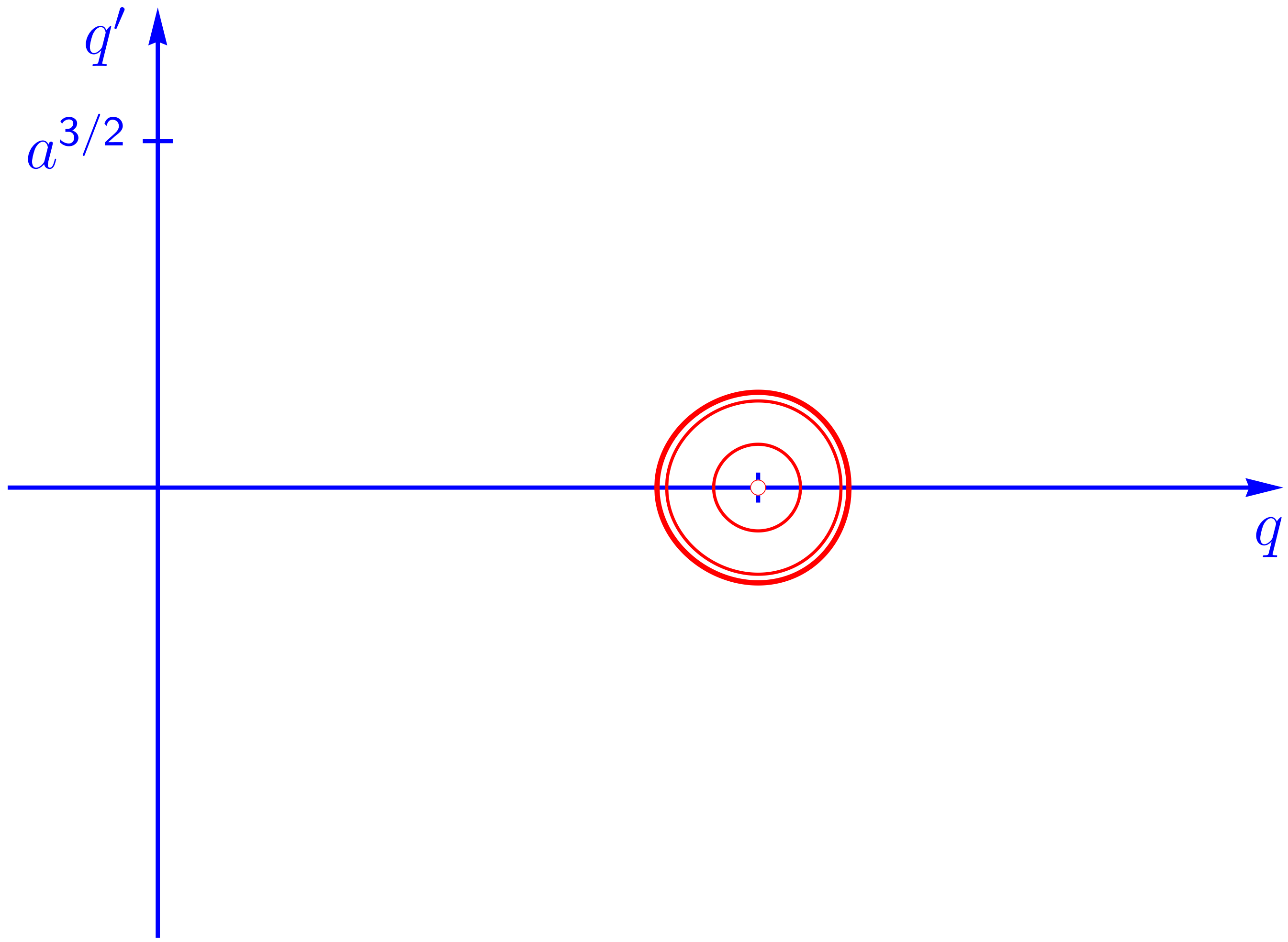

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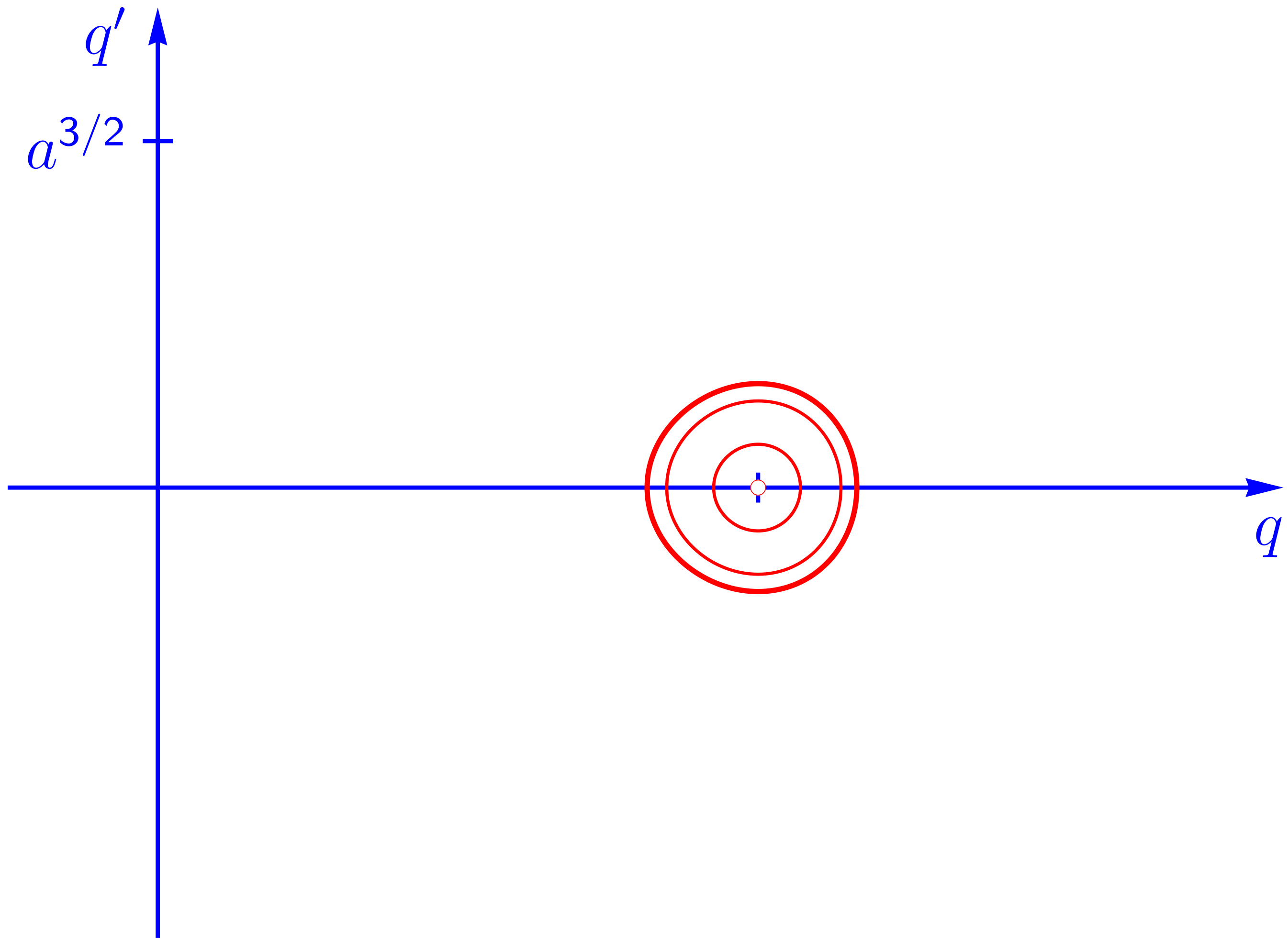

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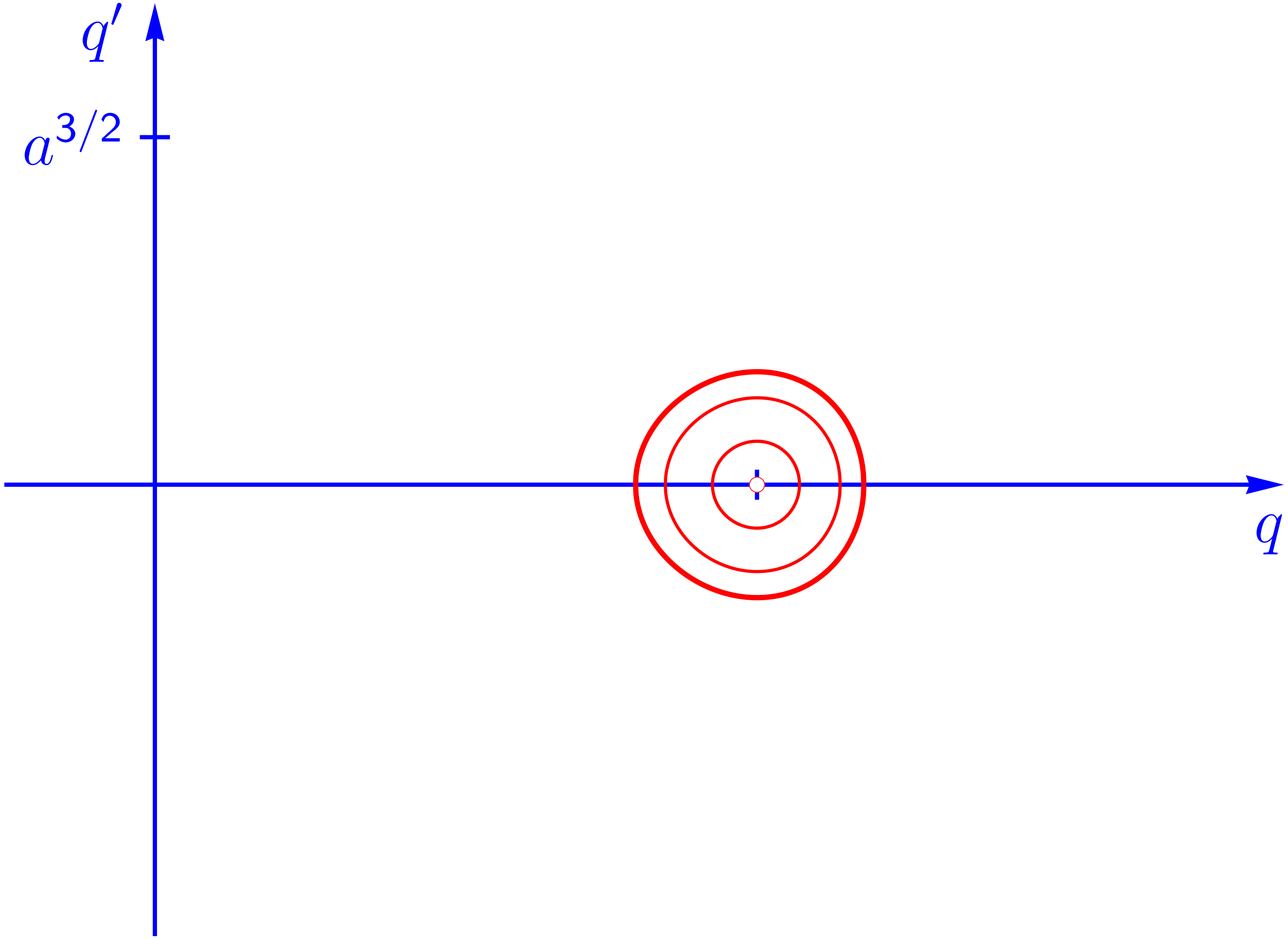

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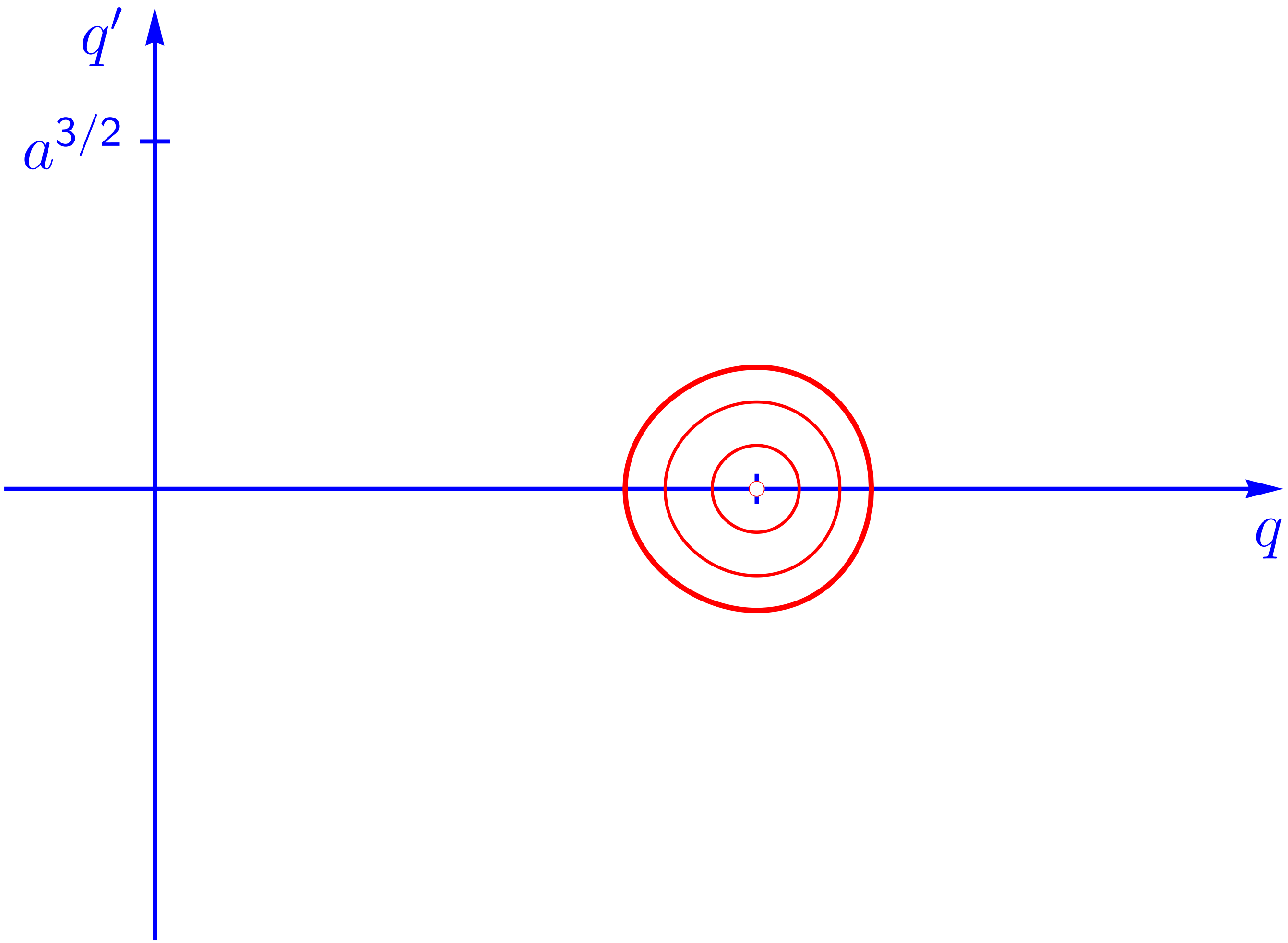

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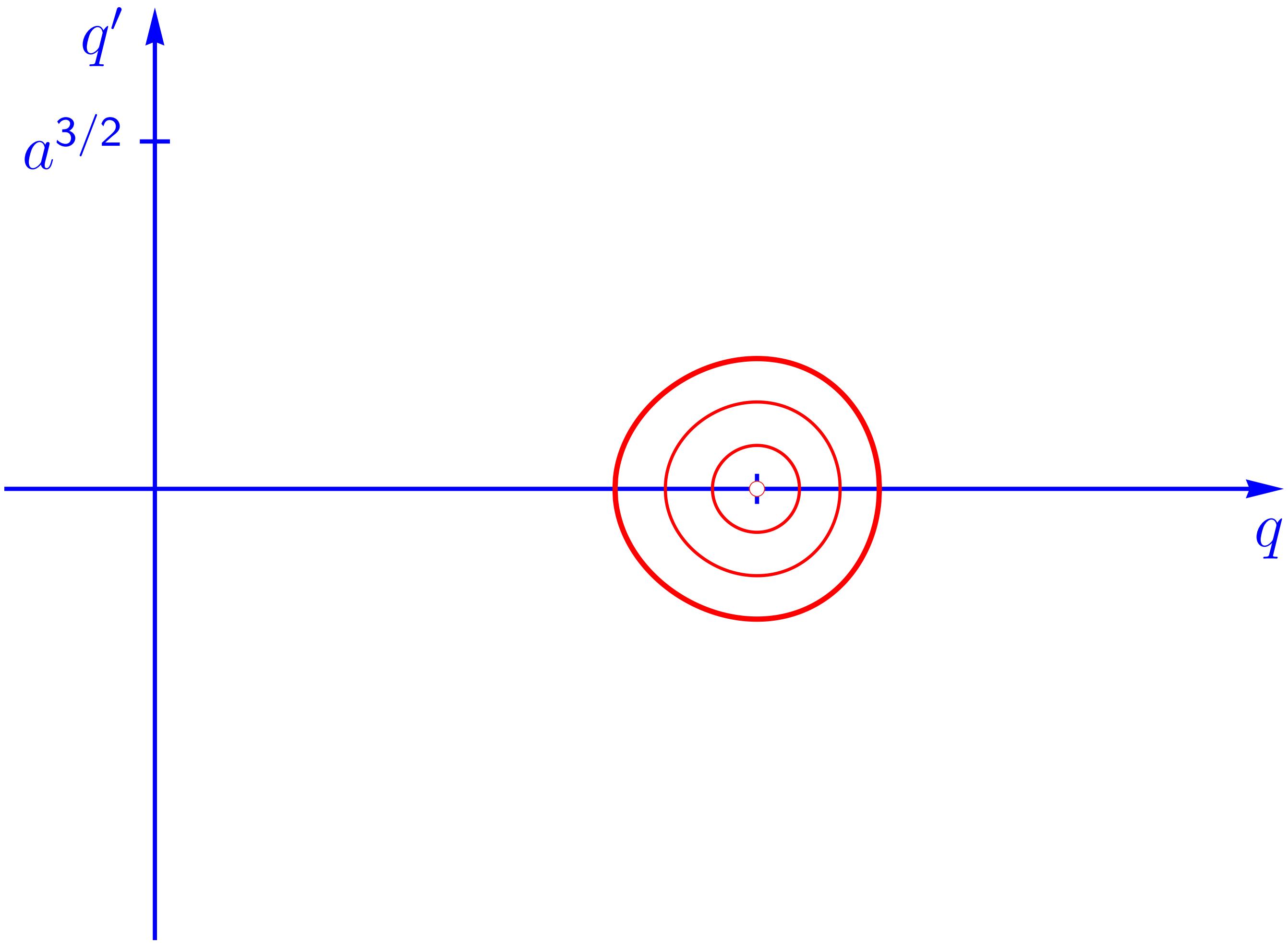

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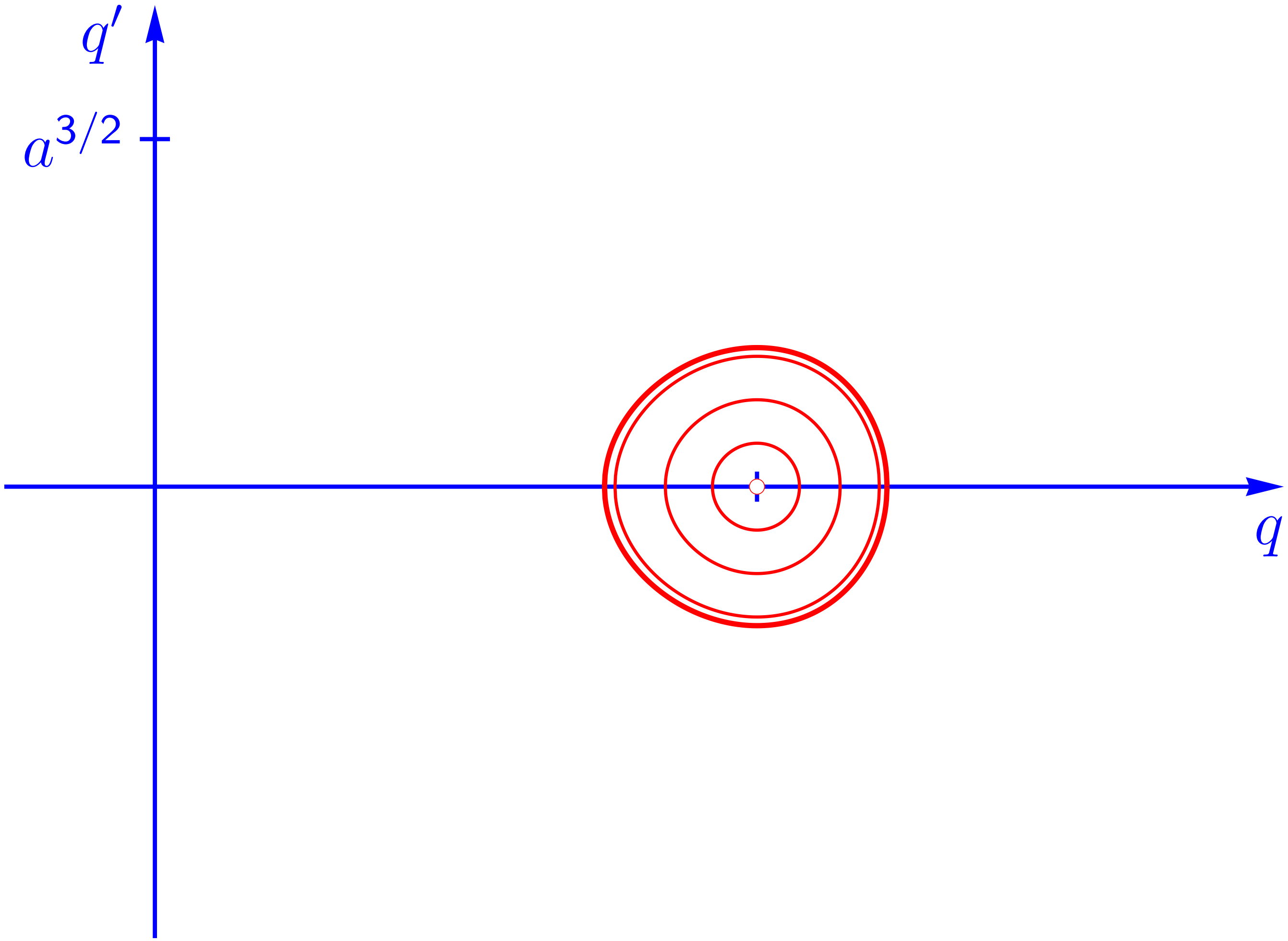

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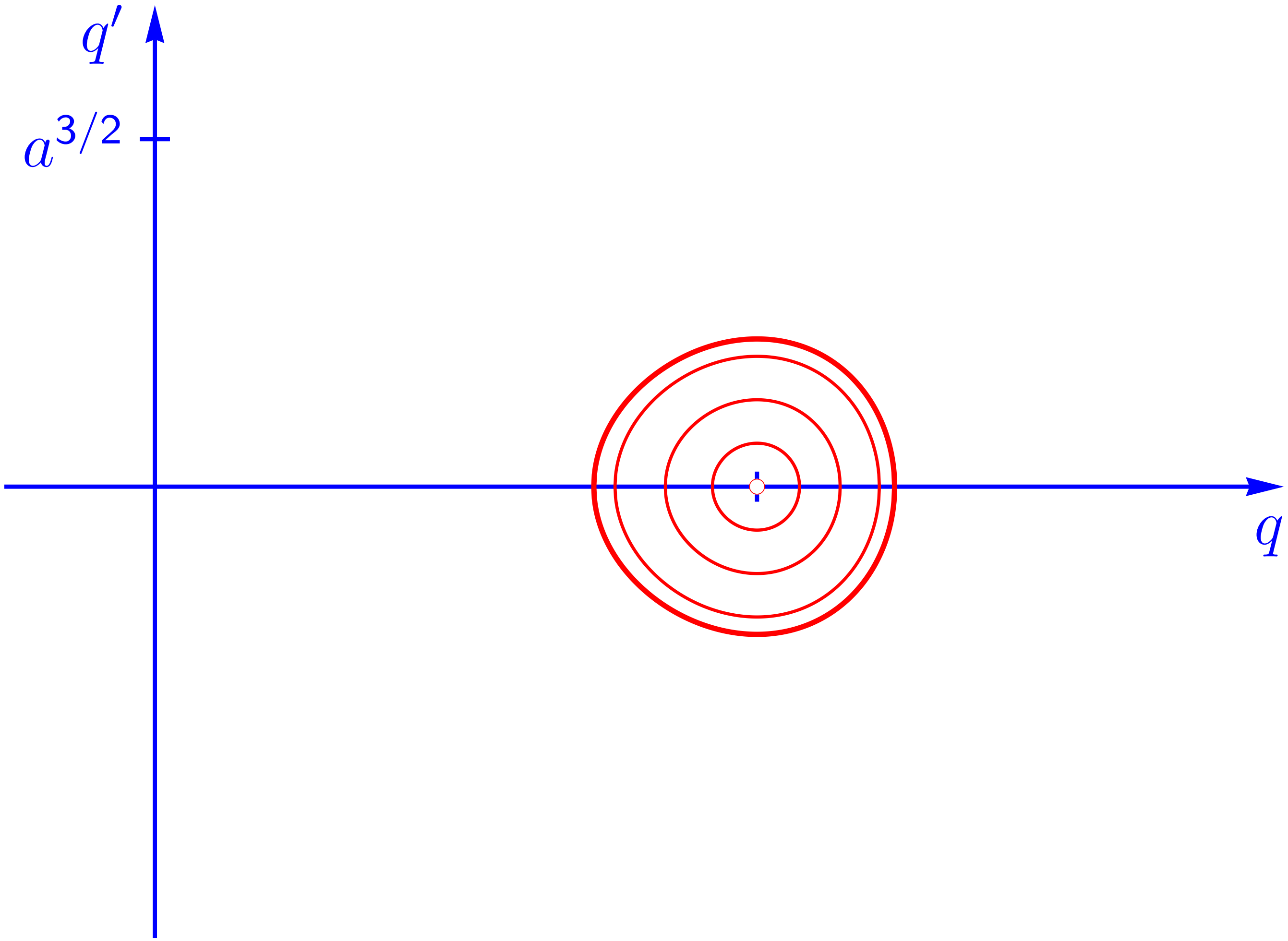

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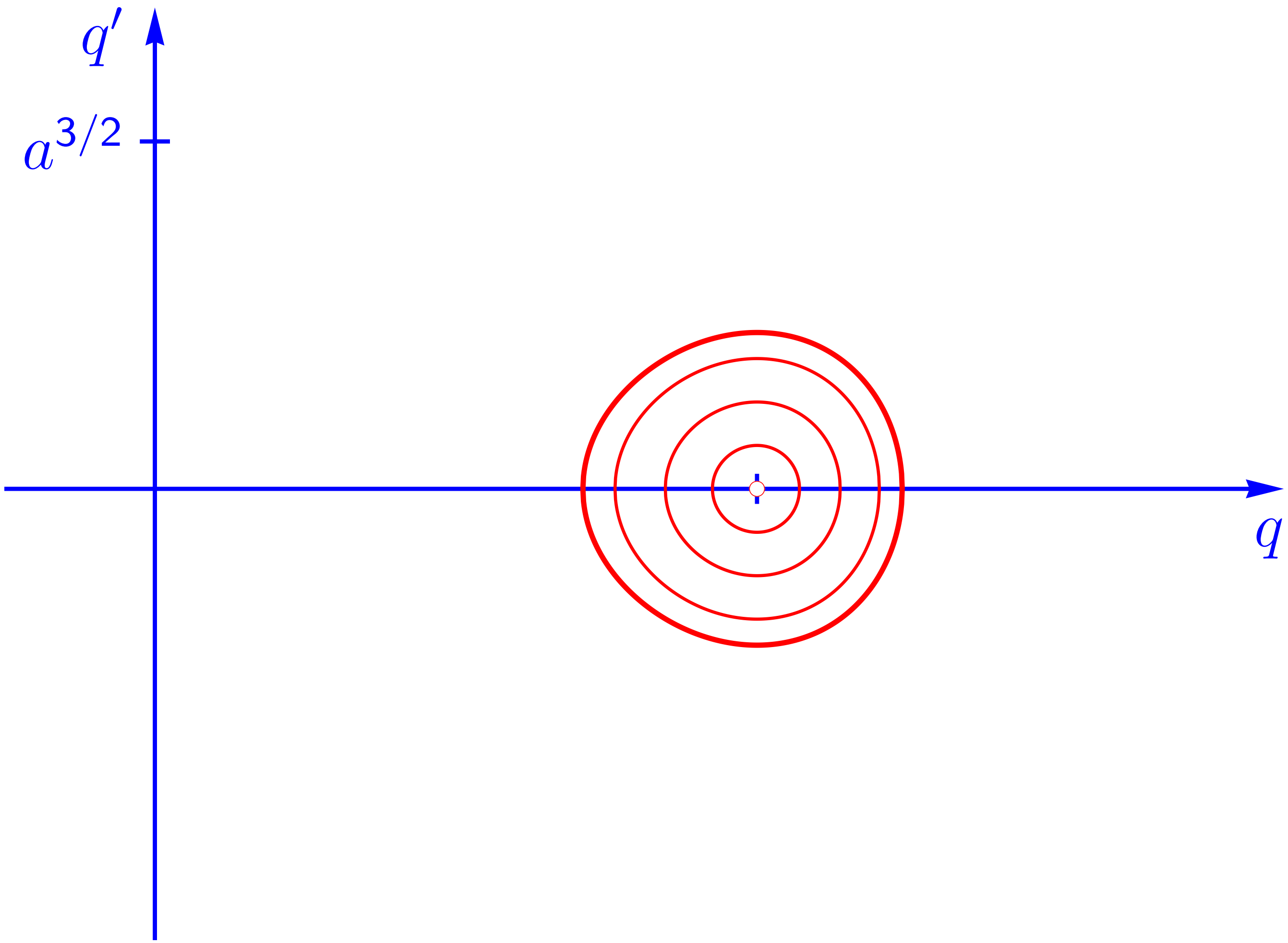

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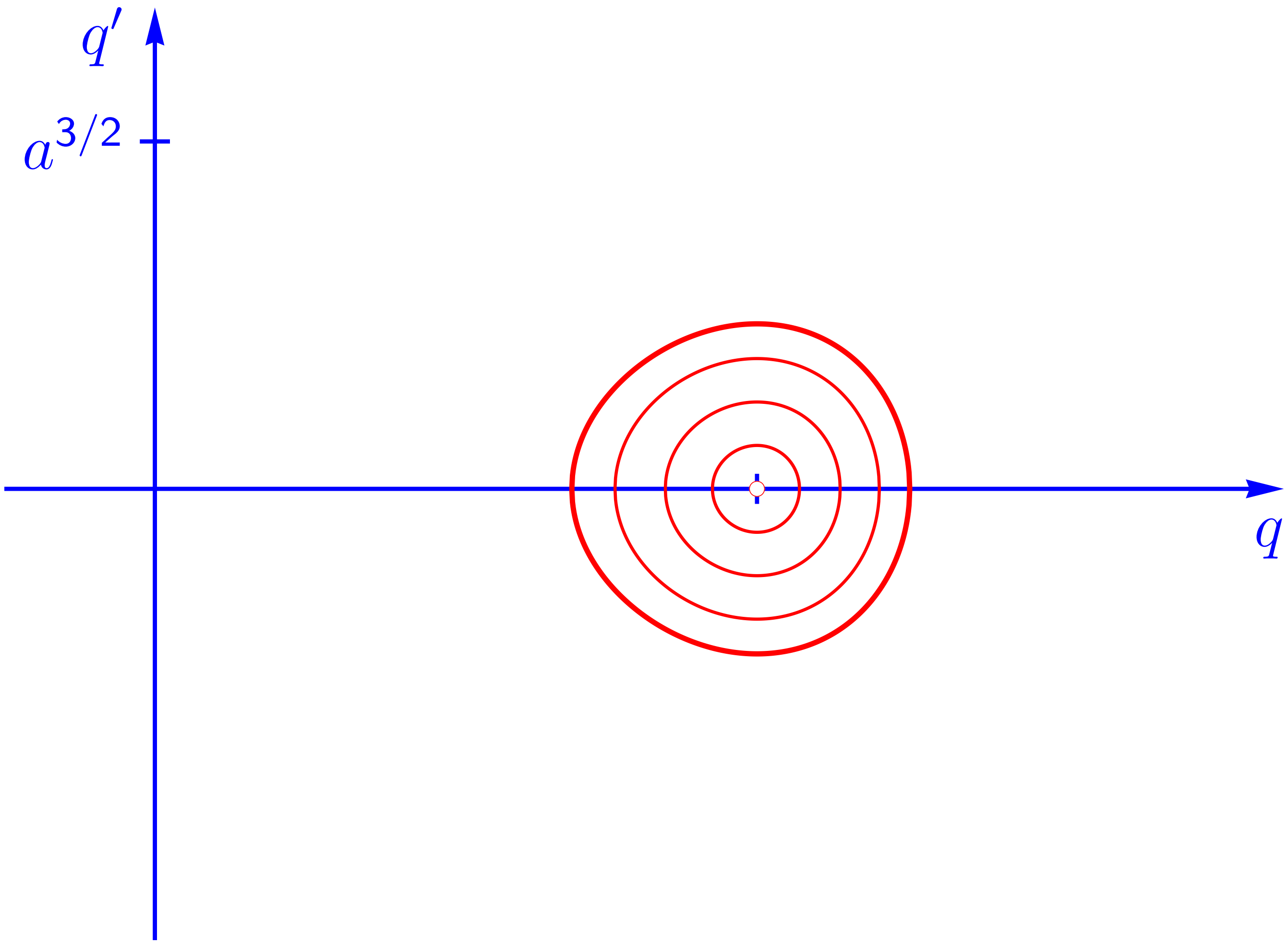

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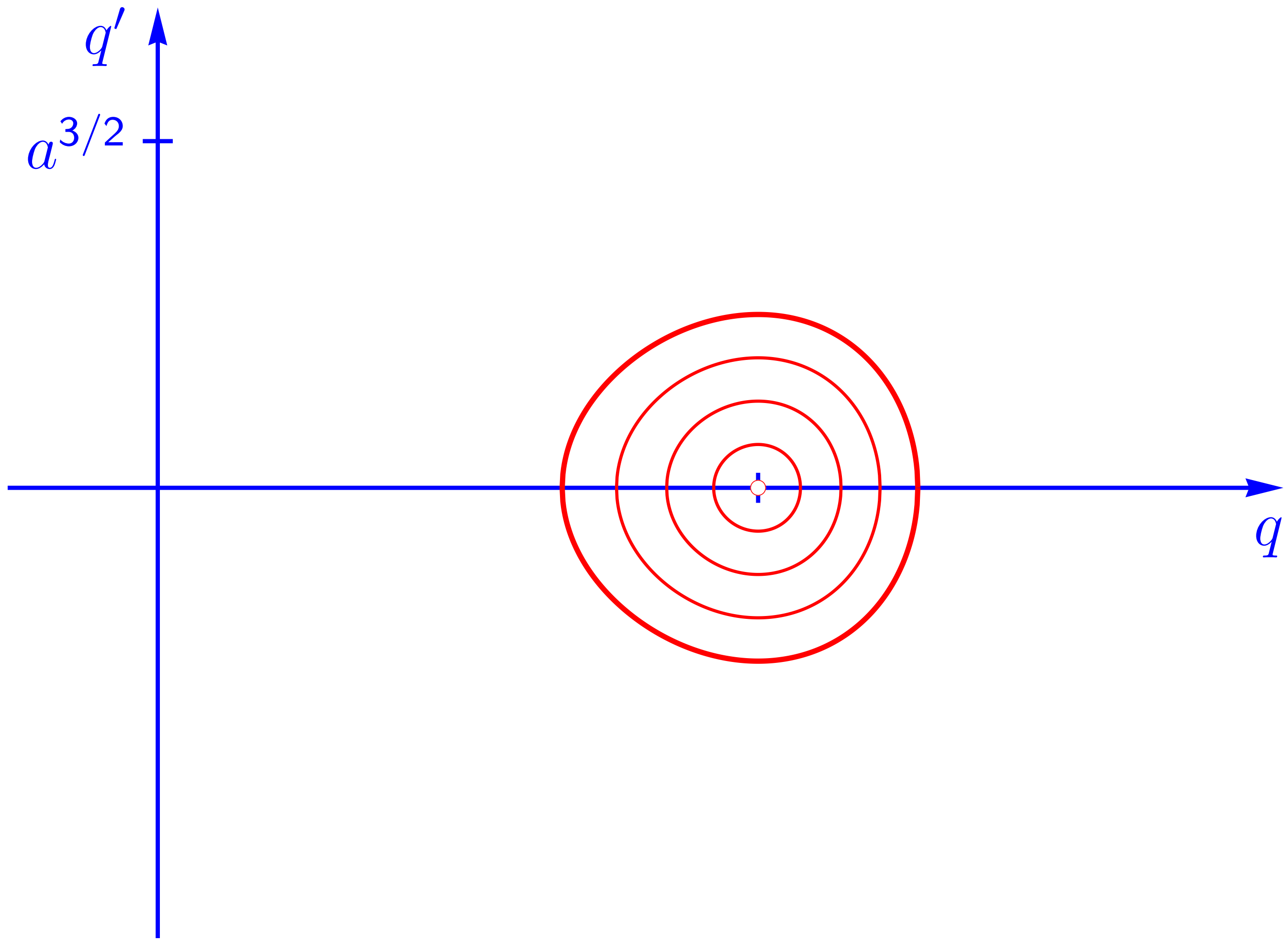

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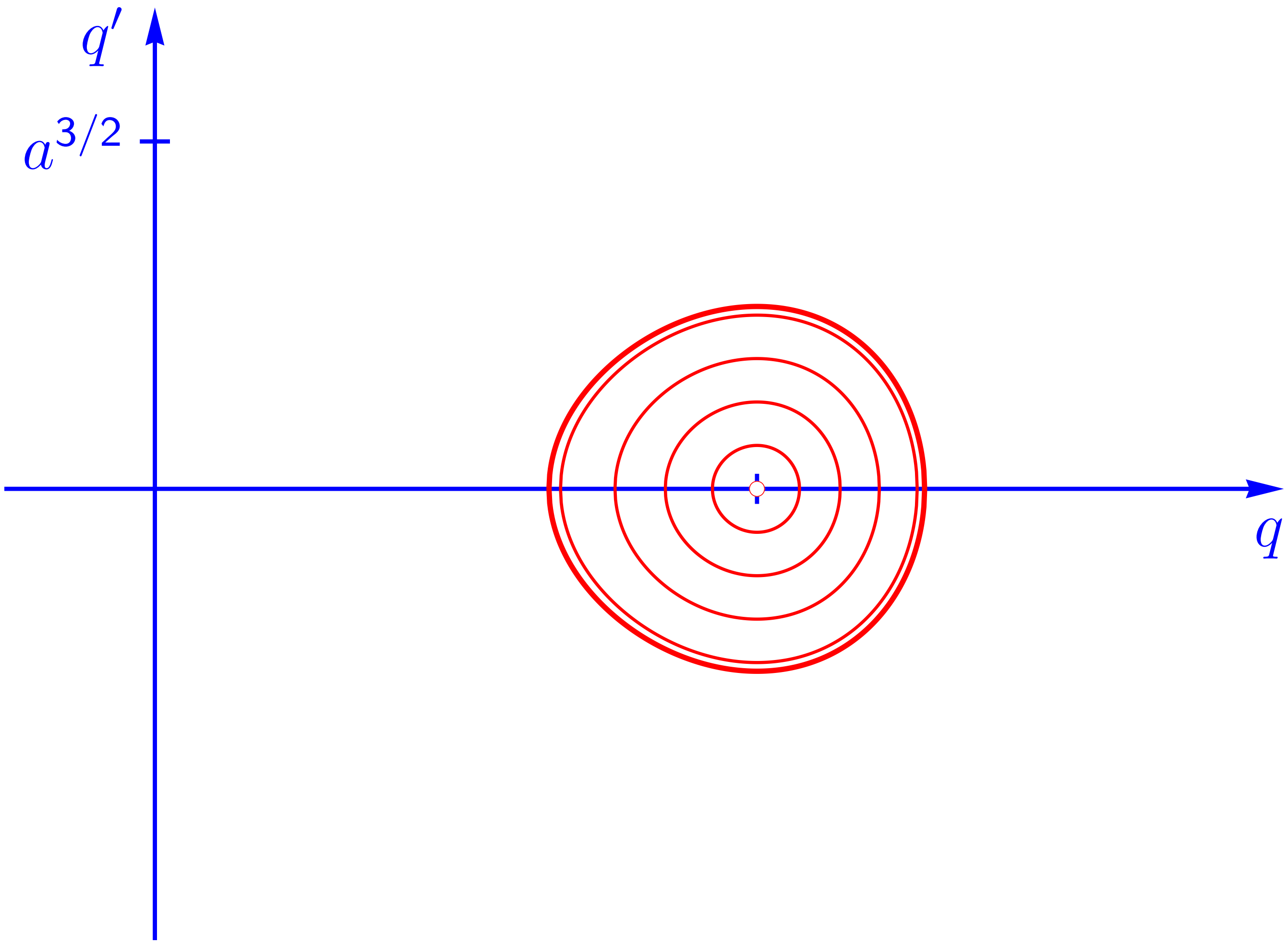

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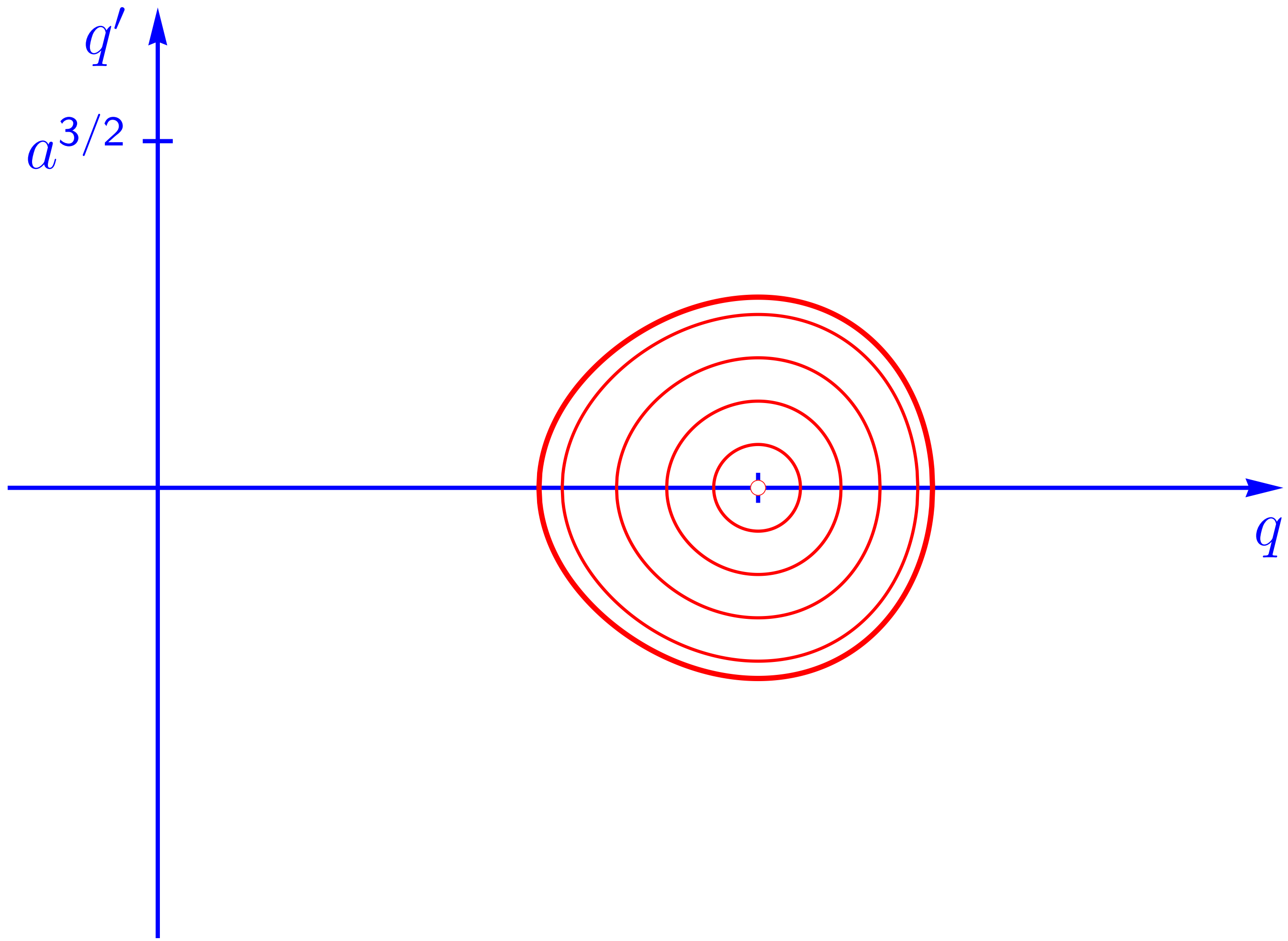

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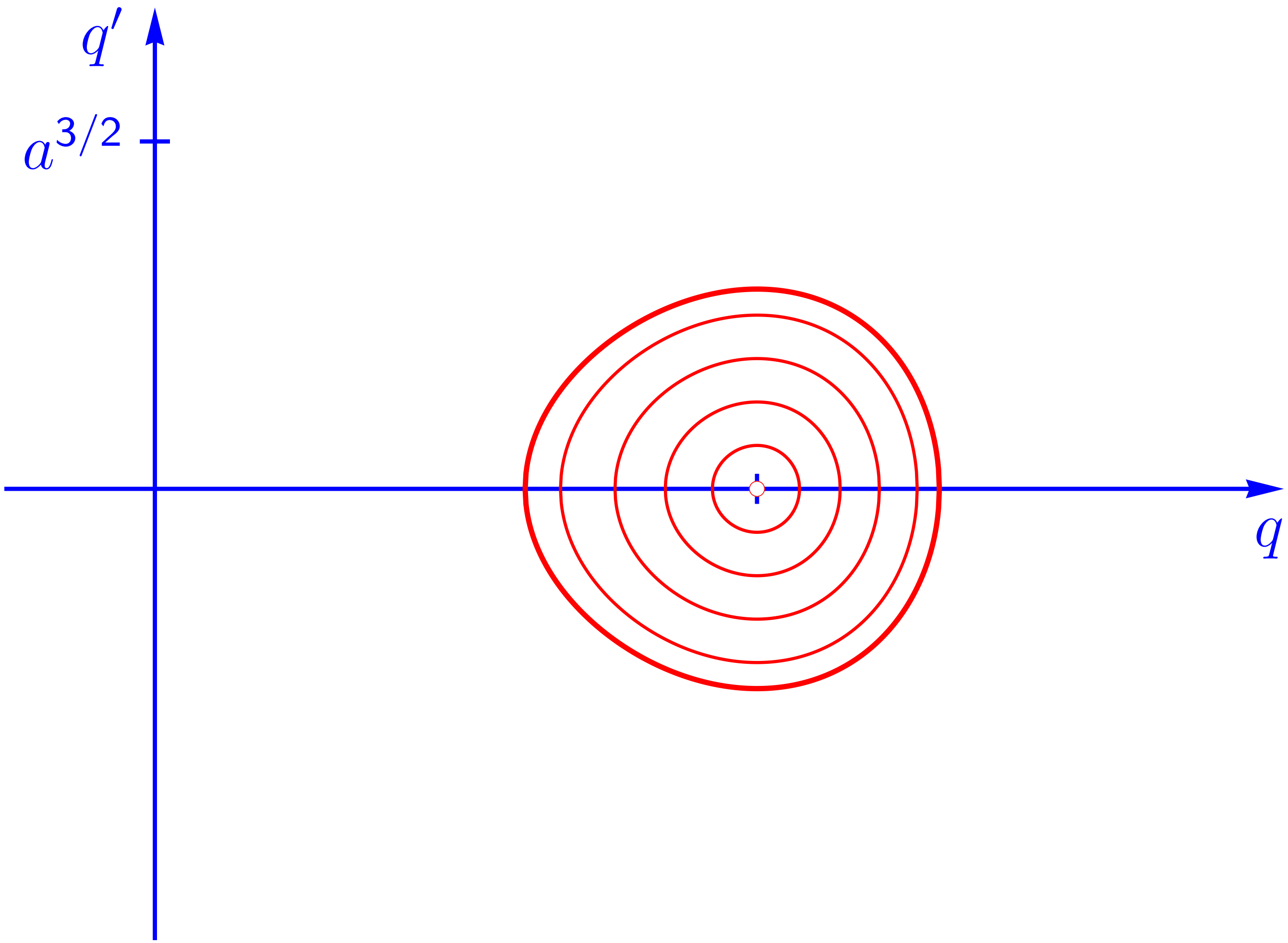

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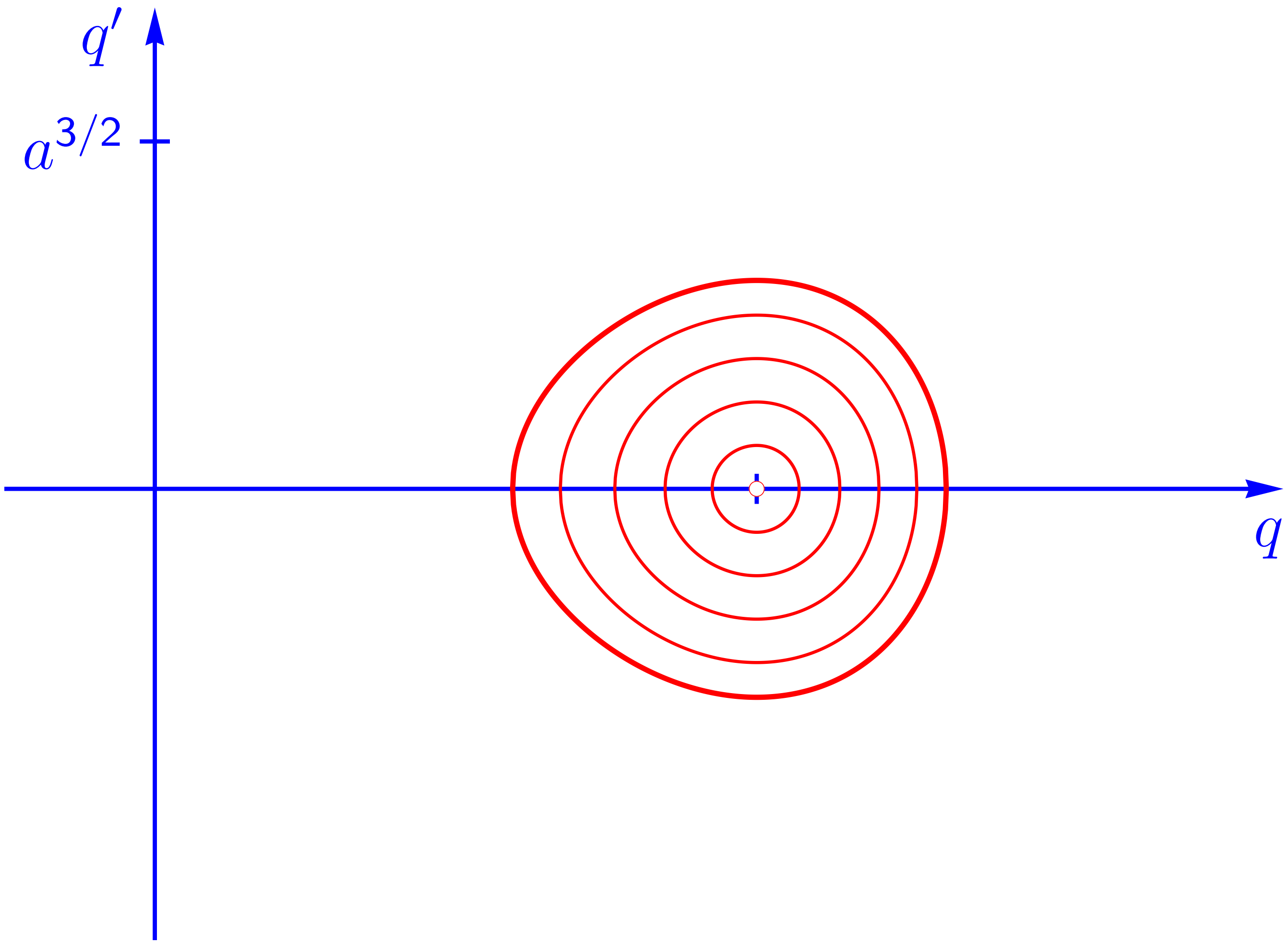

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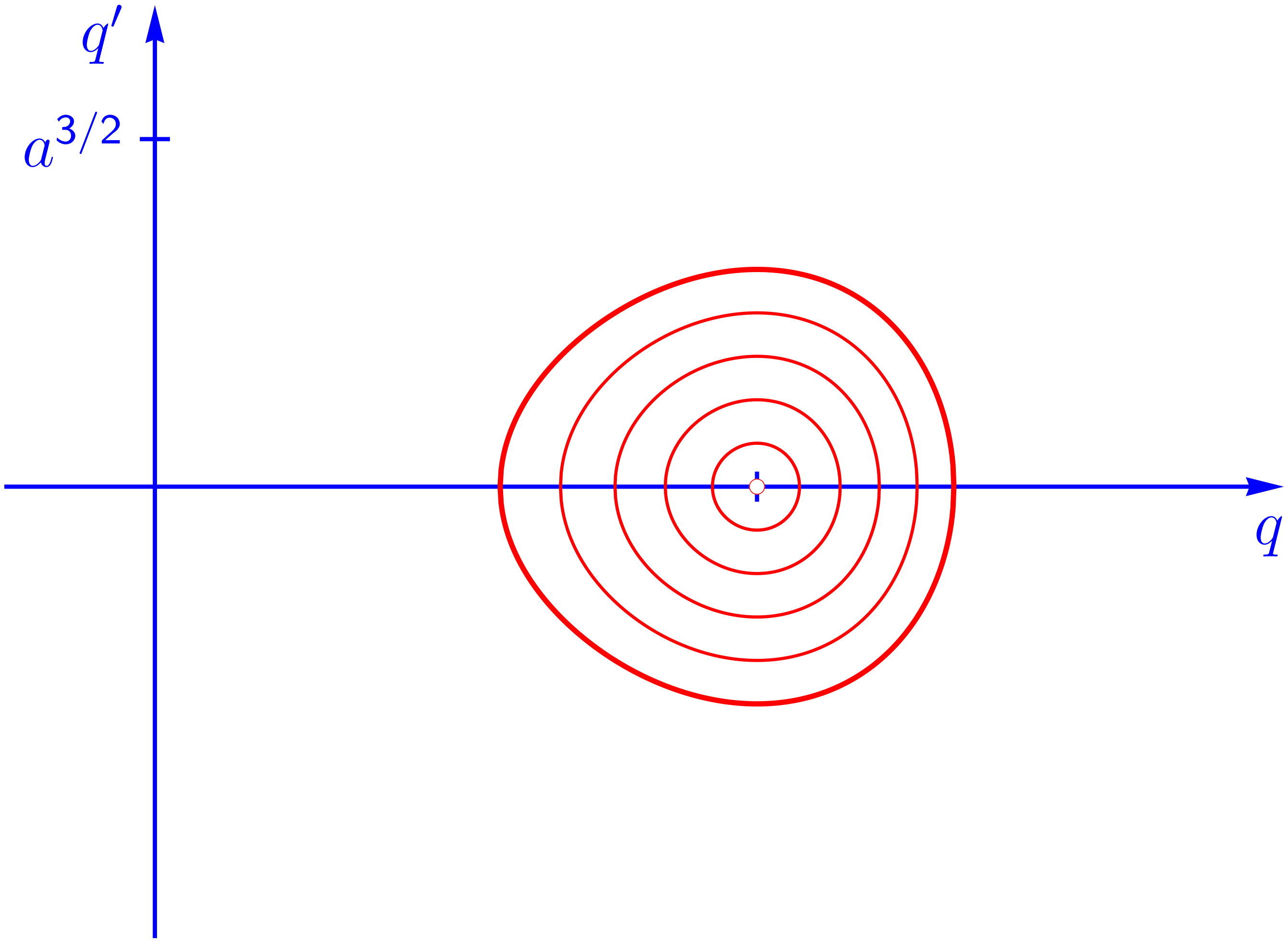

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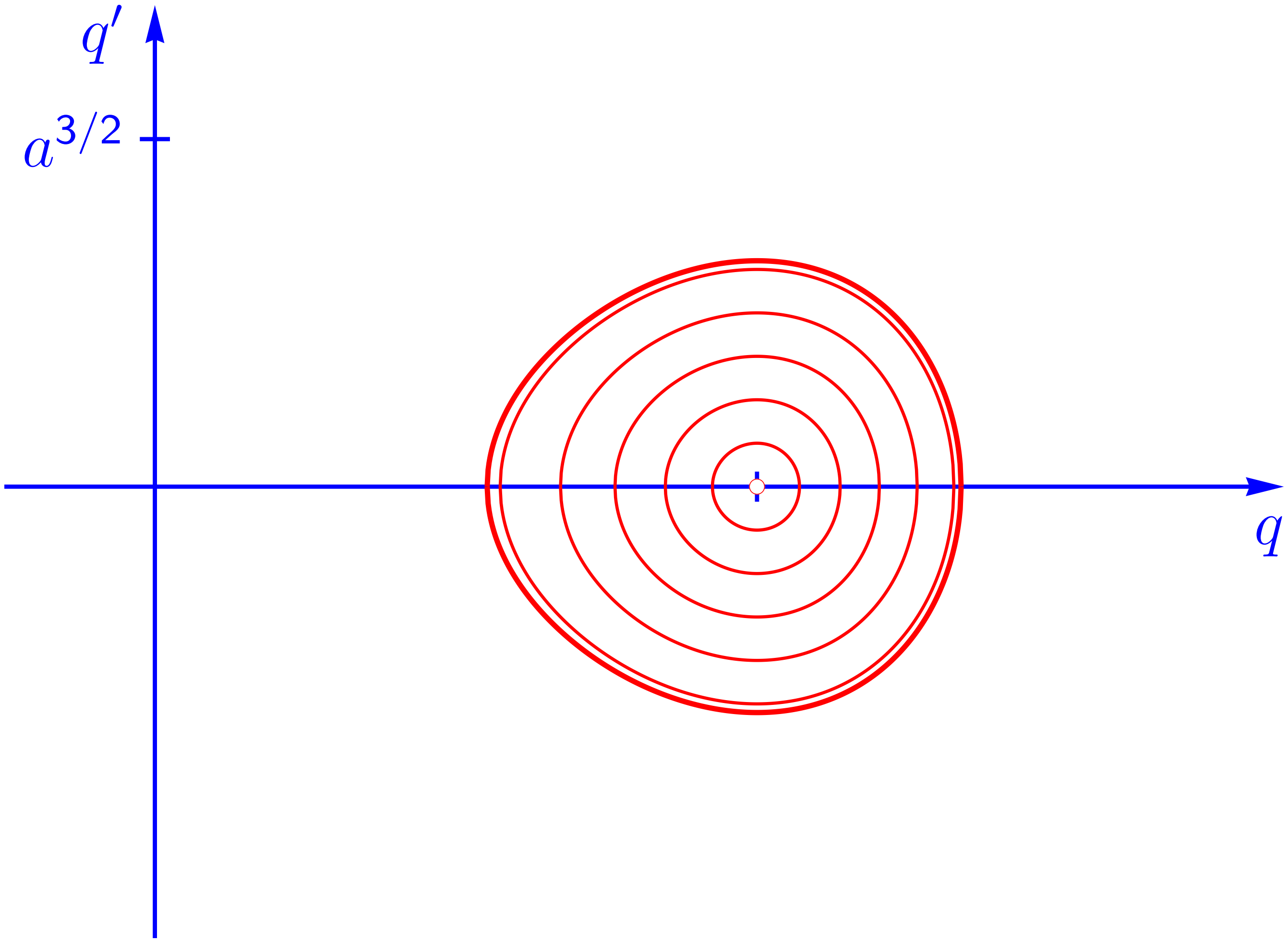

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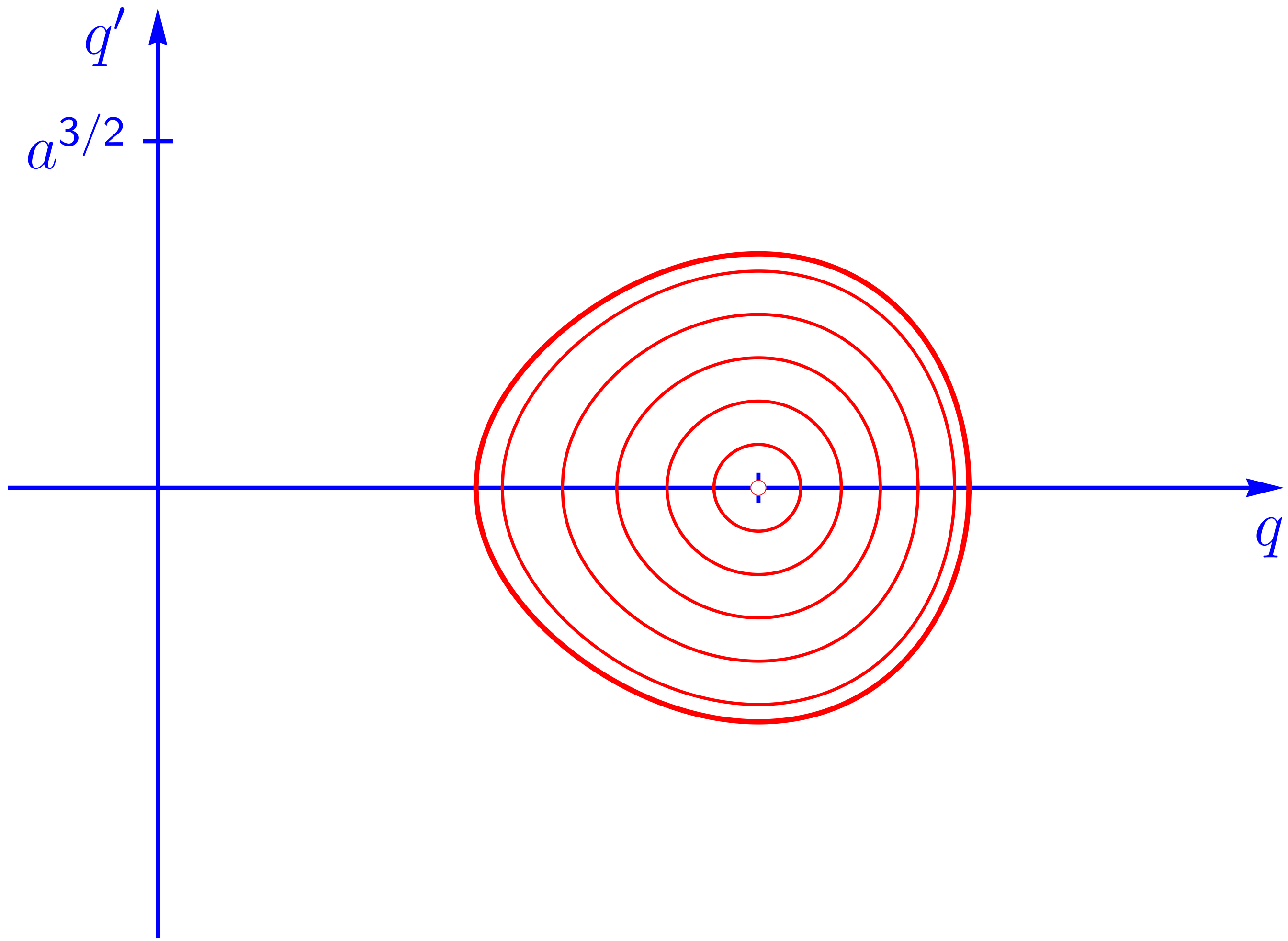

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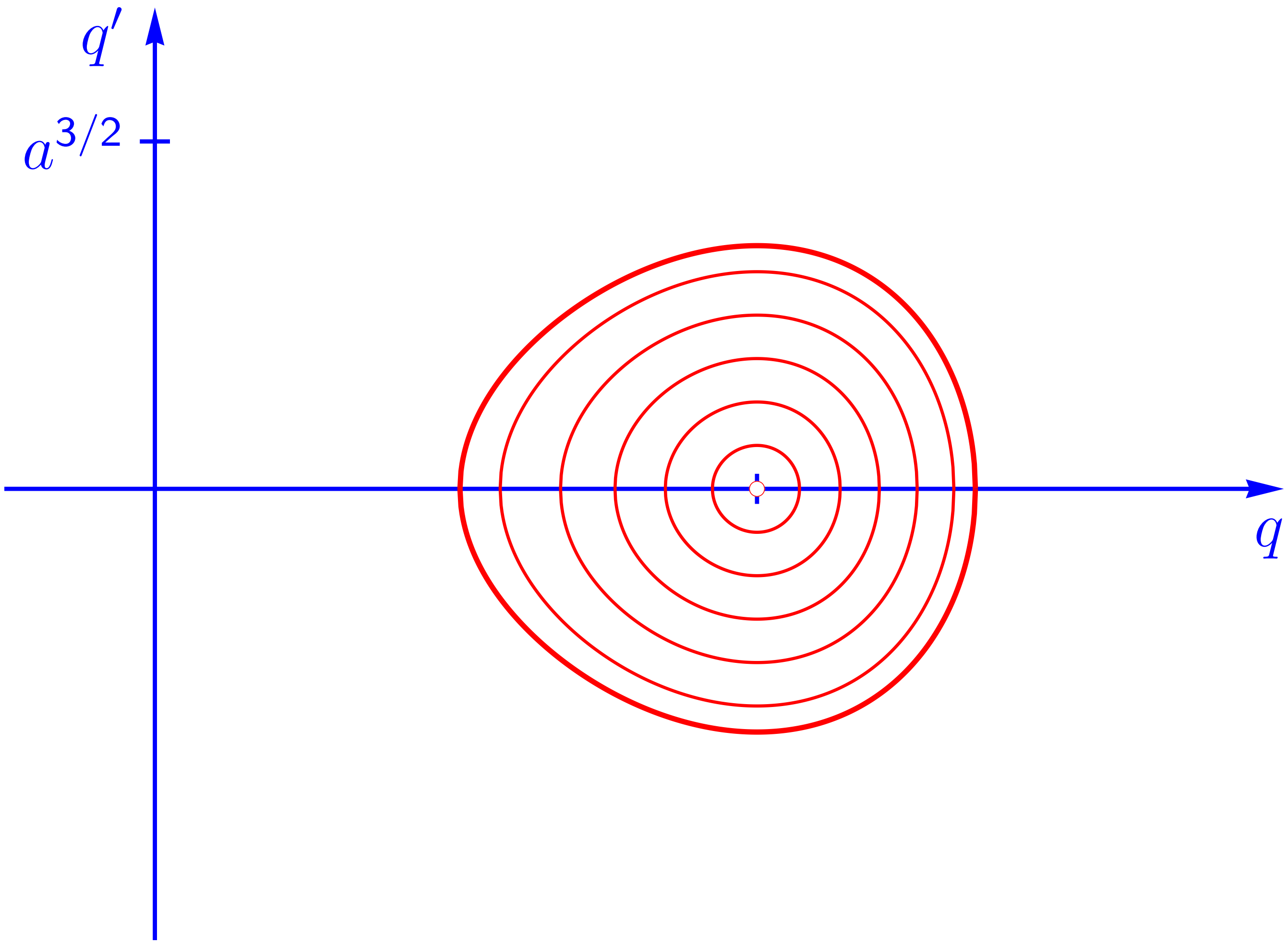

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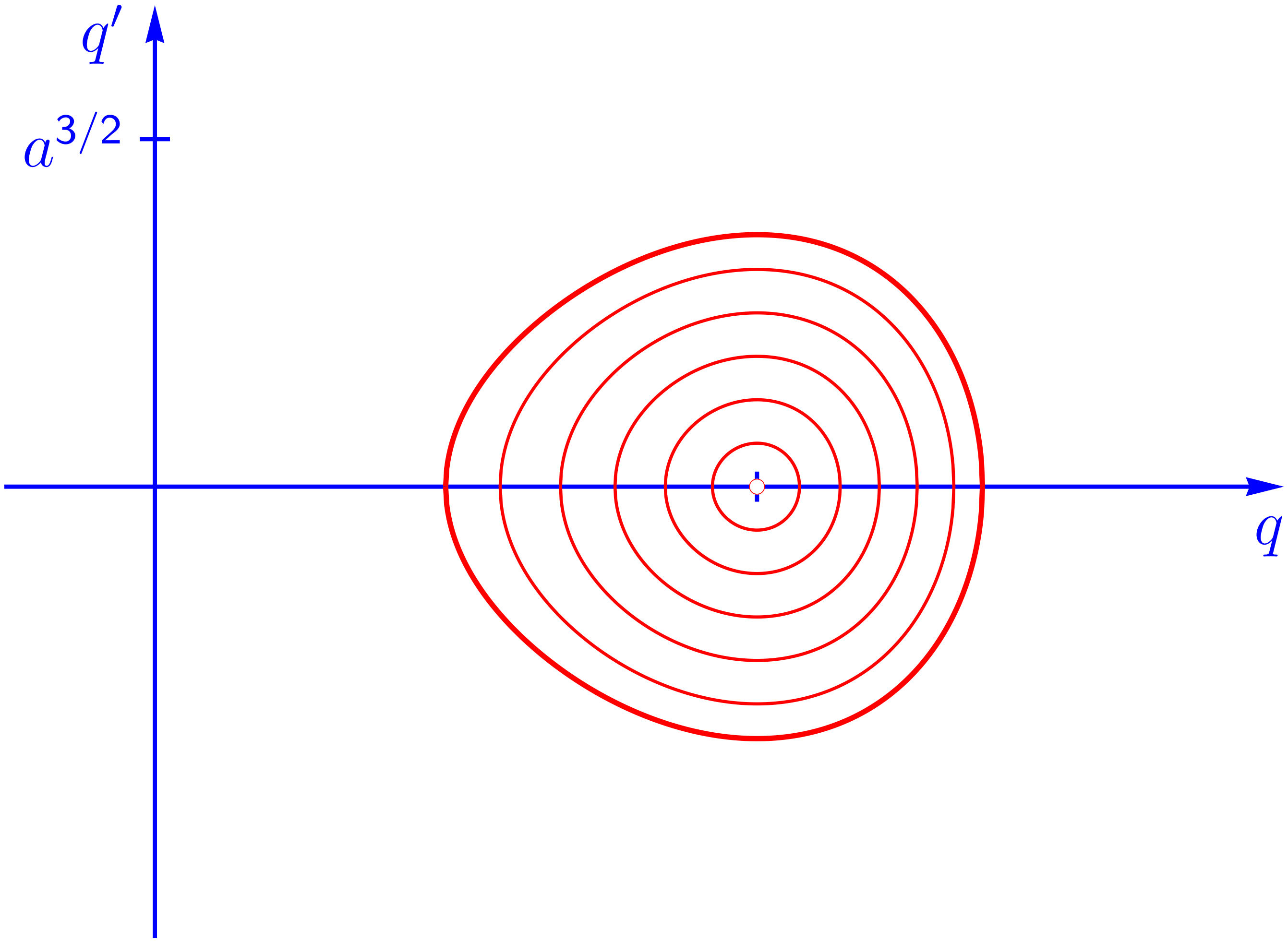

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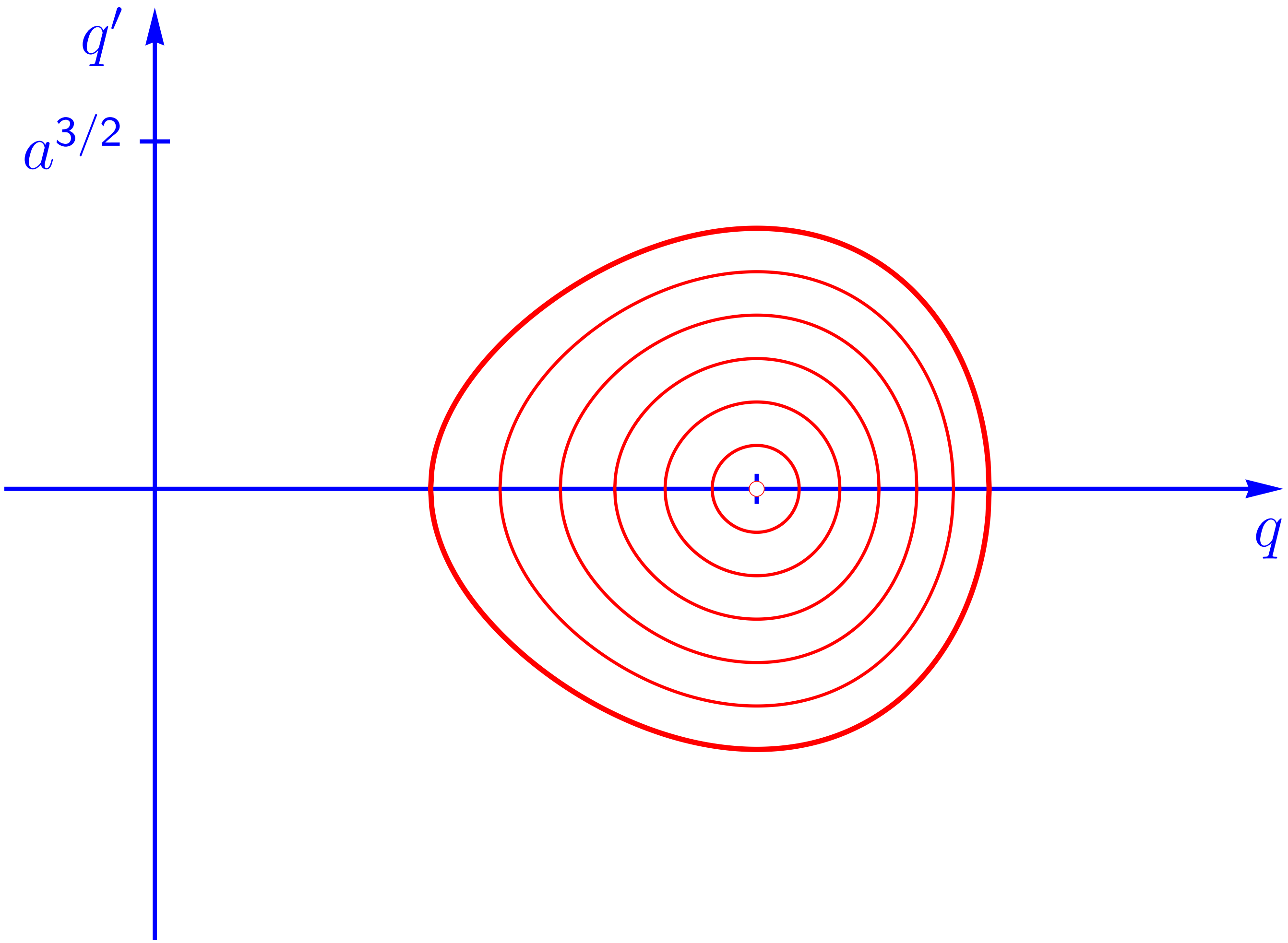

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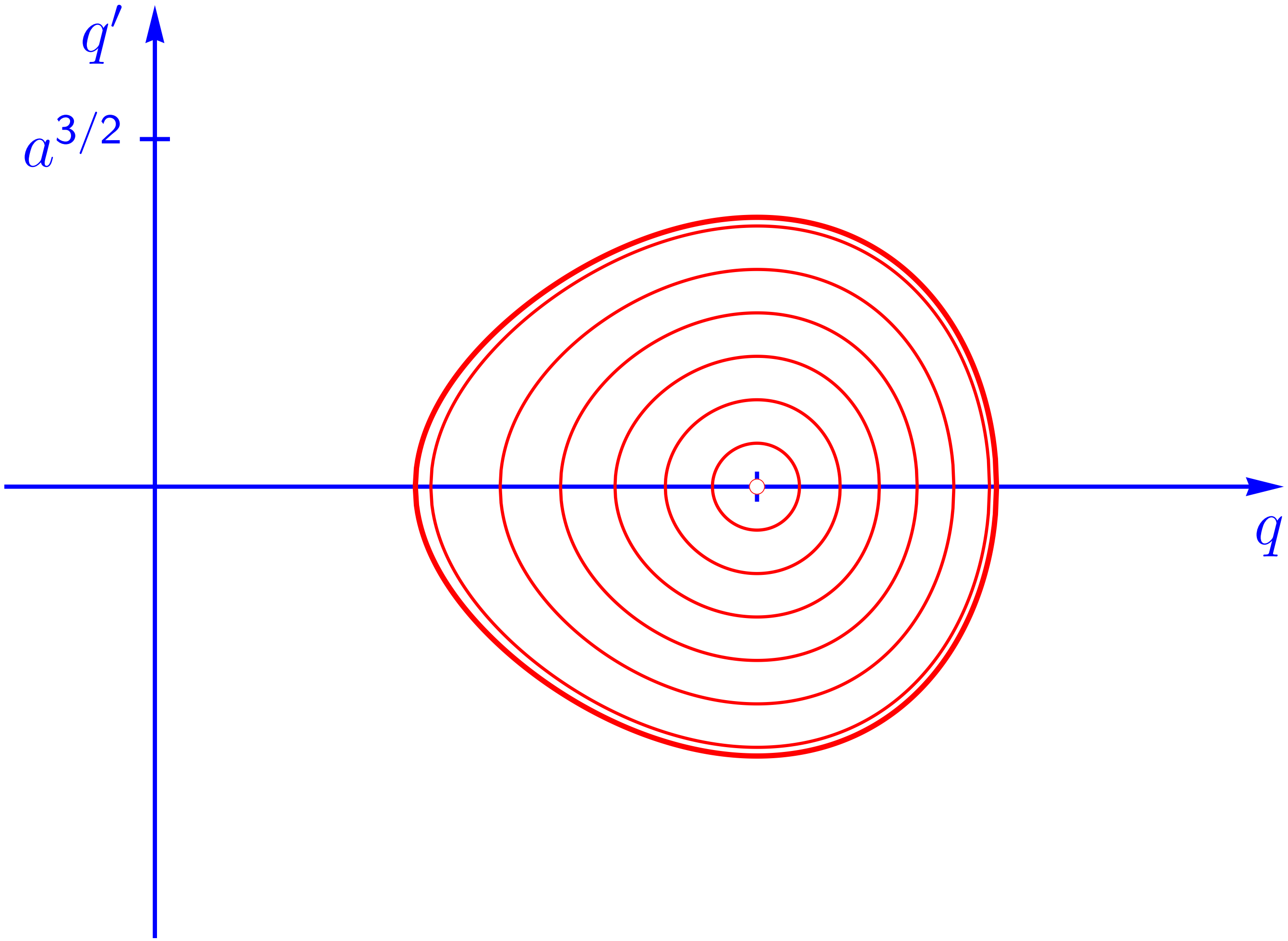

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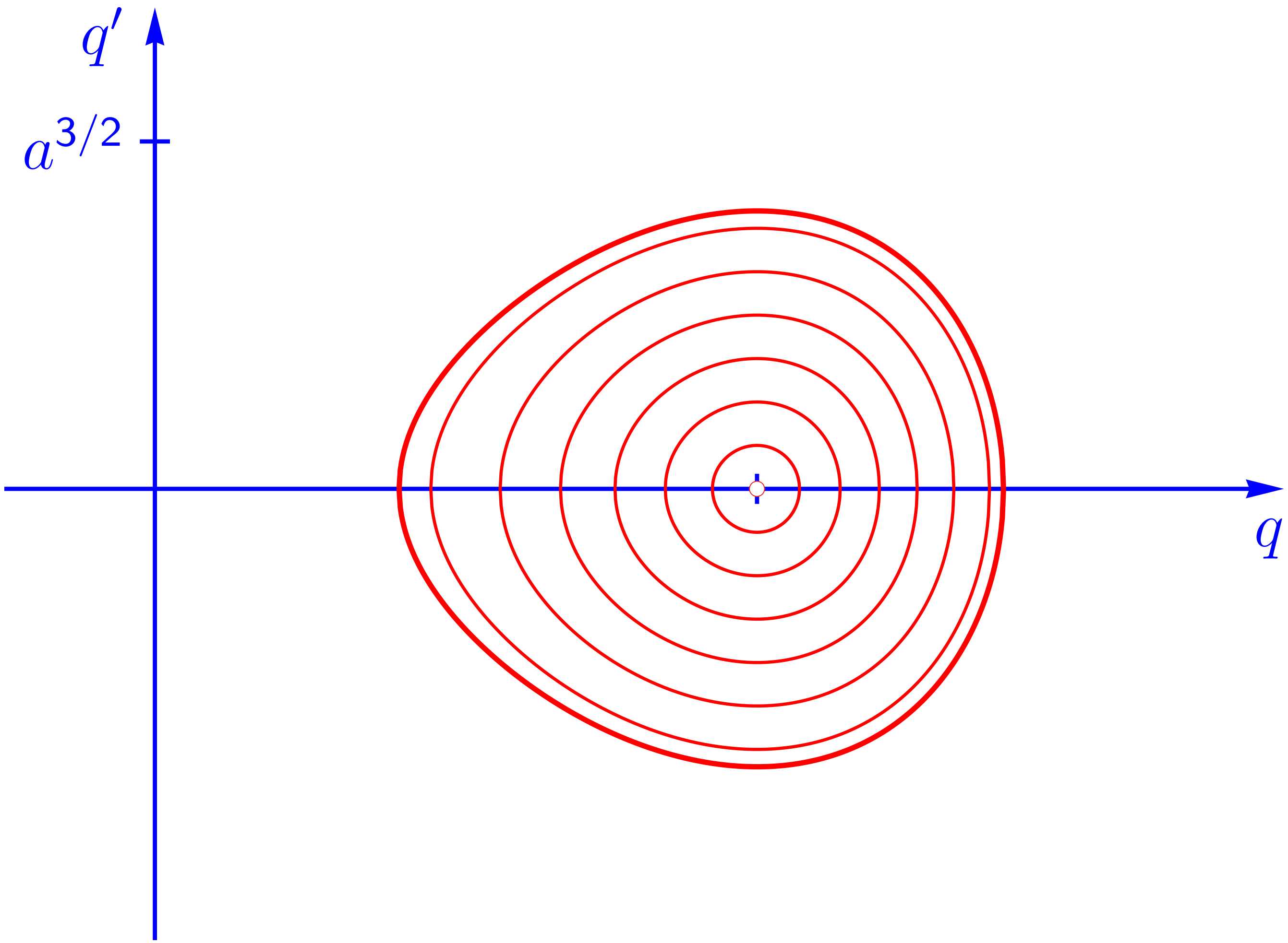

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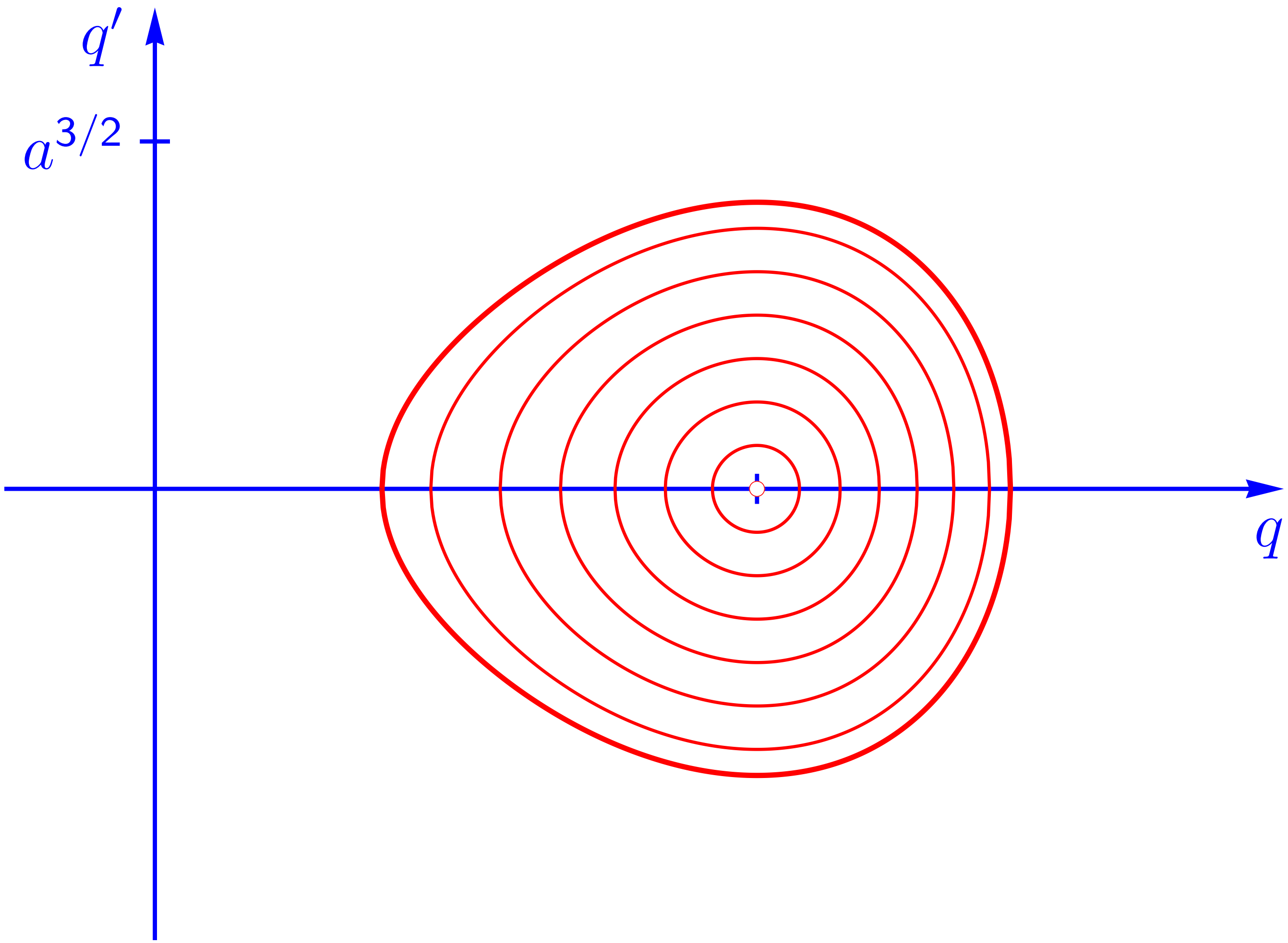

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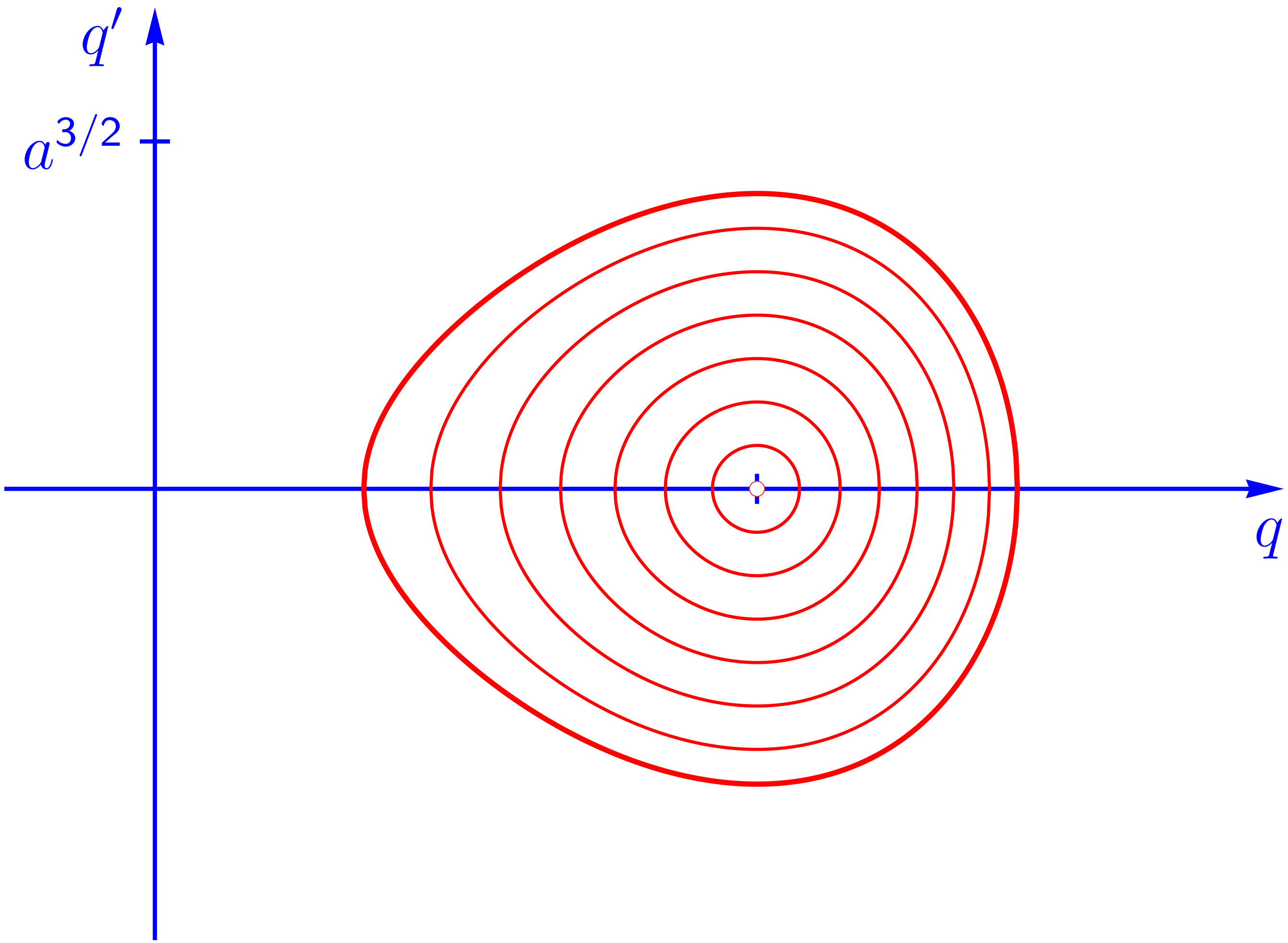

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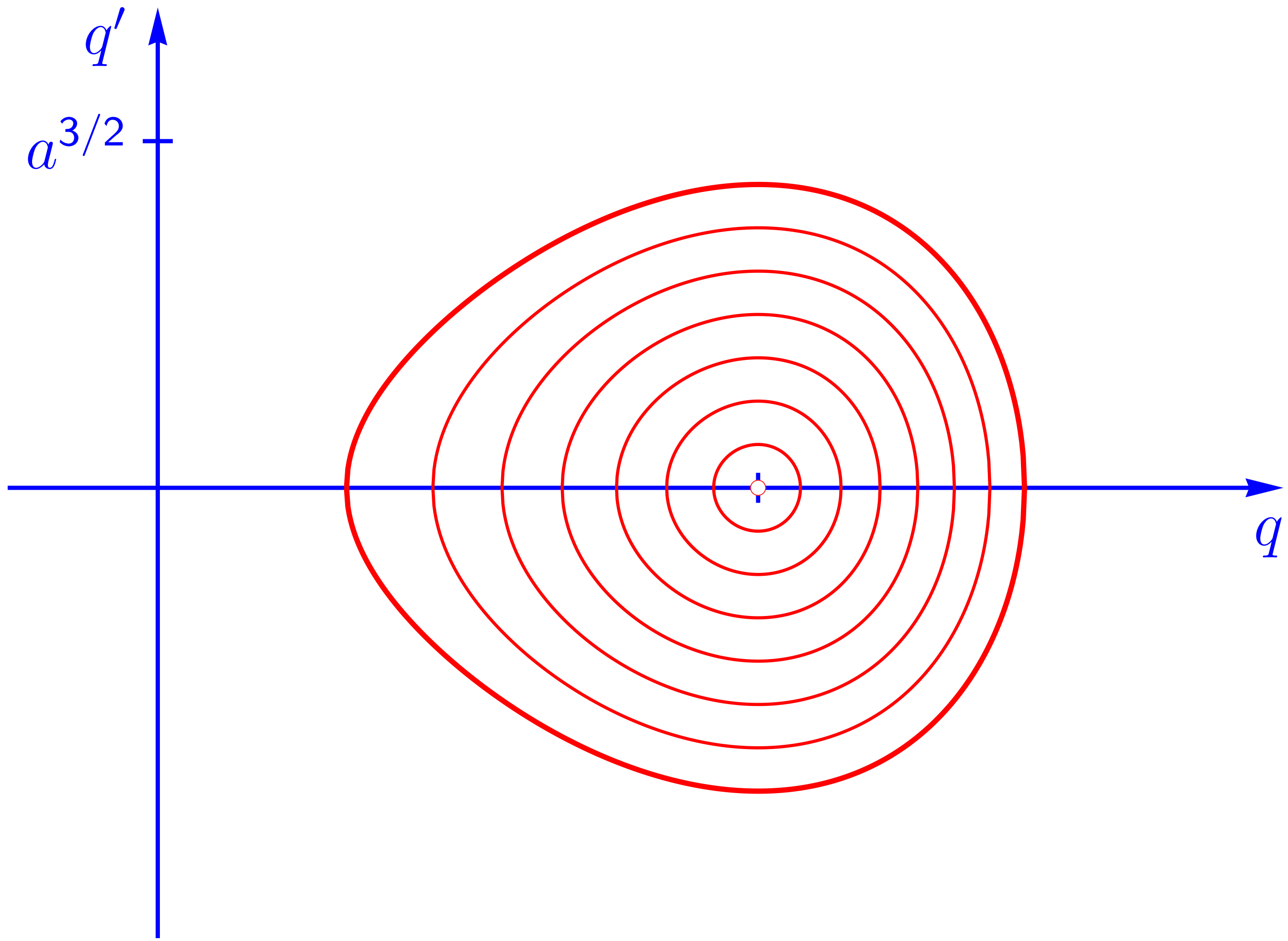

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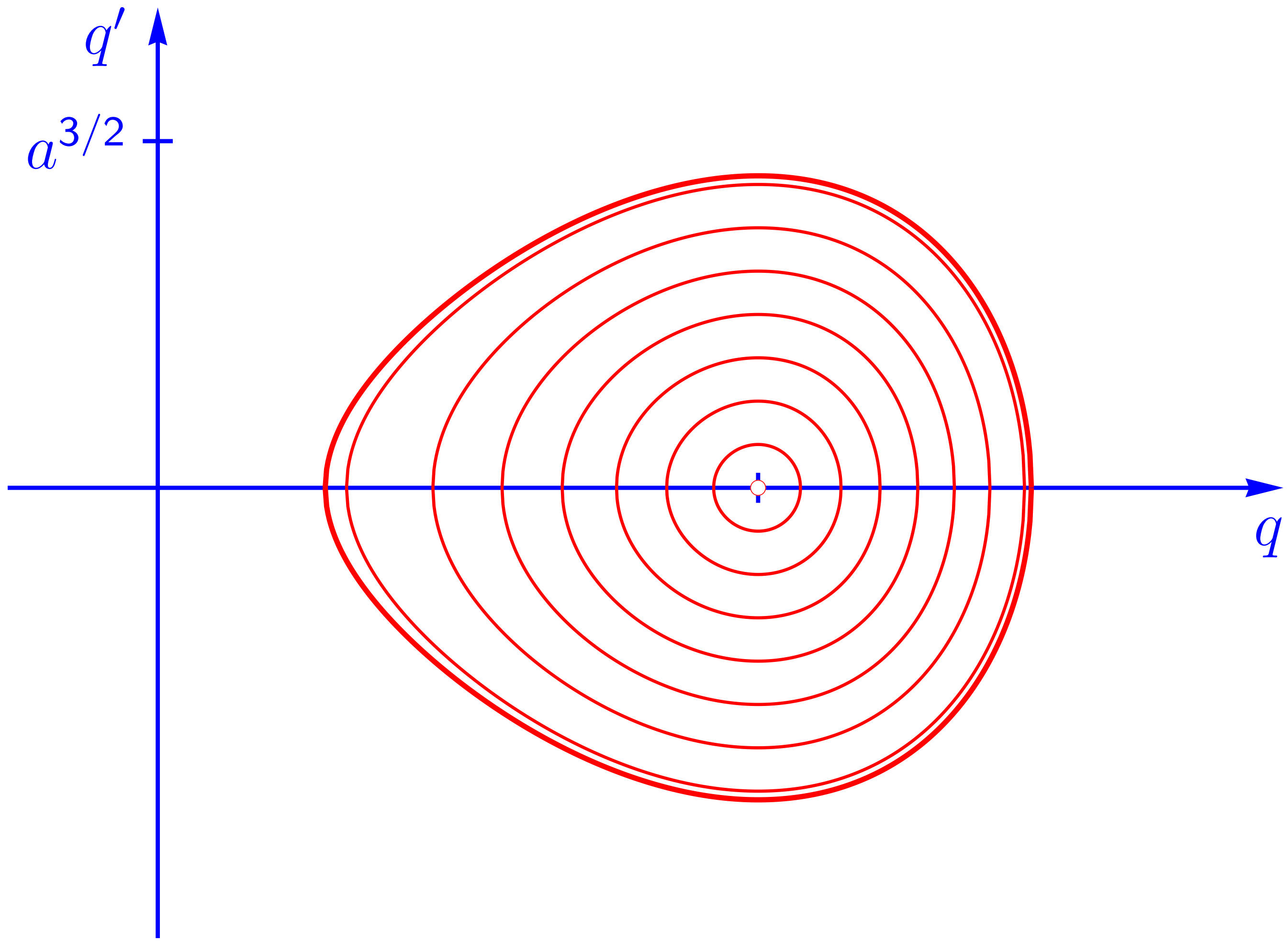

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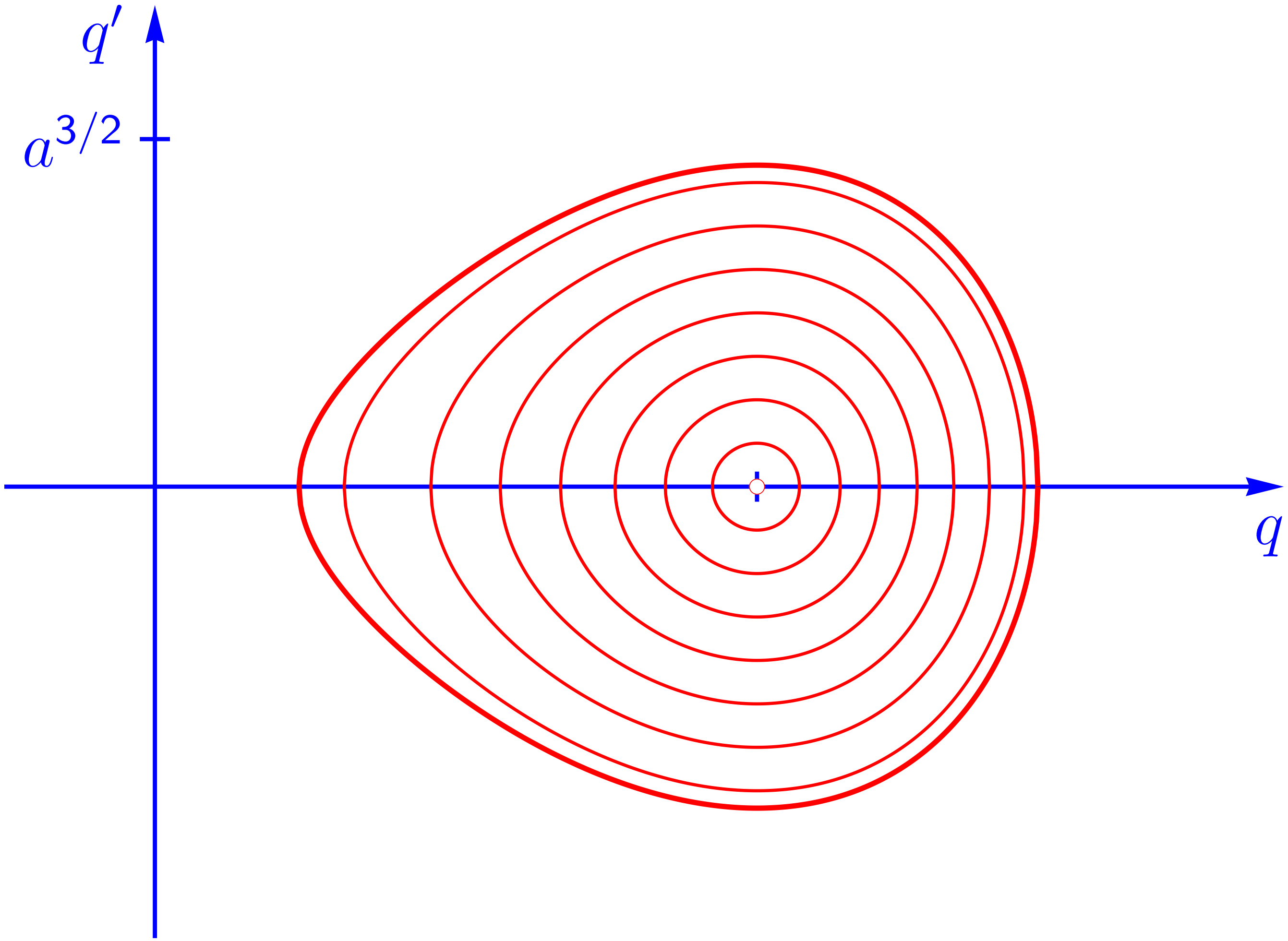

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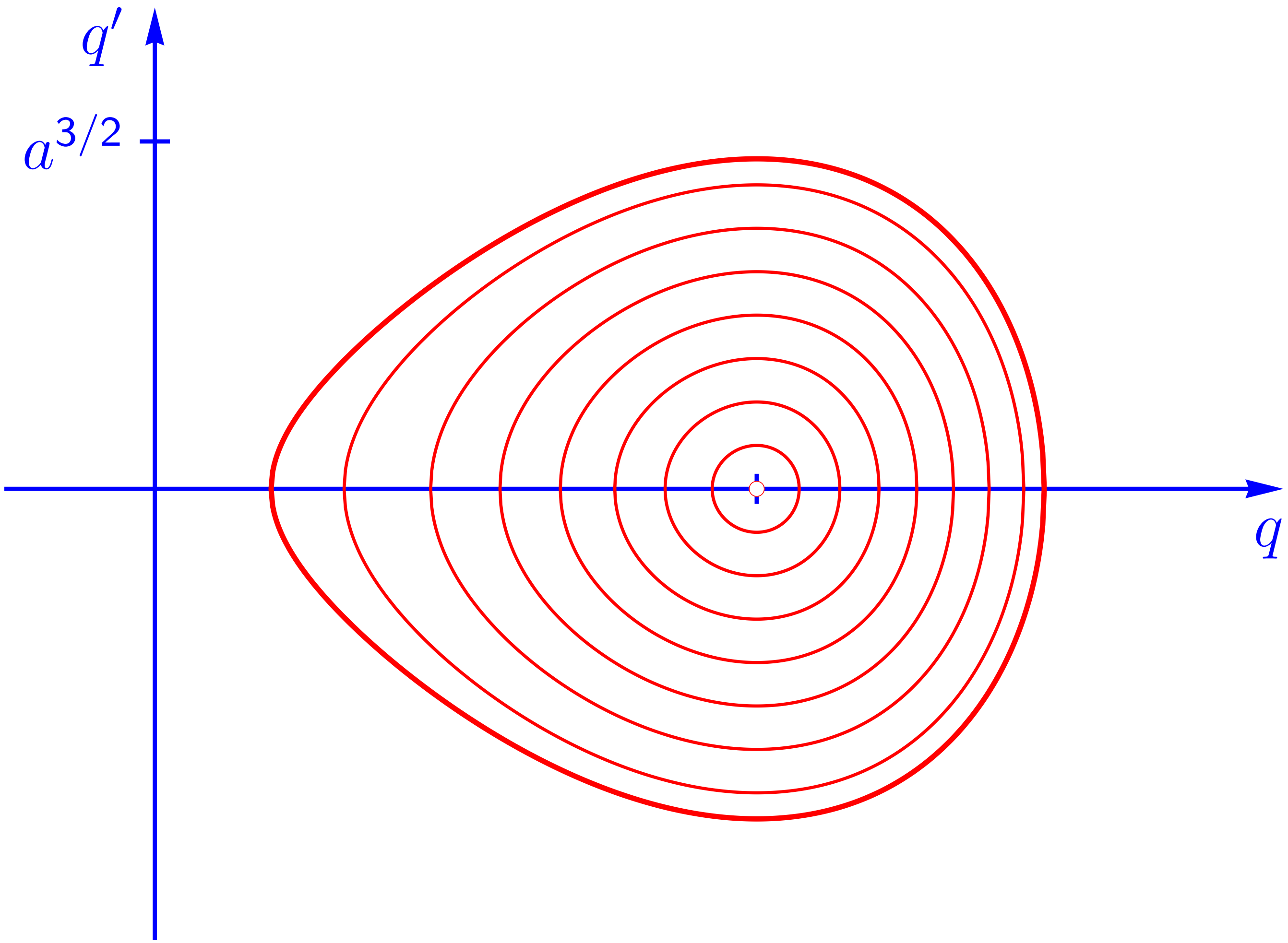

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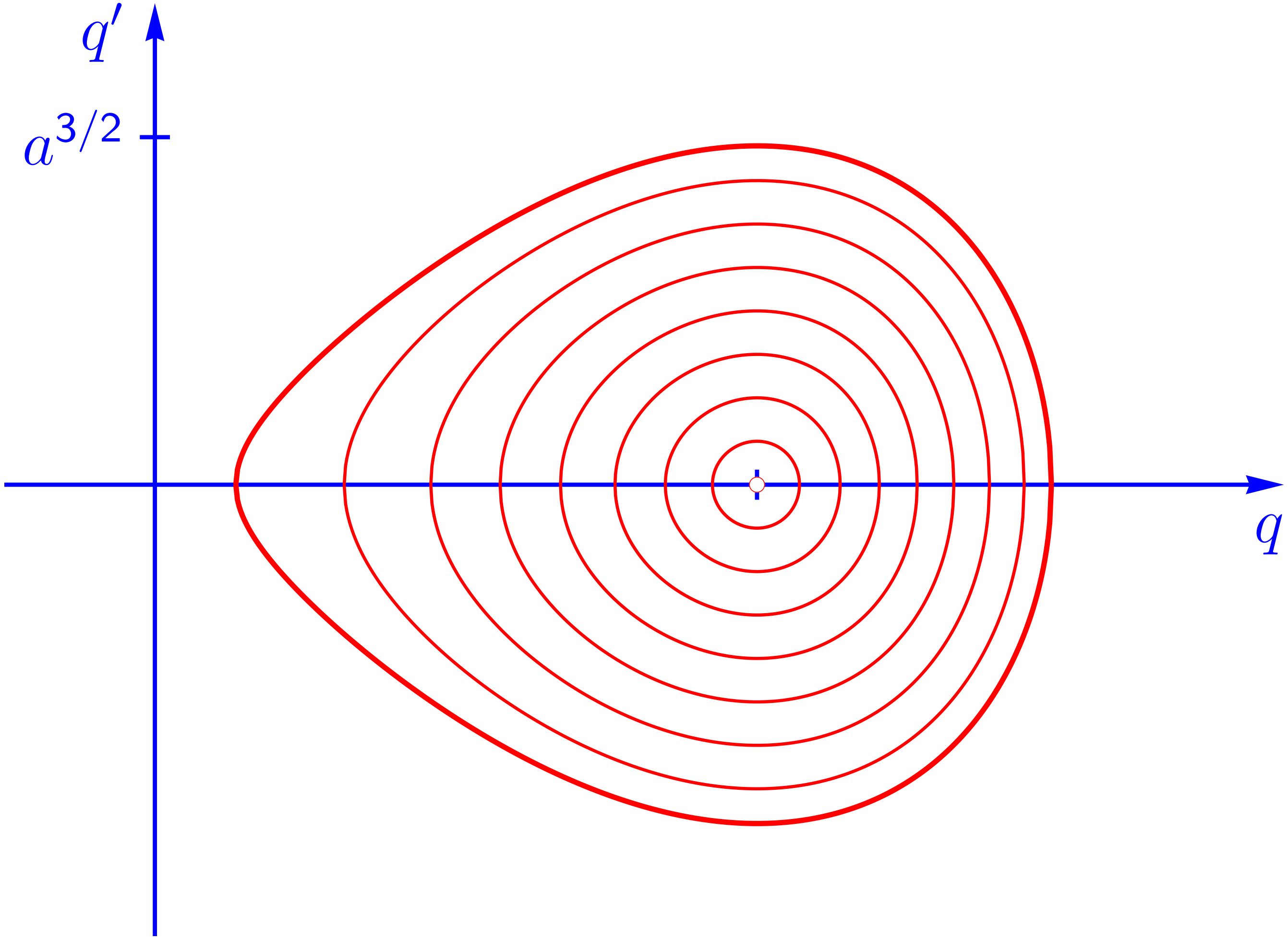

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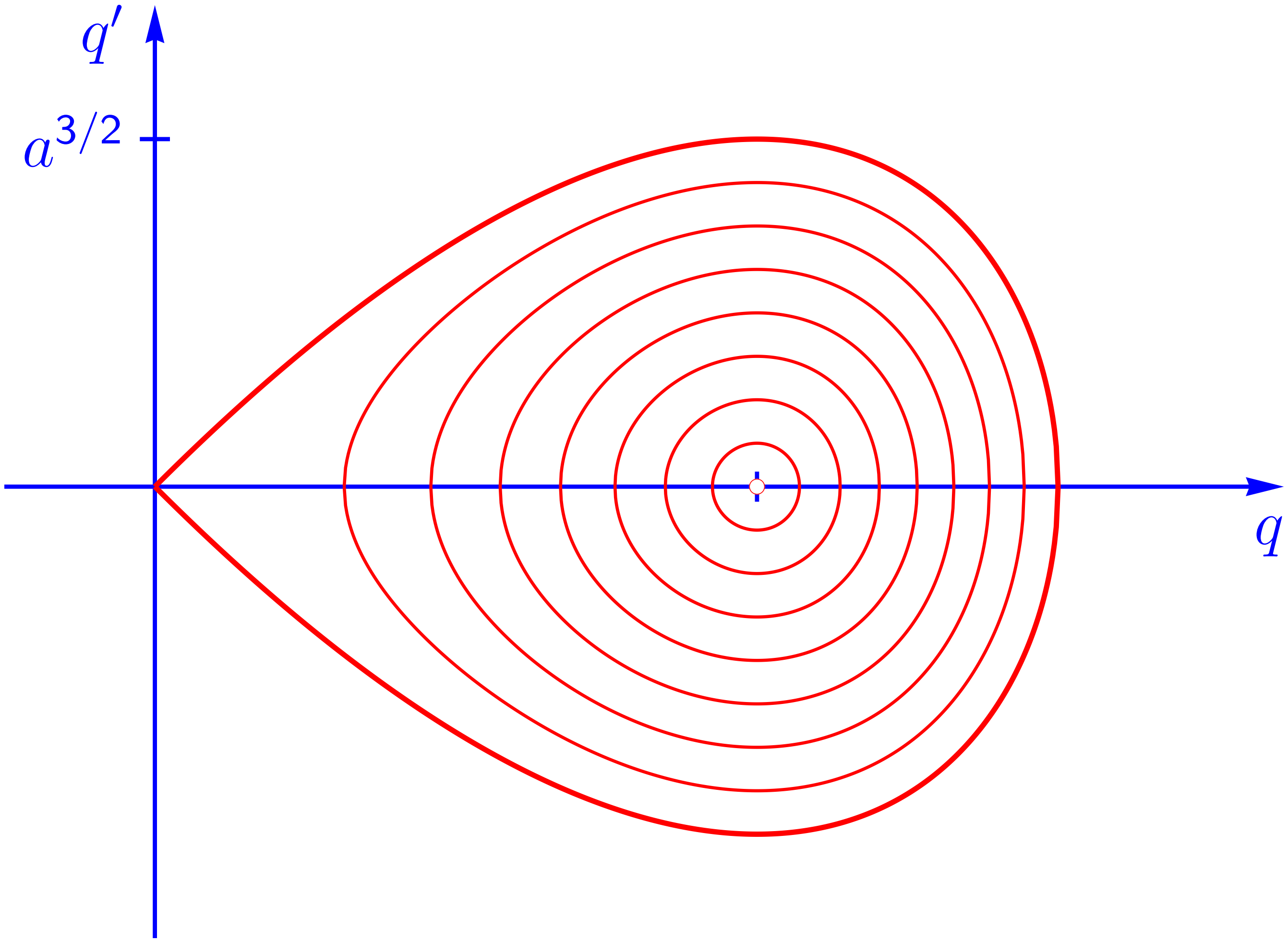

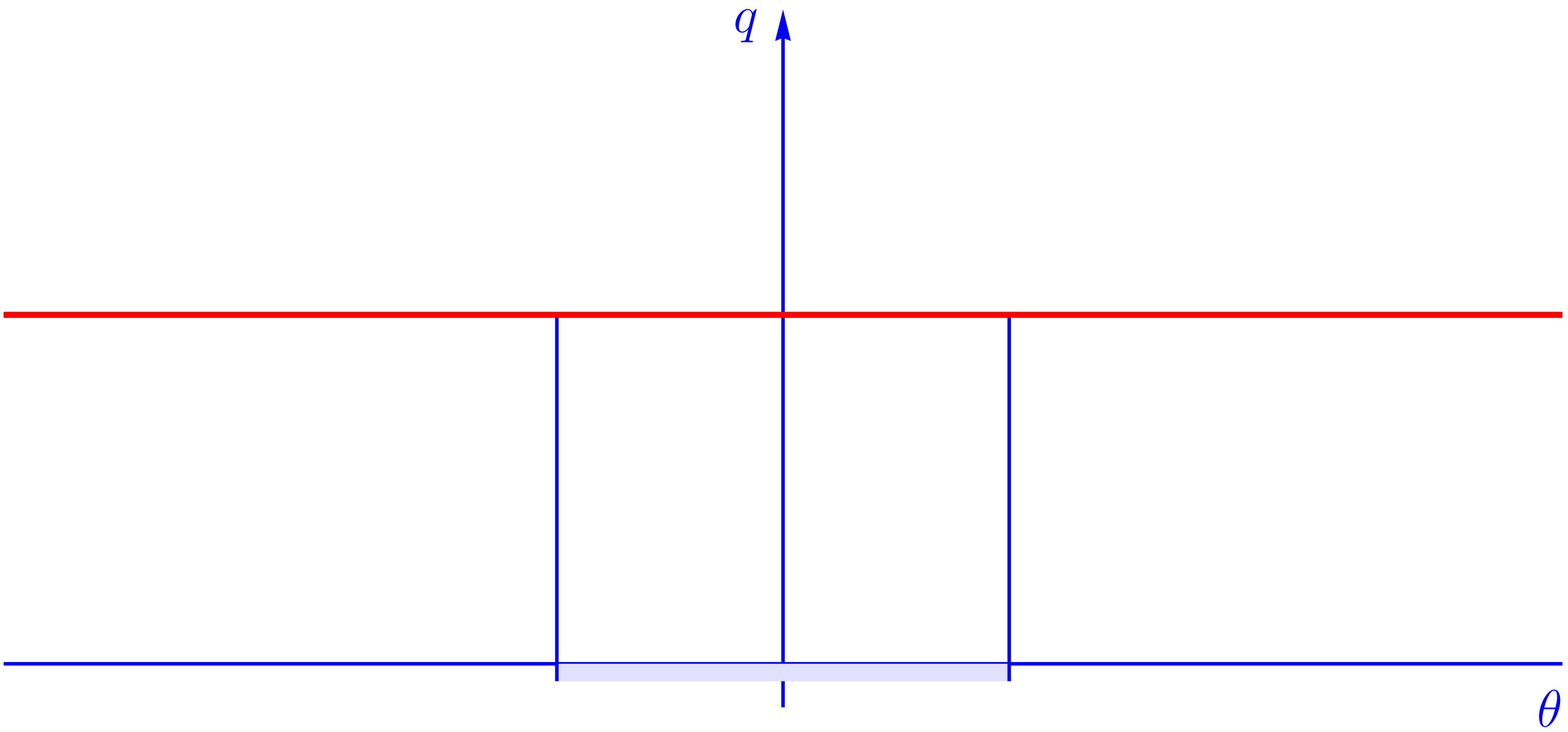
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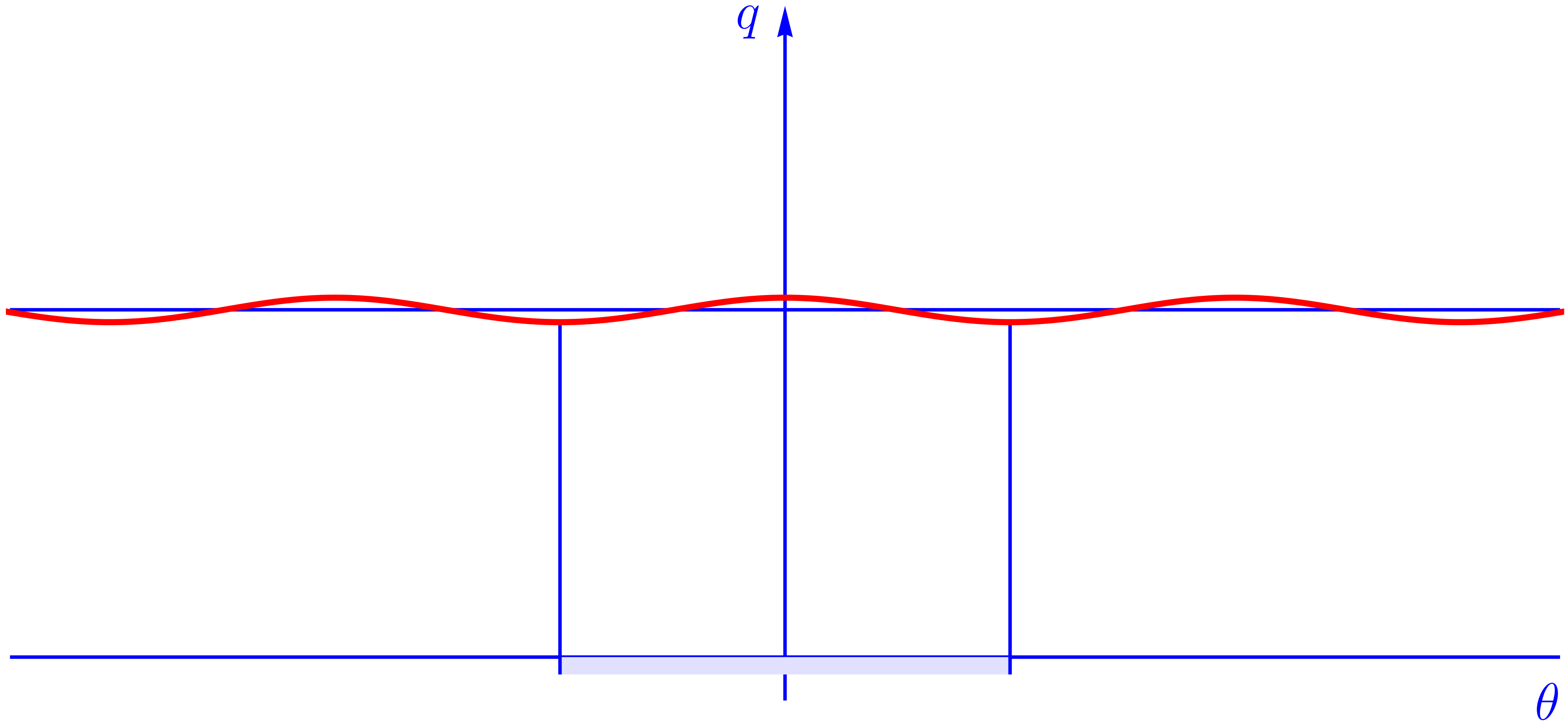
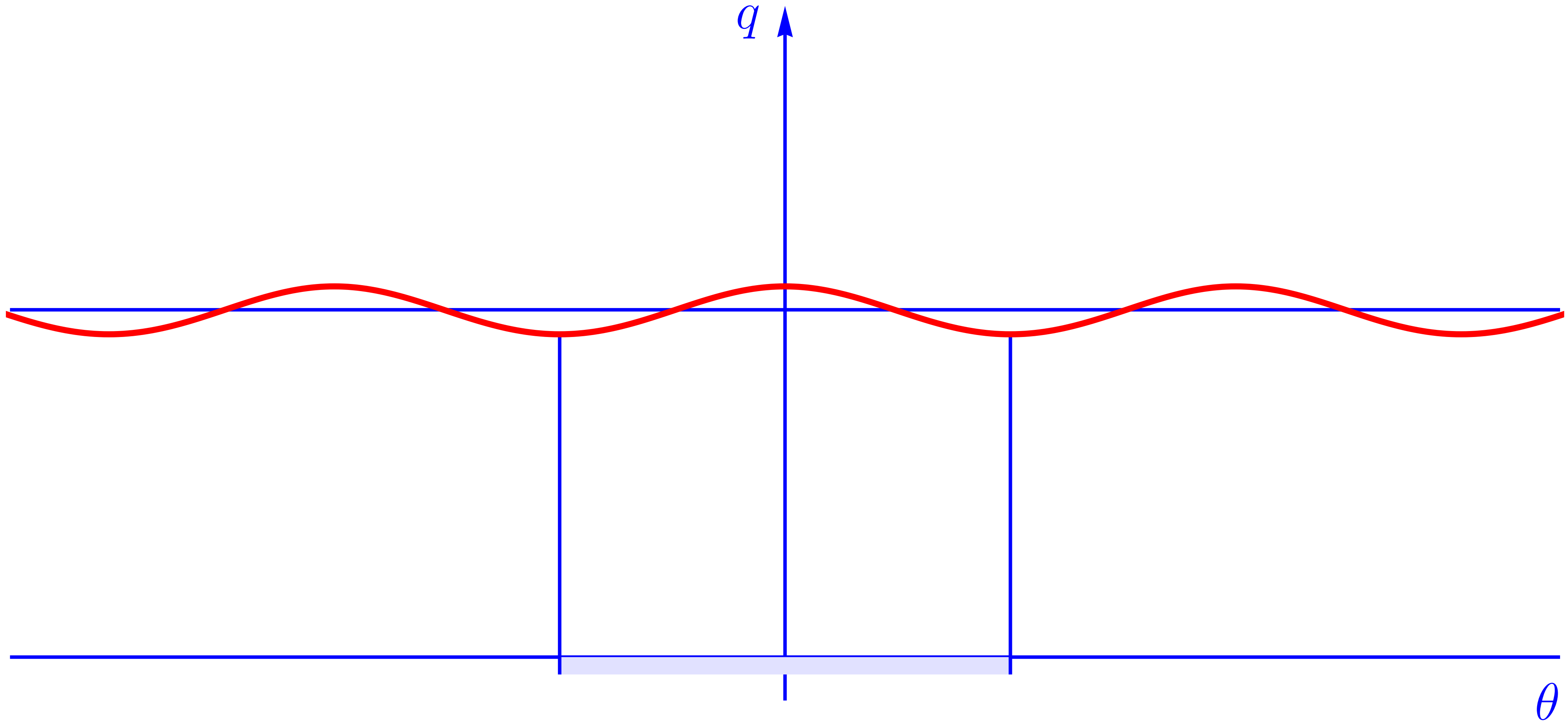
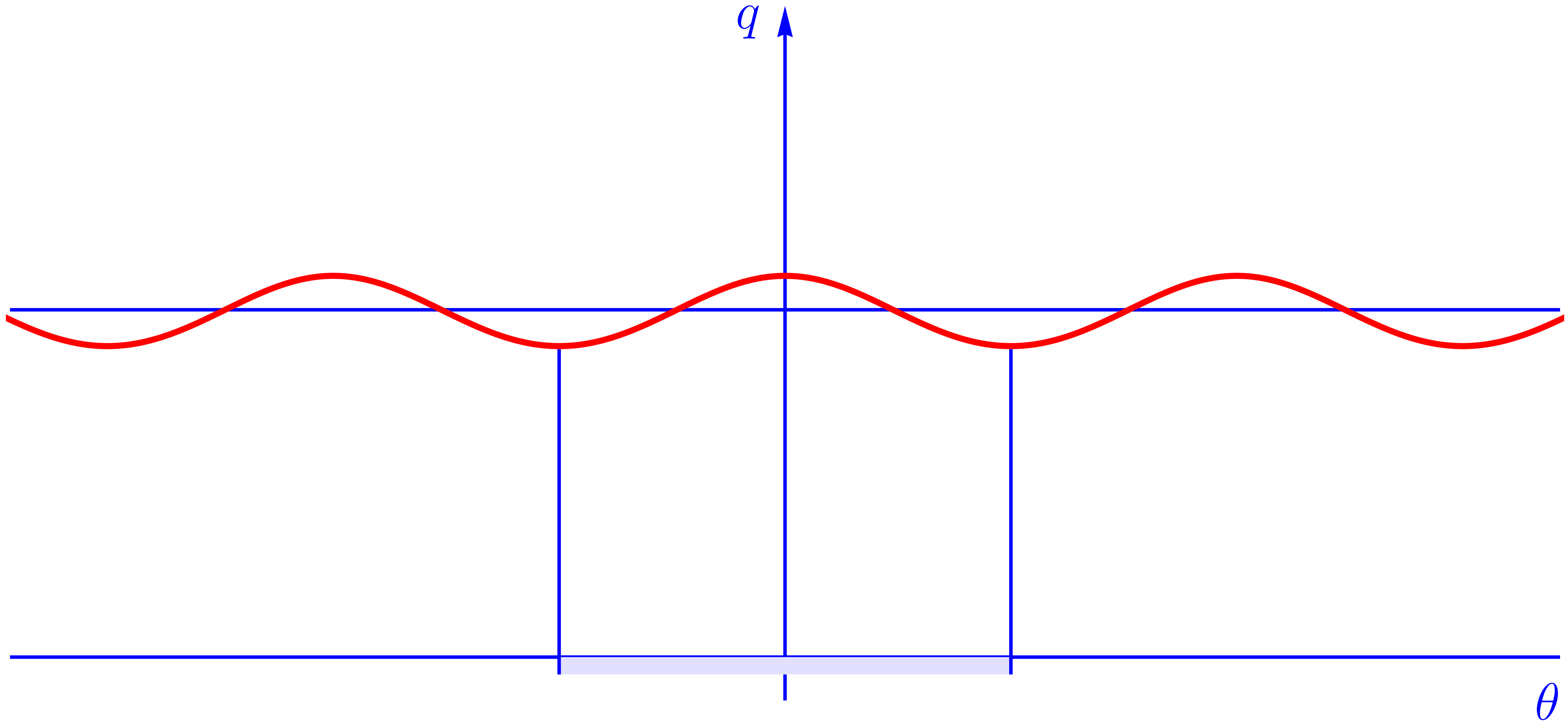
Cnoidal Waves



Cnoidal Waves

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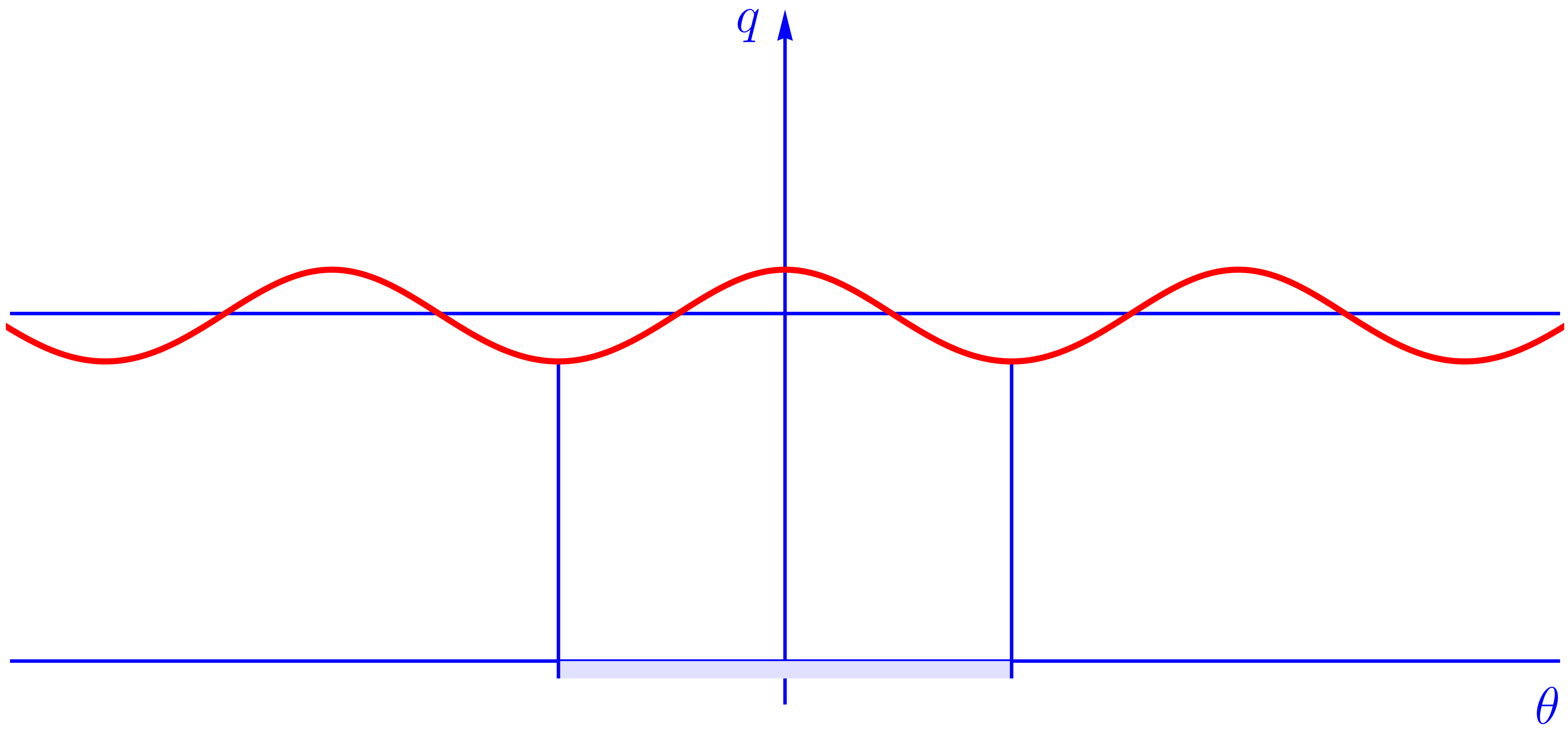
Cnoidal Waves



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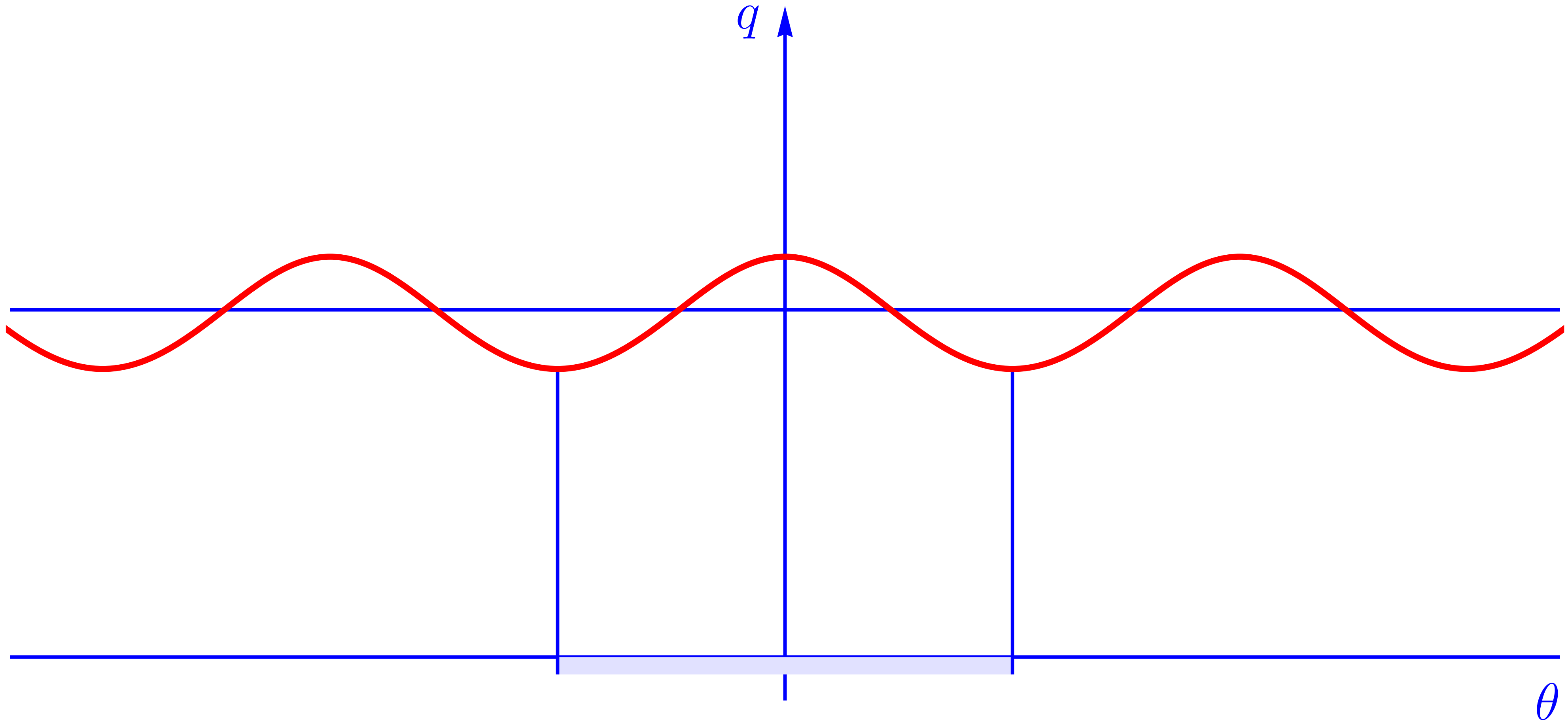

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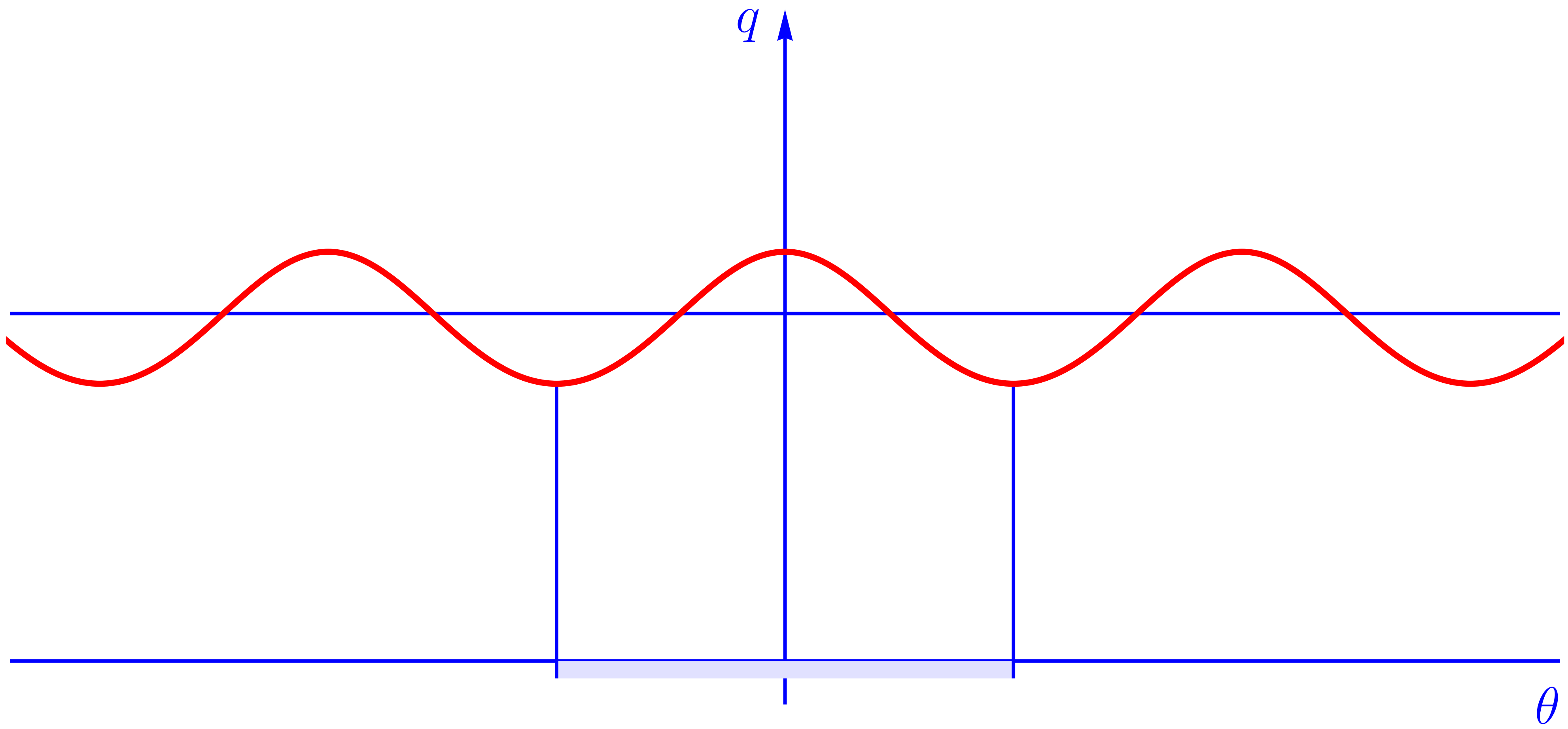

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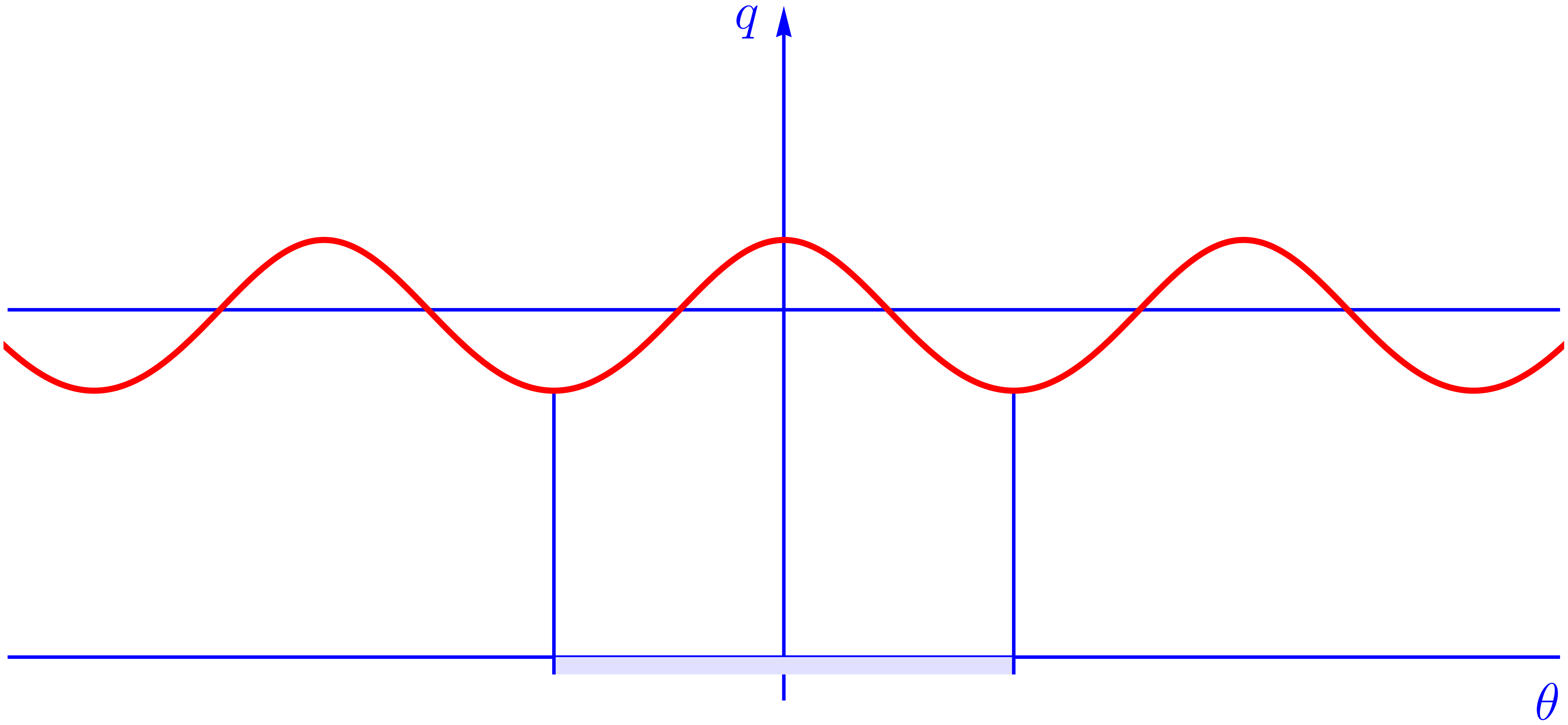
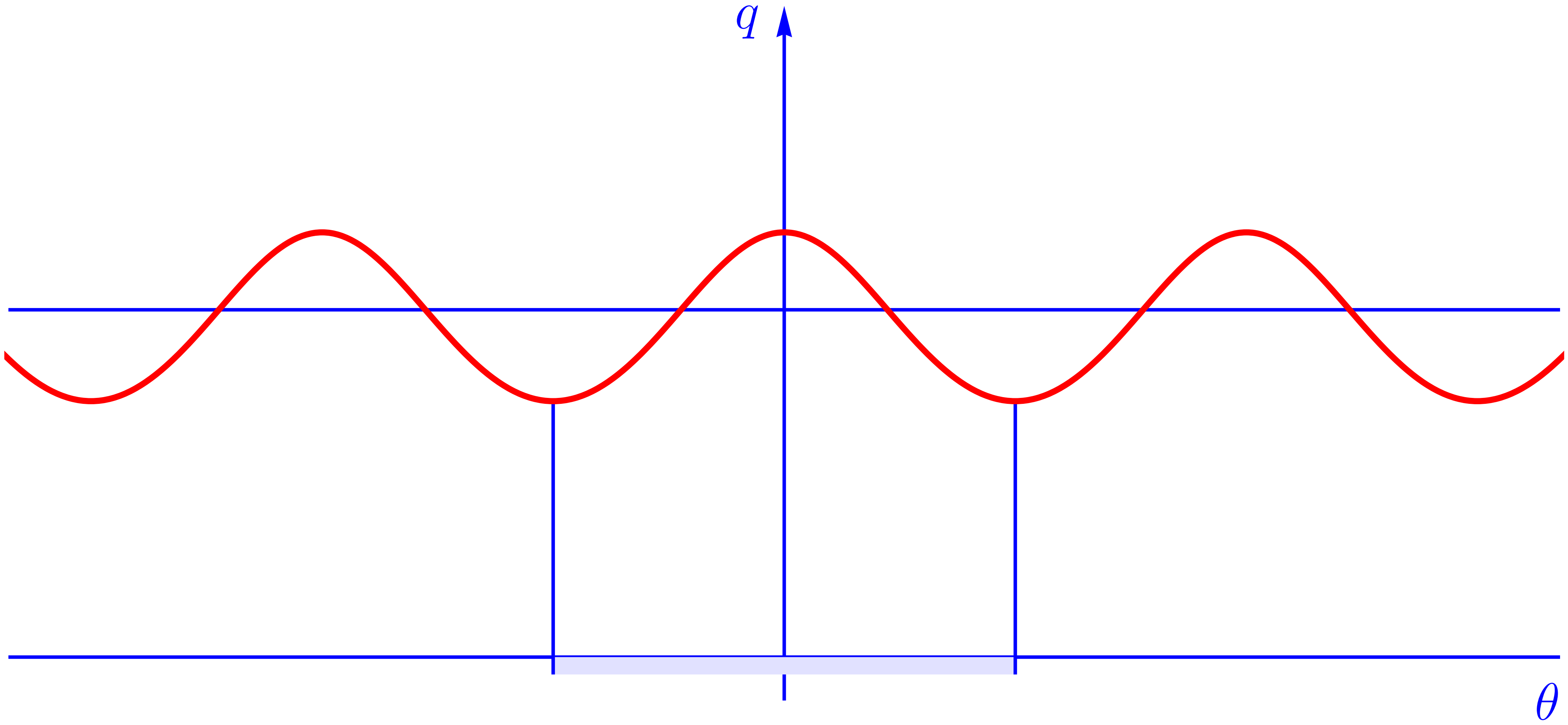


Cnoidal Waves



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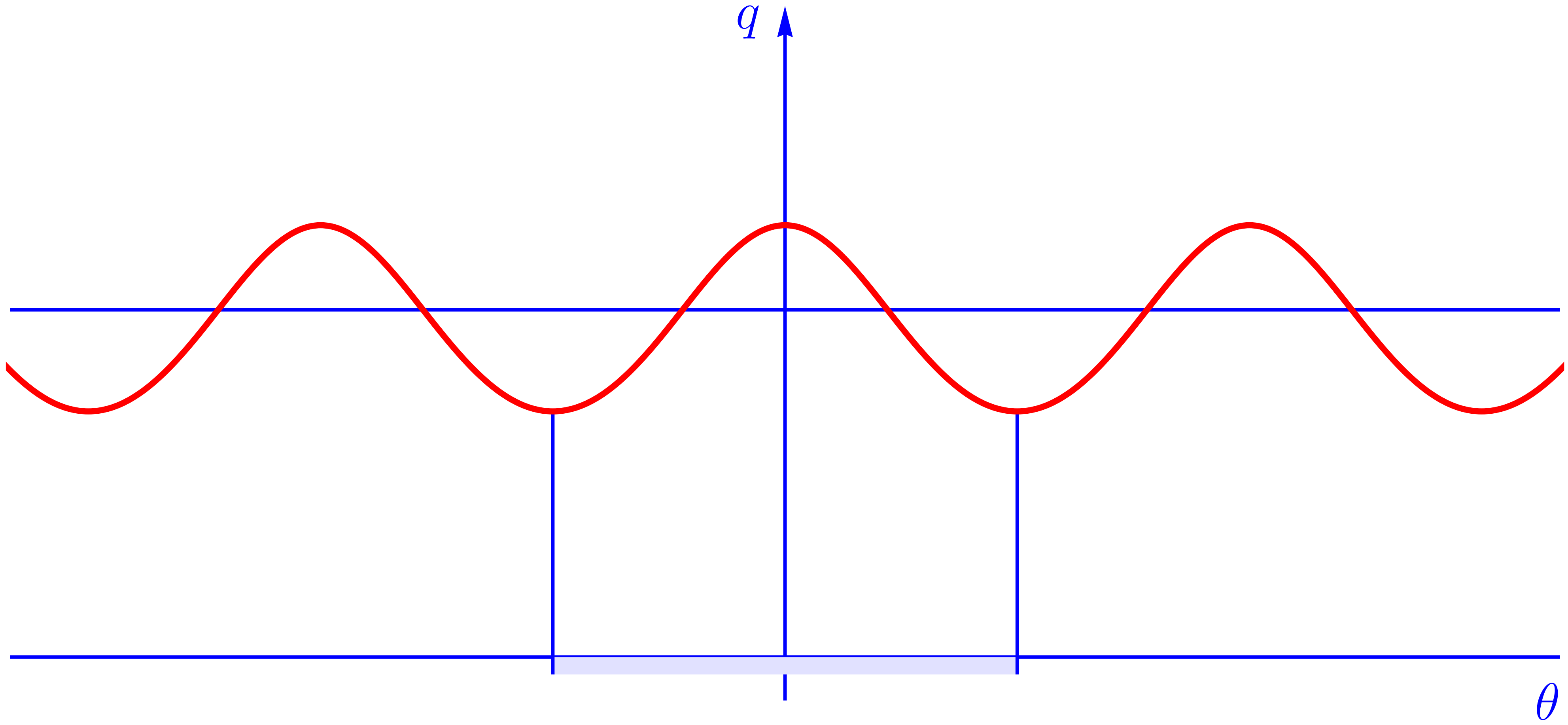

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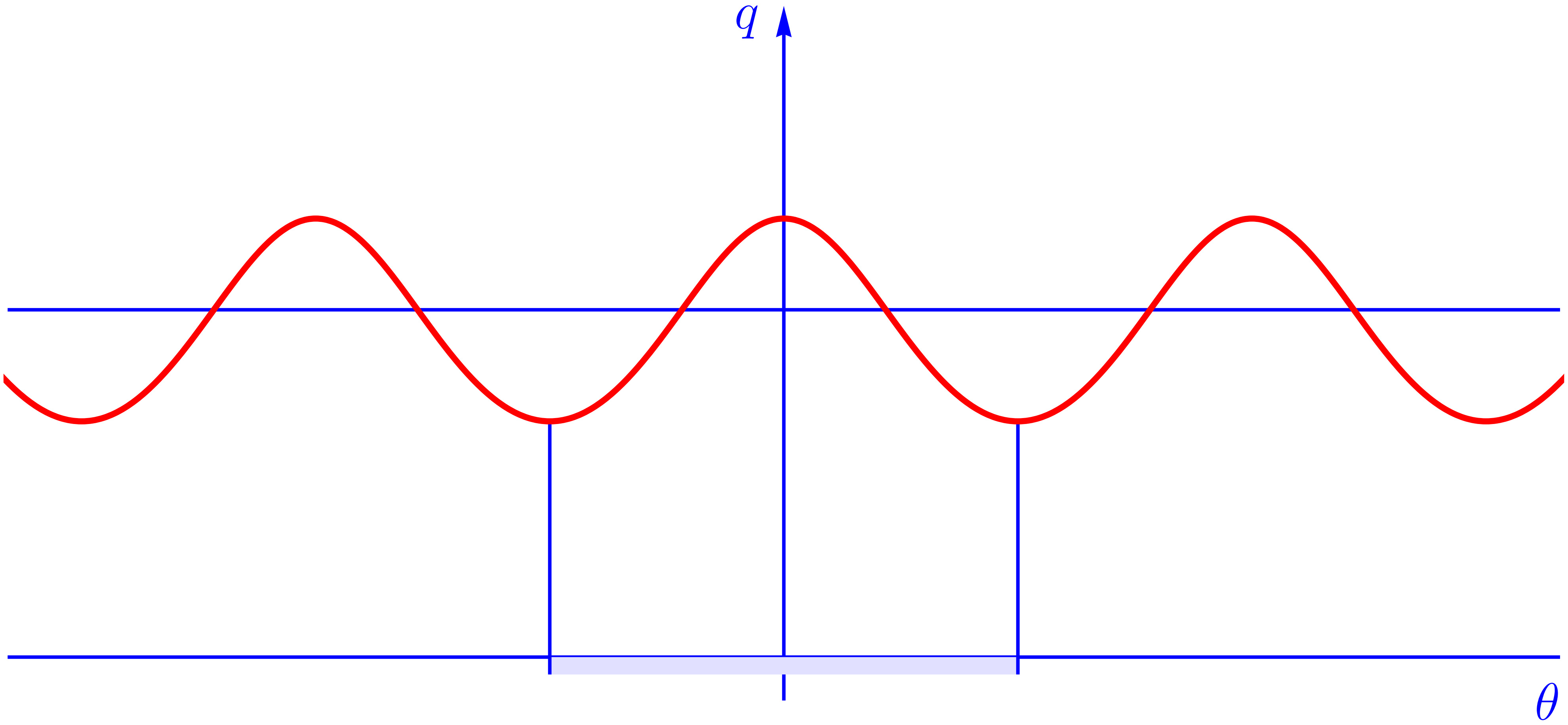

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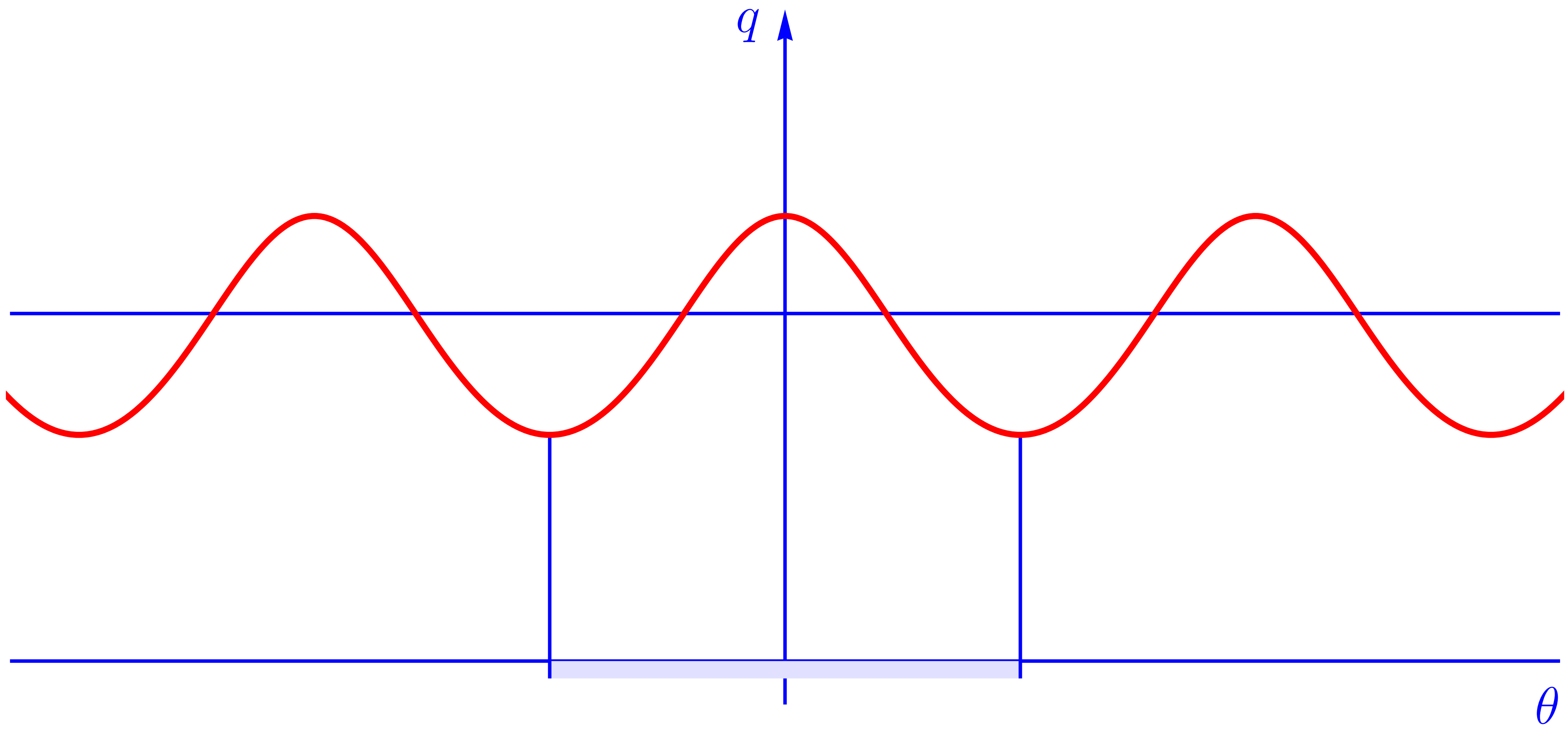

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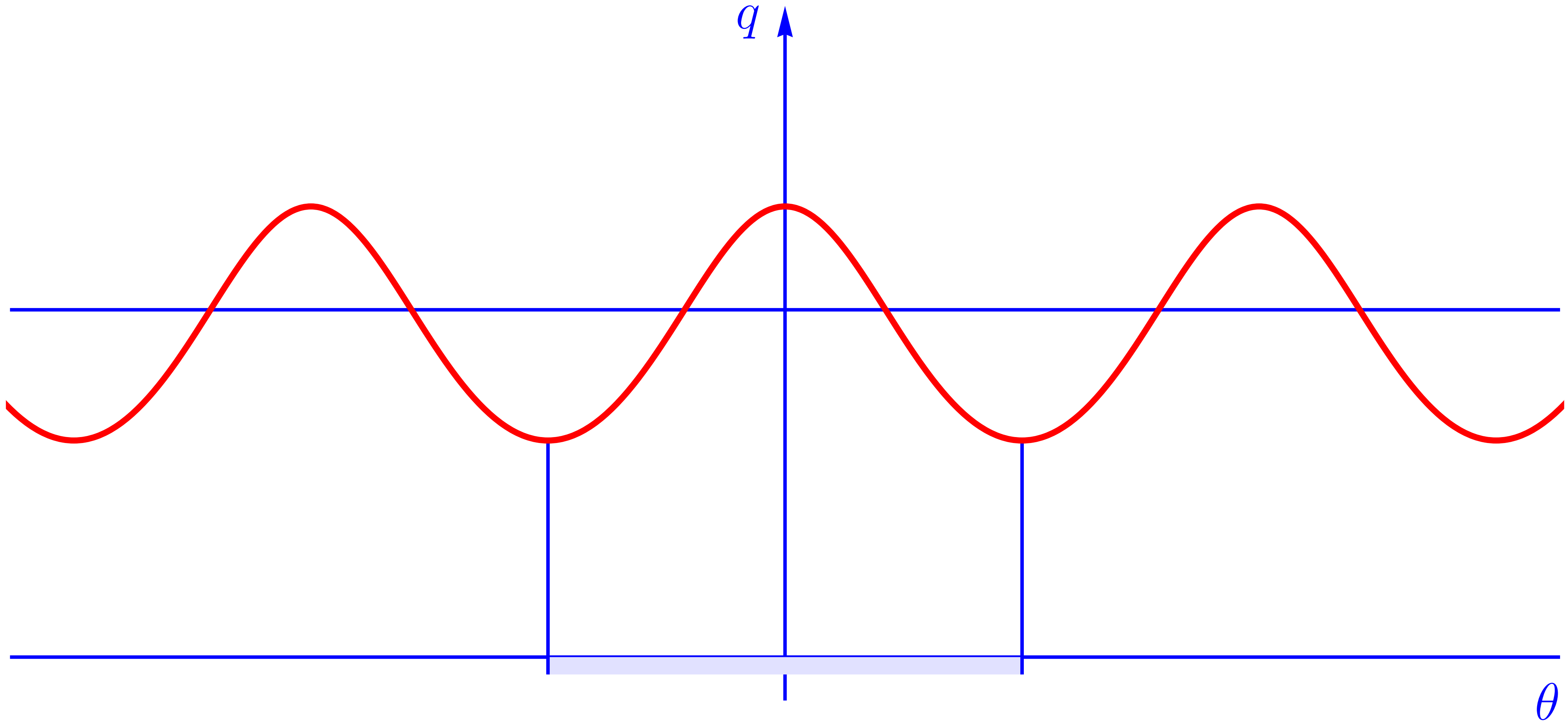

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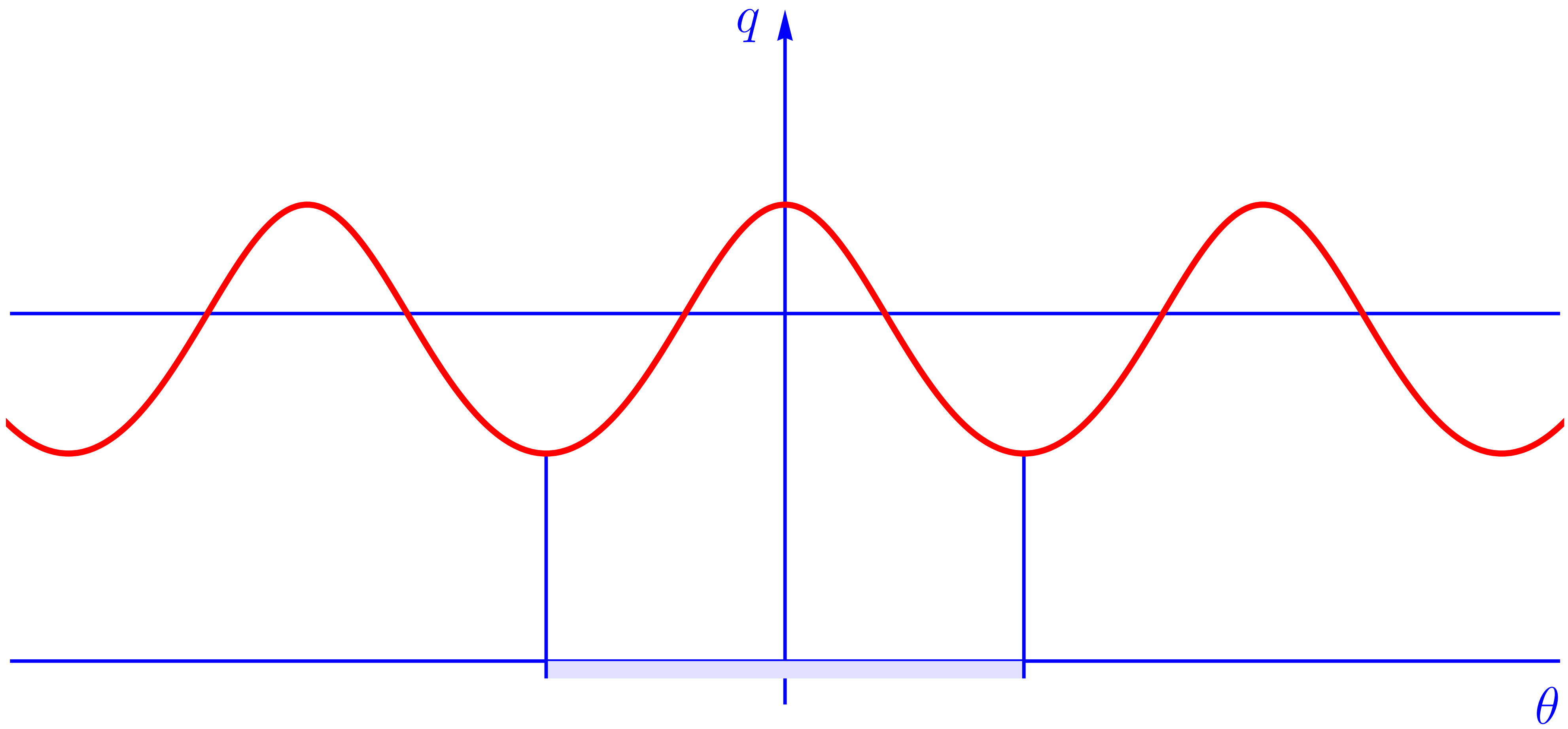

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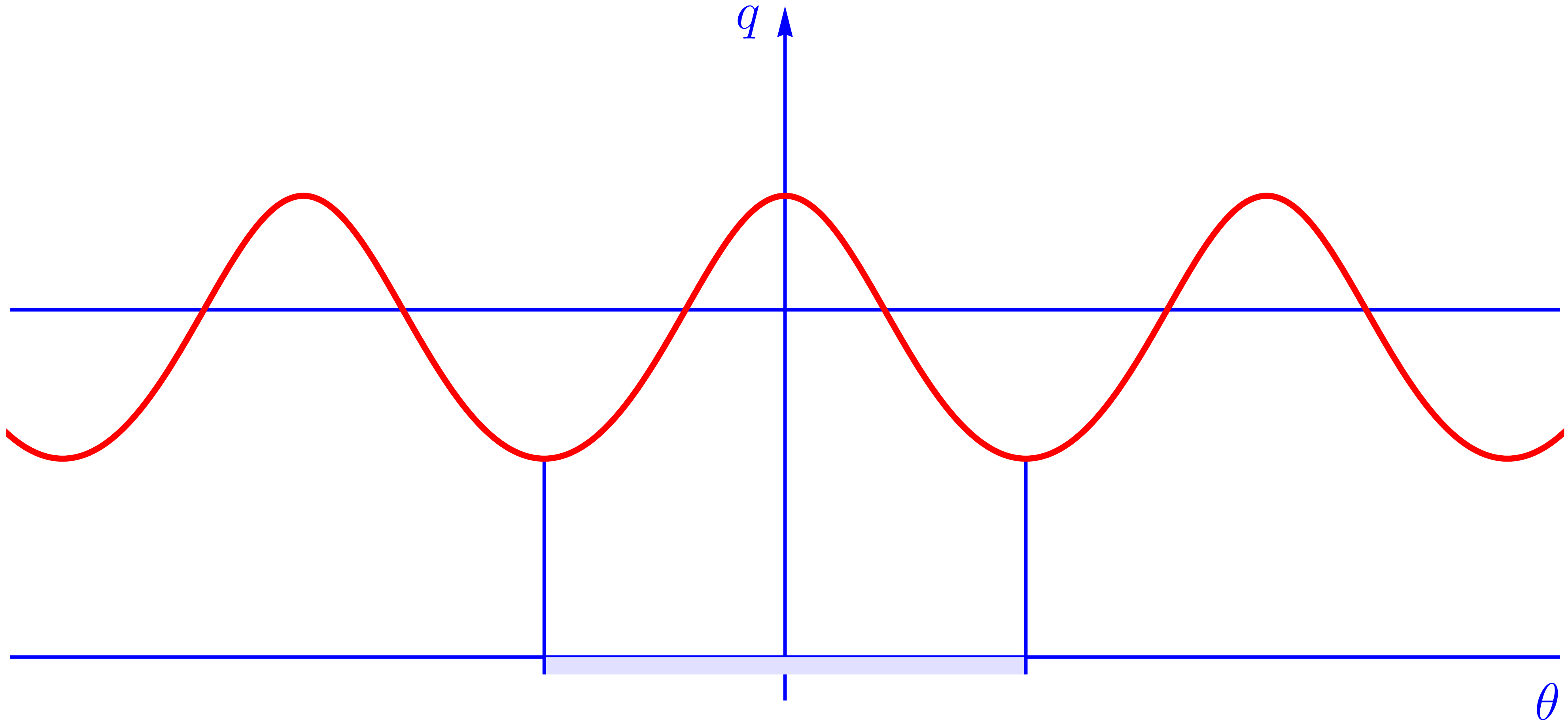

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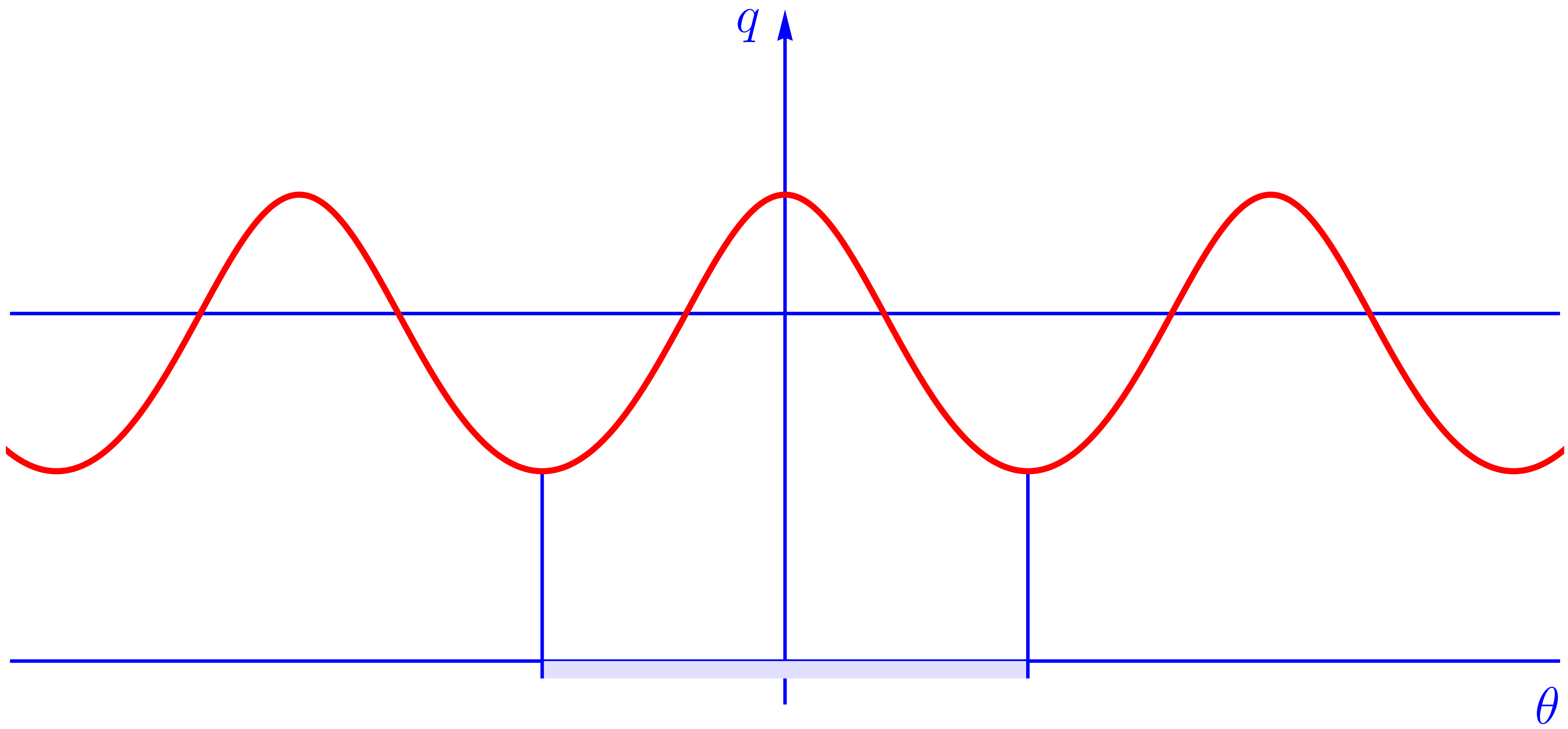

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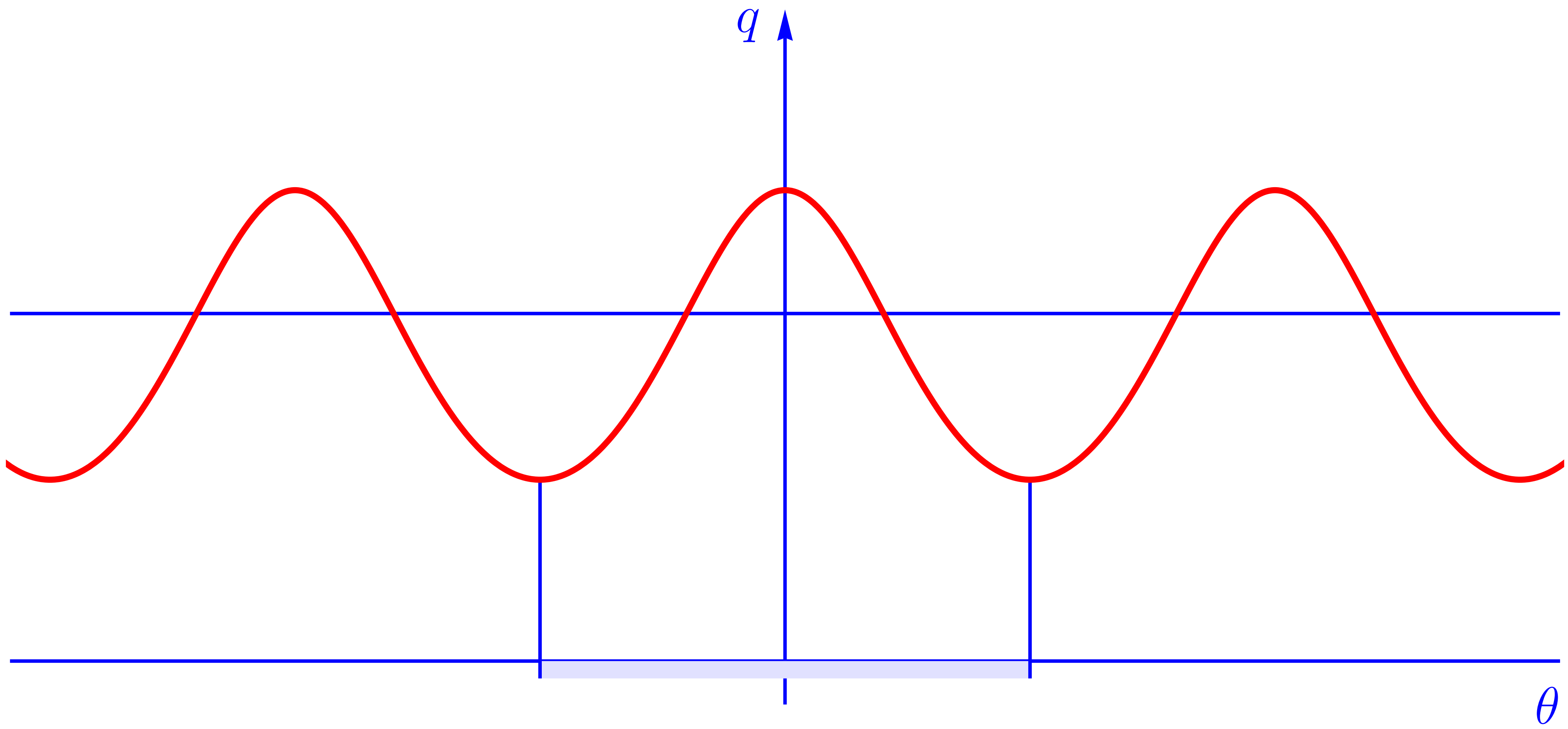

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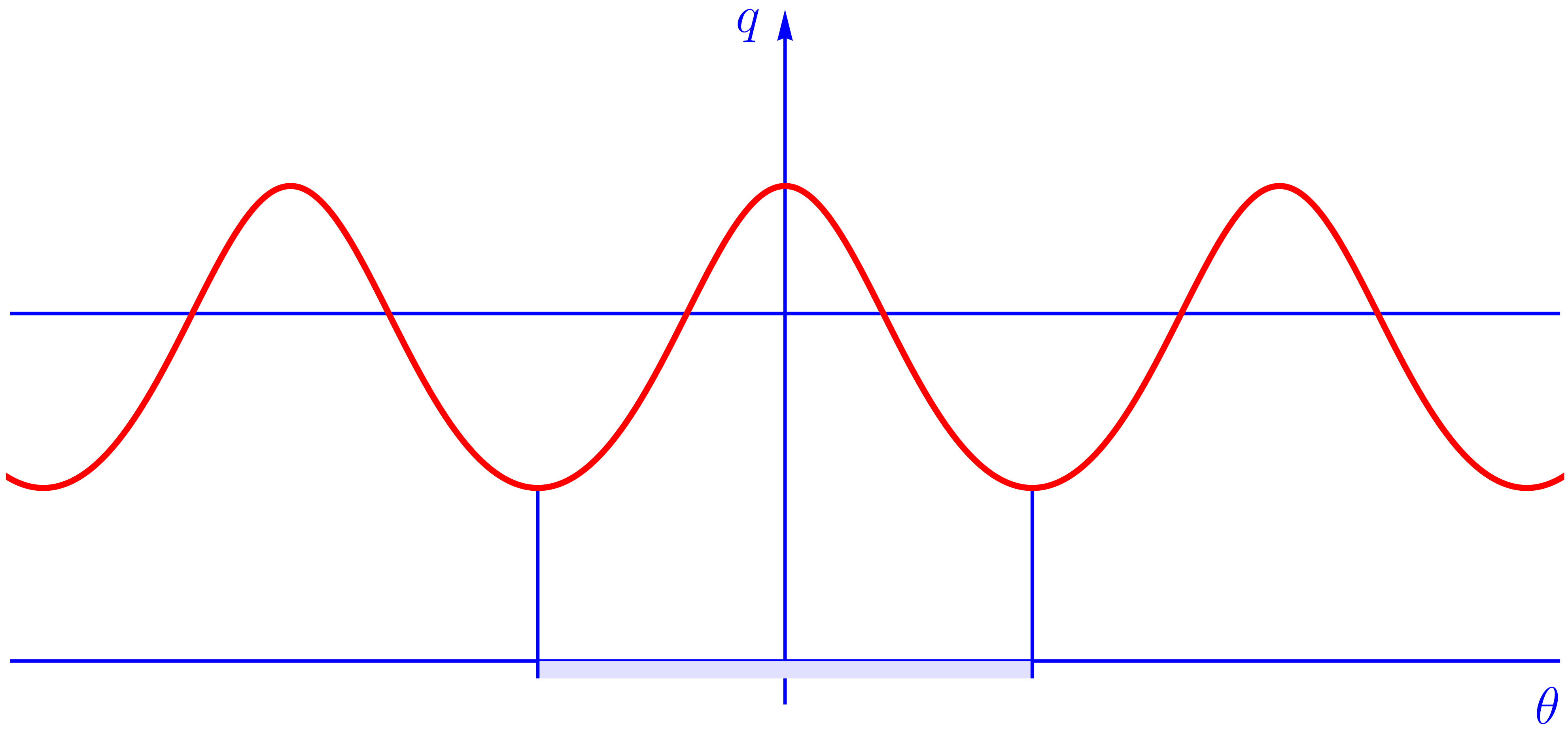

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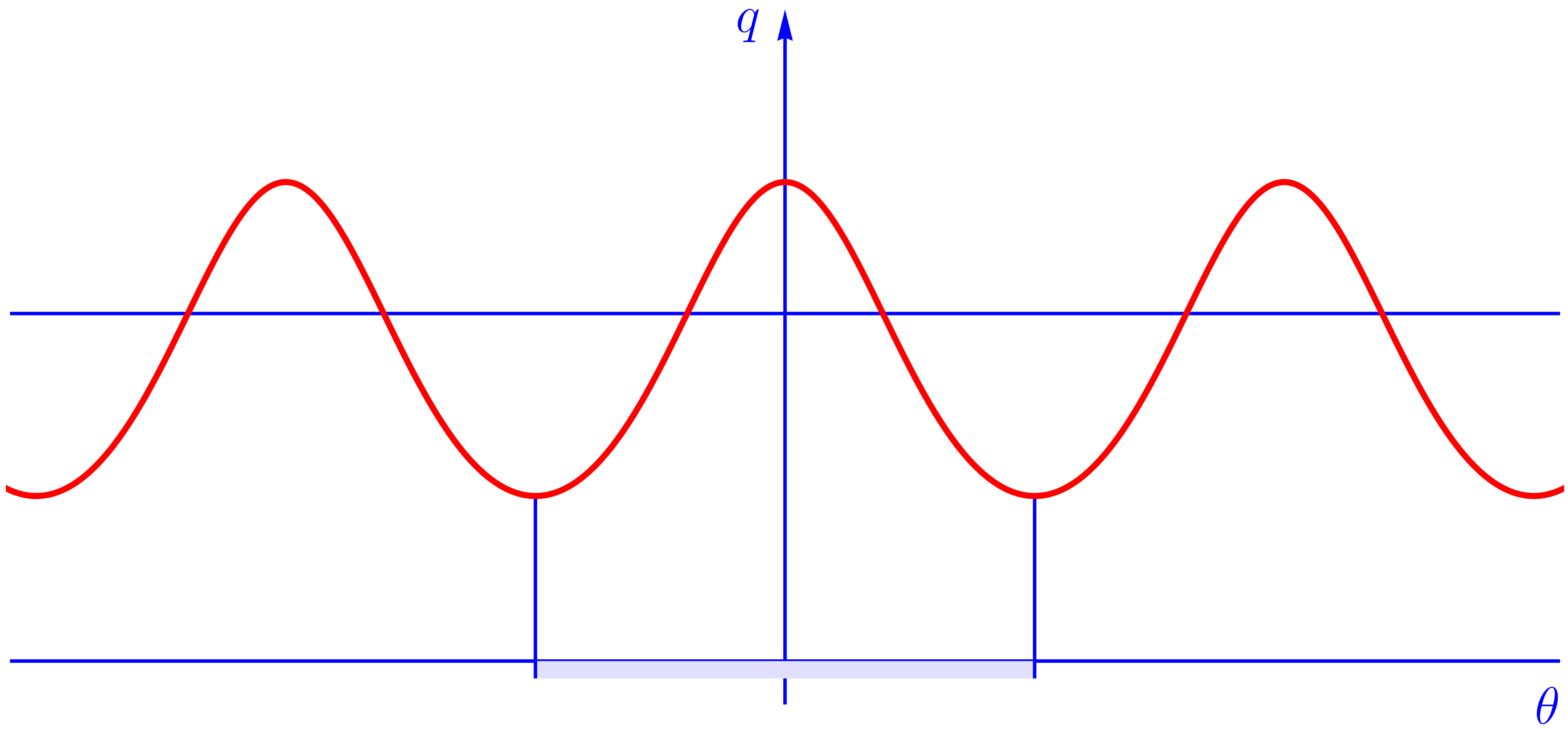

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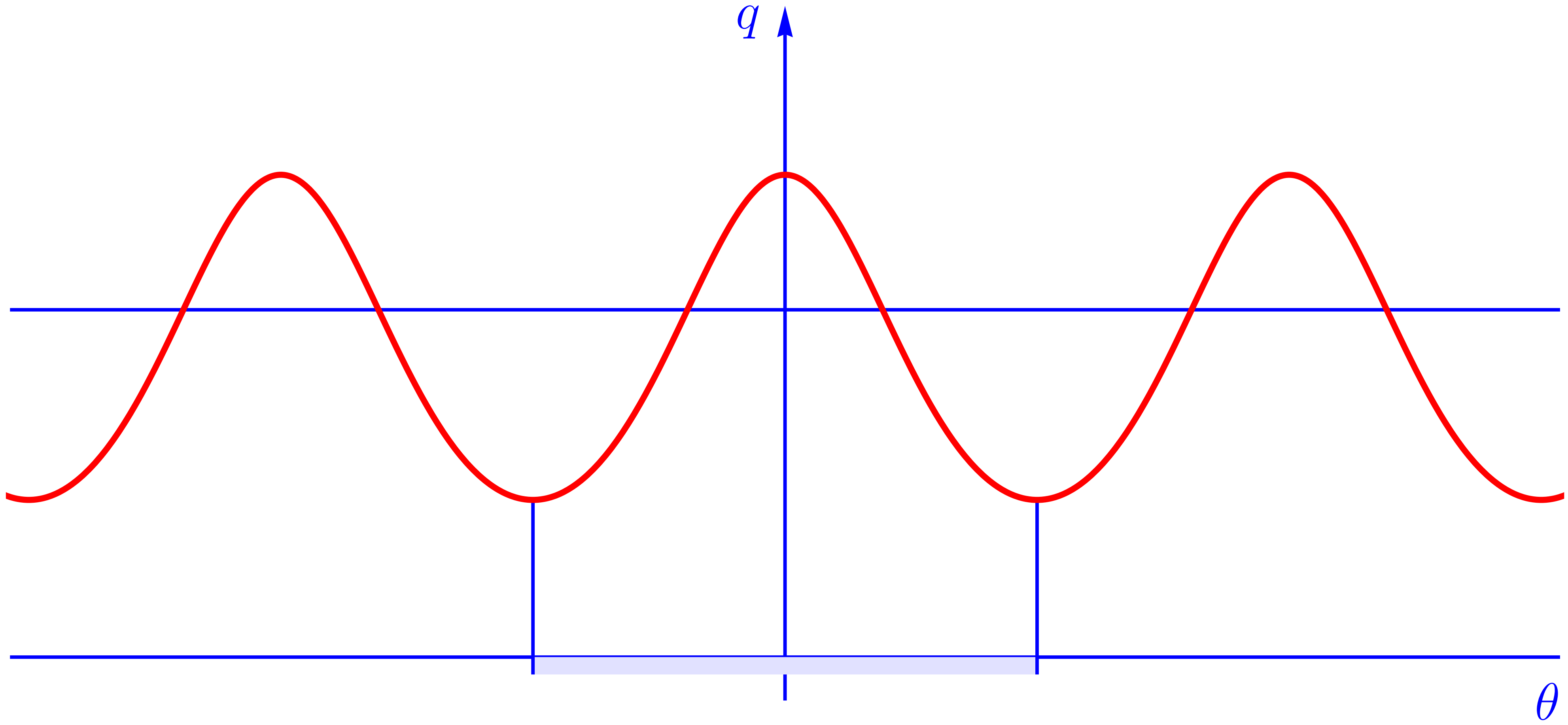

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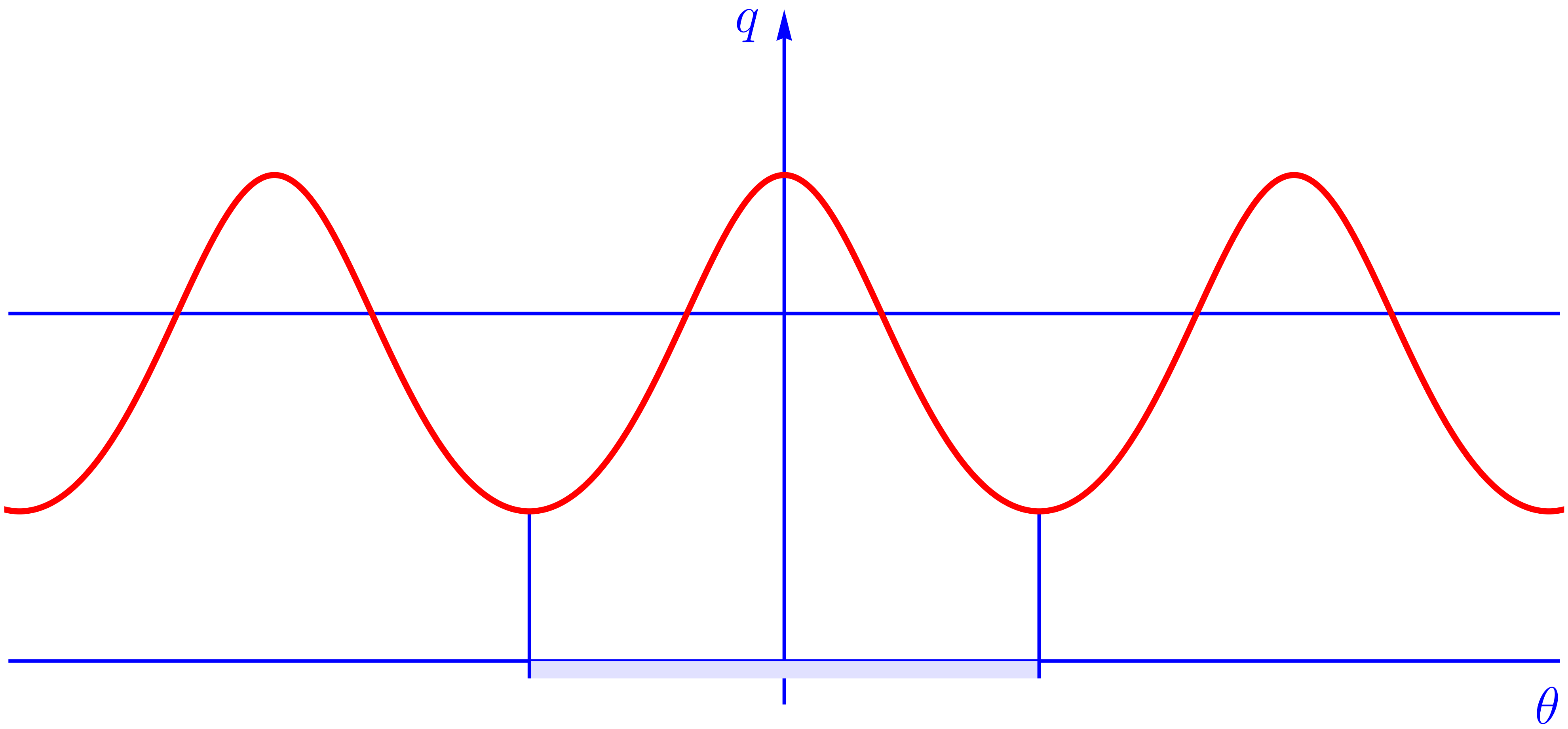

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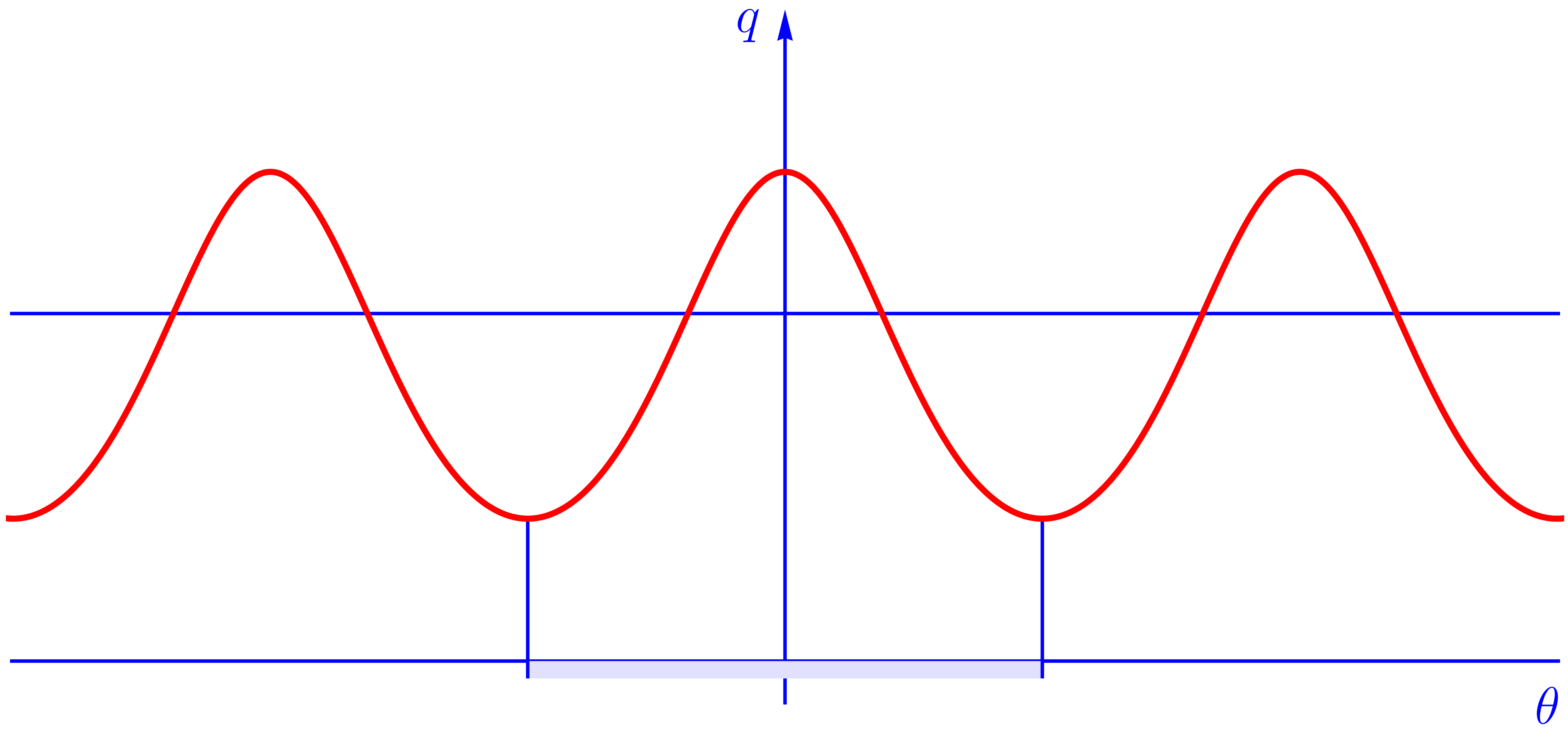

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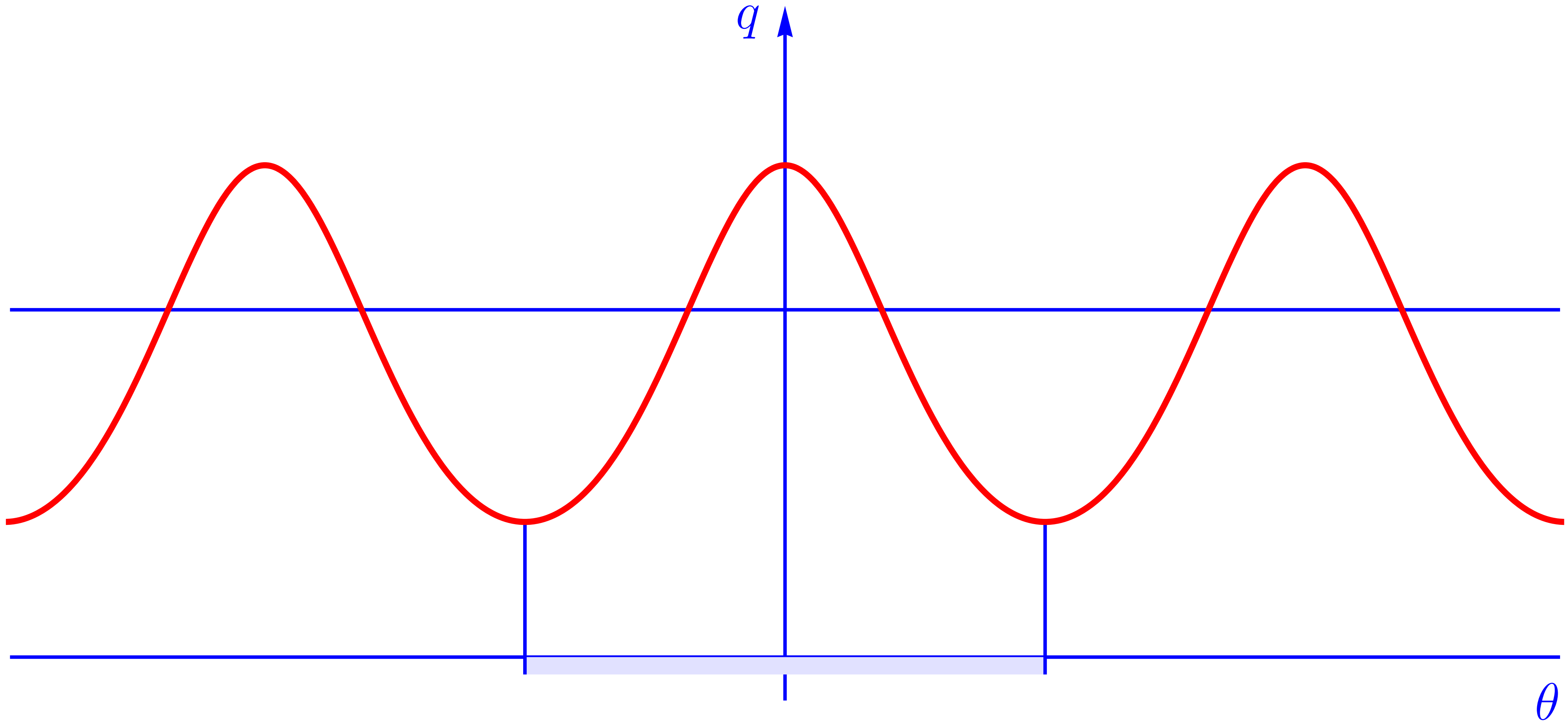

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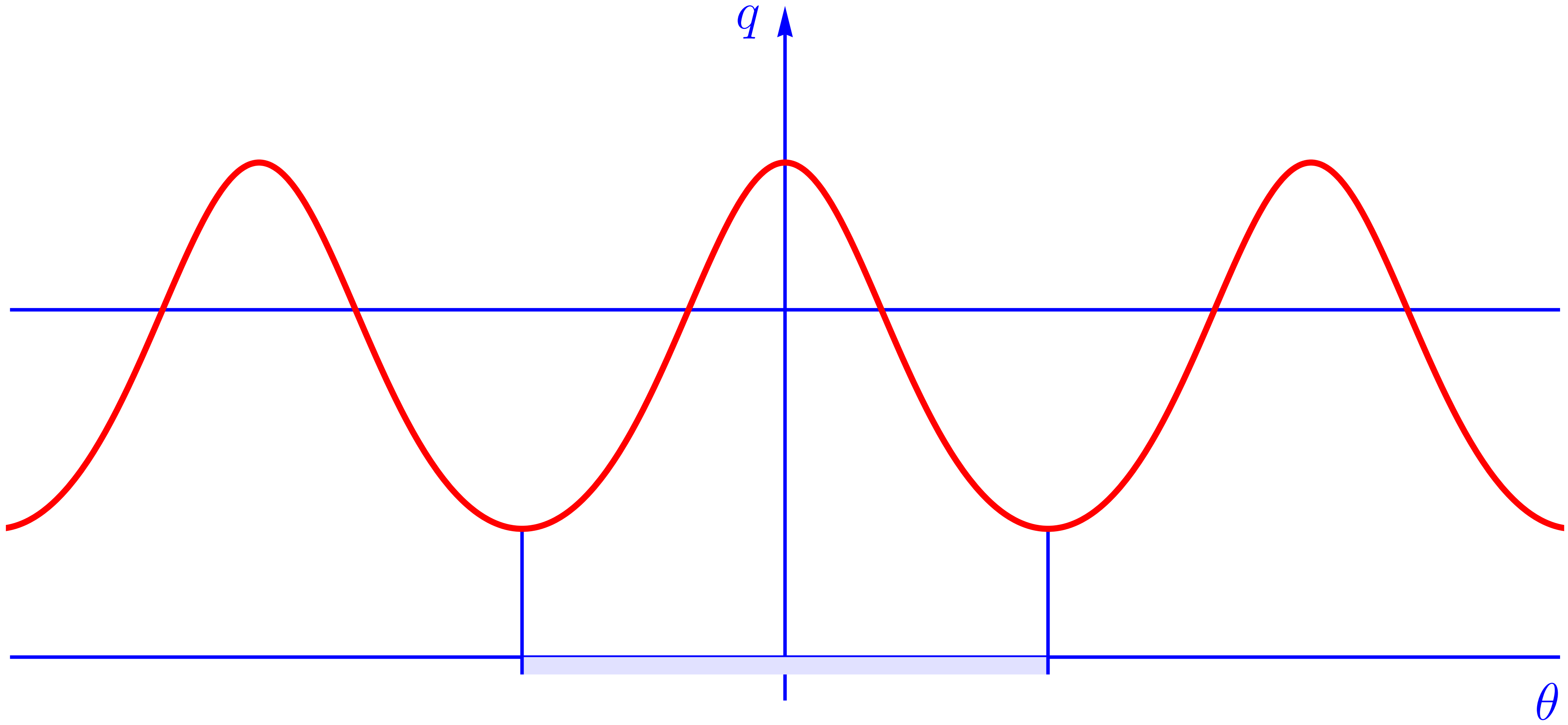

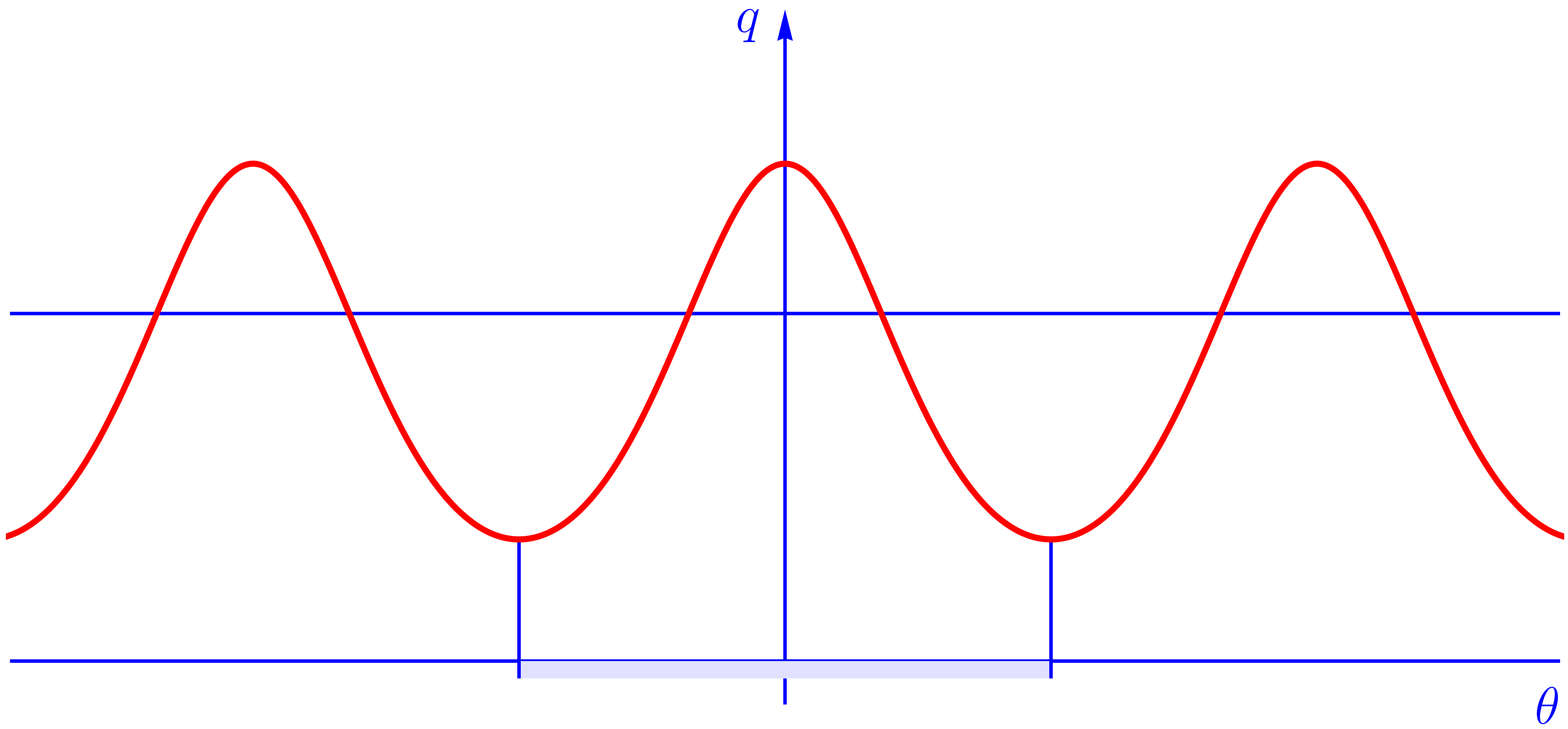

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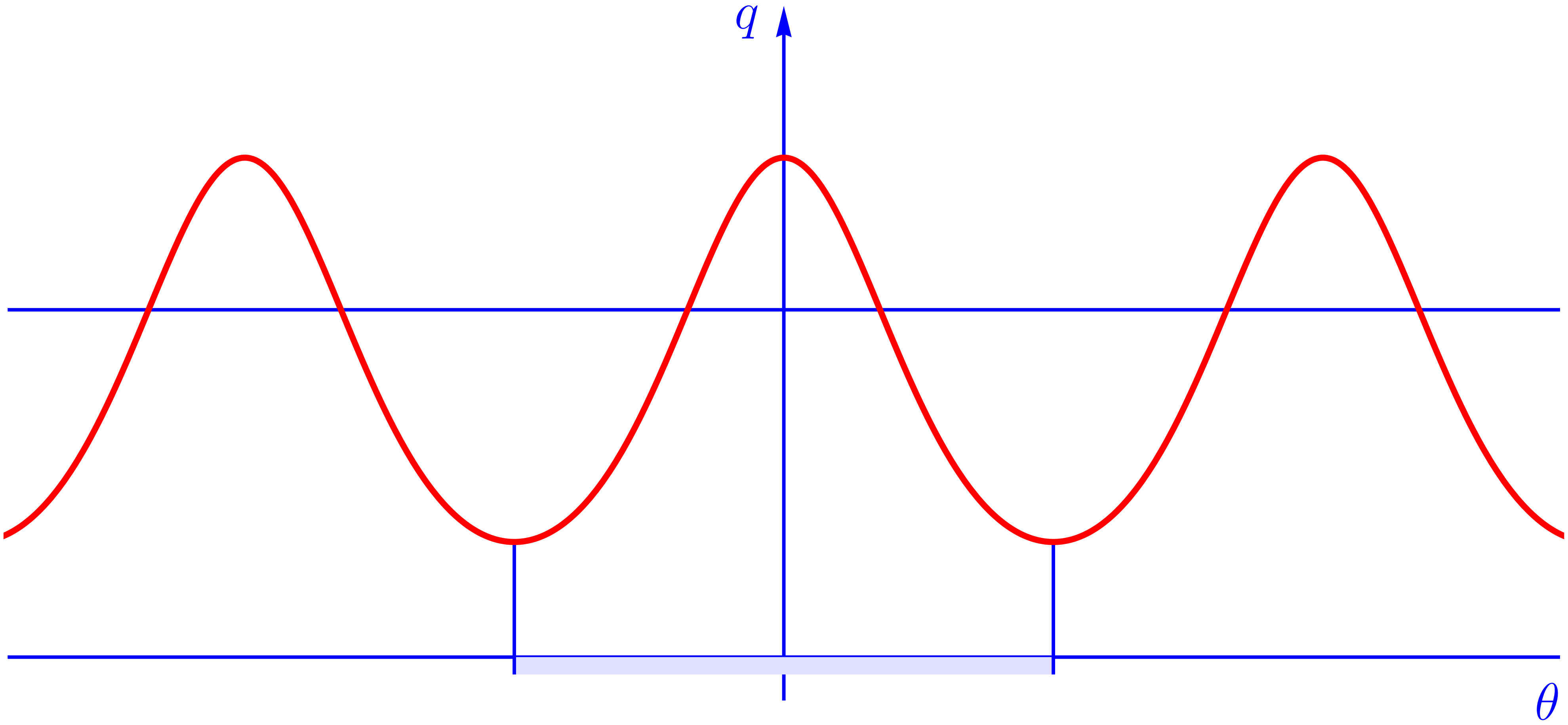

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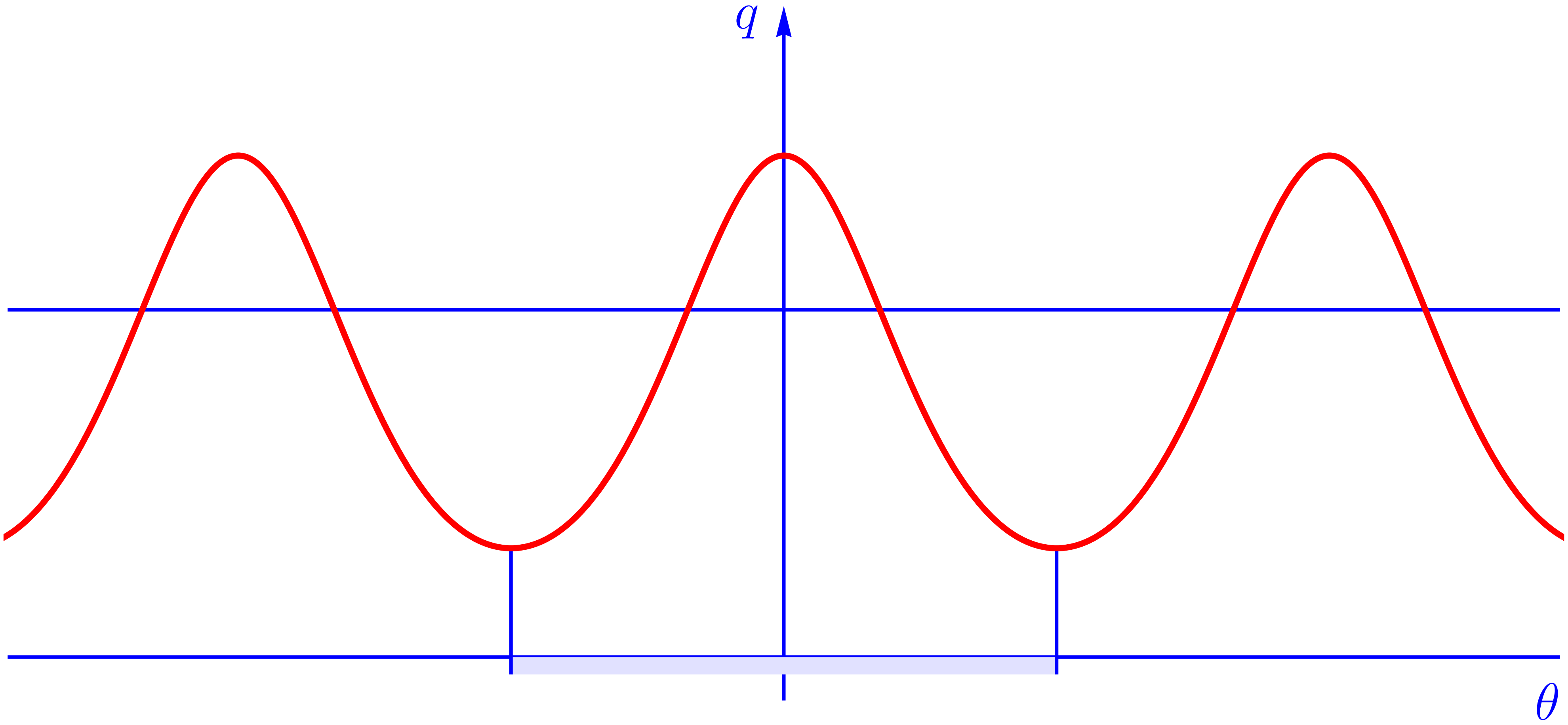

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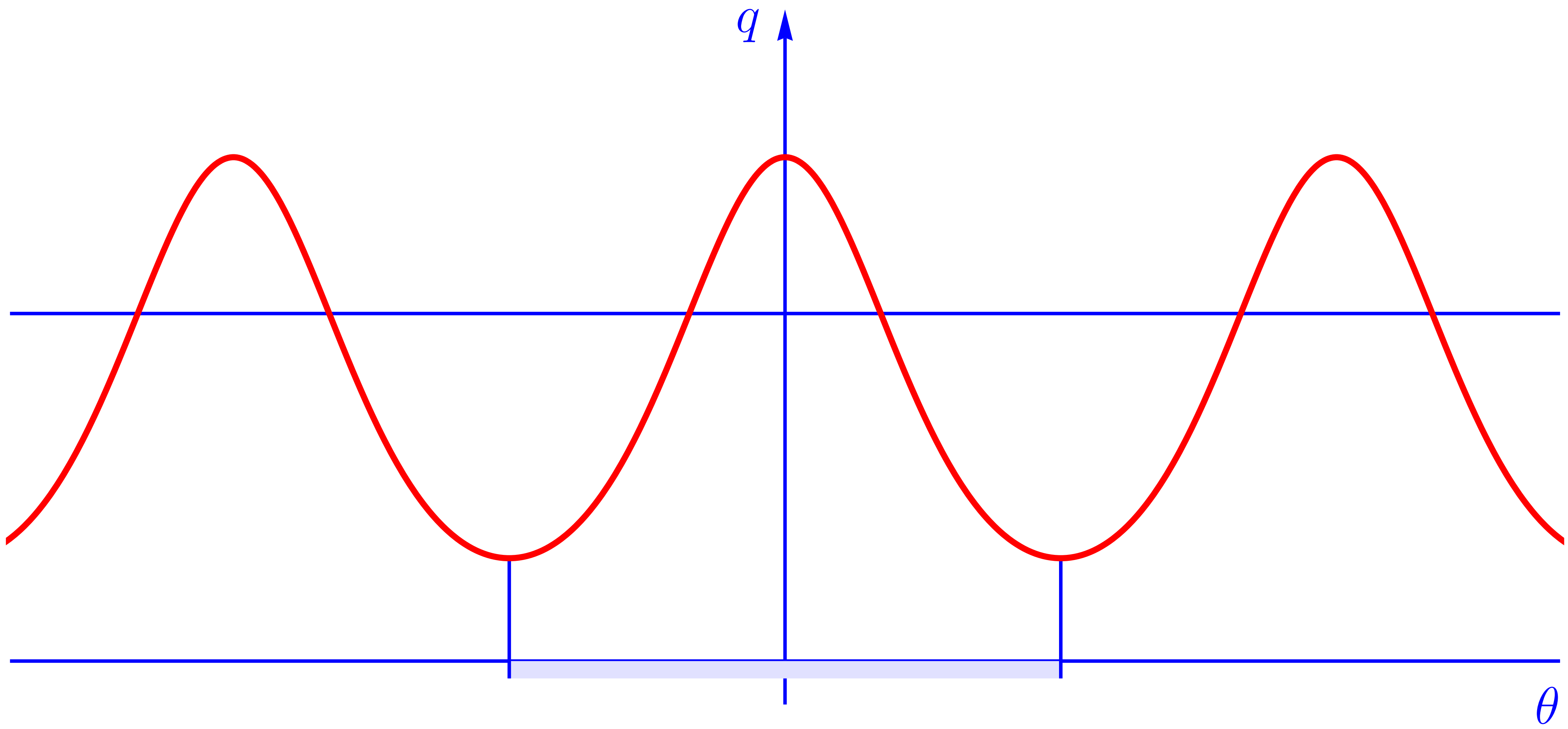

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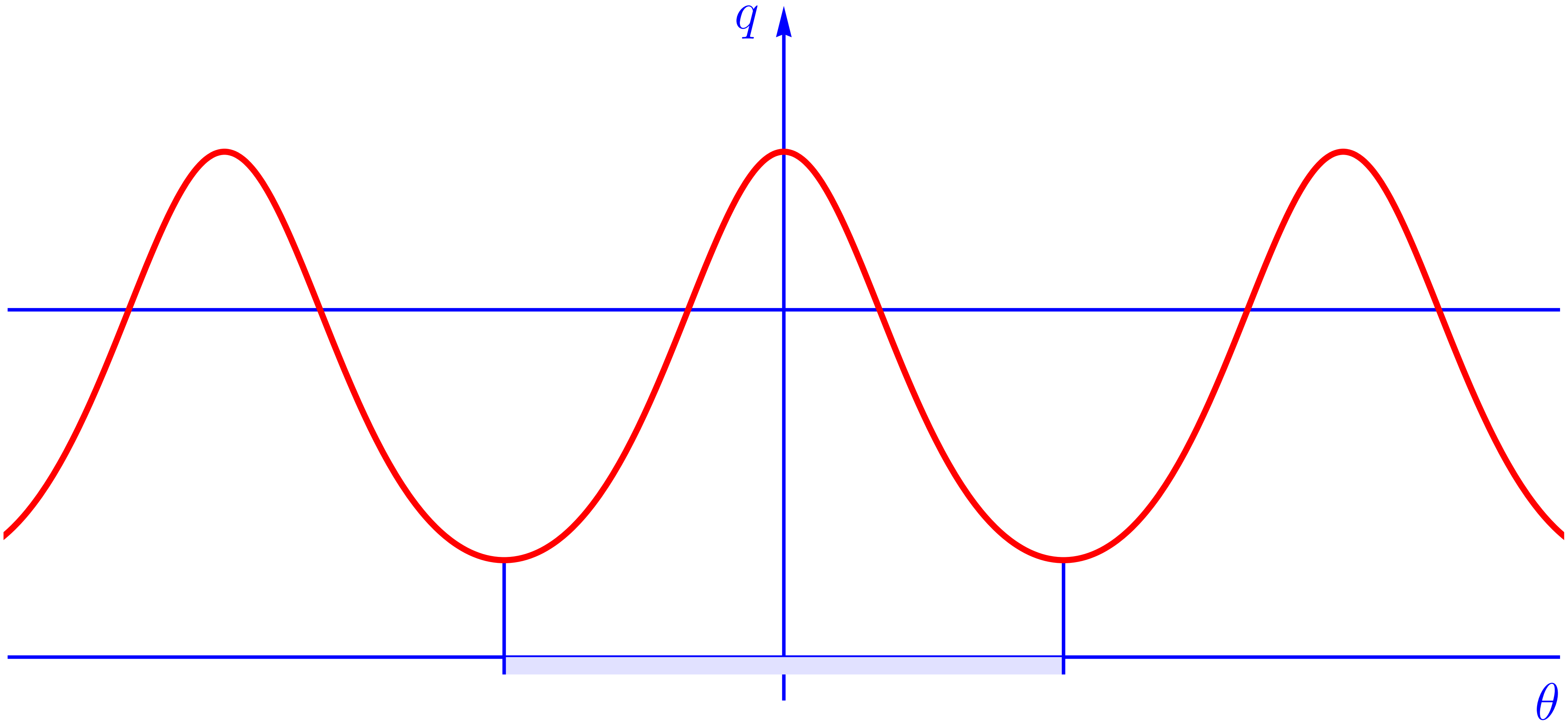

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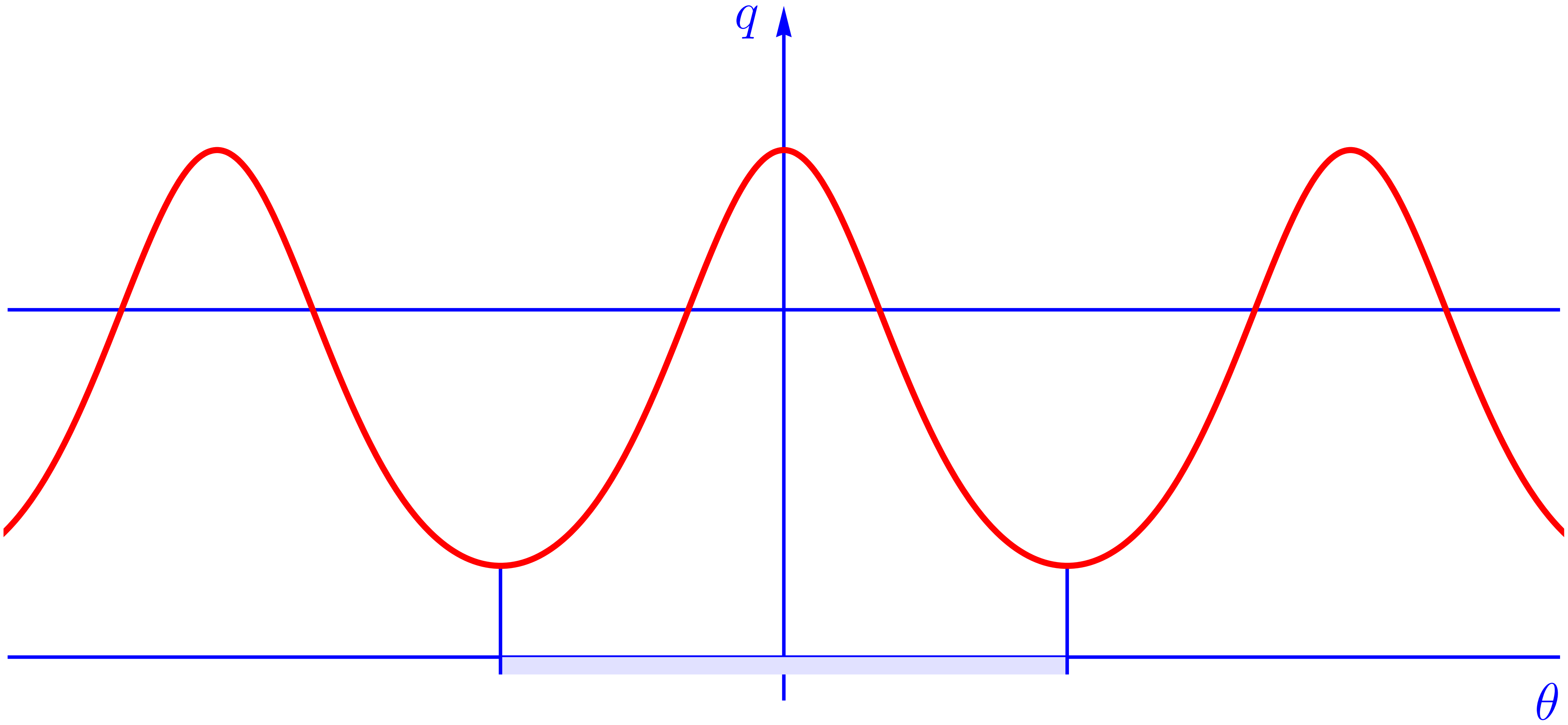

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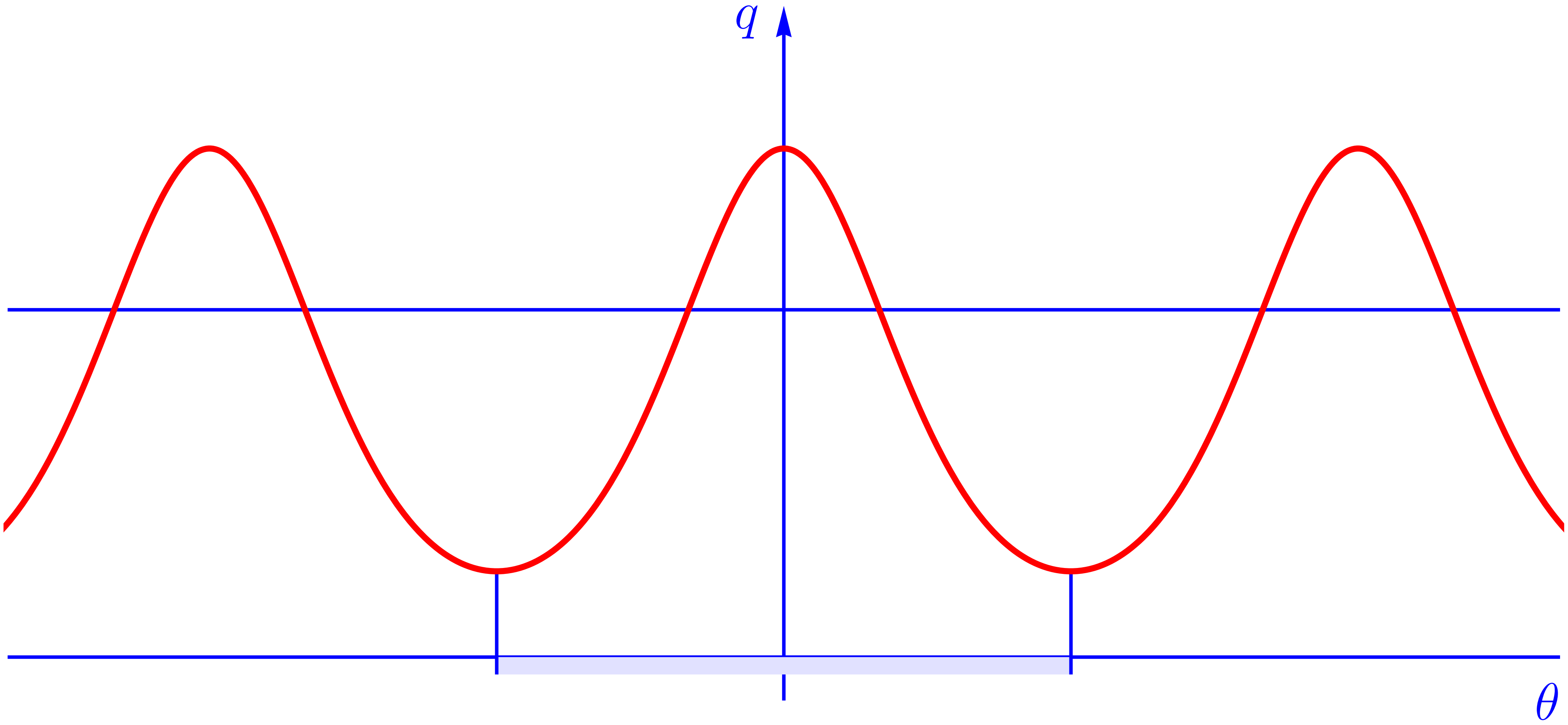

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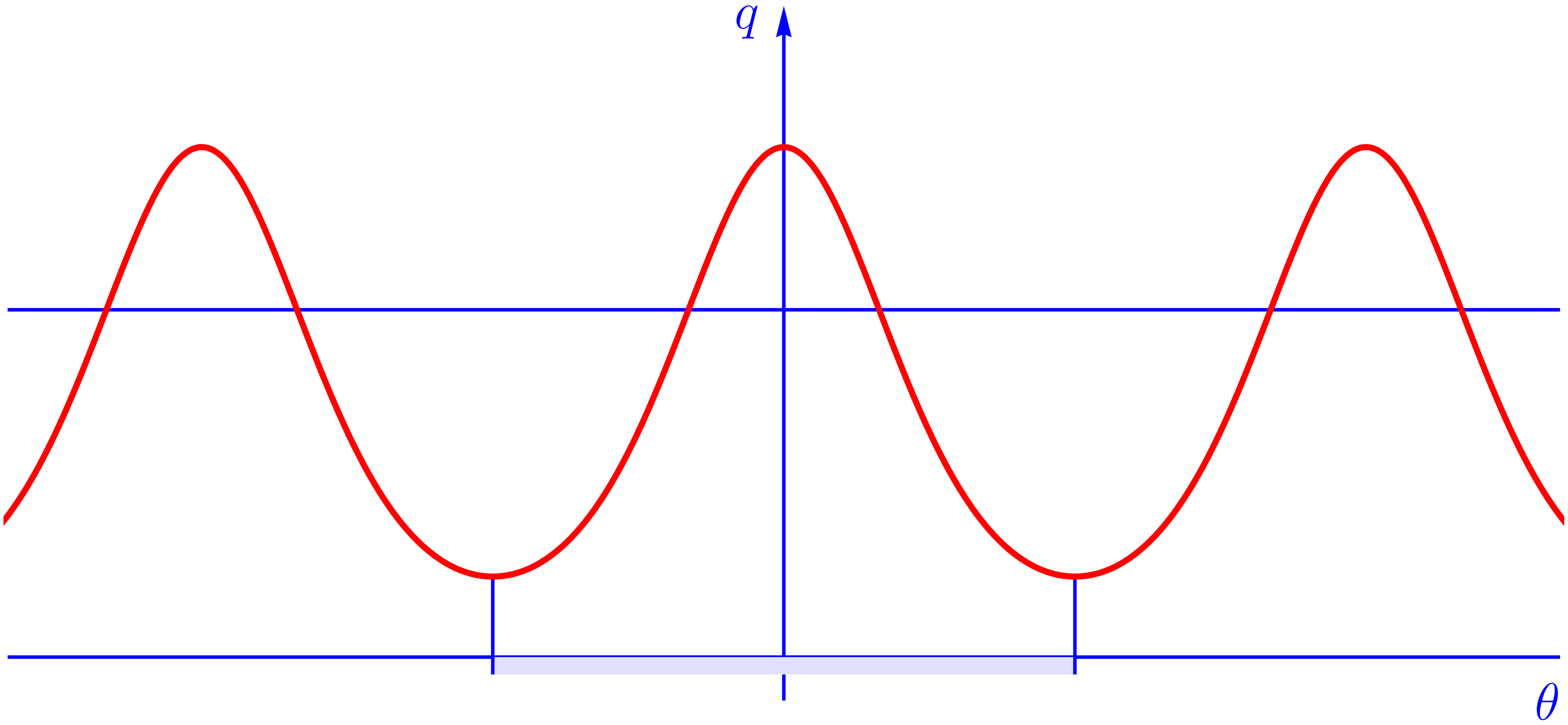

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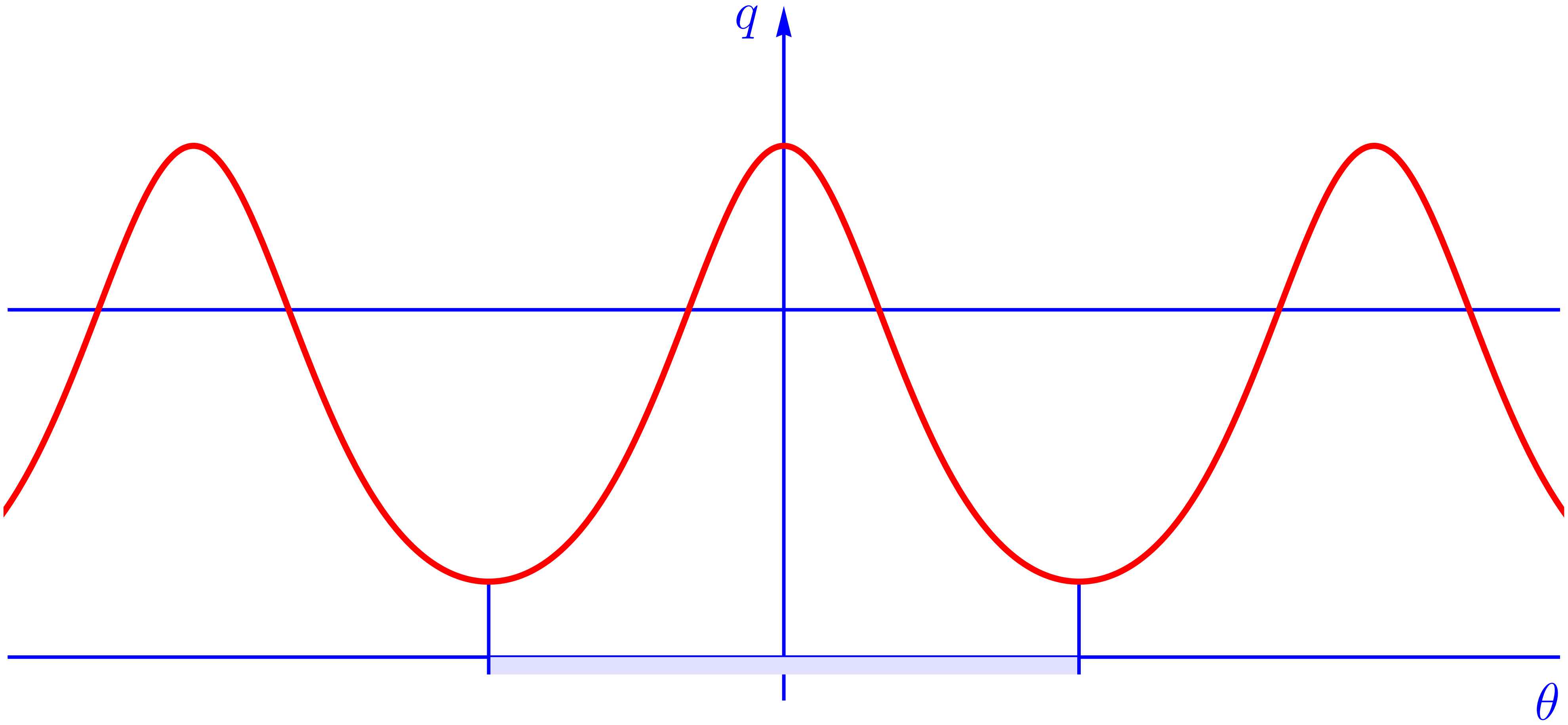

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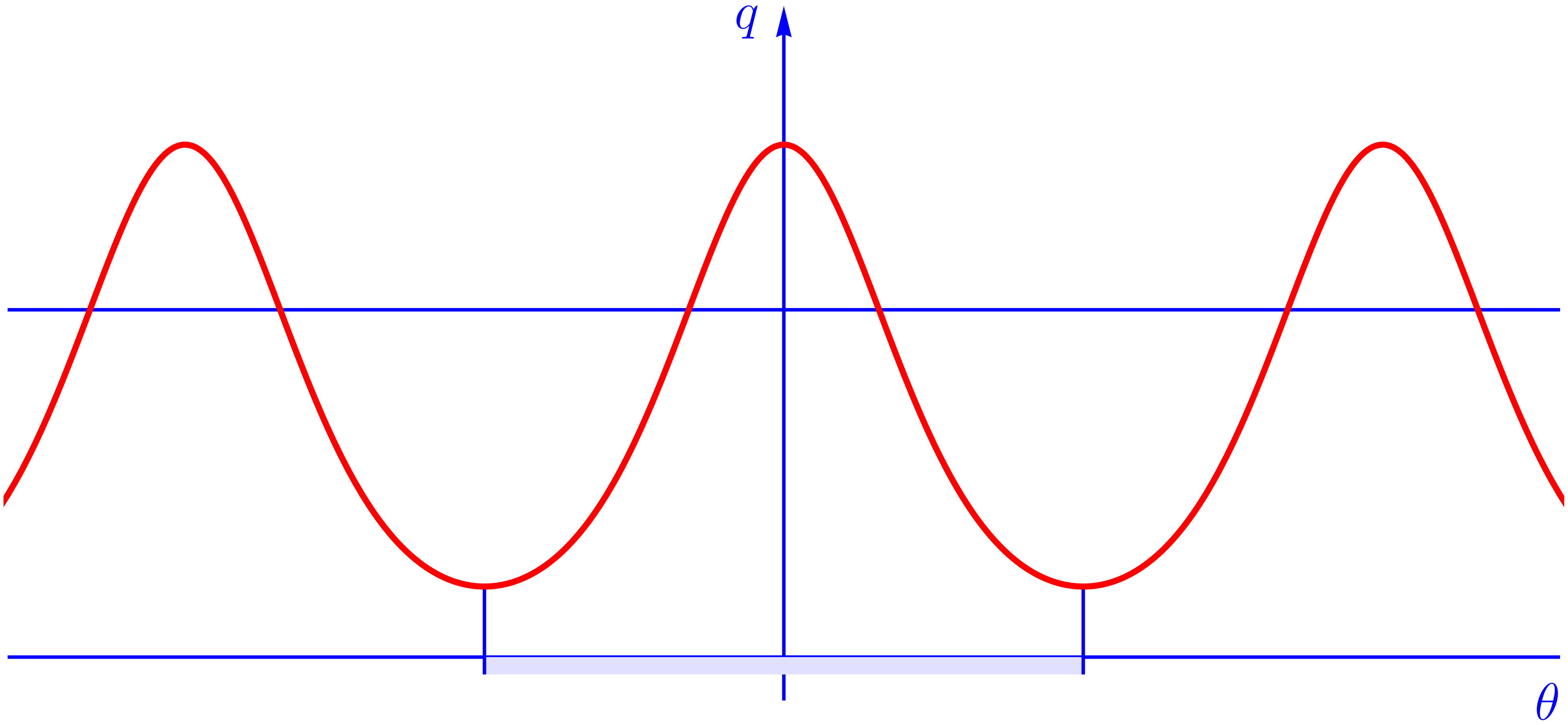

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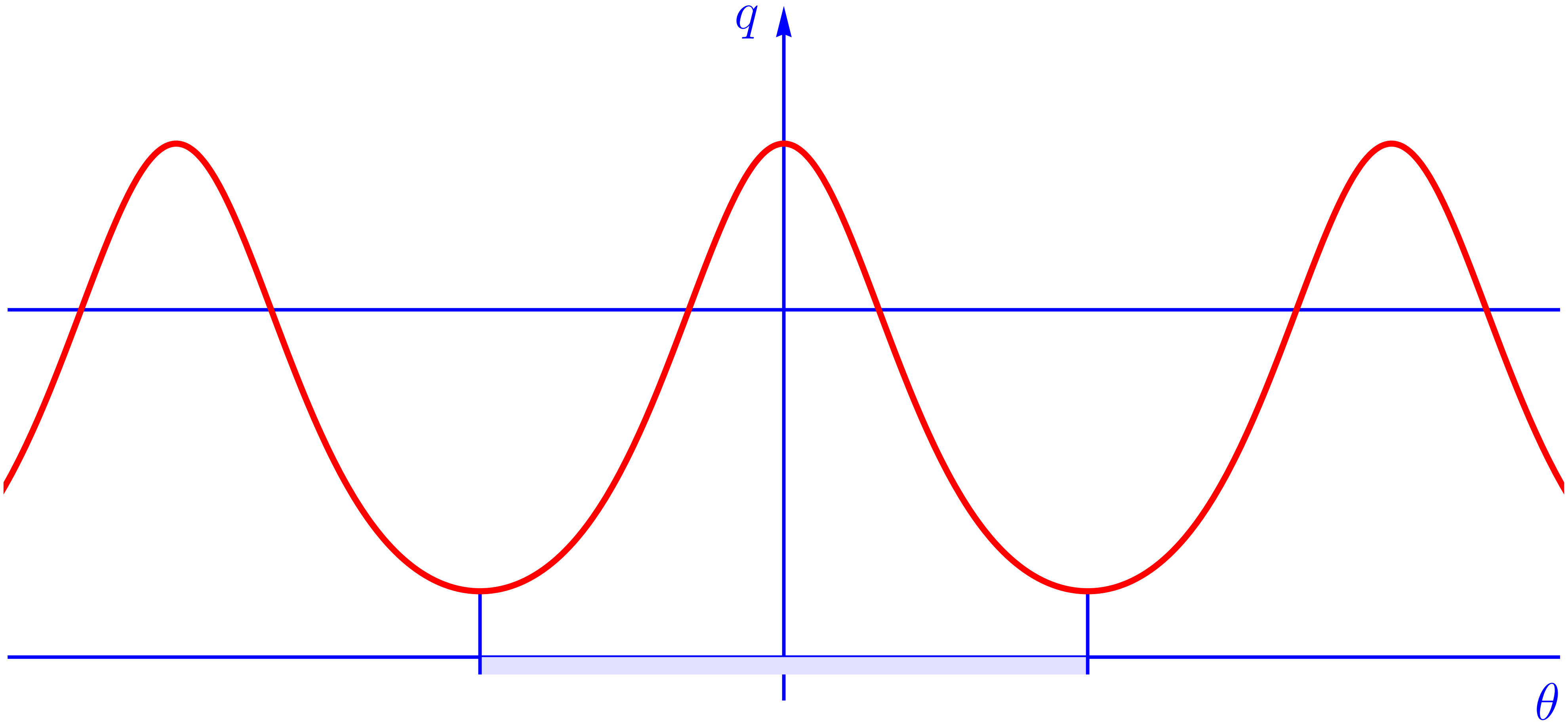

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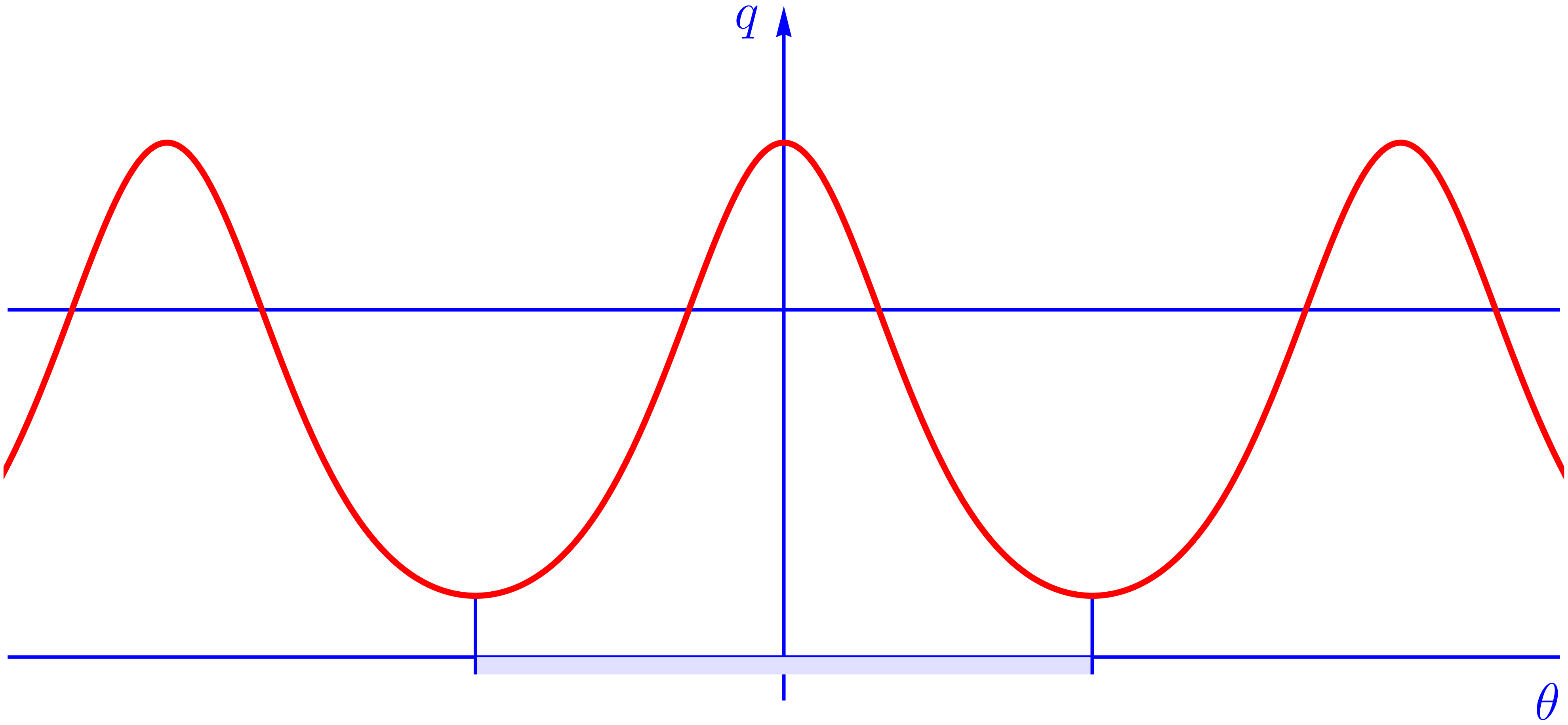

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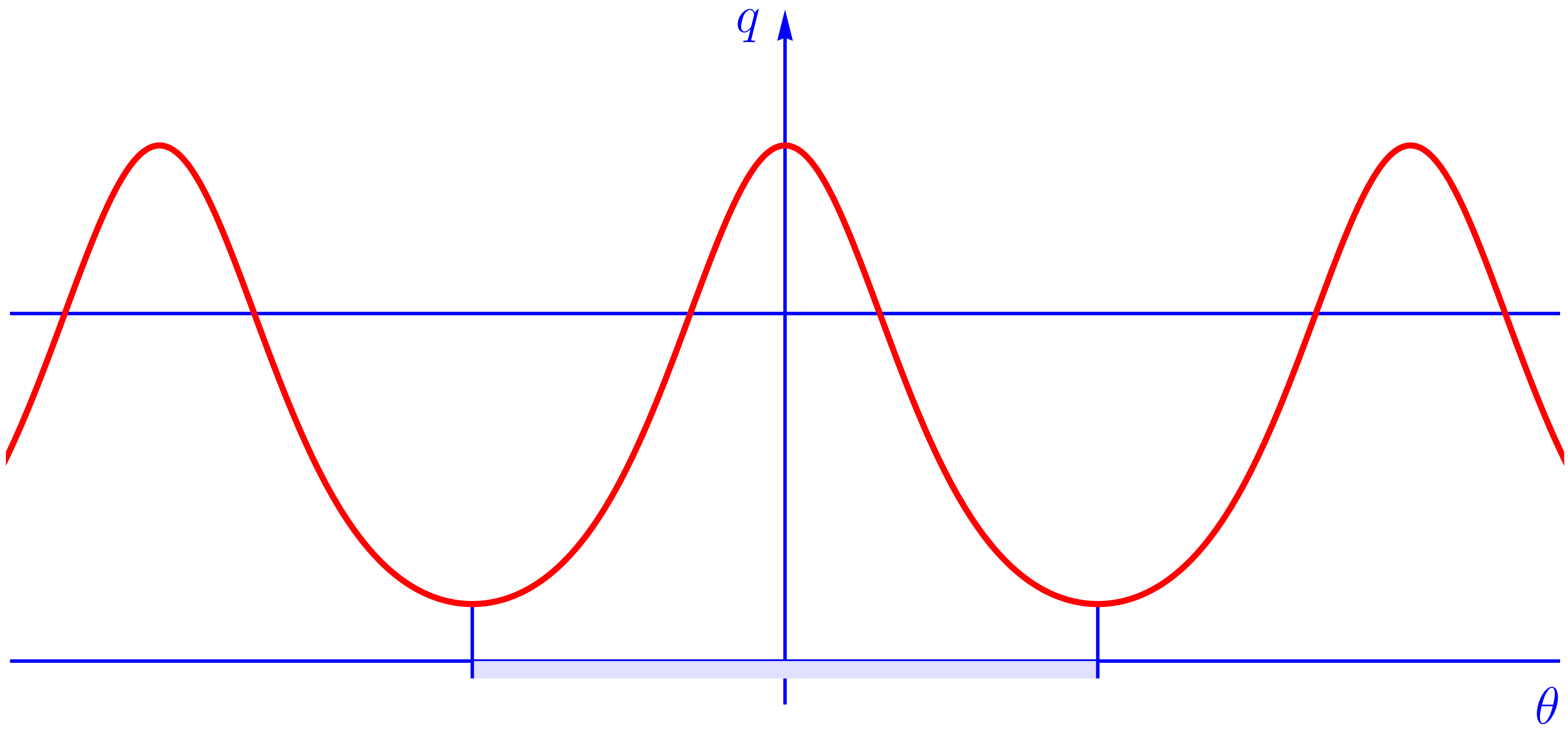

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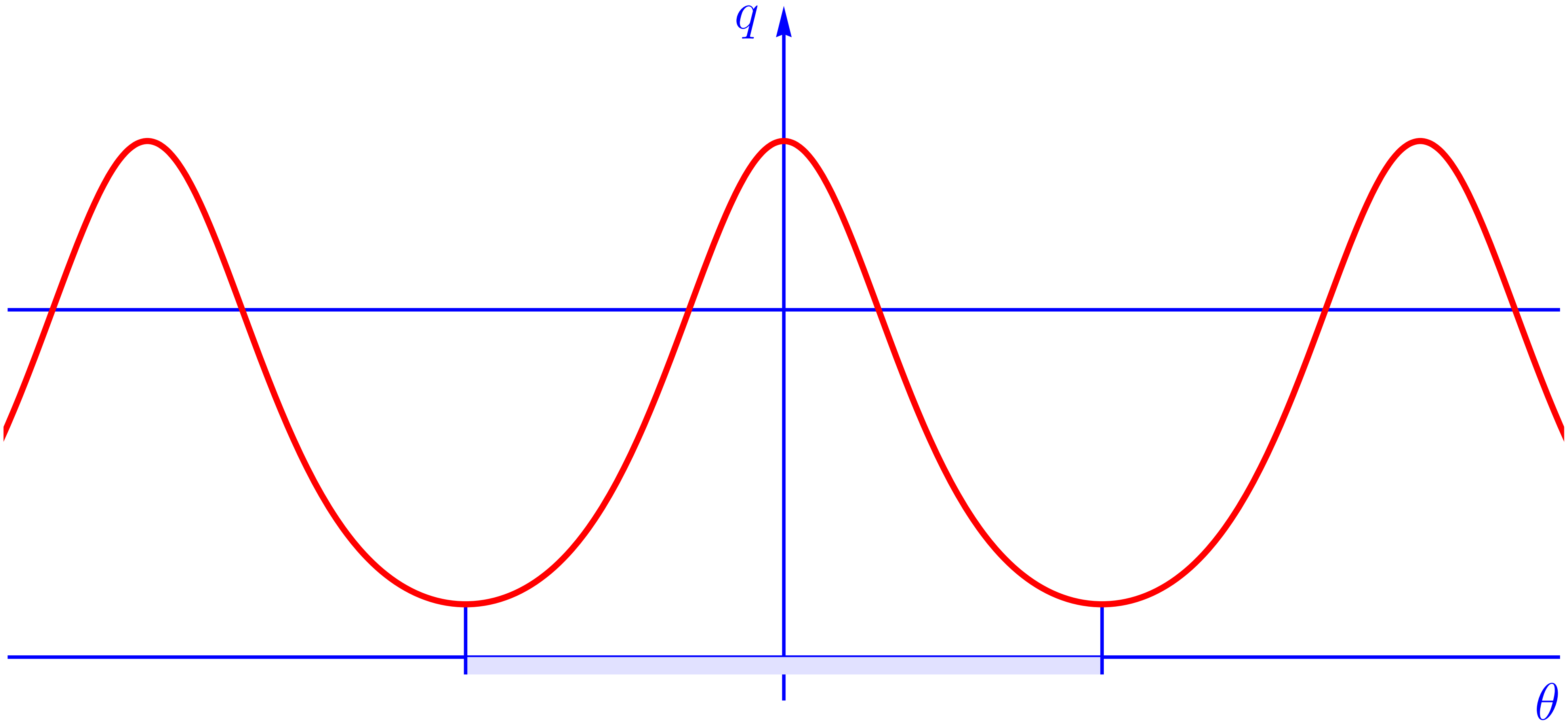

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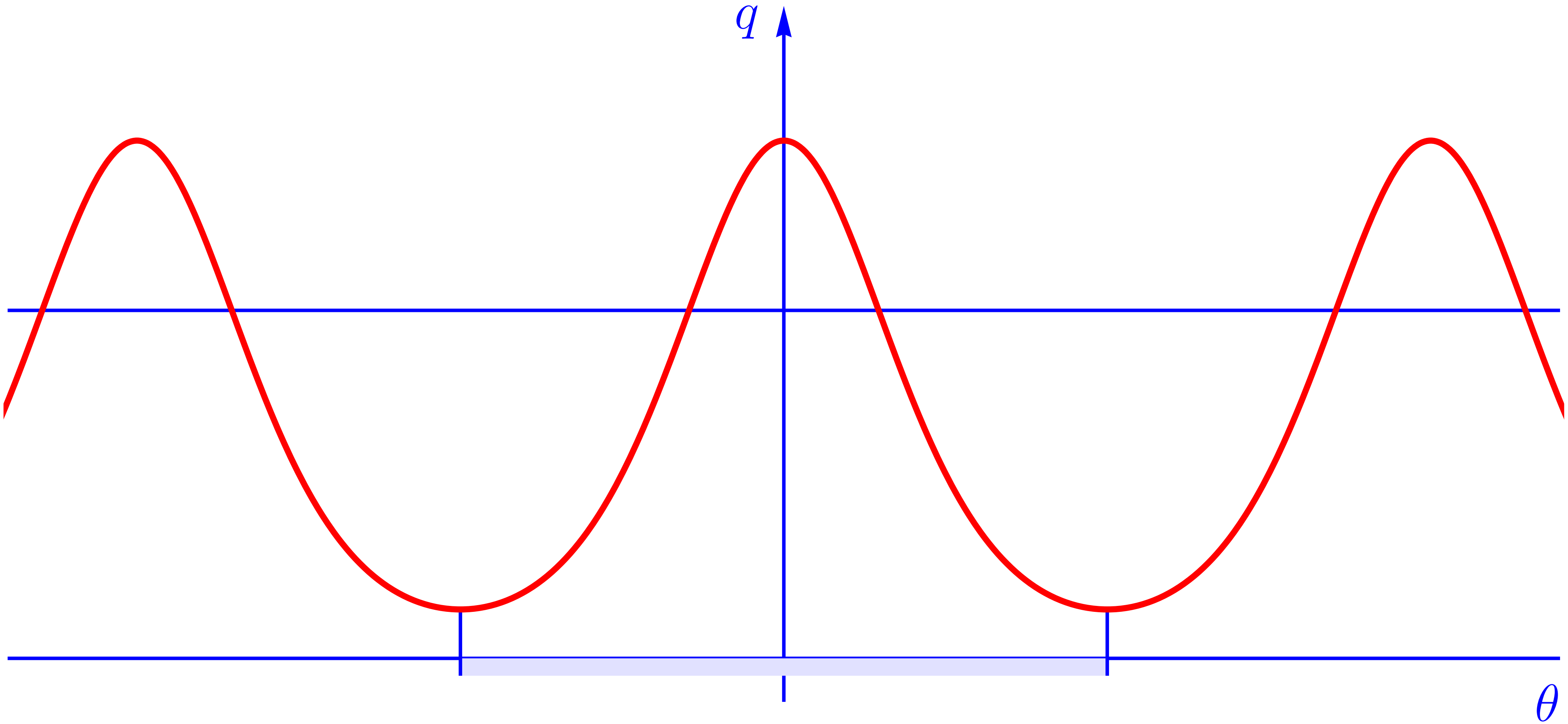

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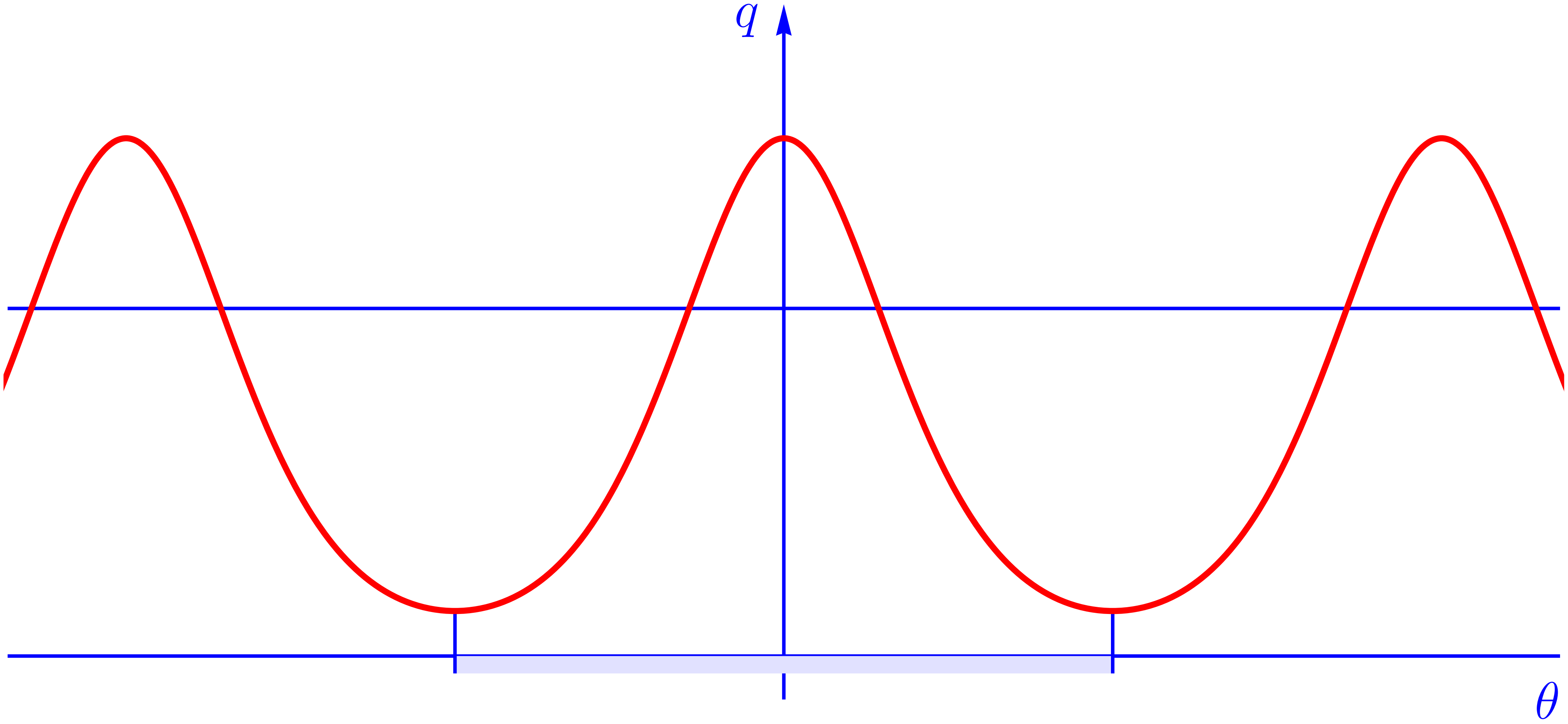

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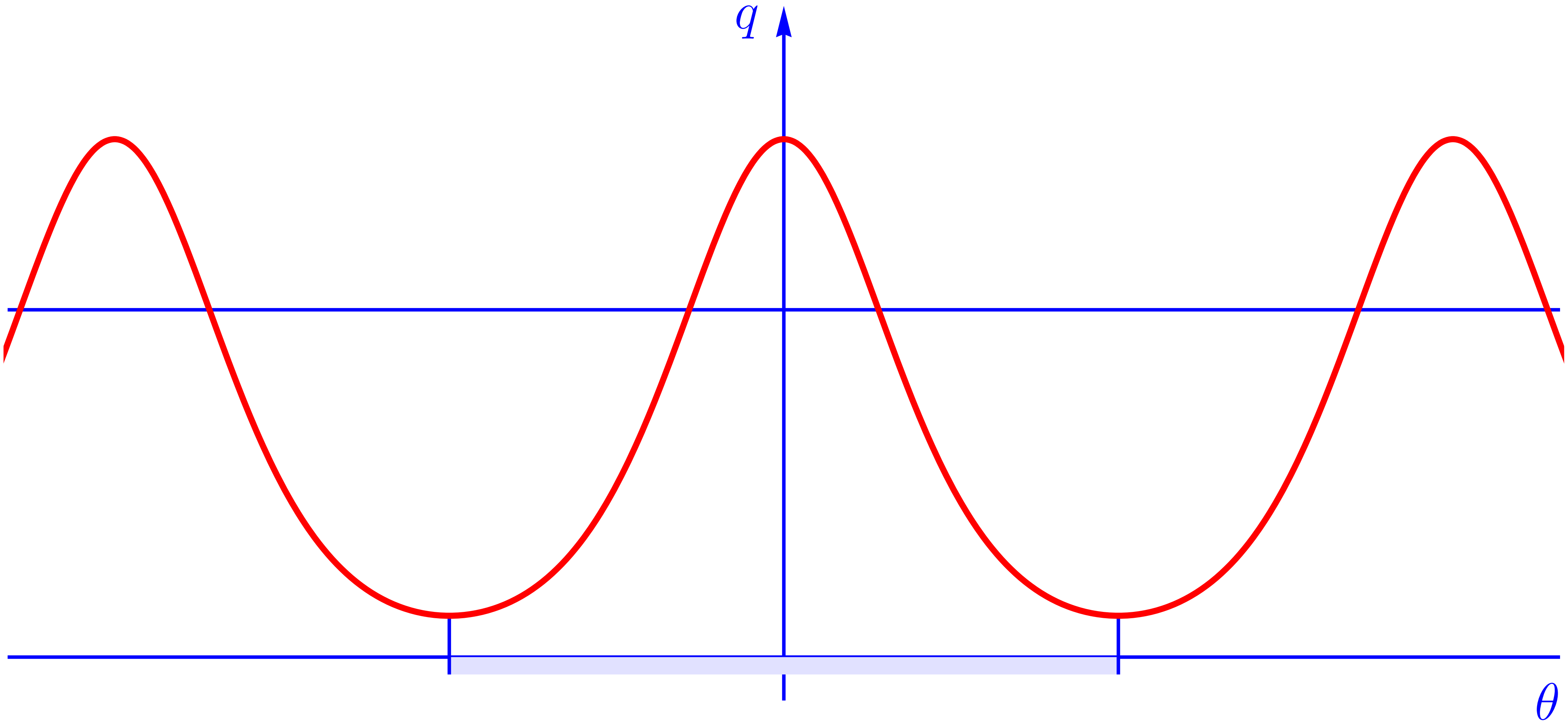

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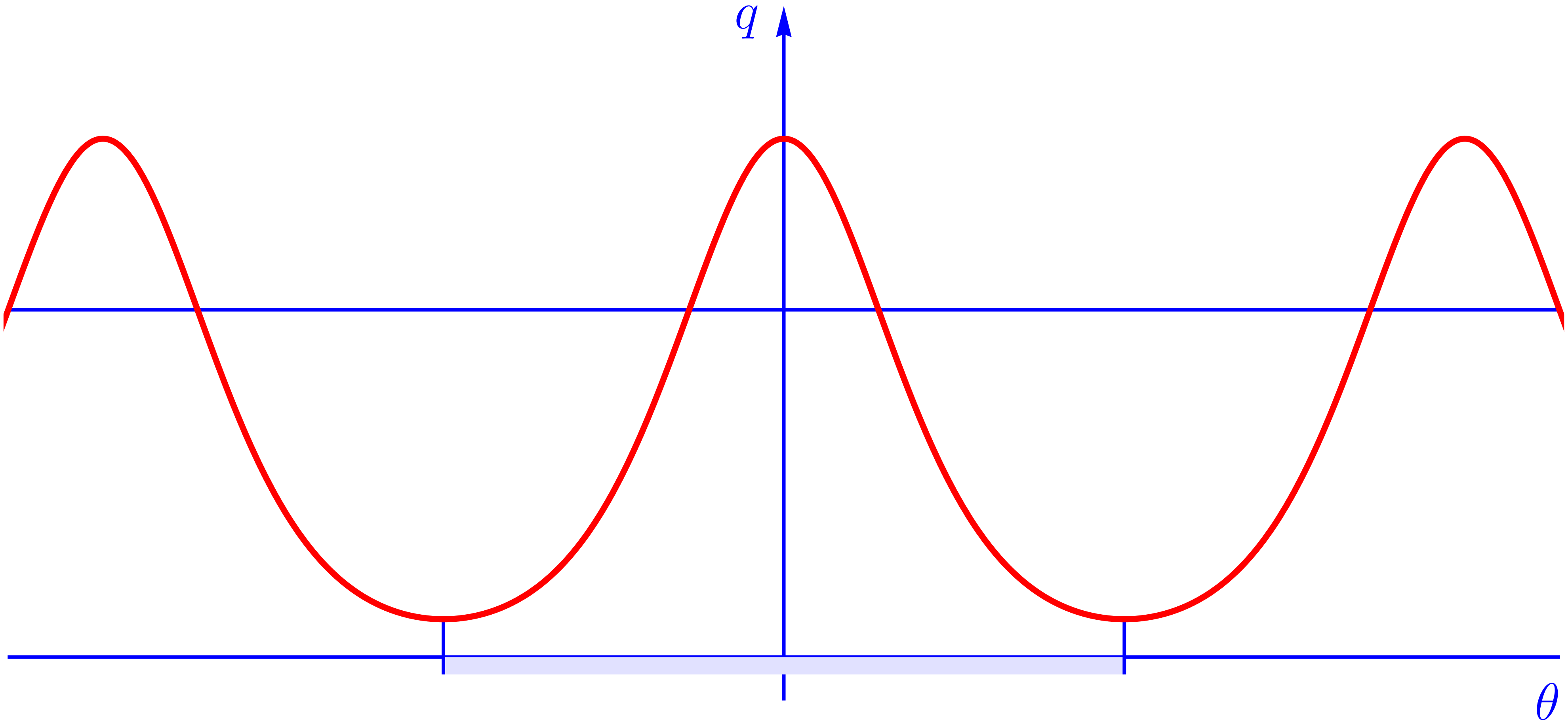

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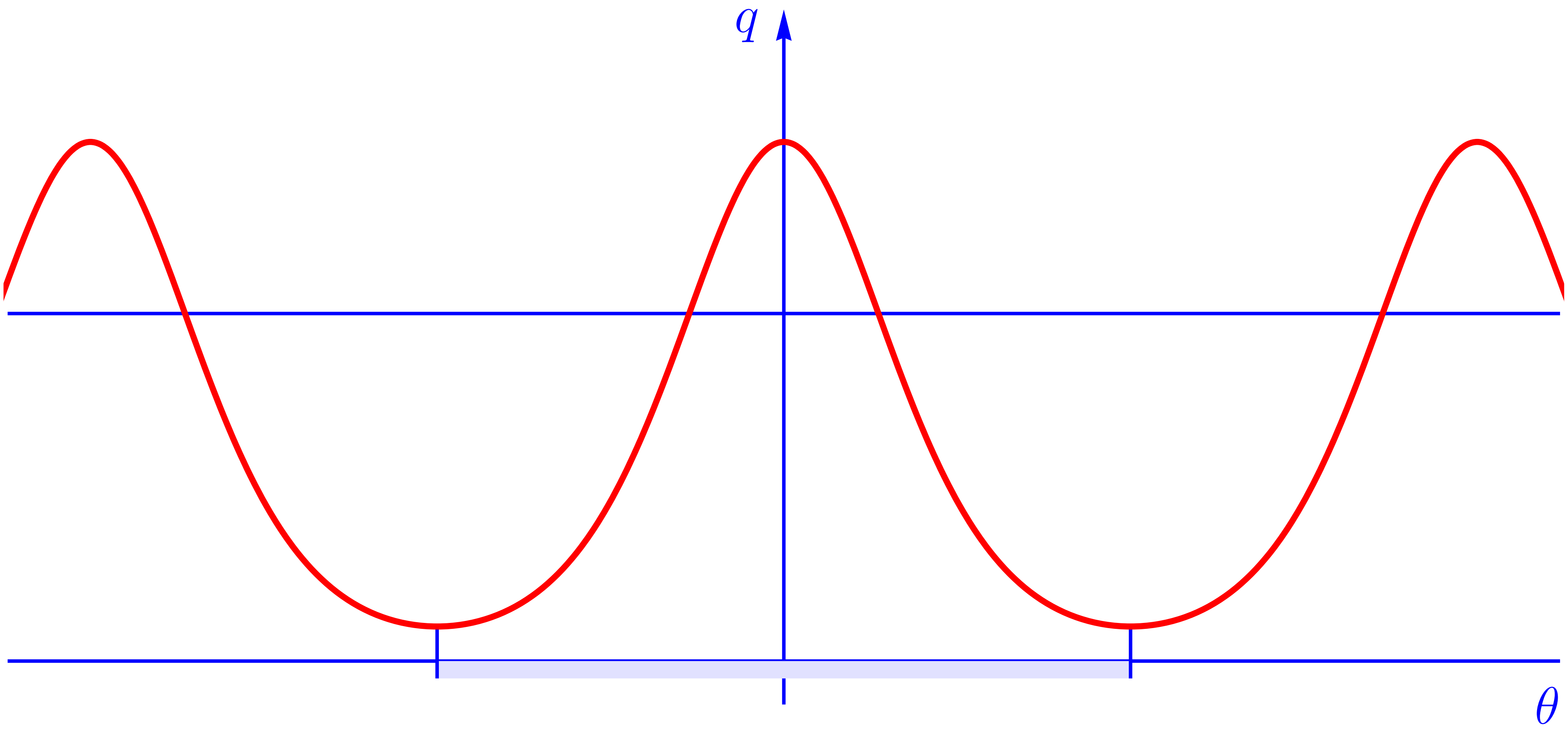

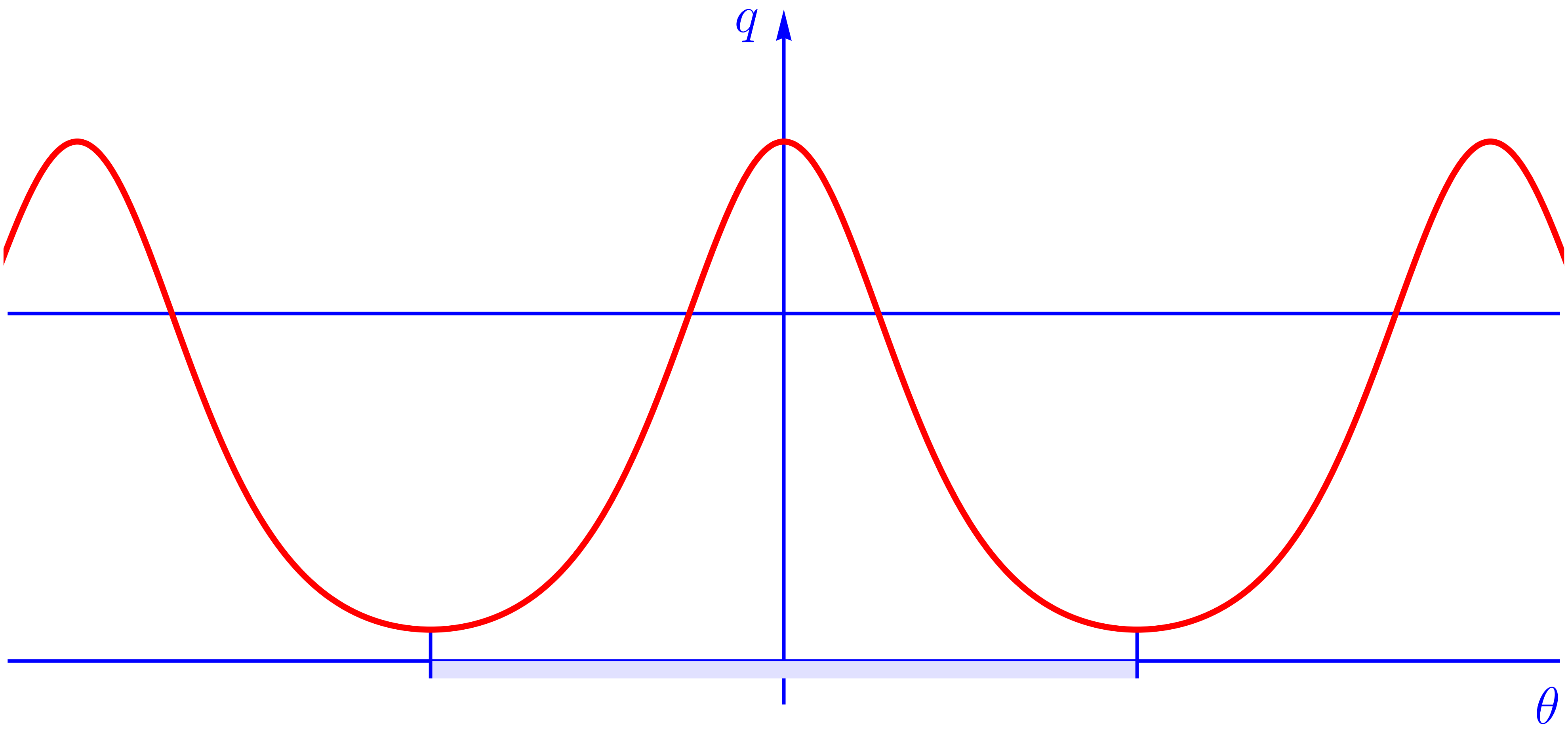
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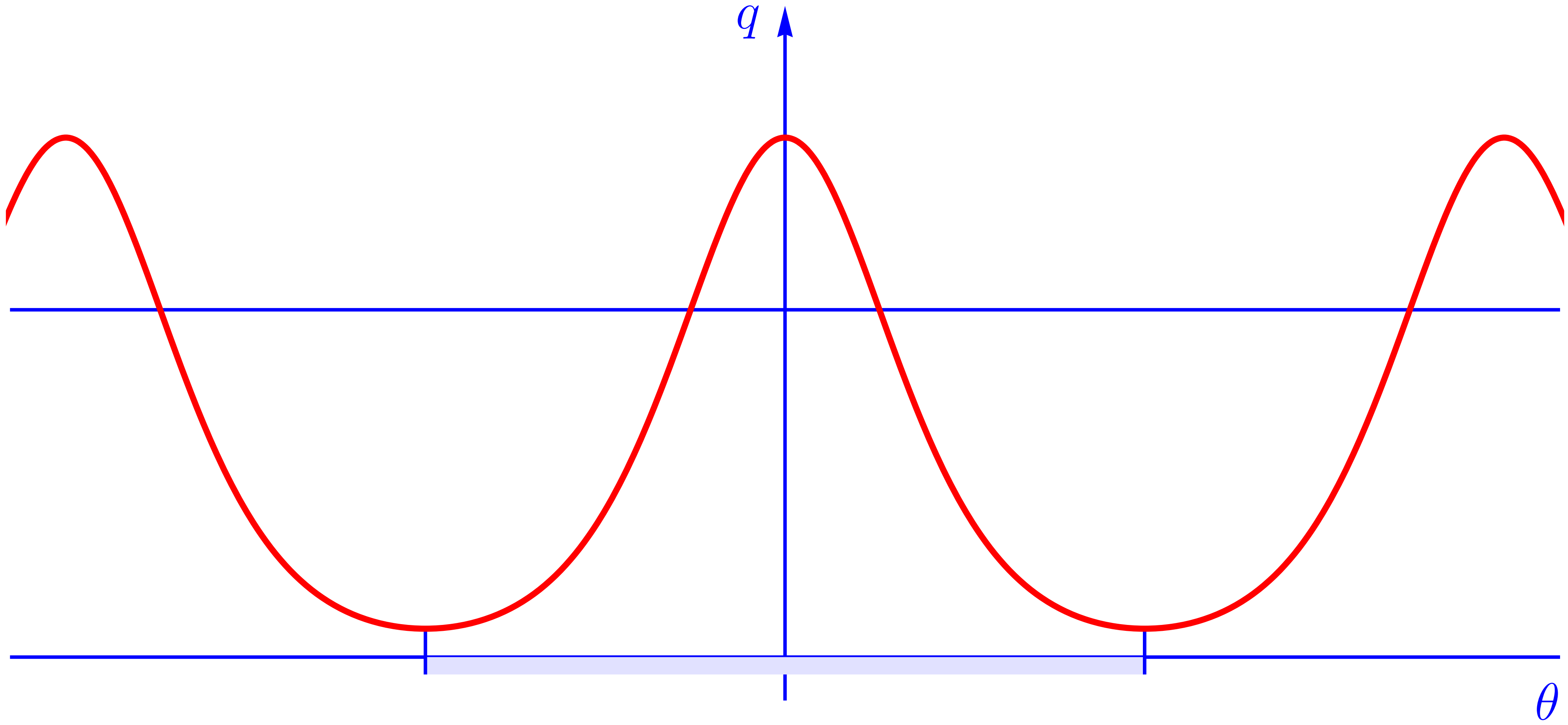

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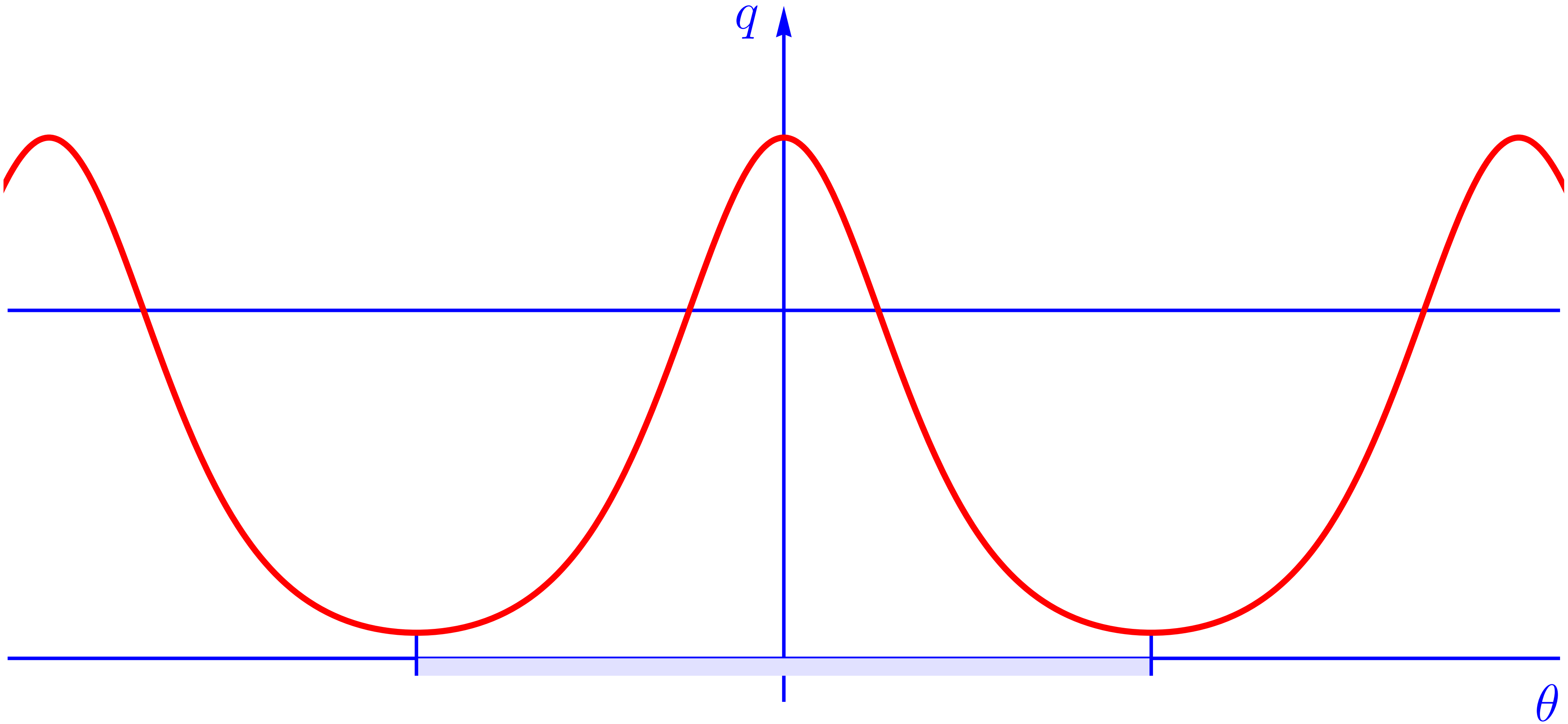

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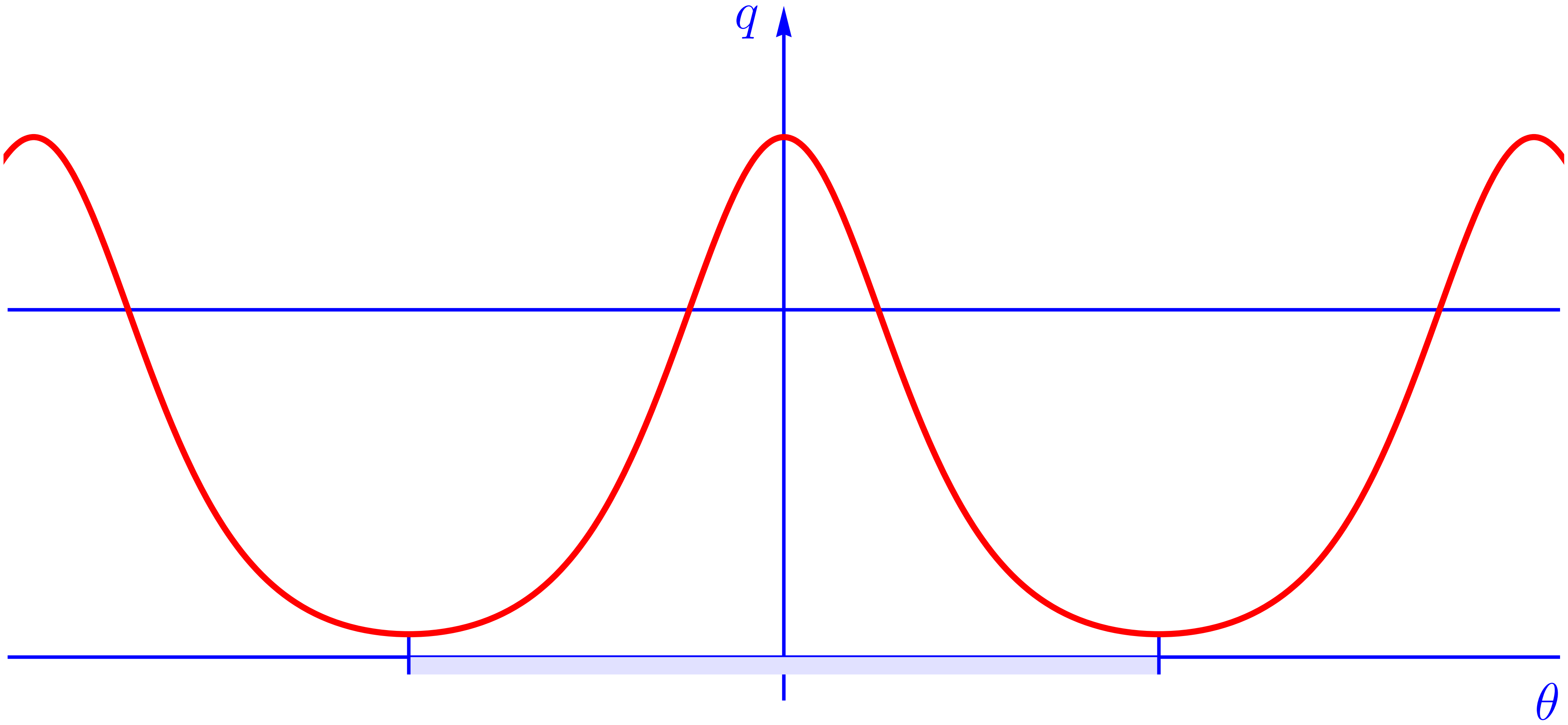

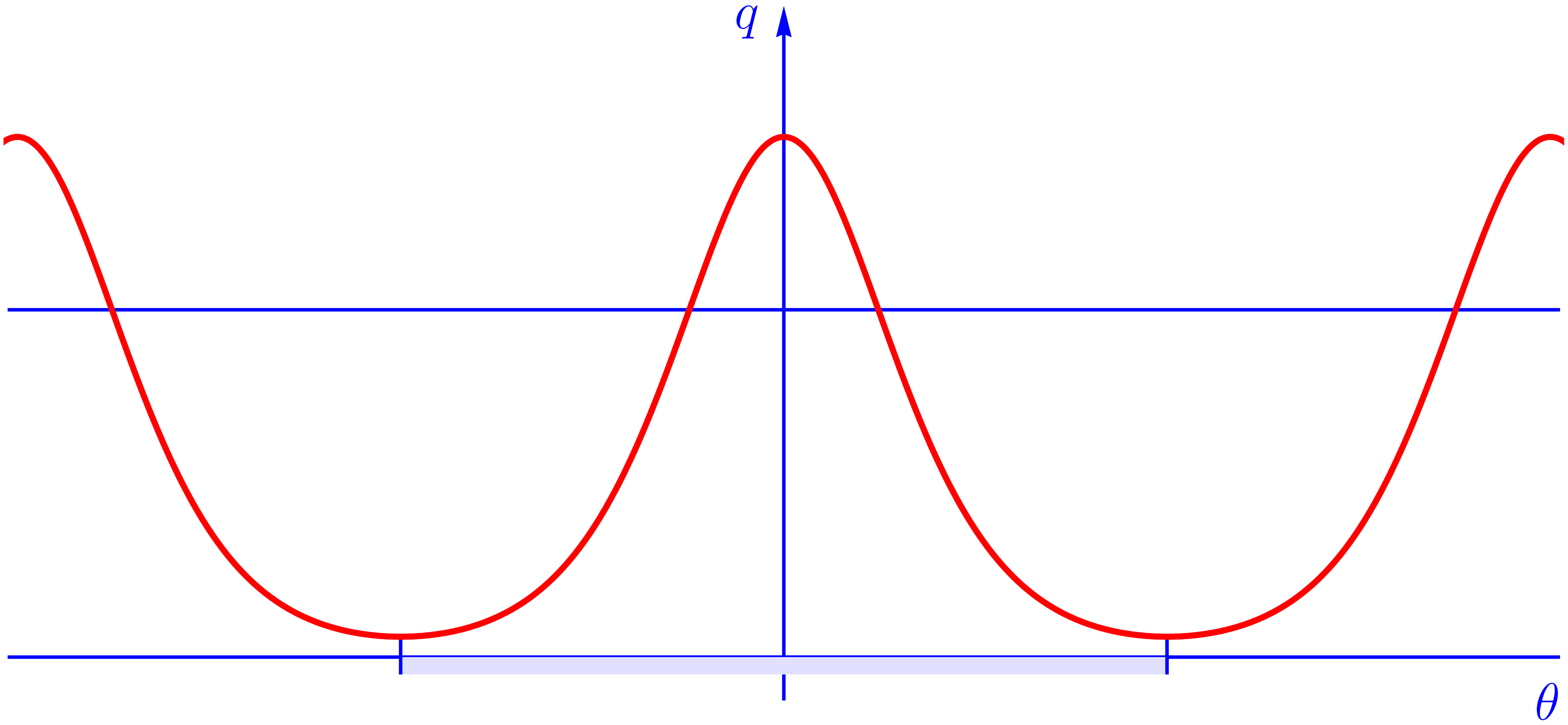
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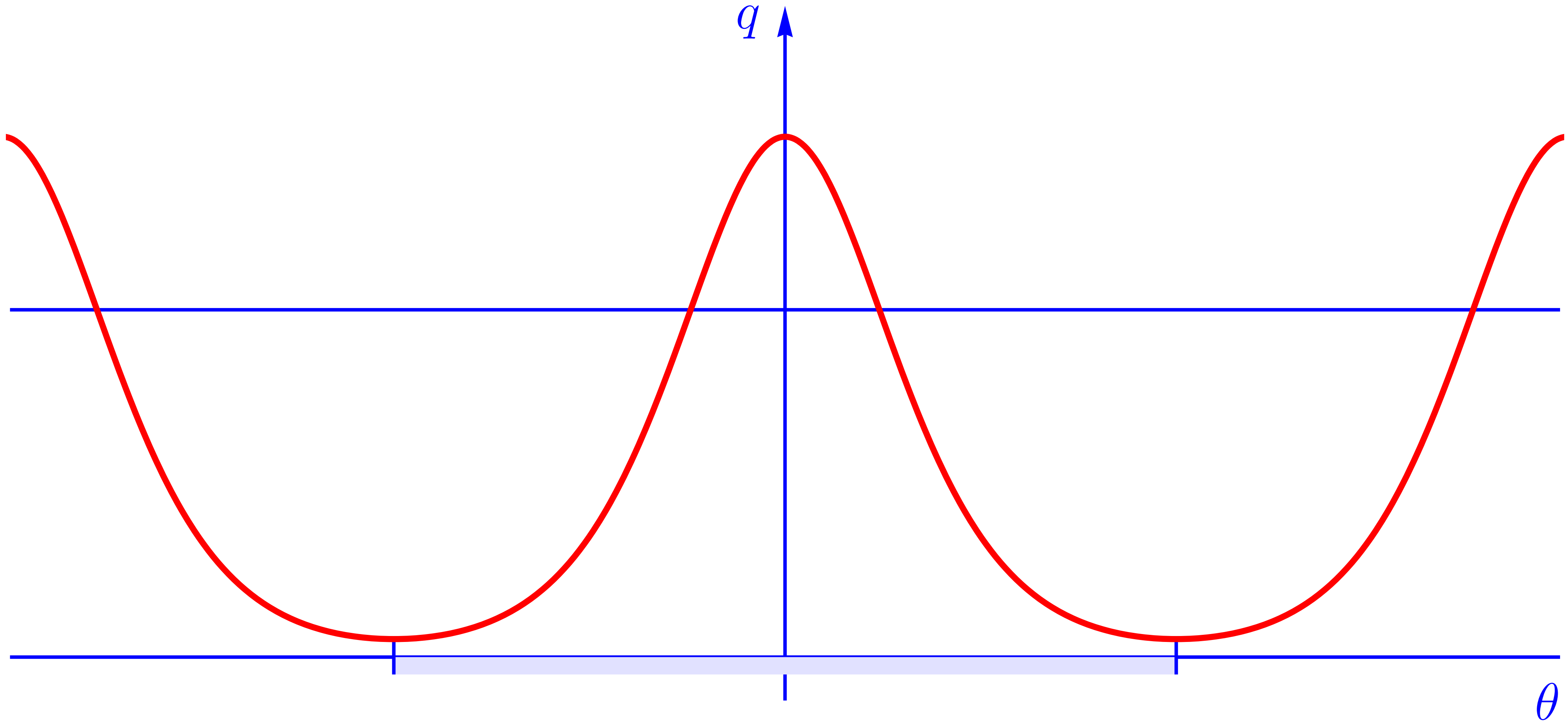

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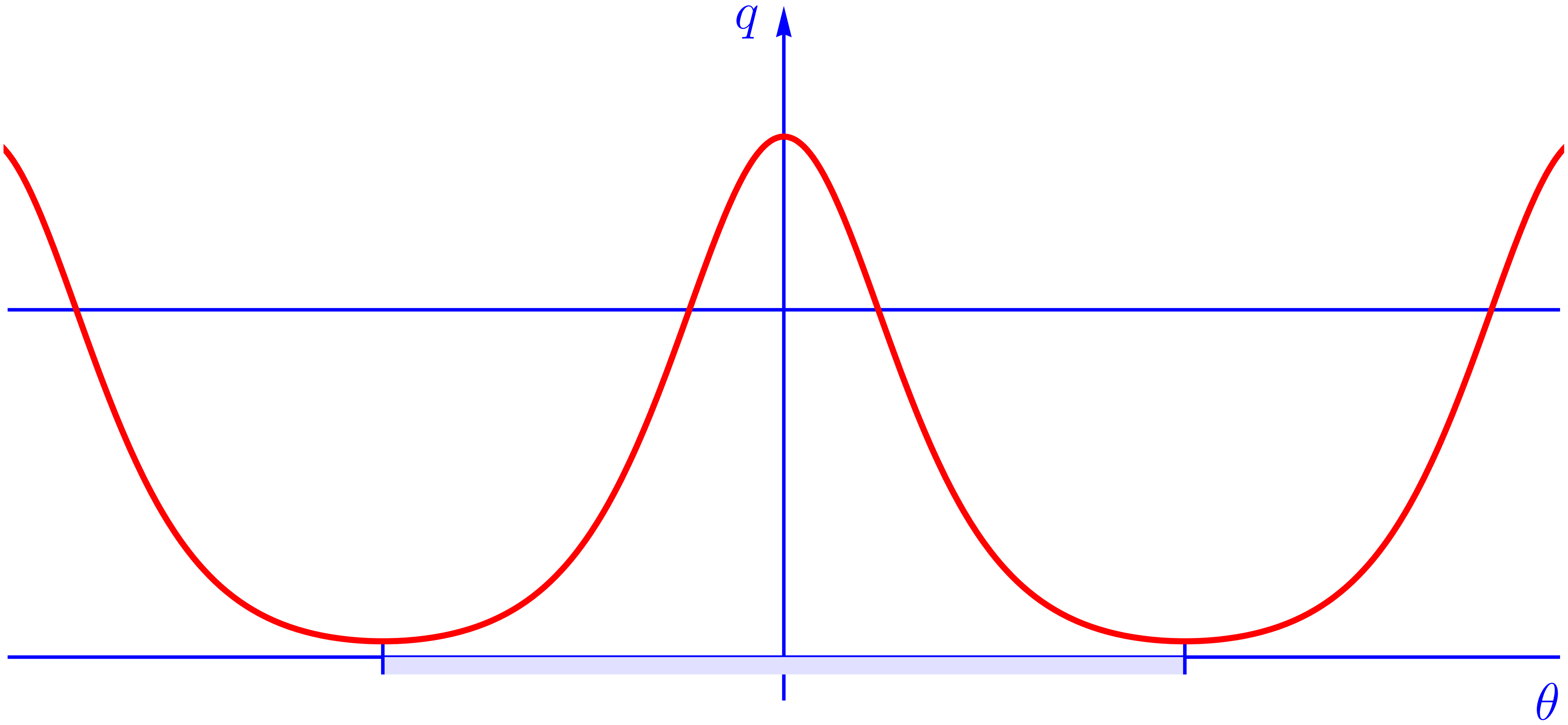

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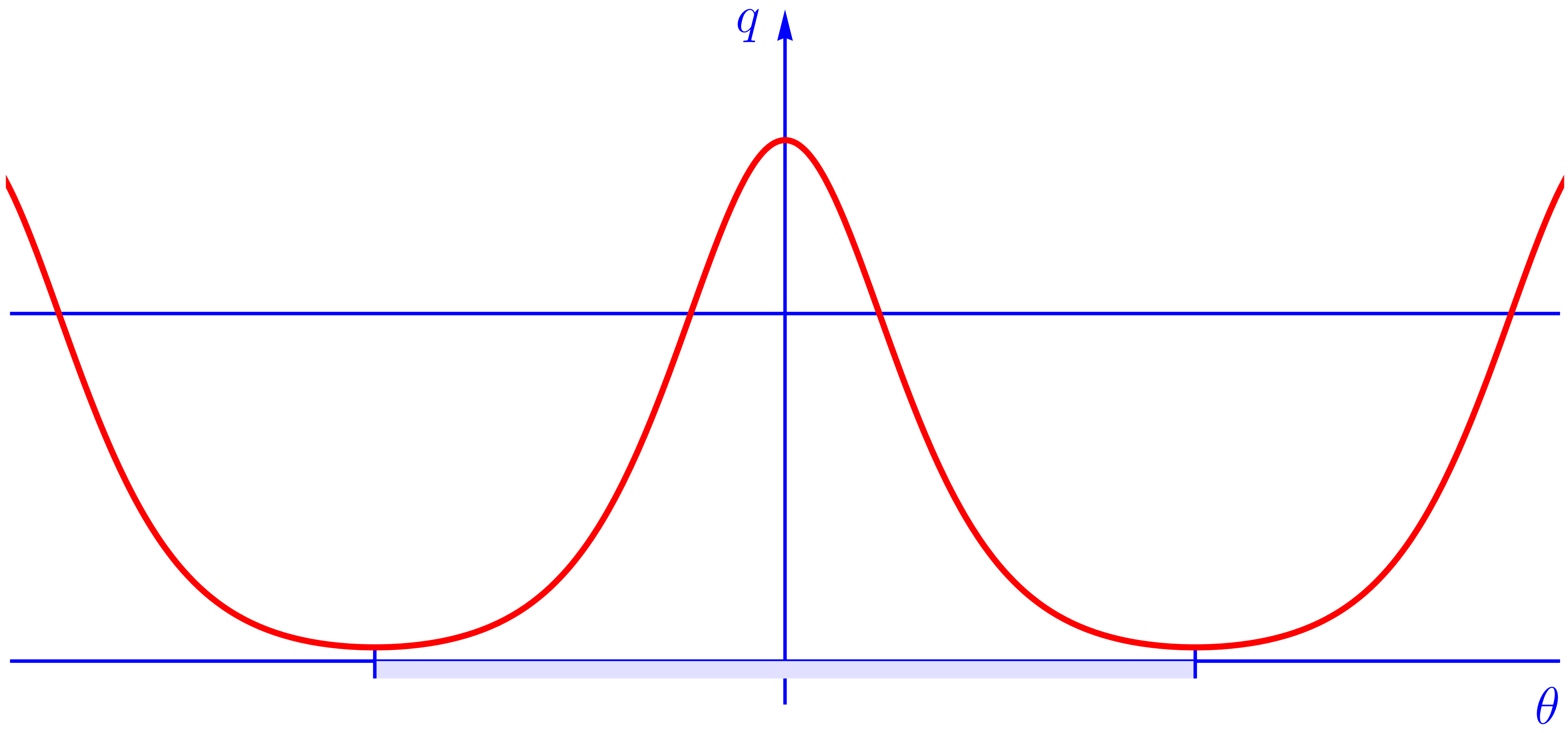

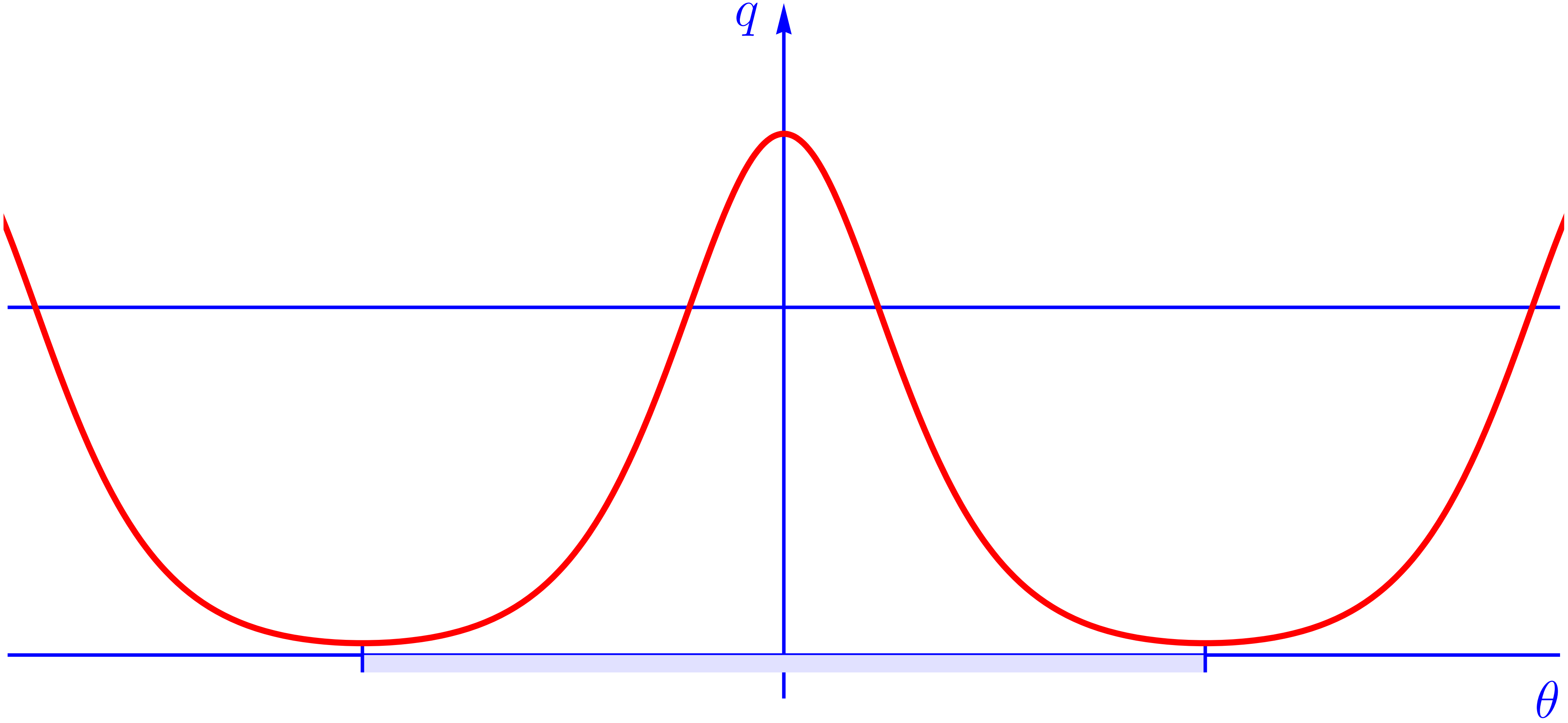

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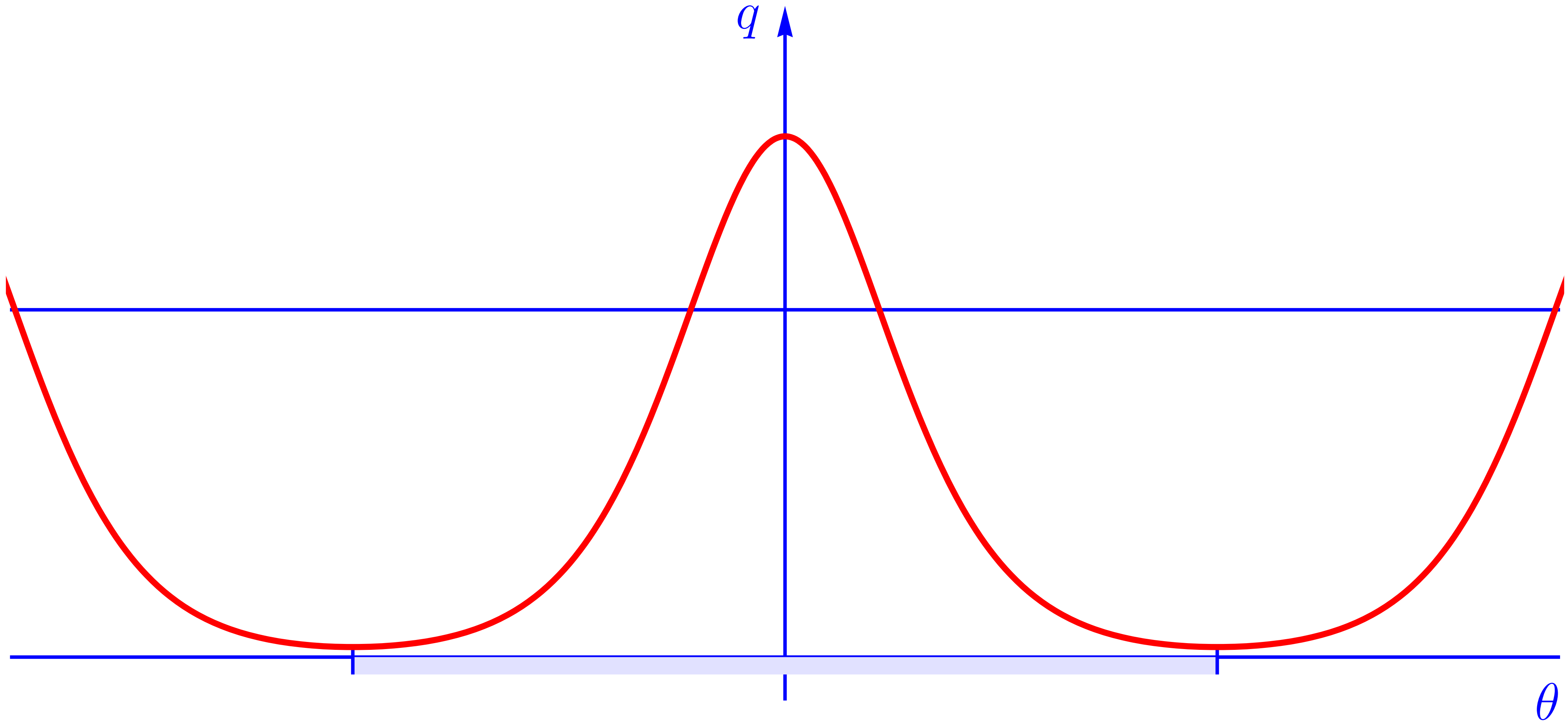

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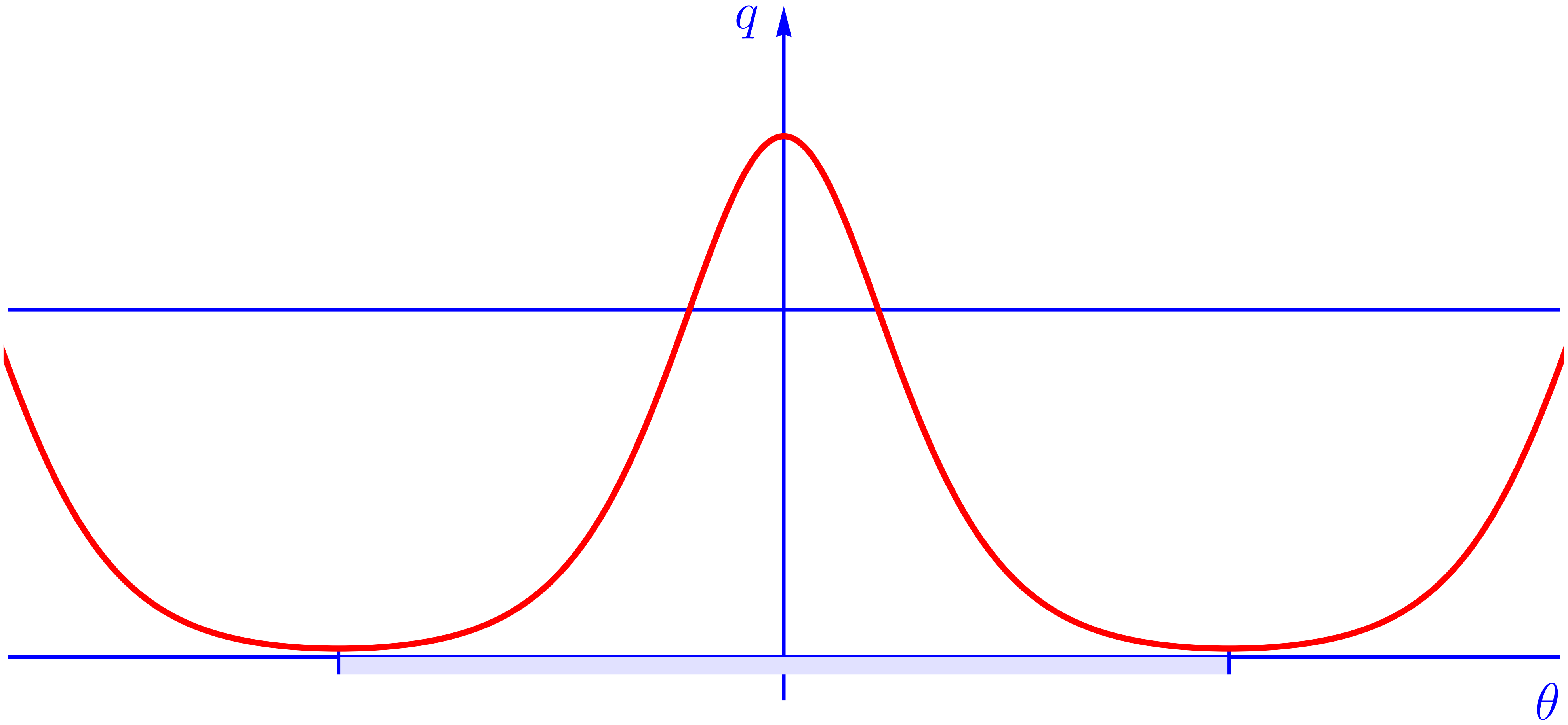

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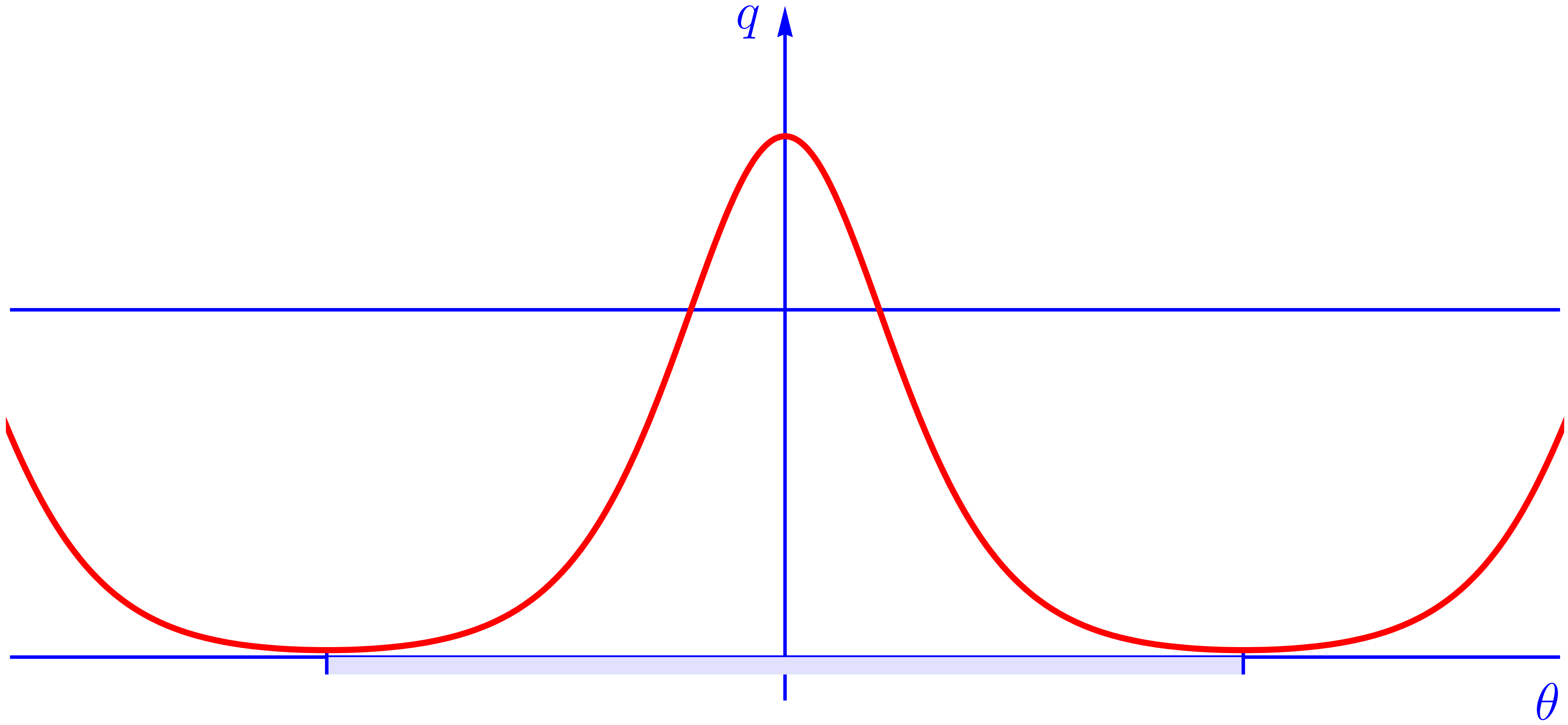

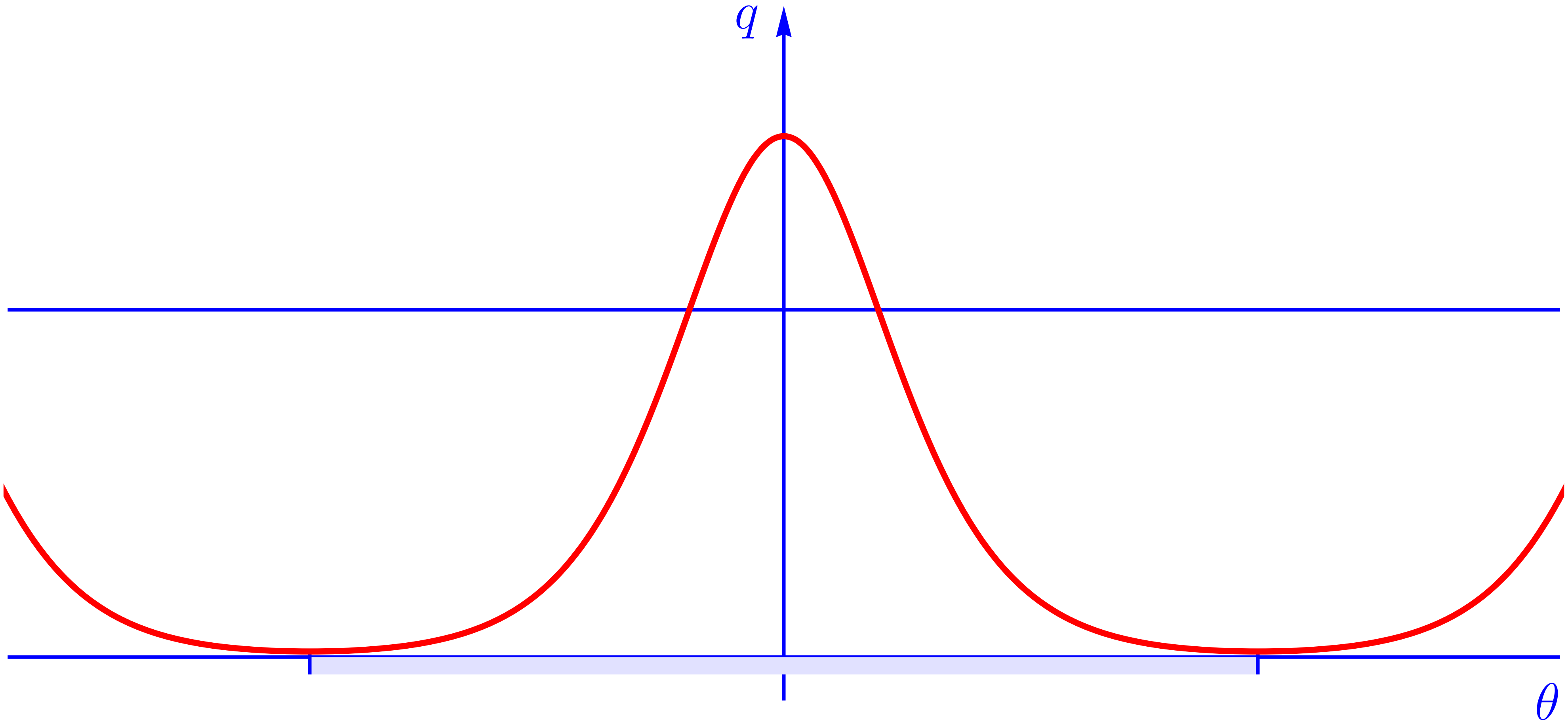
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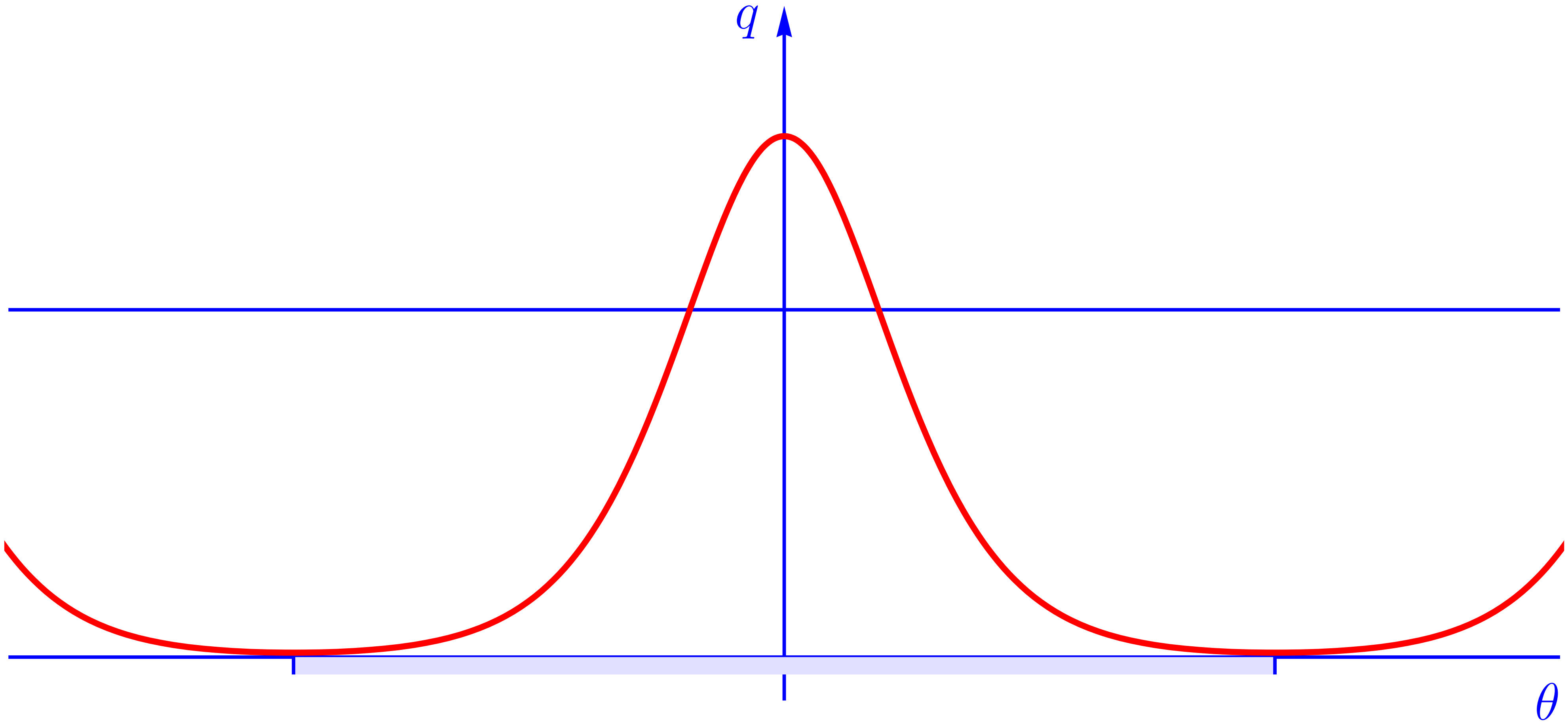

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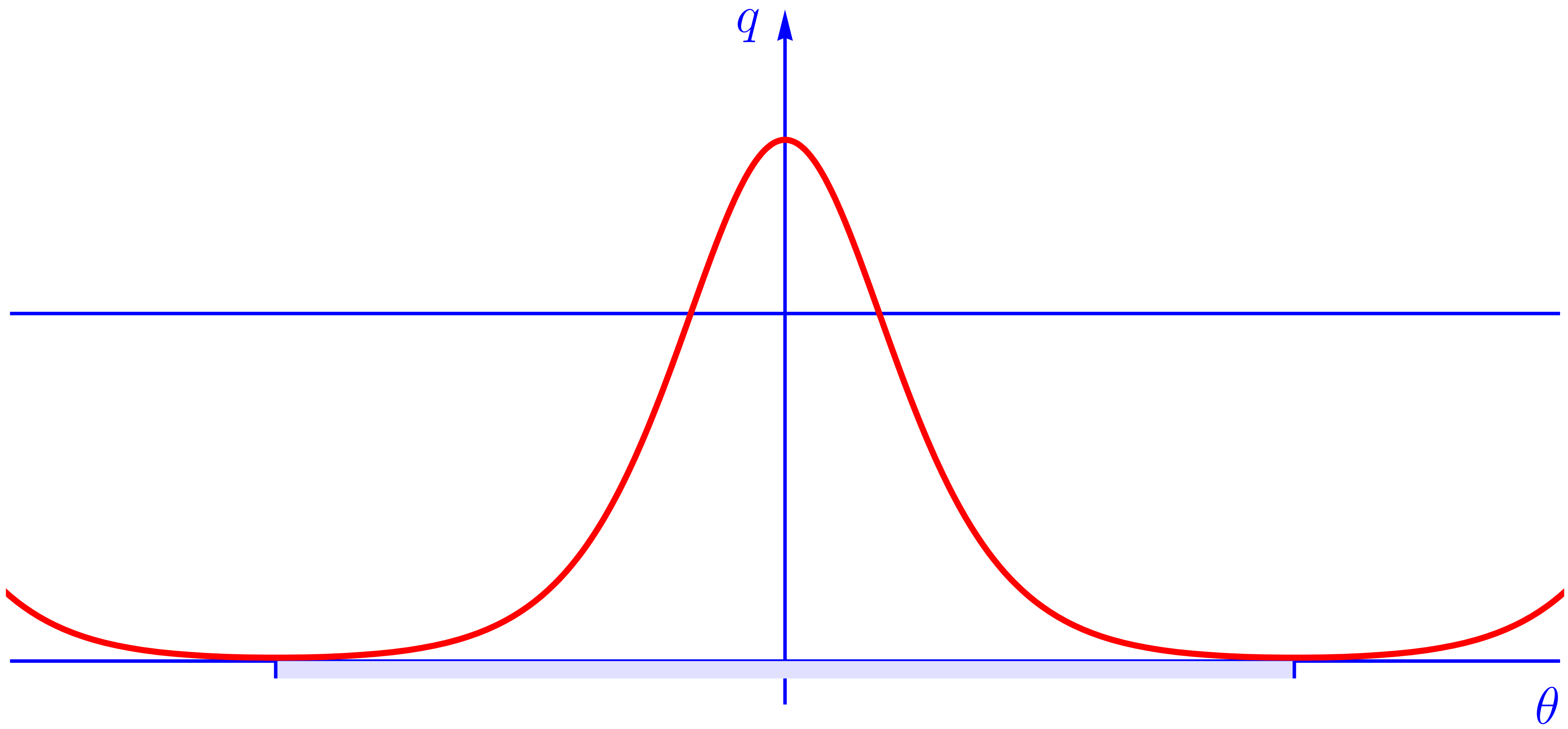

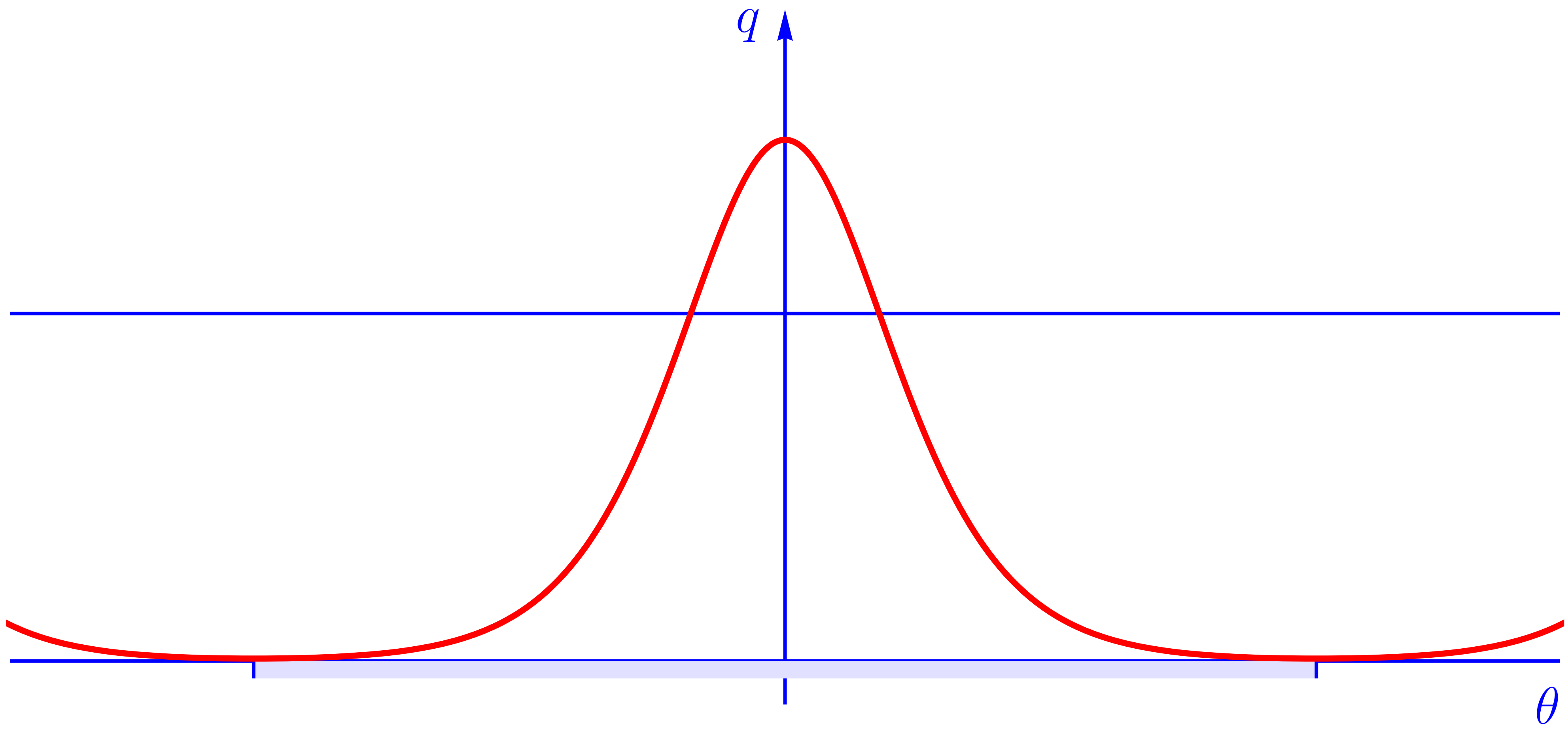
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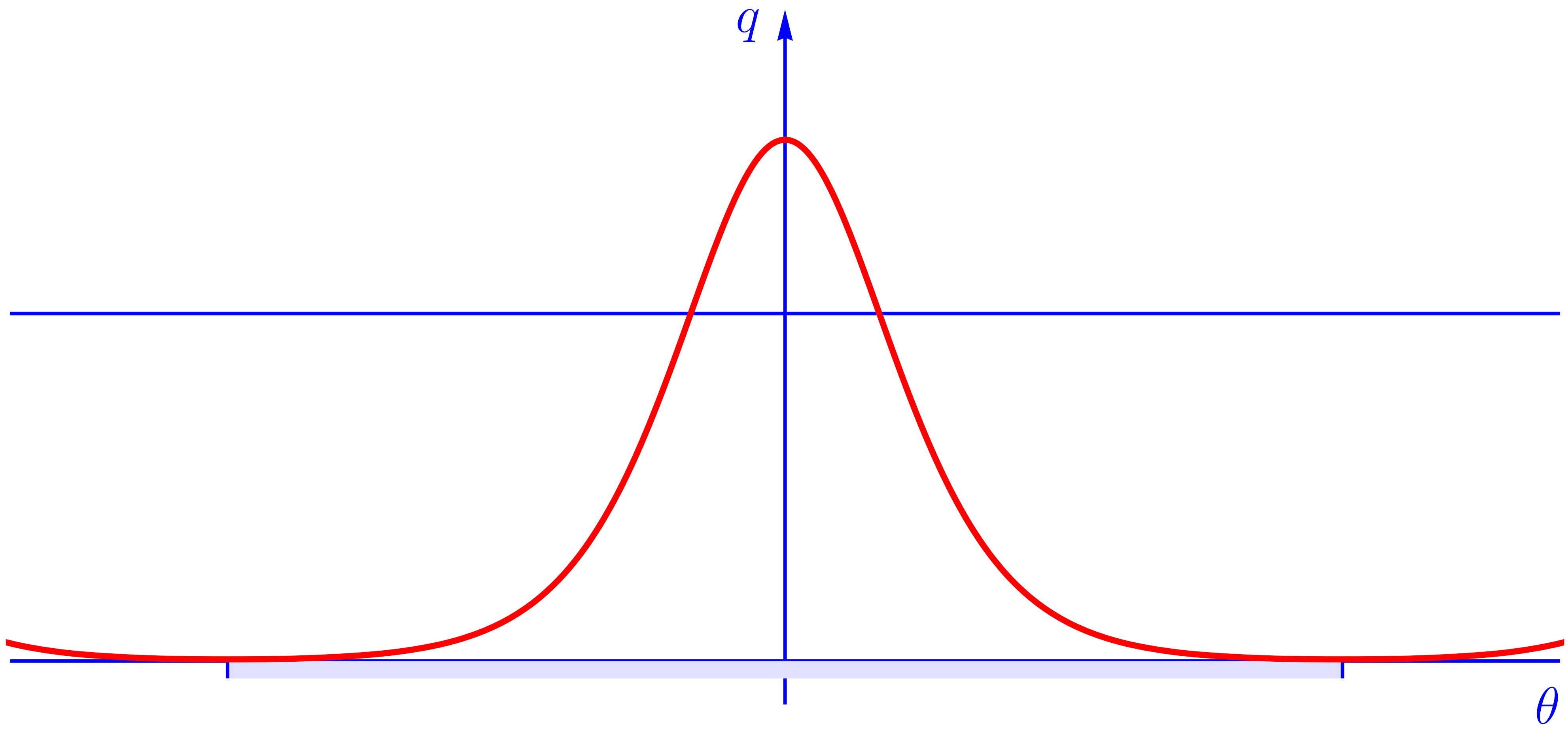

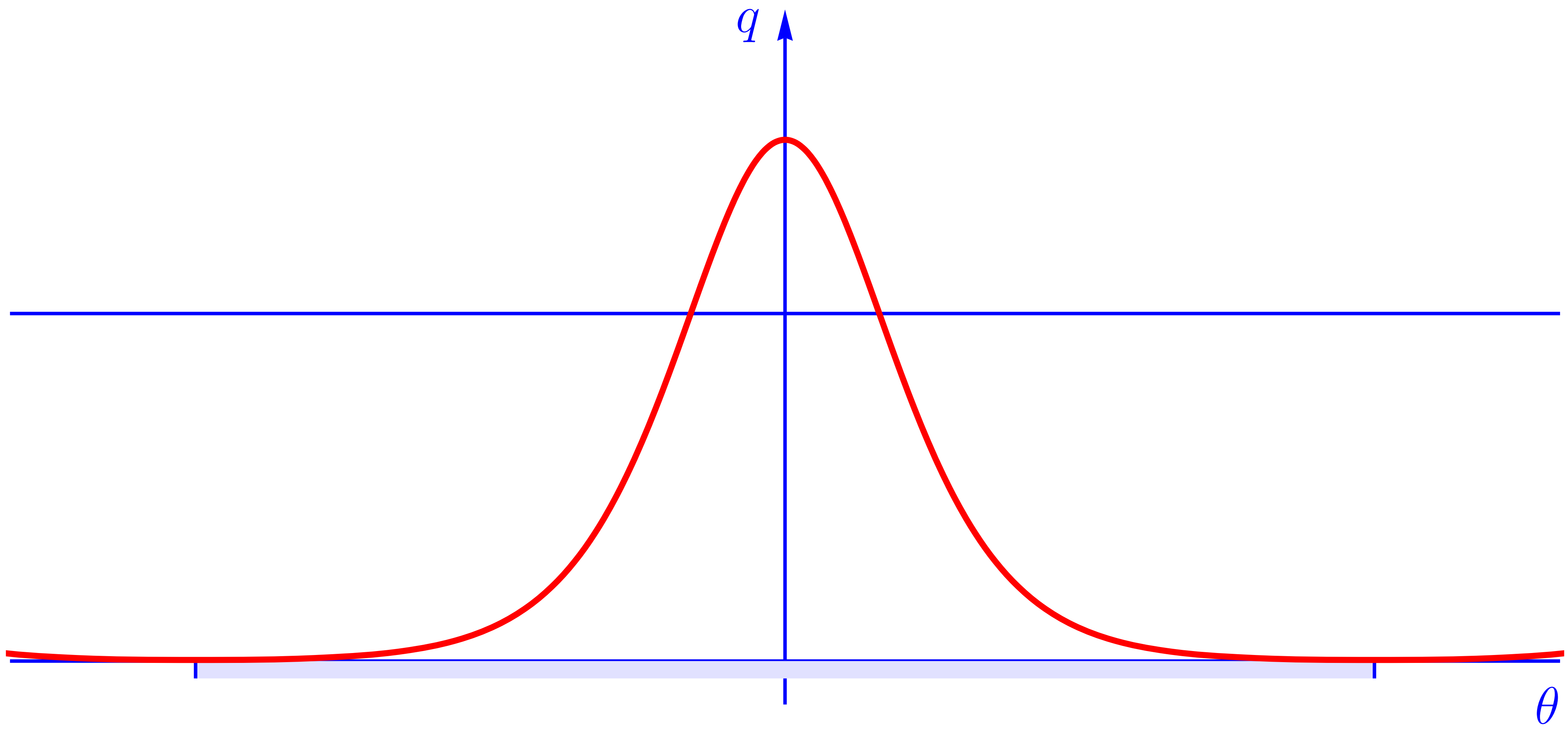
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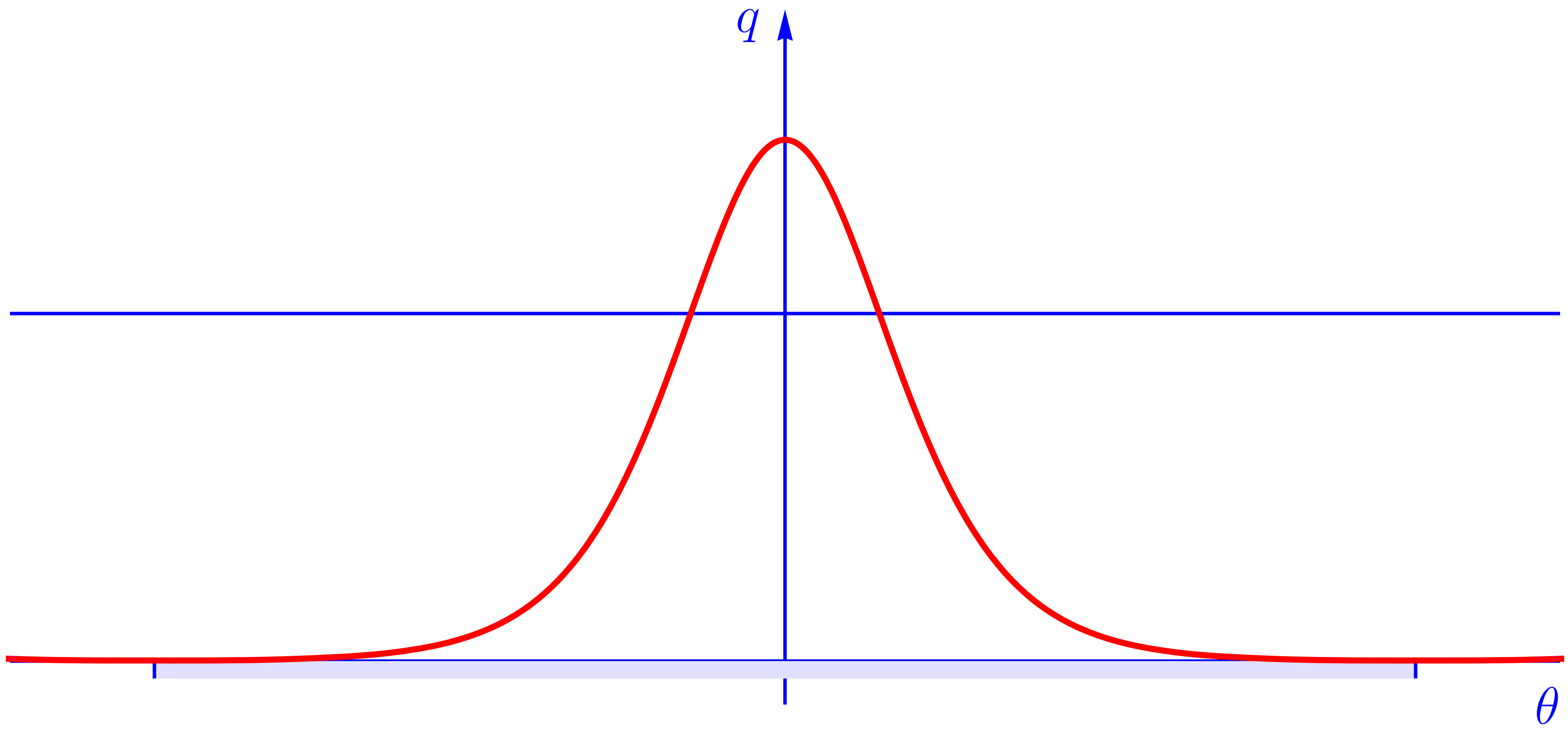

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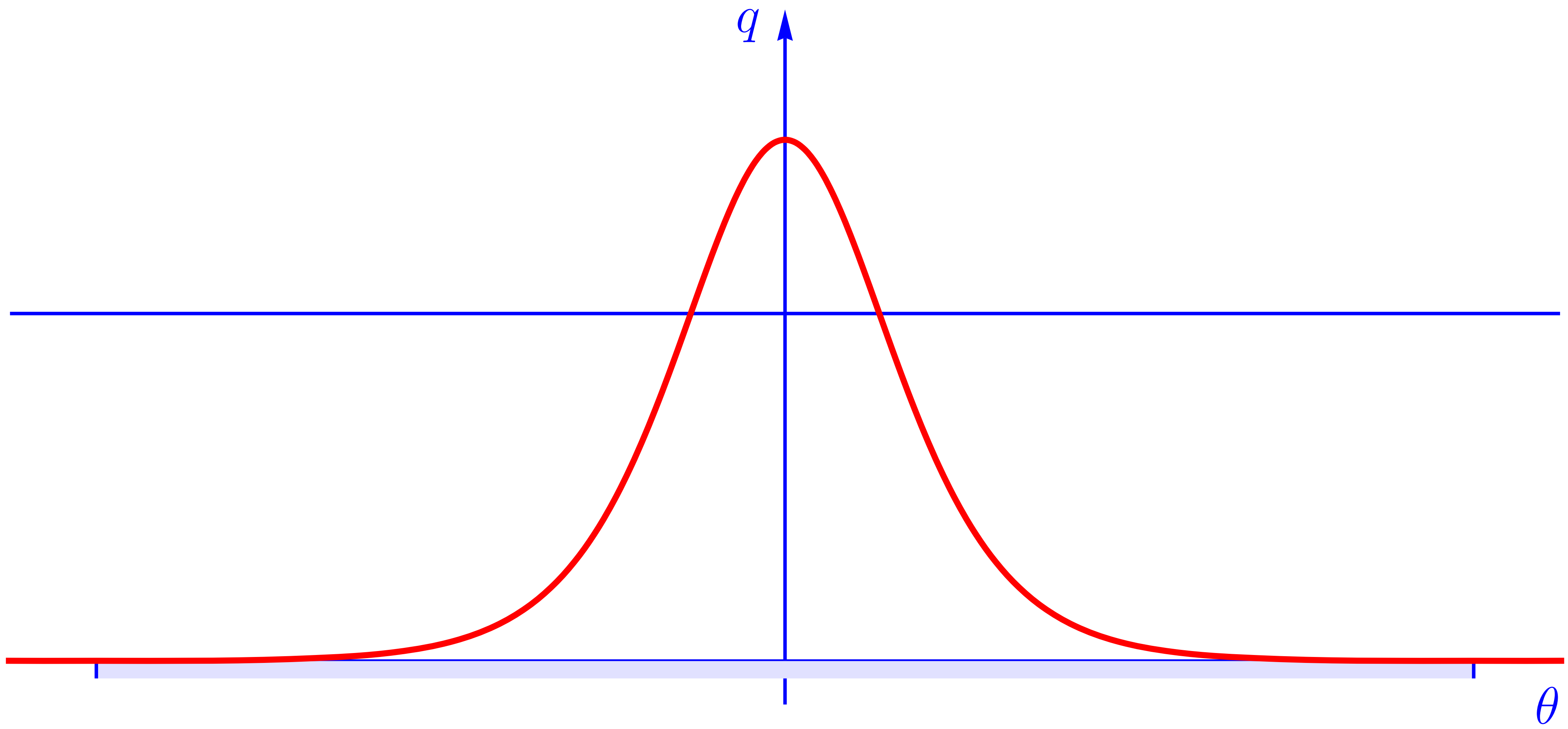

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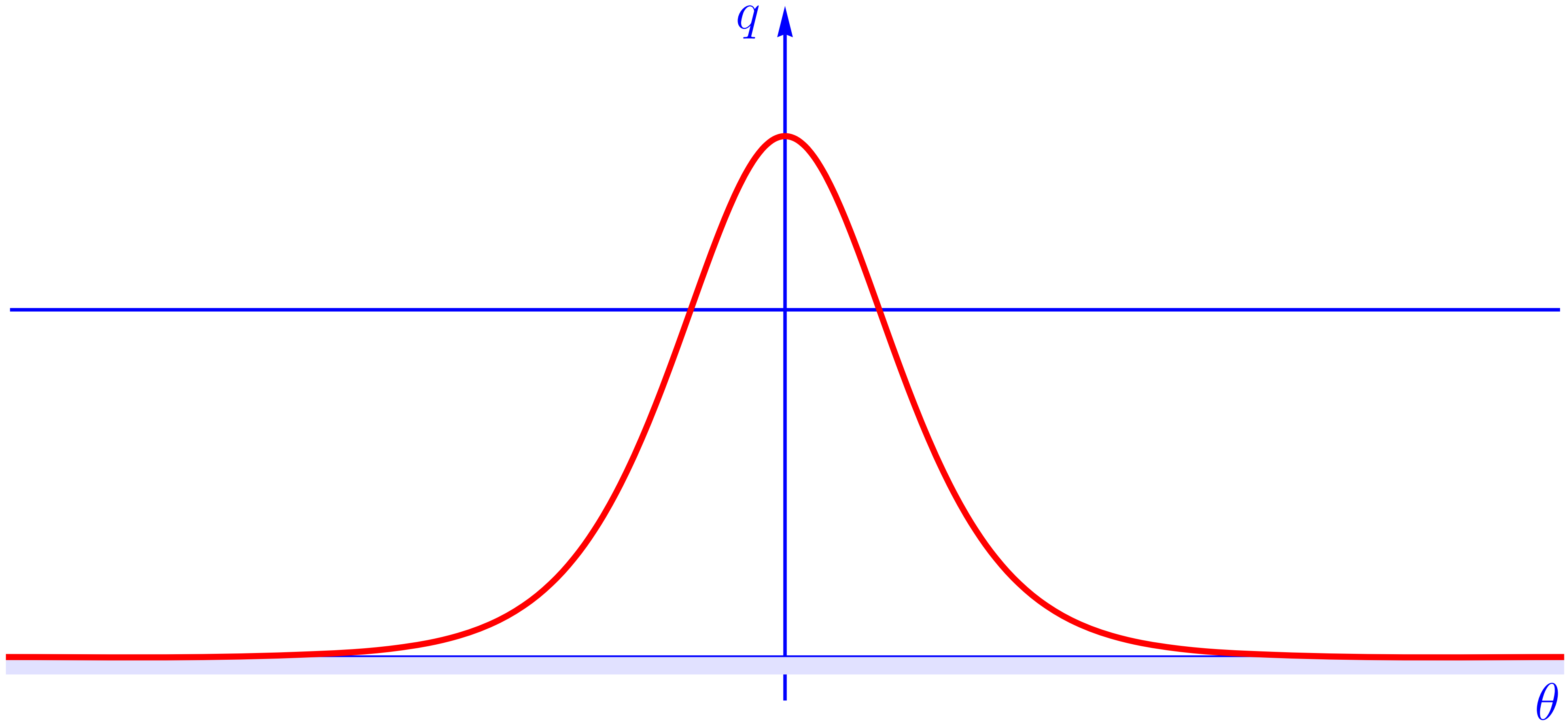

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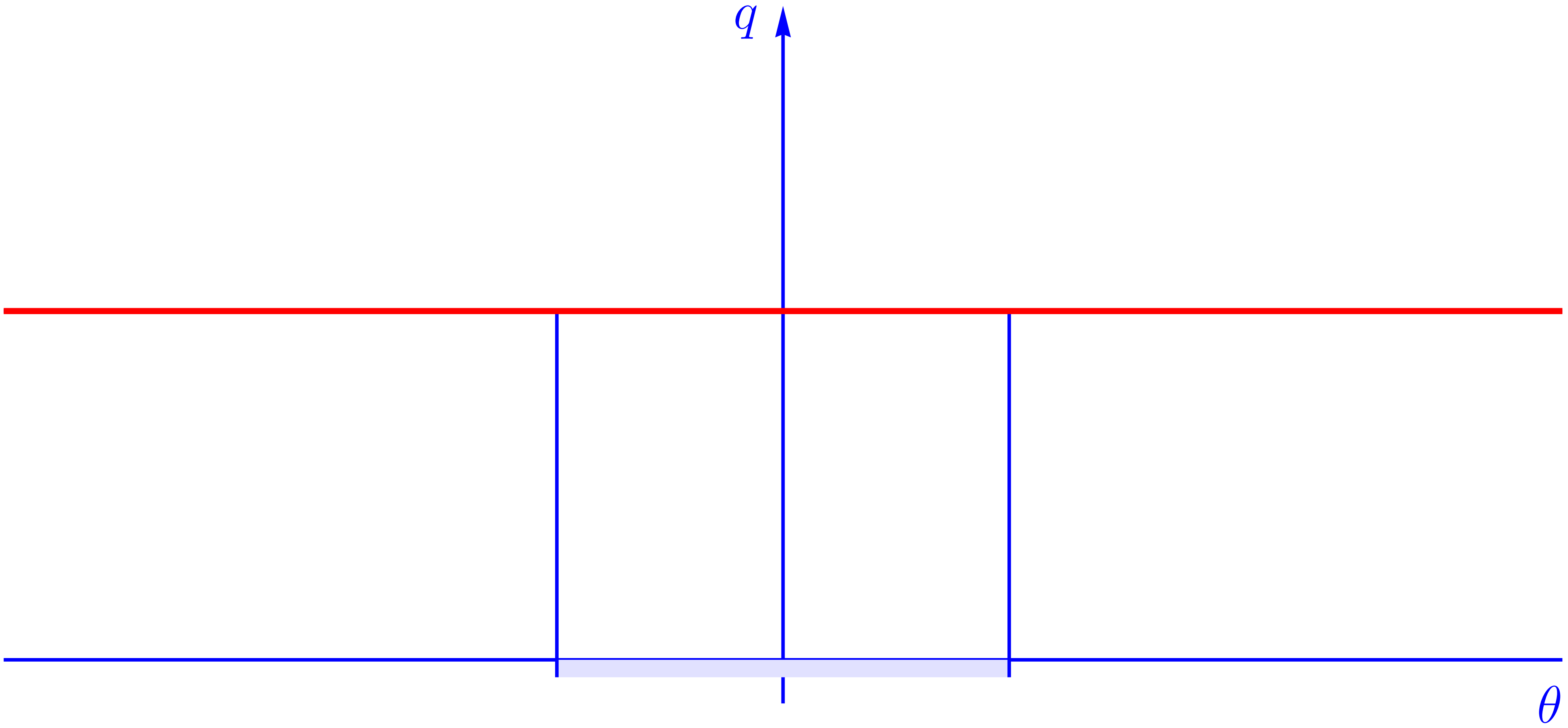

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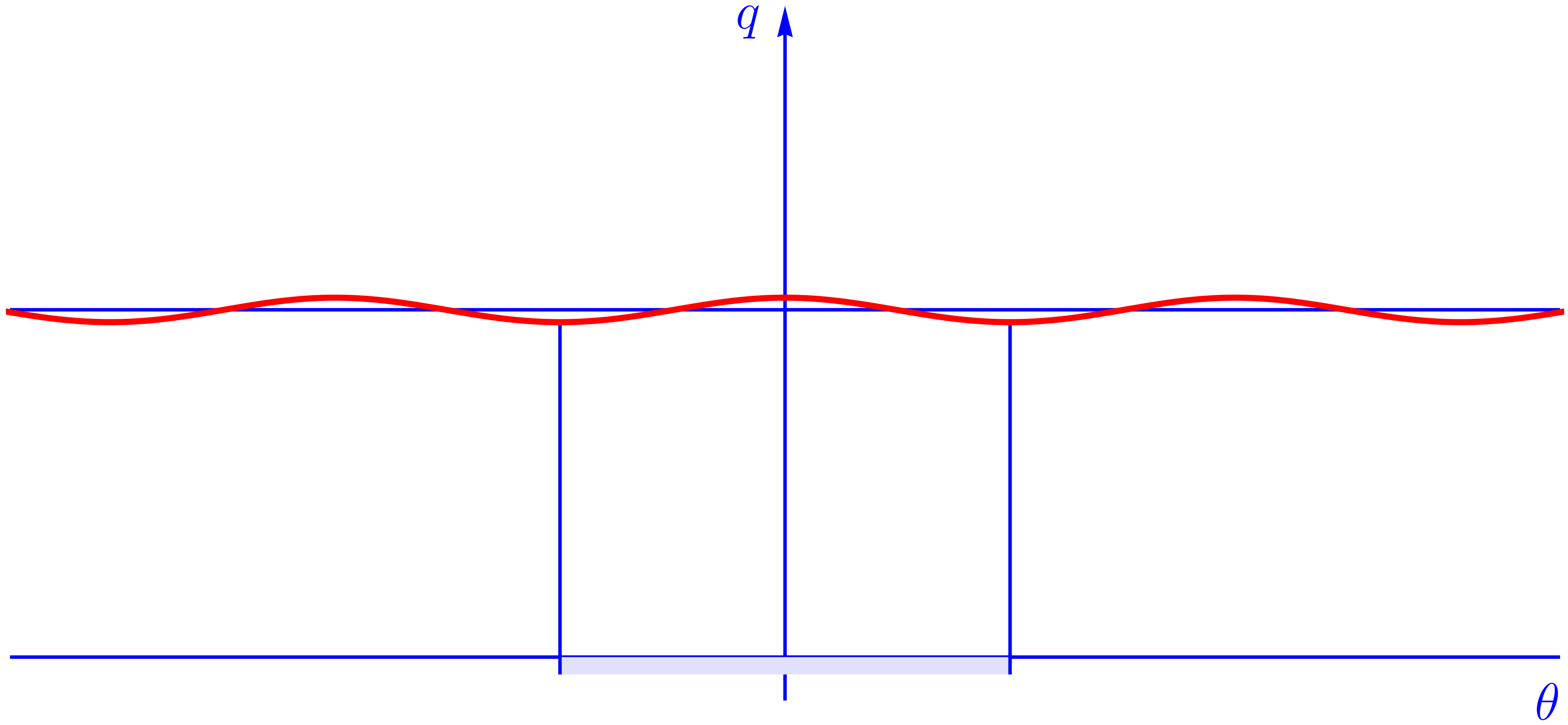

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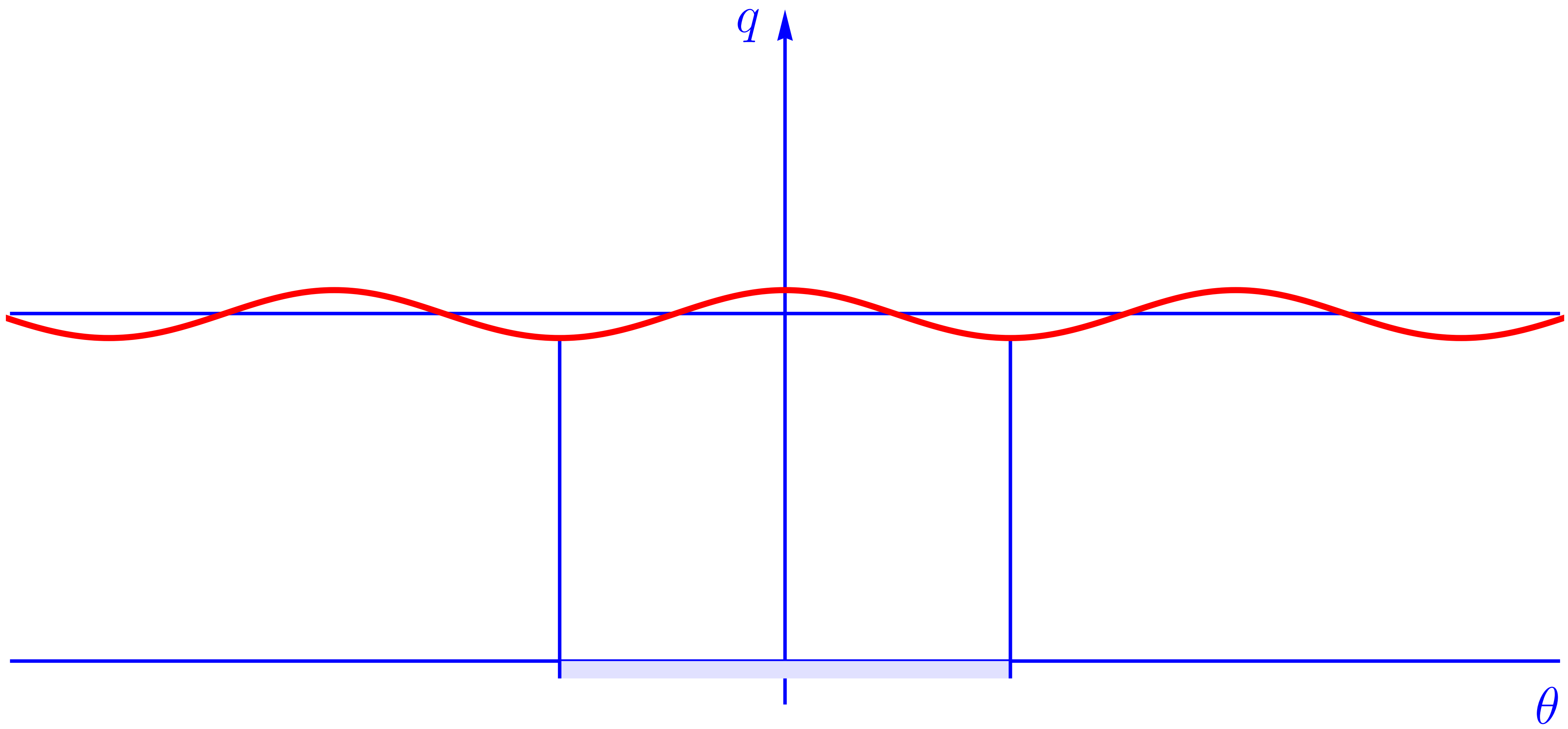

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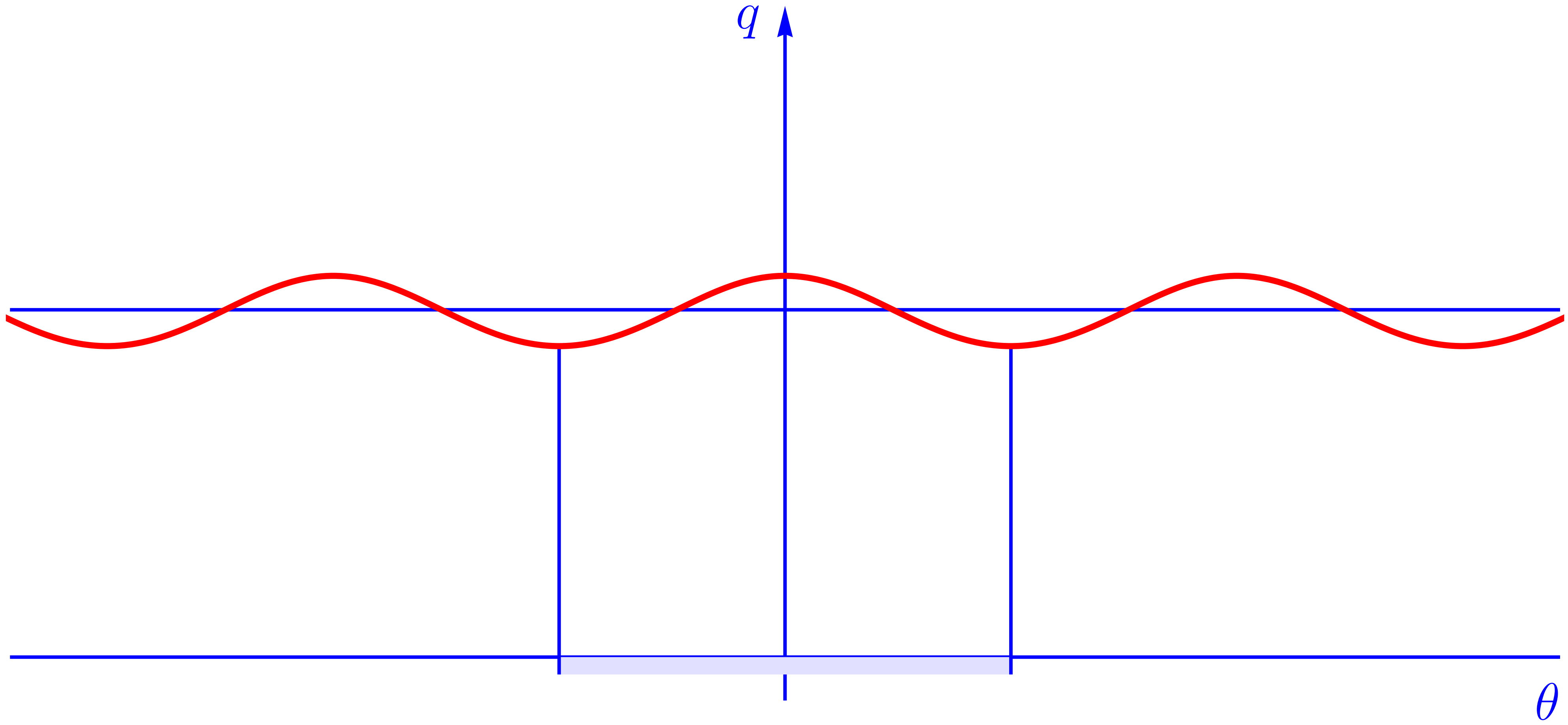

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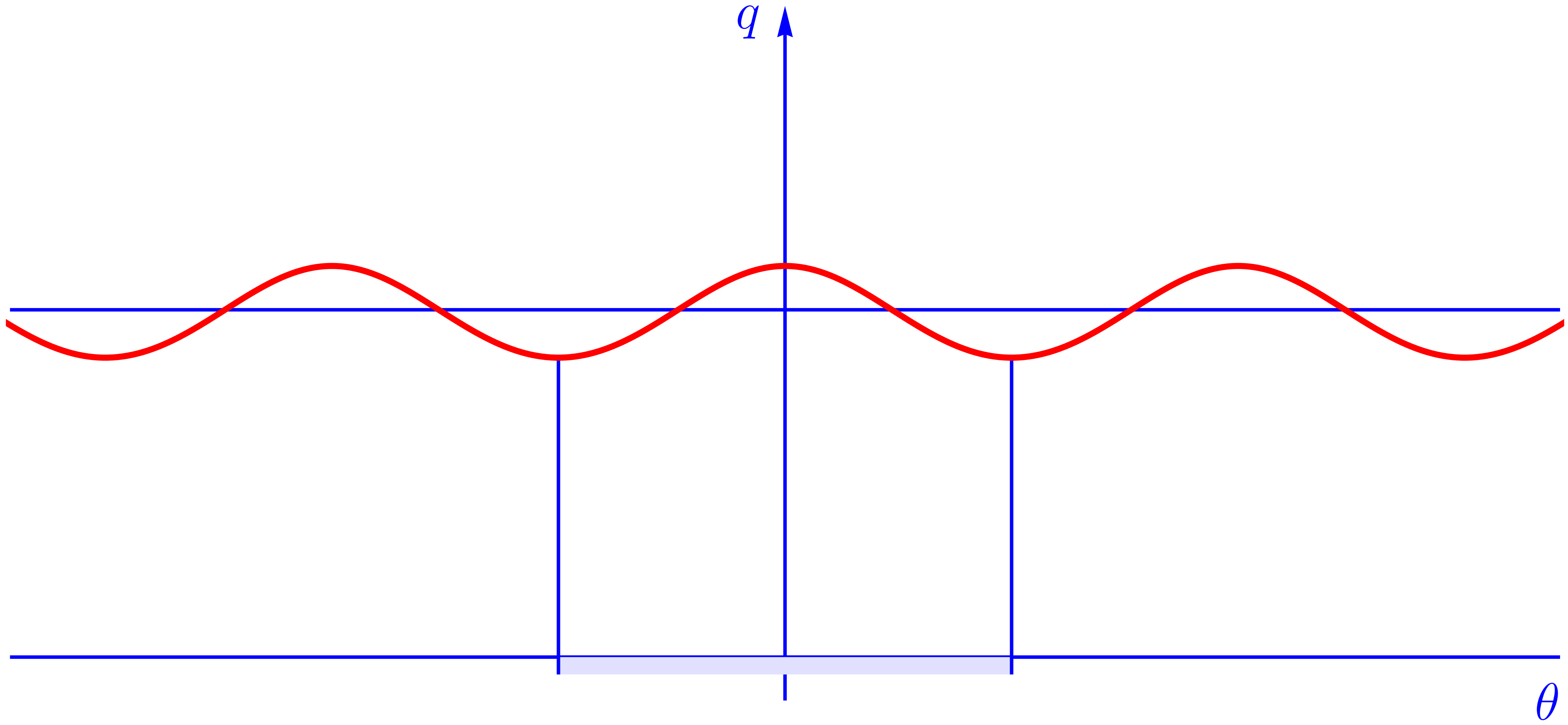

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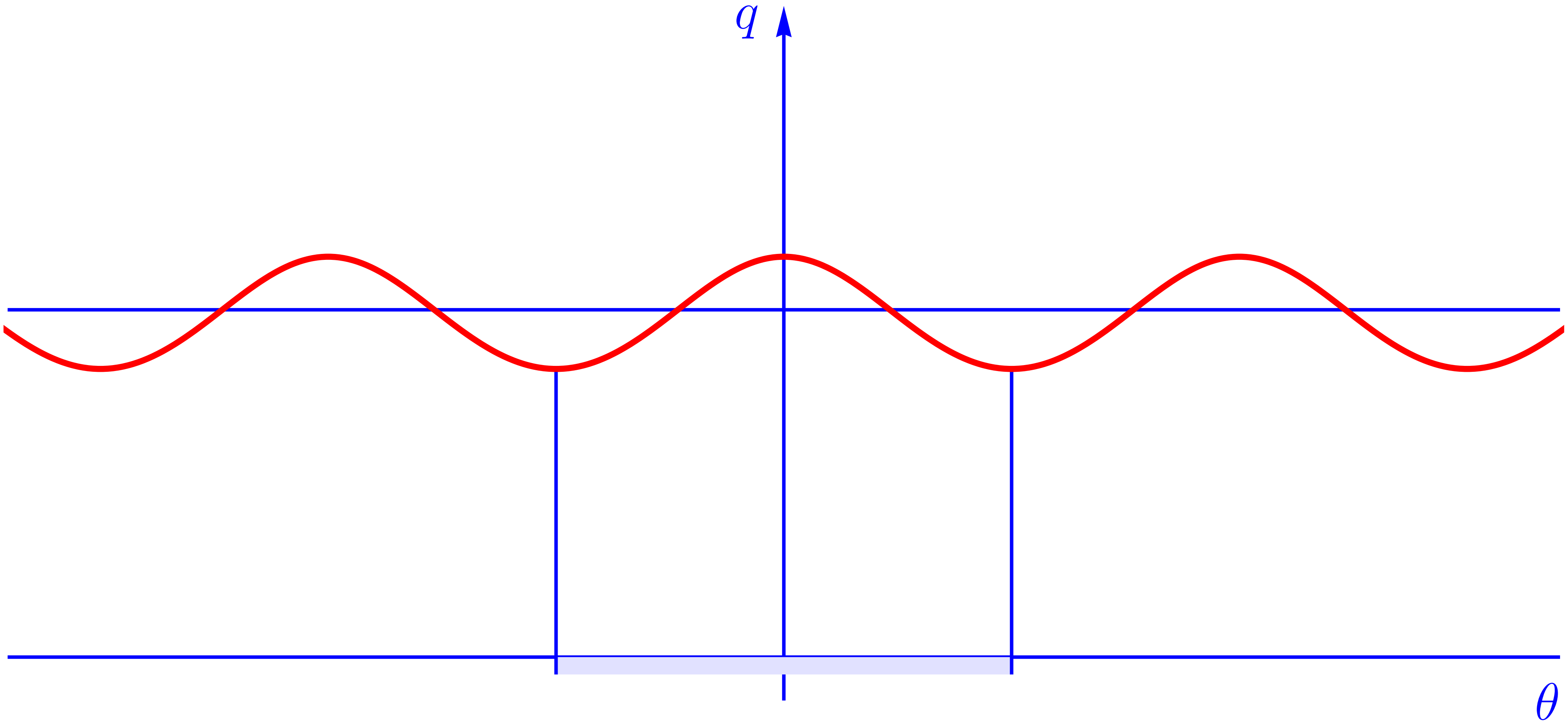

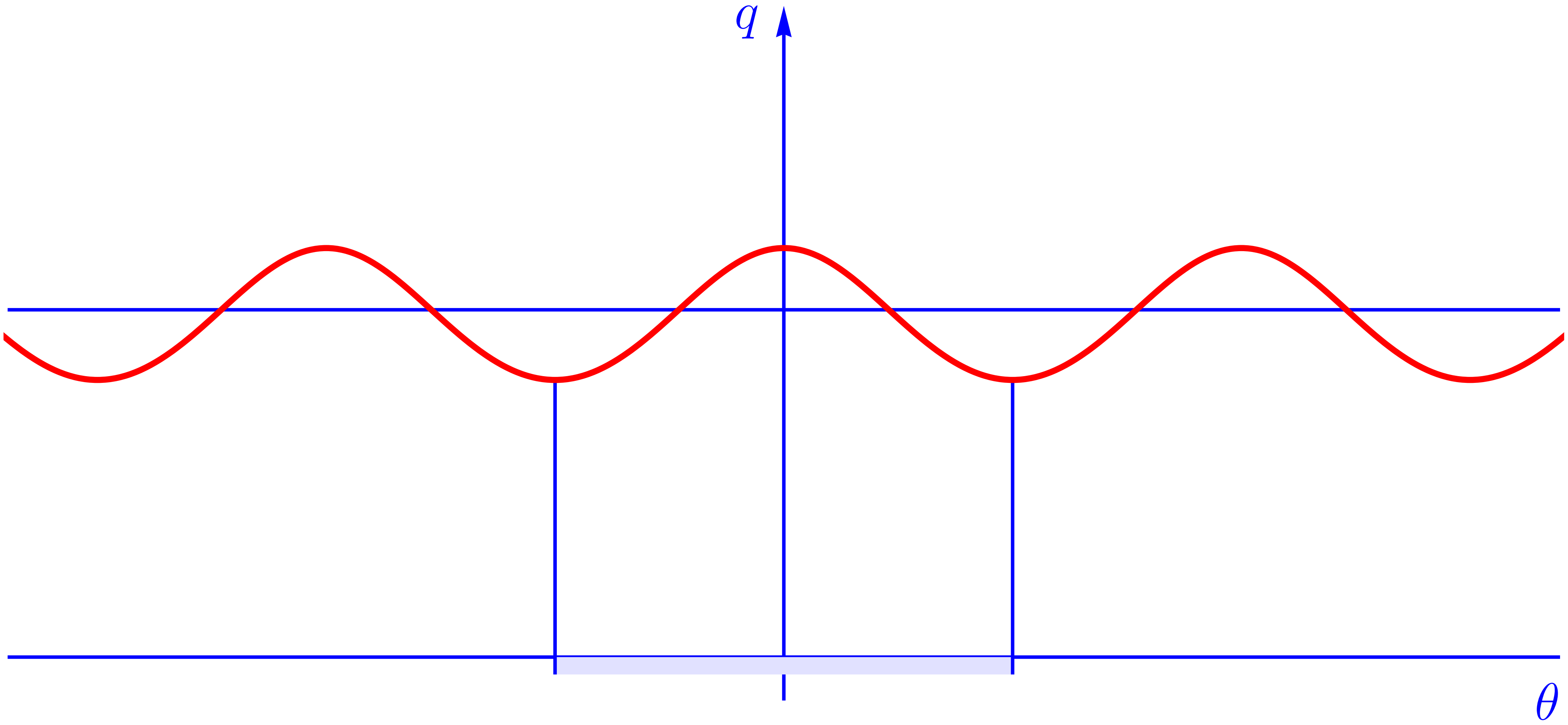
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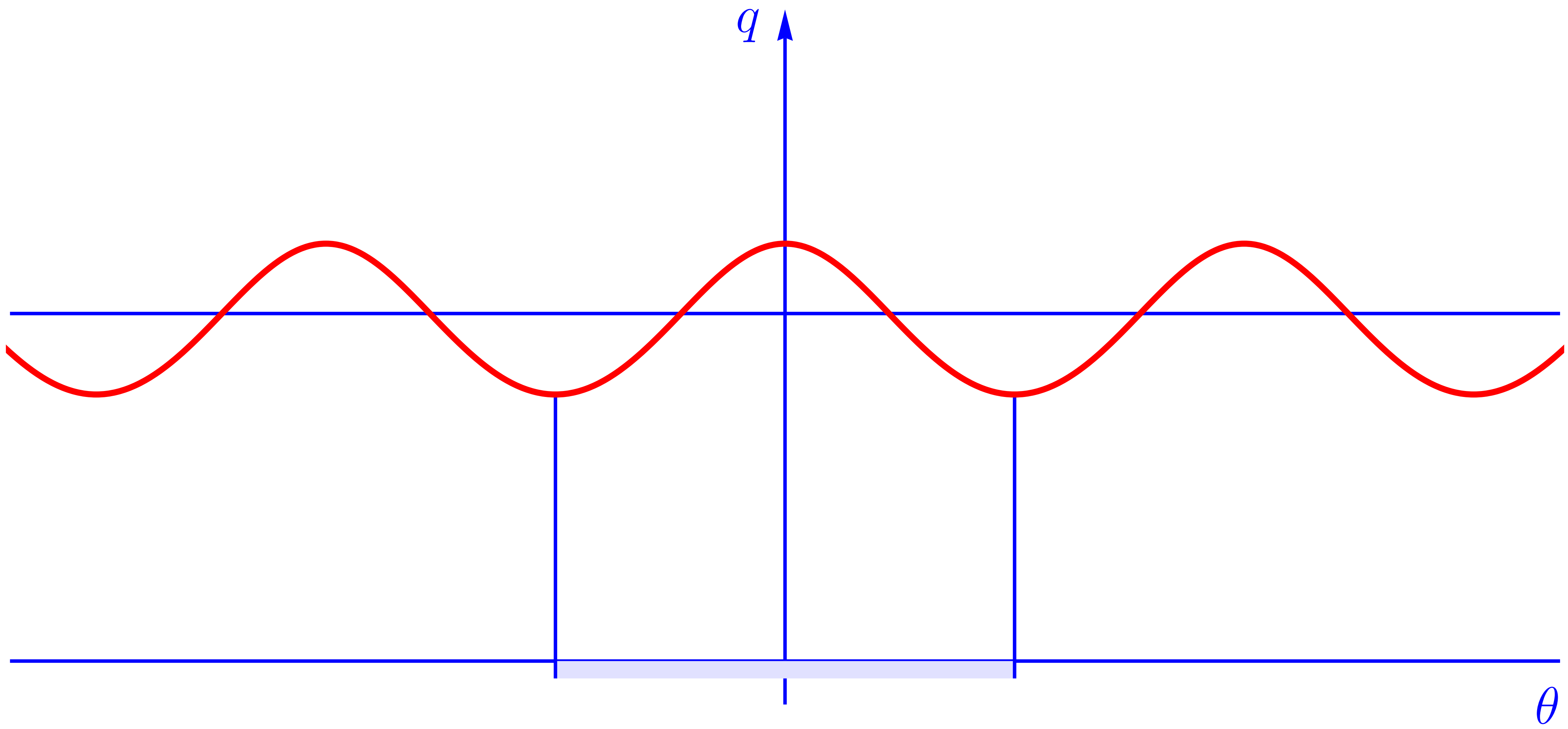

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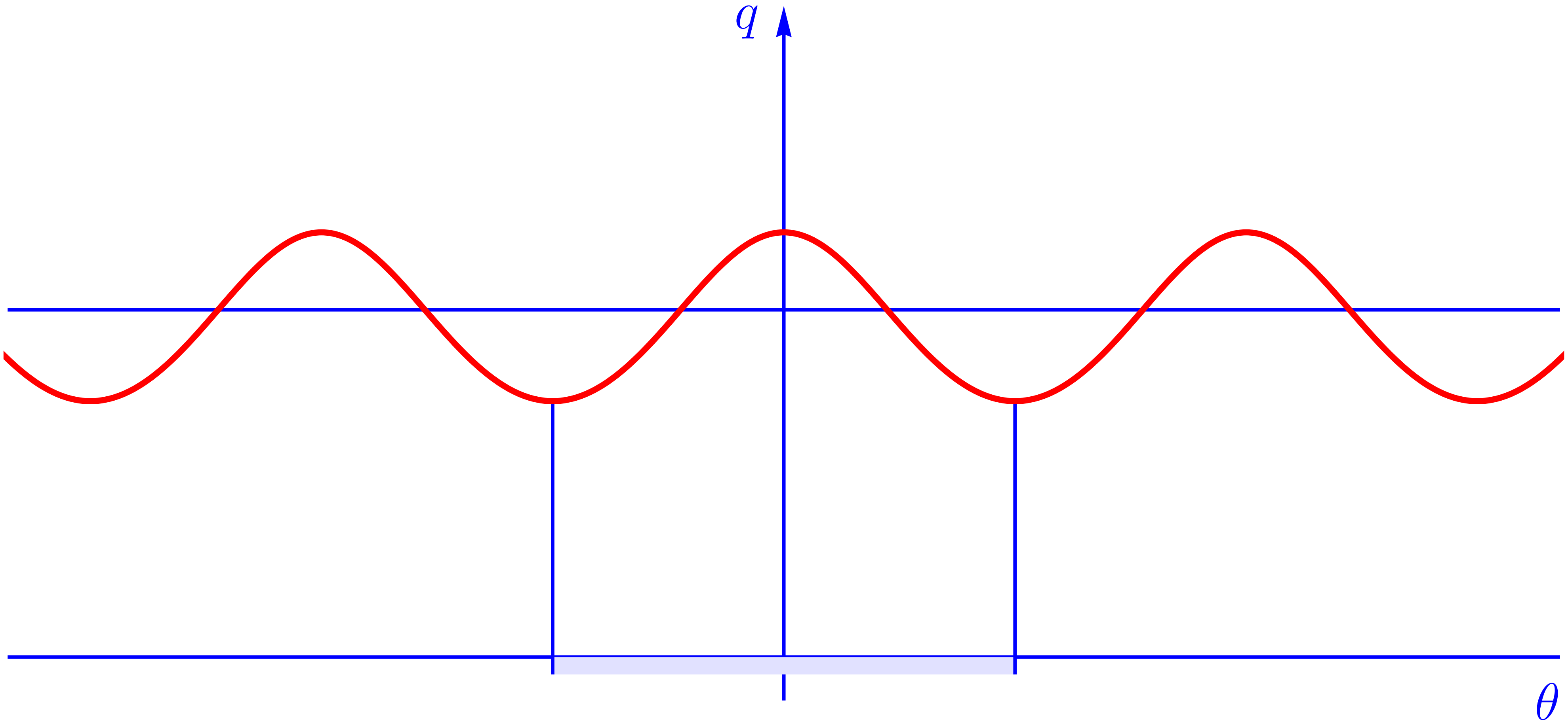

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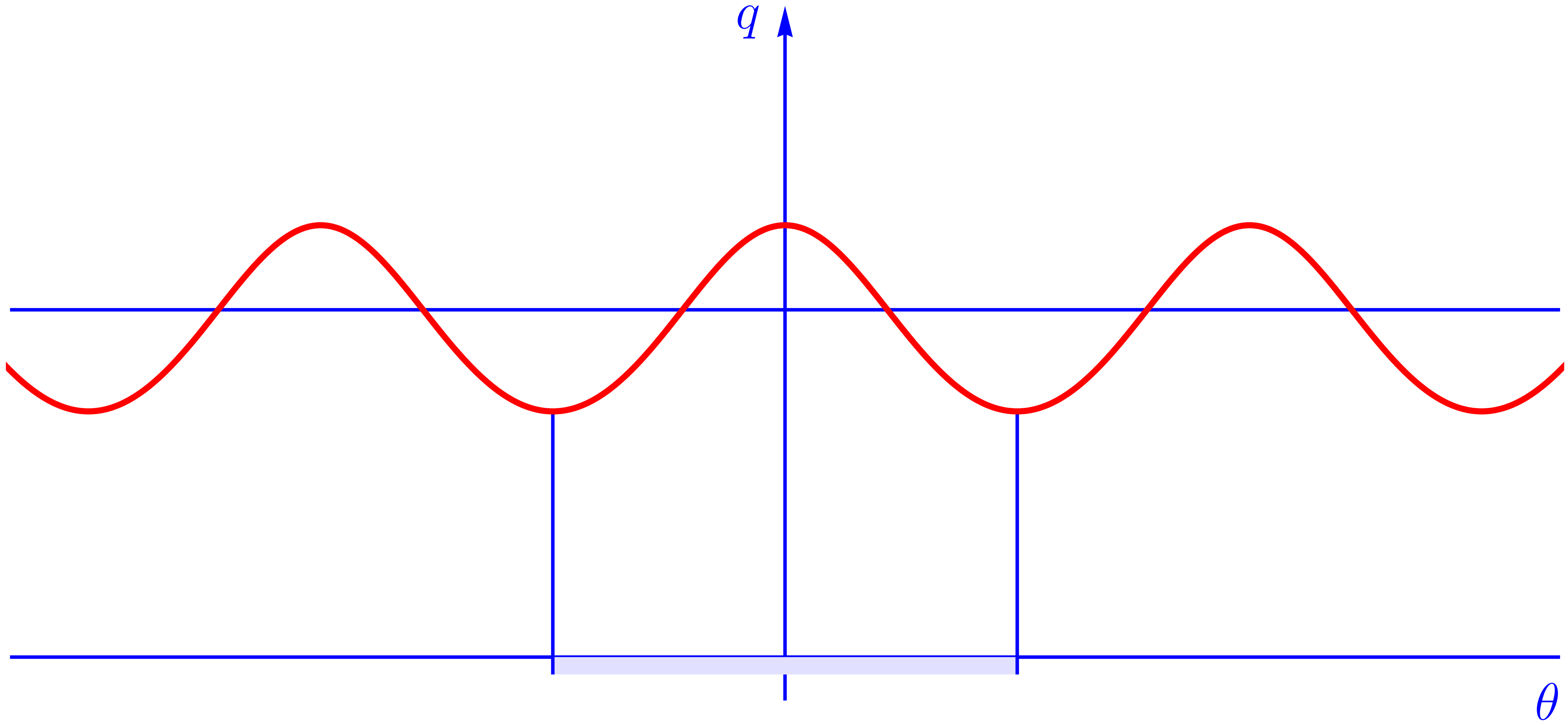

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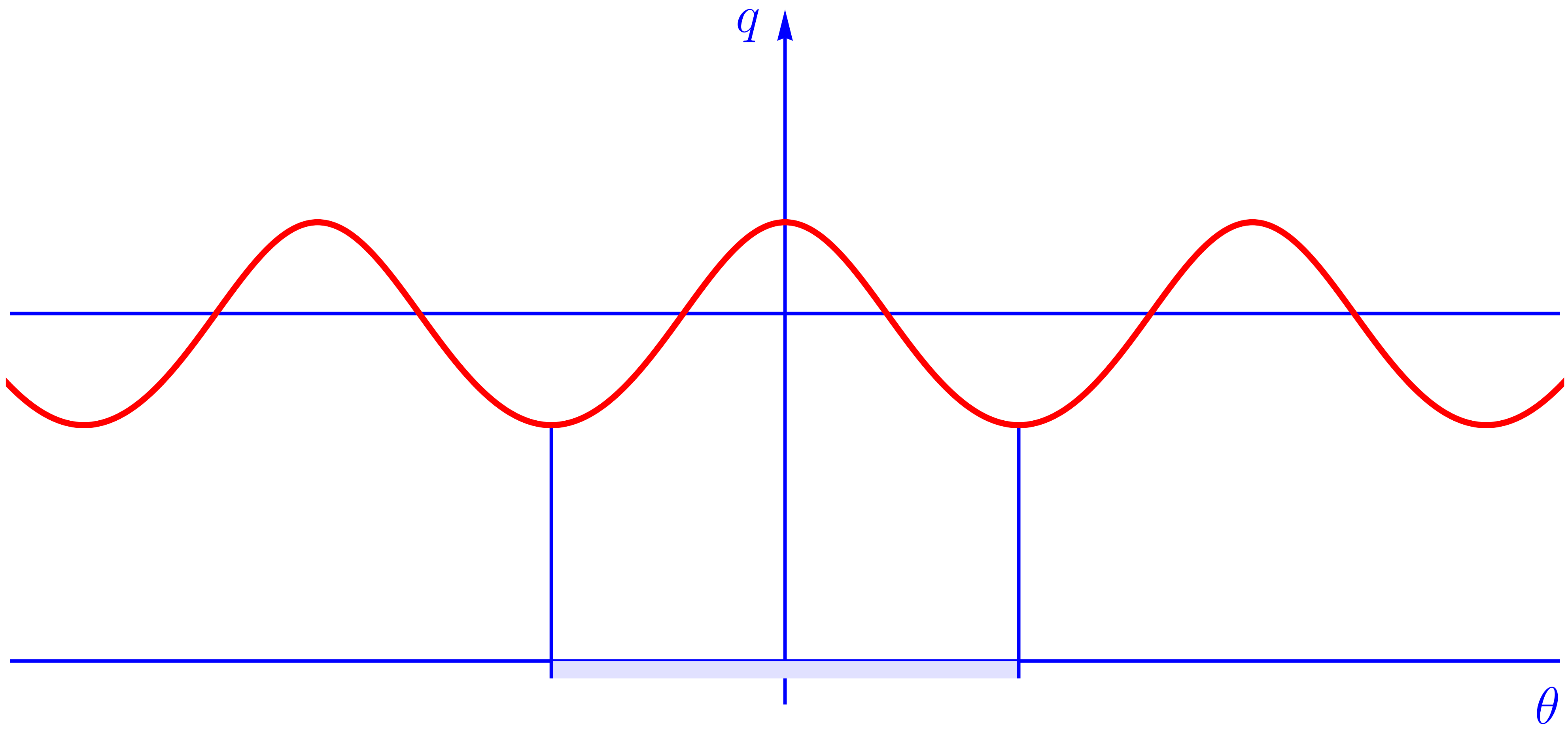

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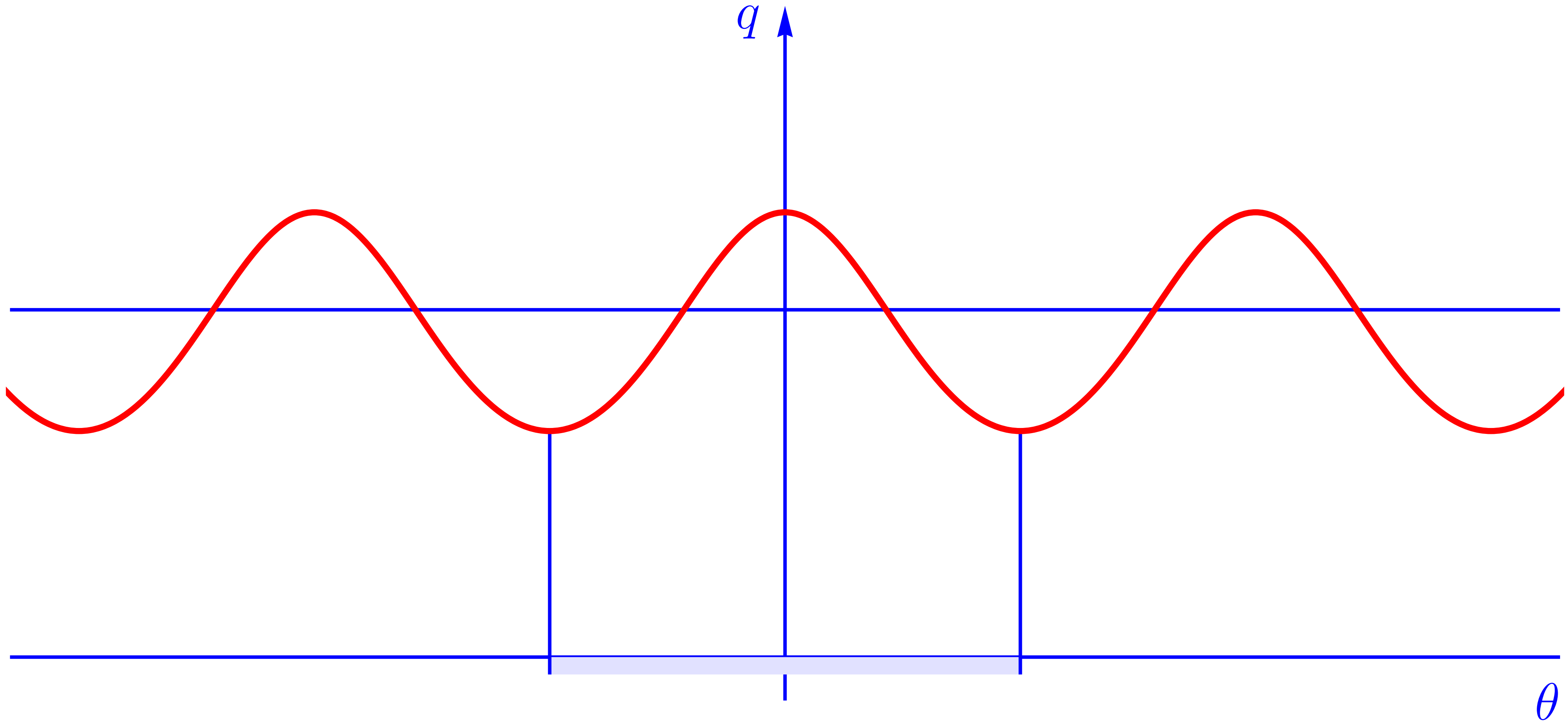

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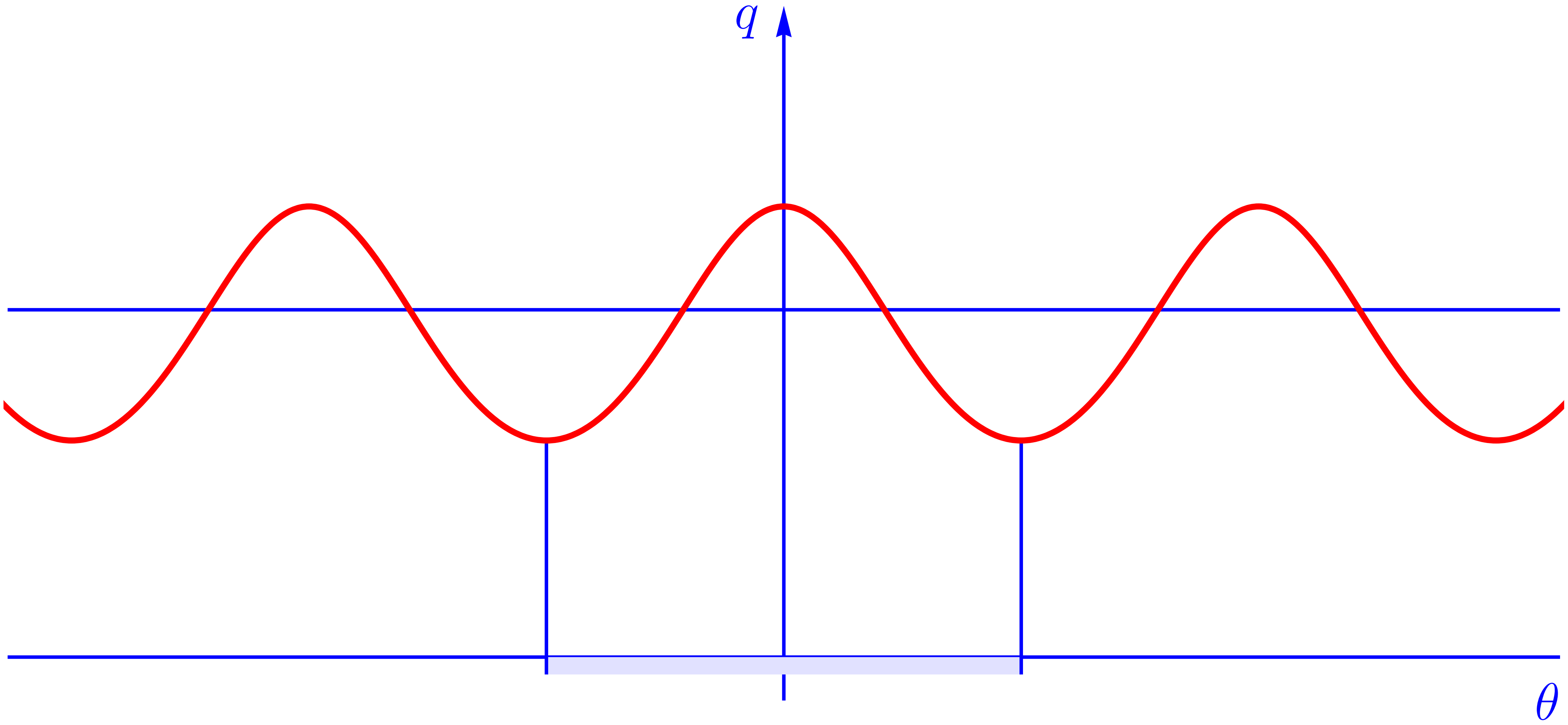

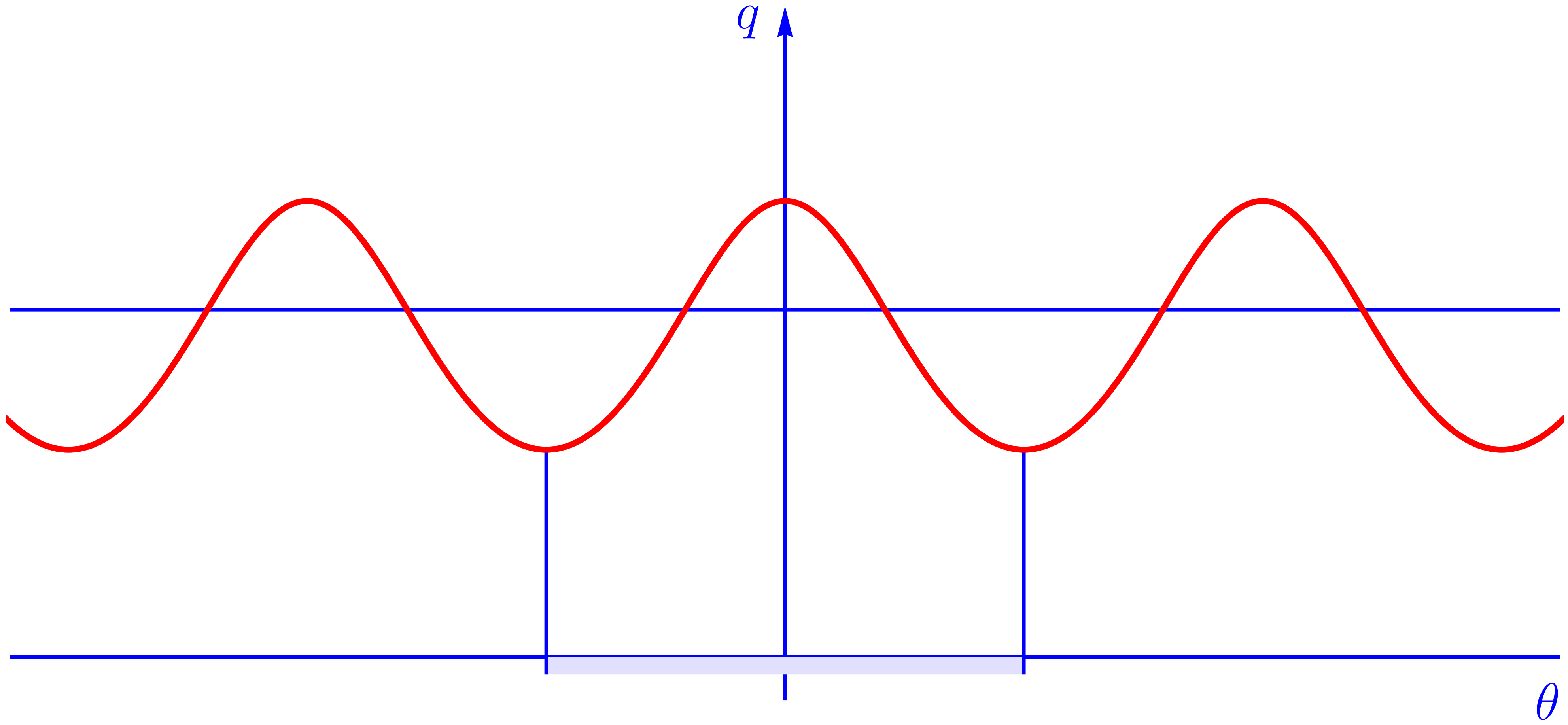
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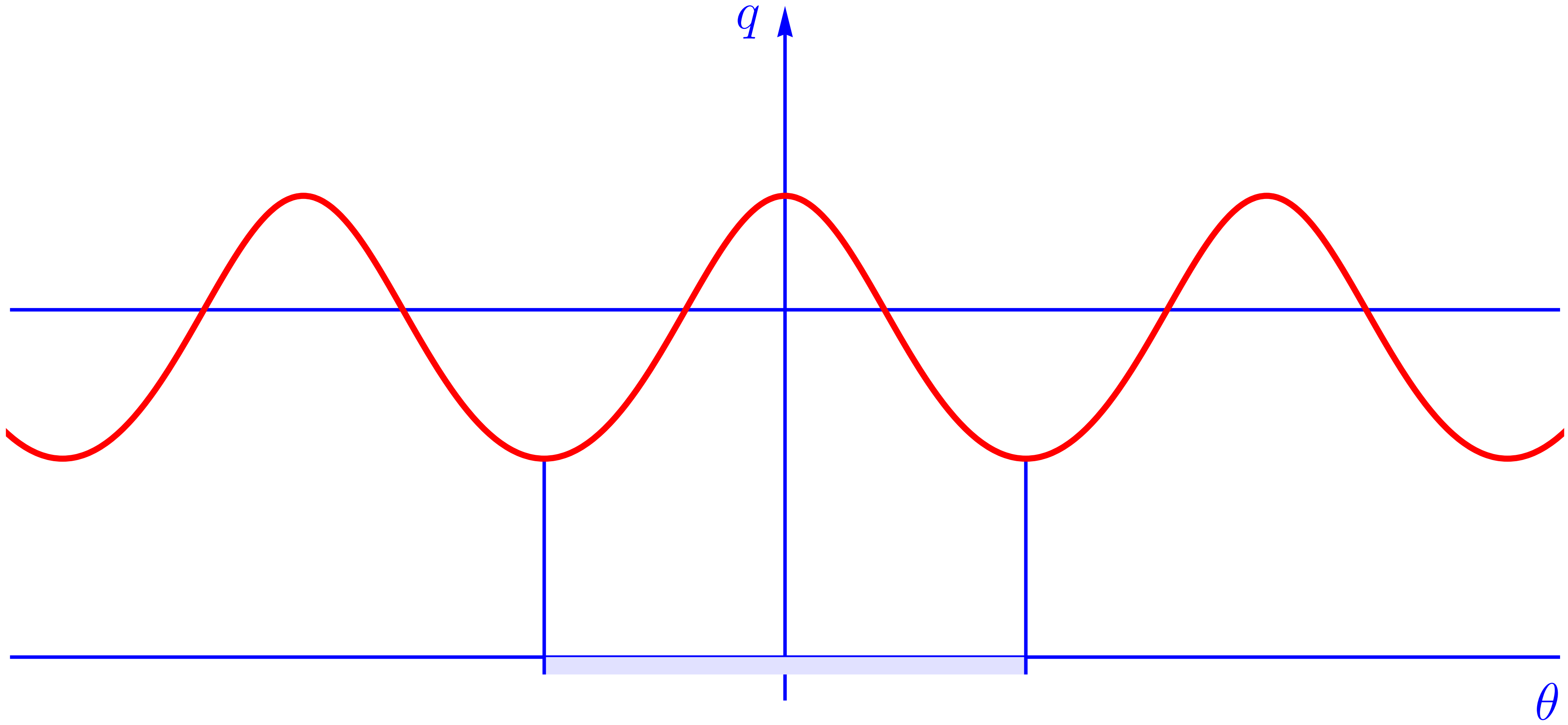

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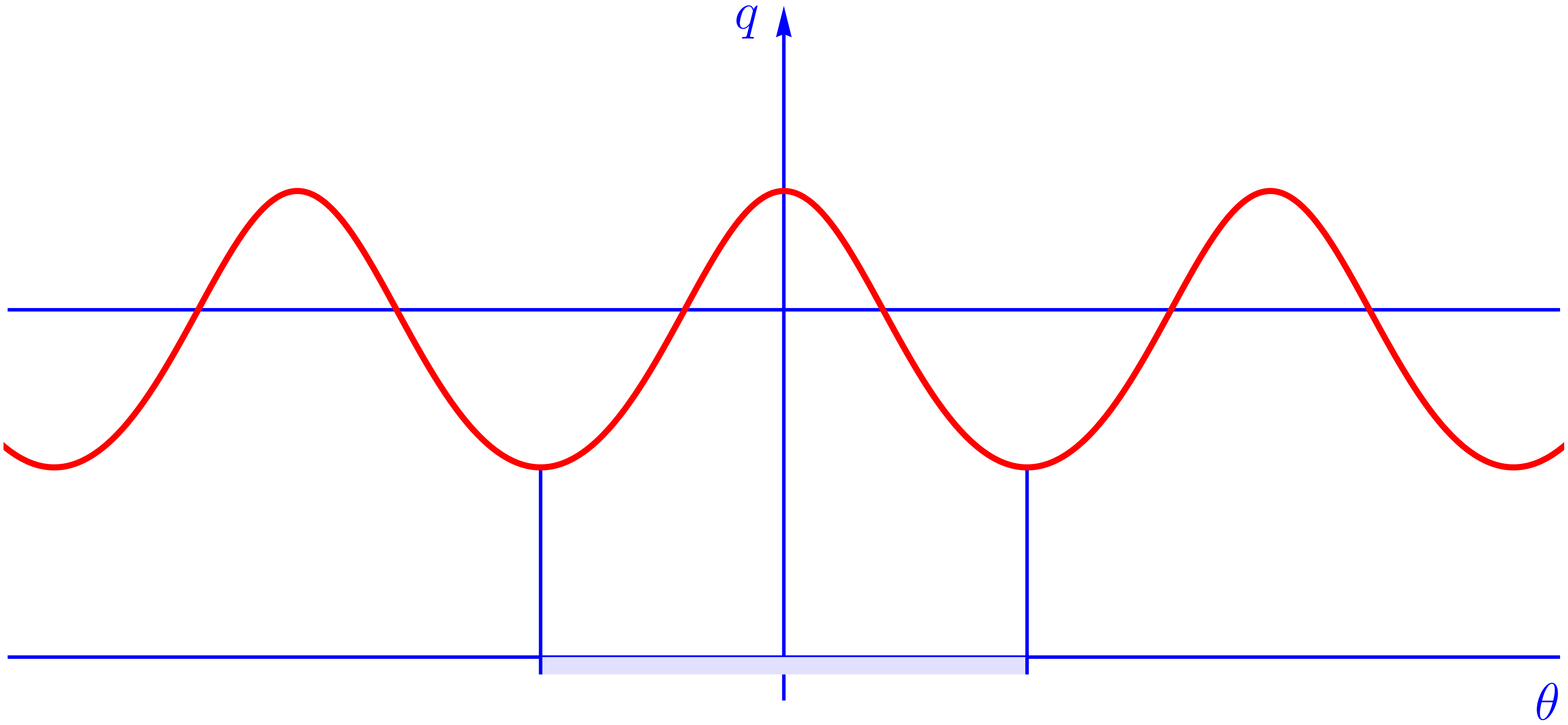

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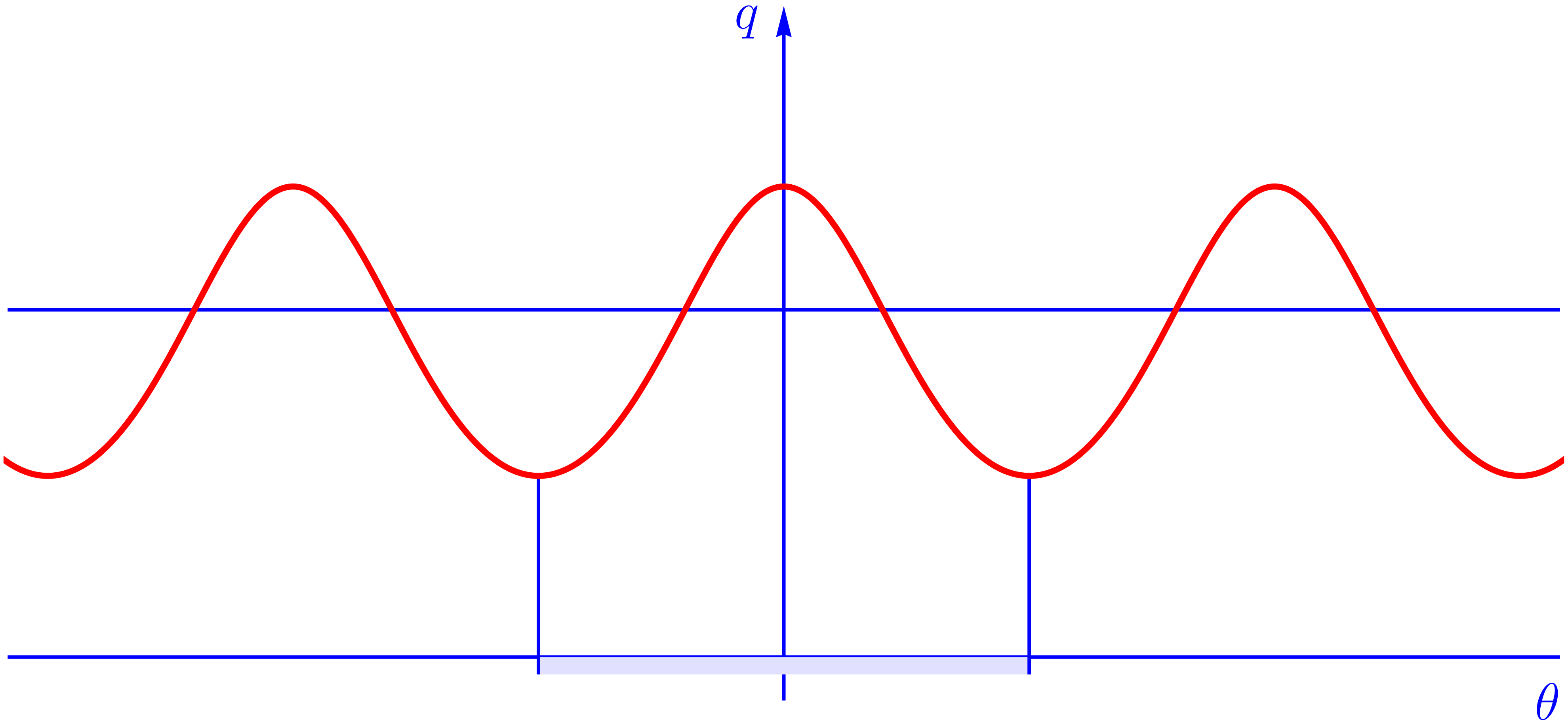
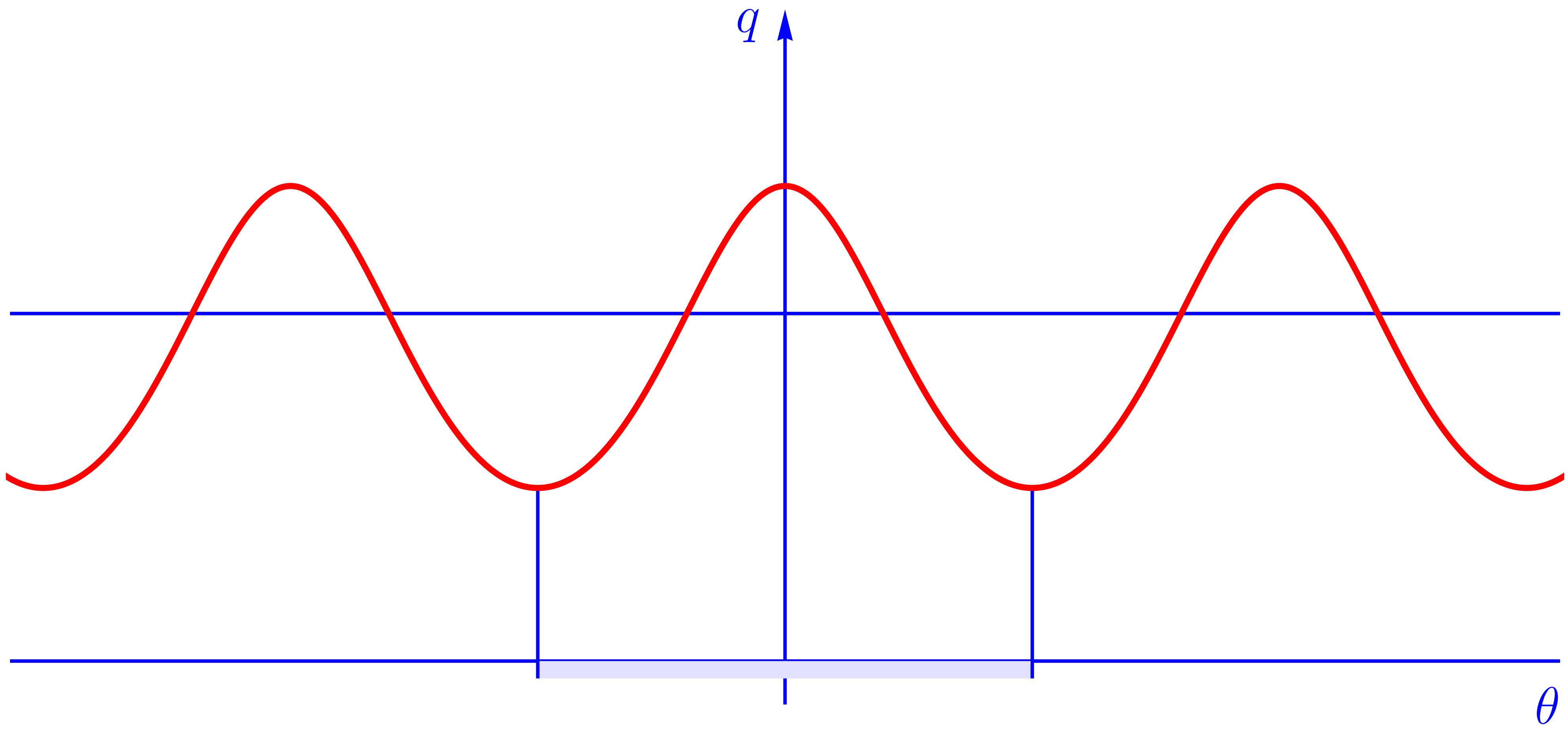
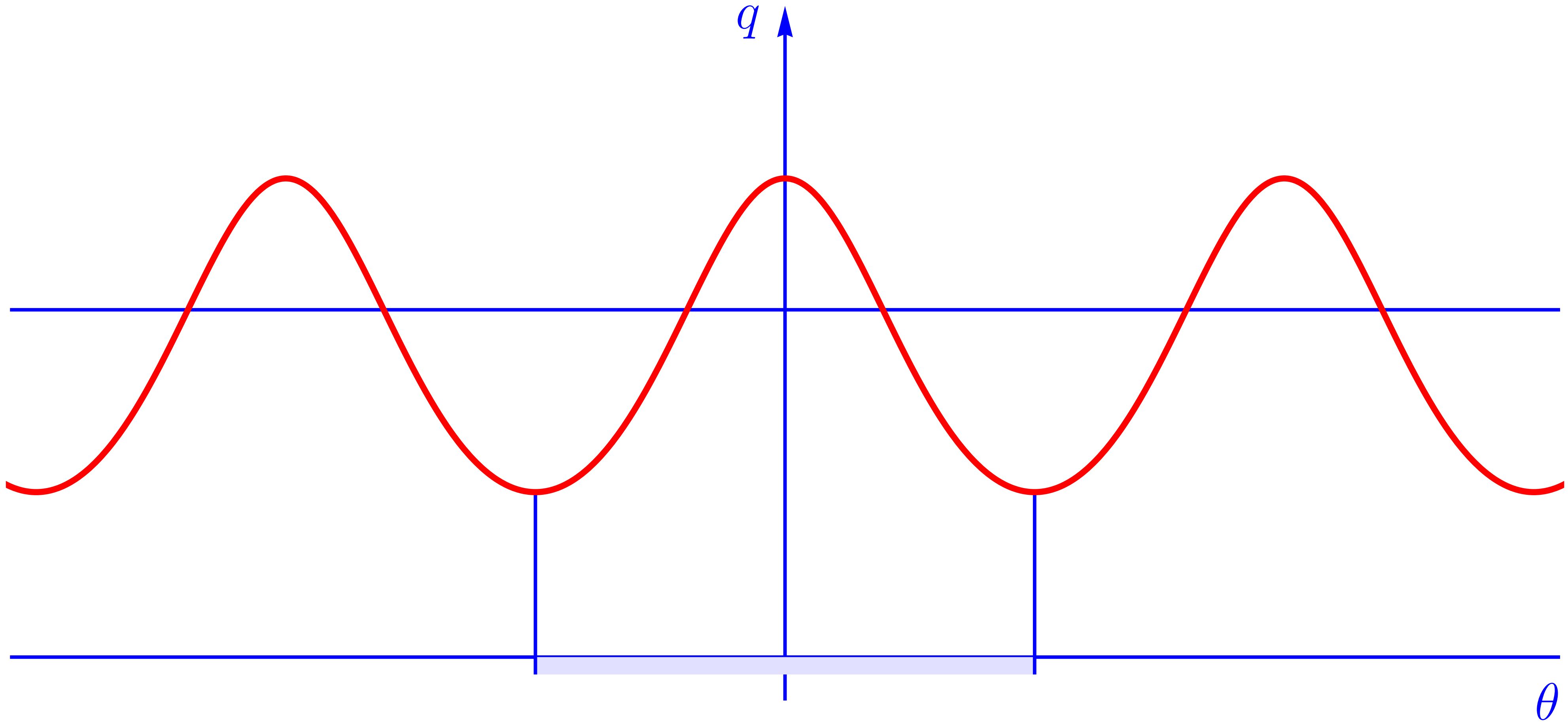


Cnoidal Waves



Cnoidal Waves



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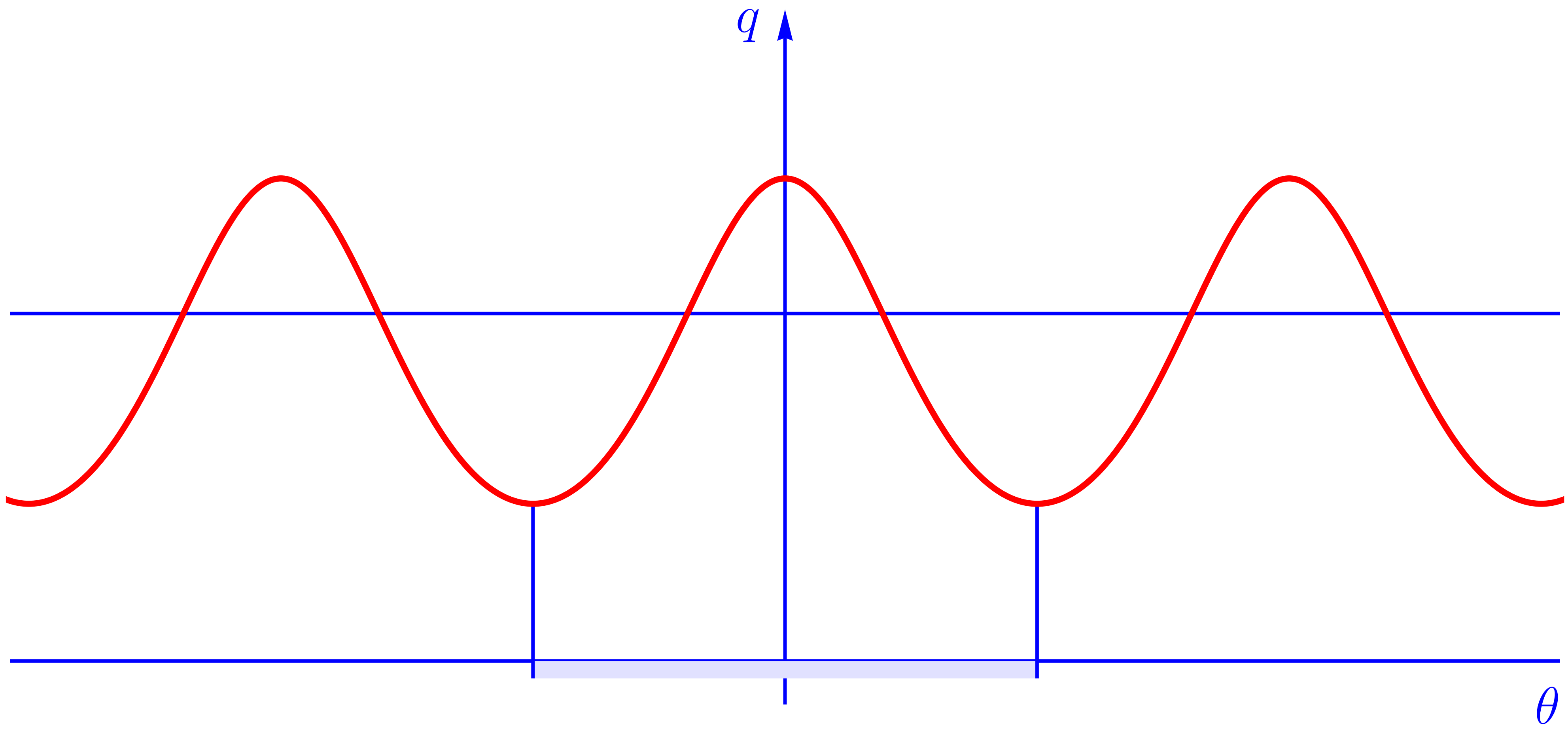
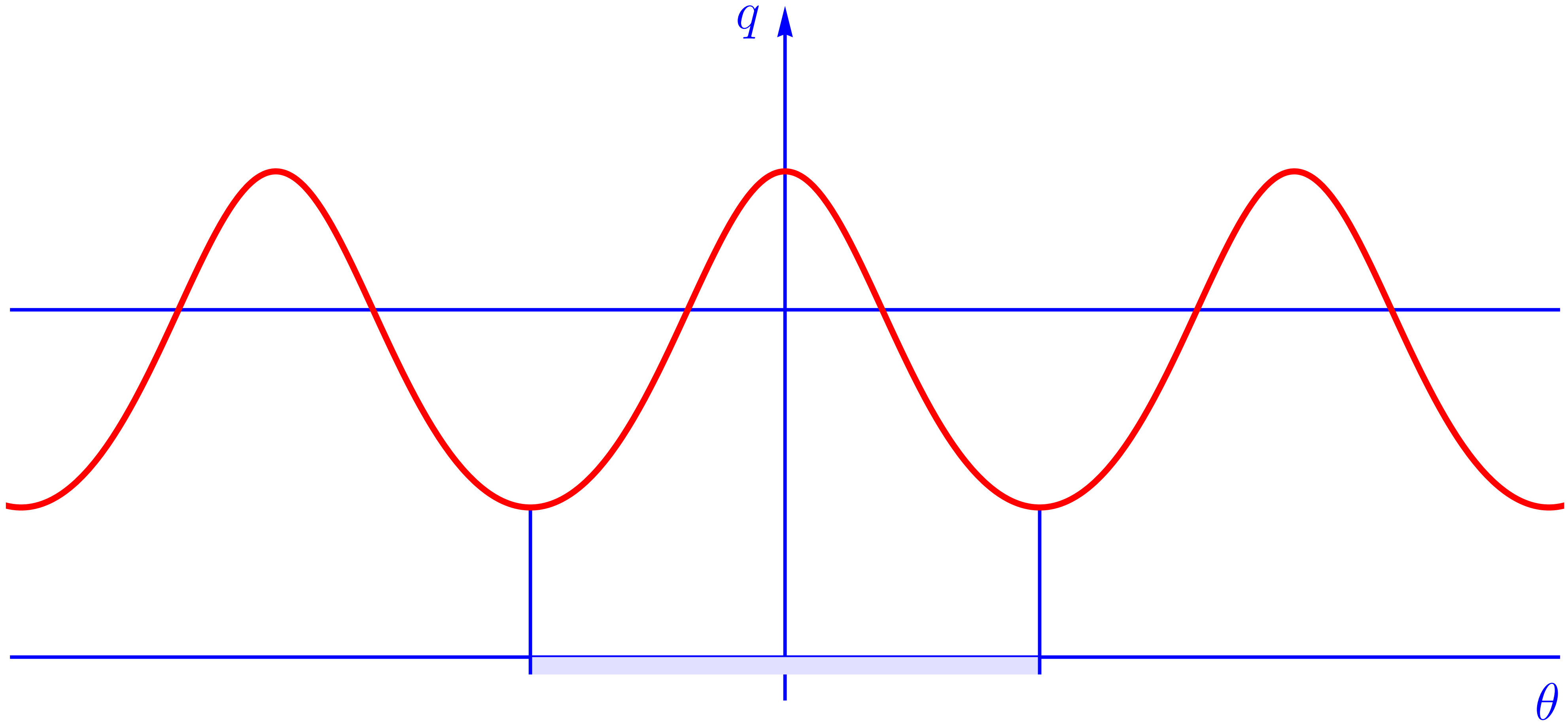


Cnoidal Waves



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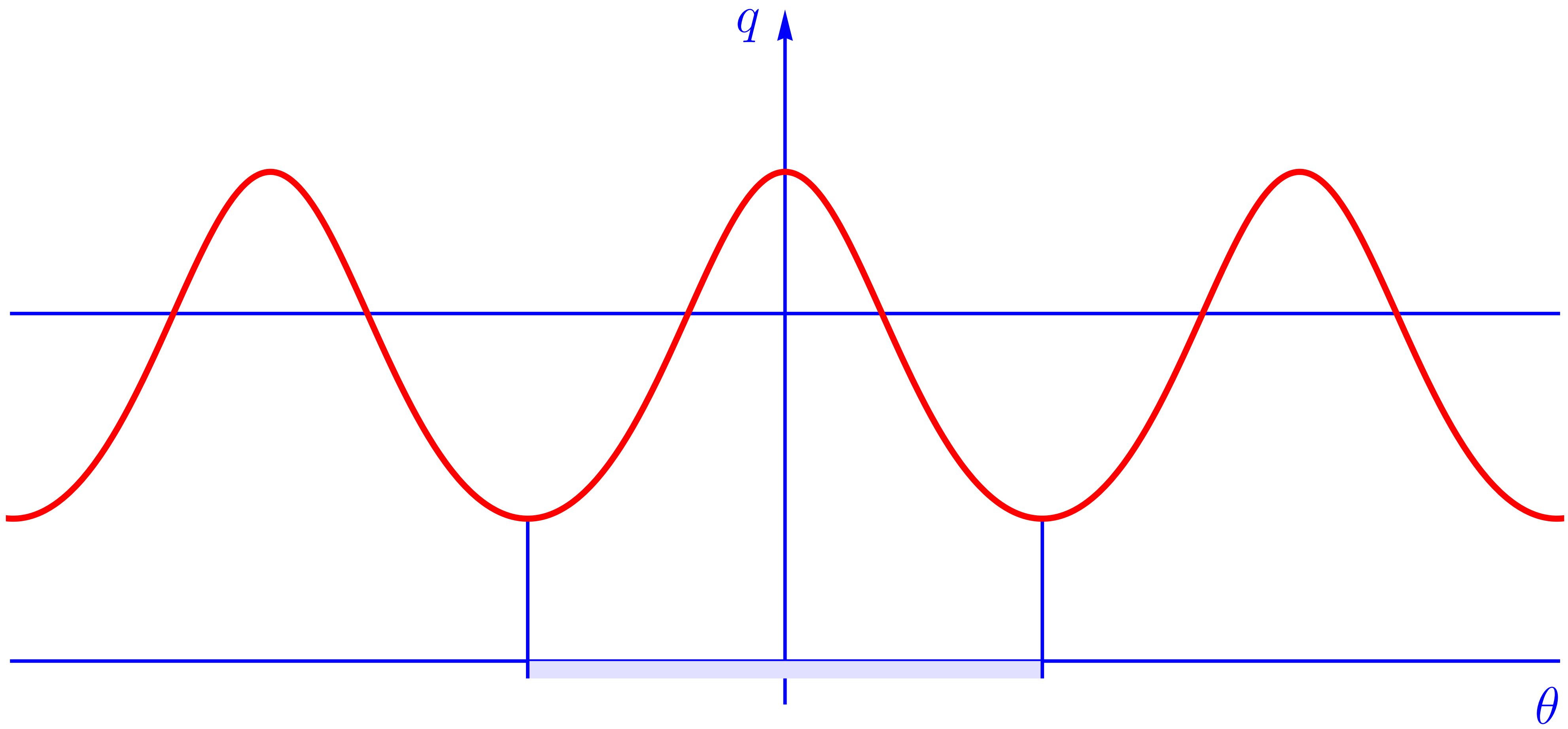

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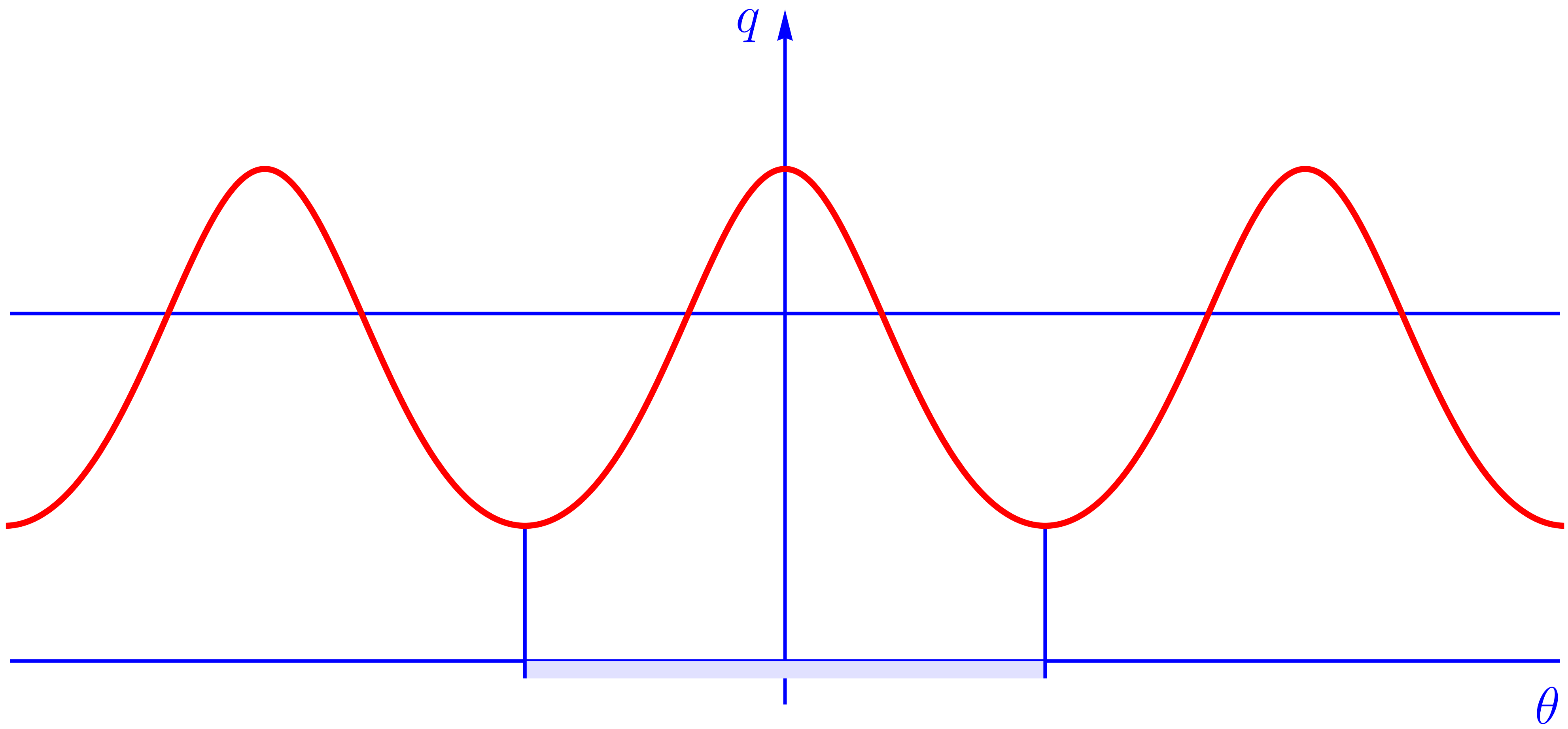

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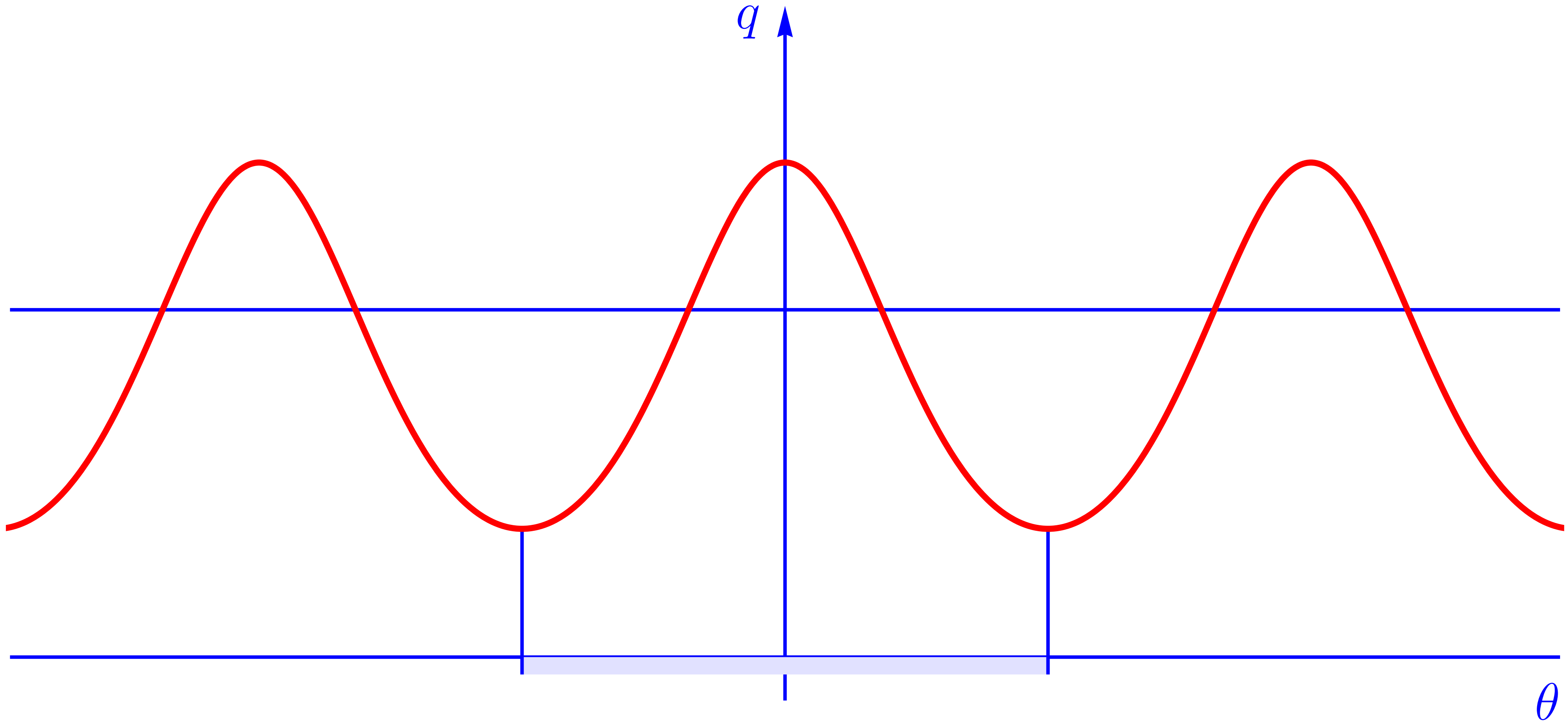

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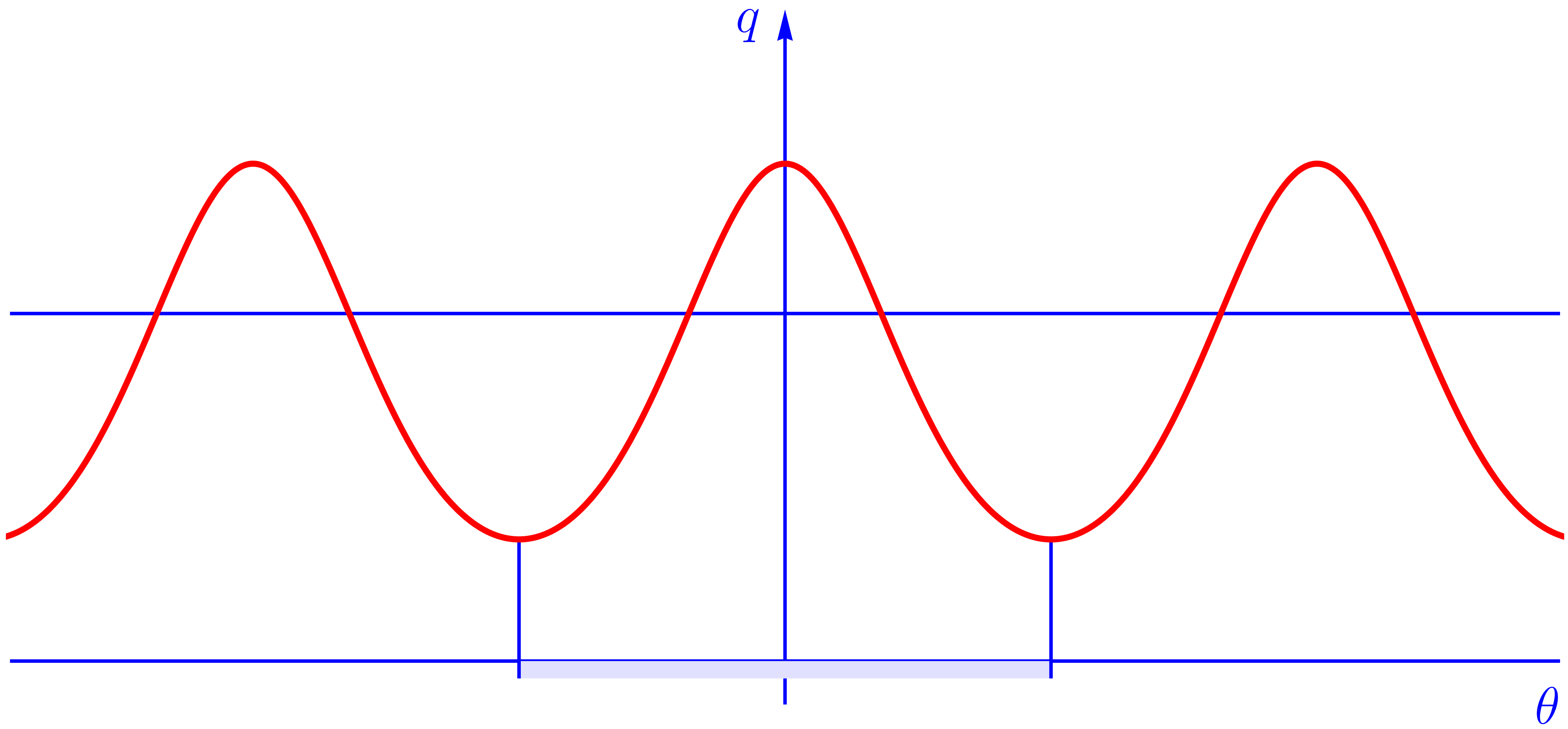

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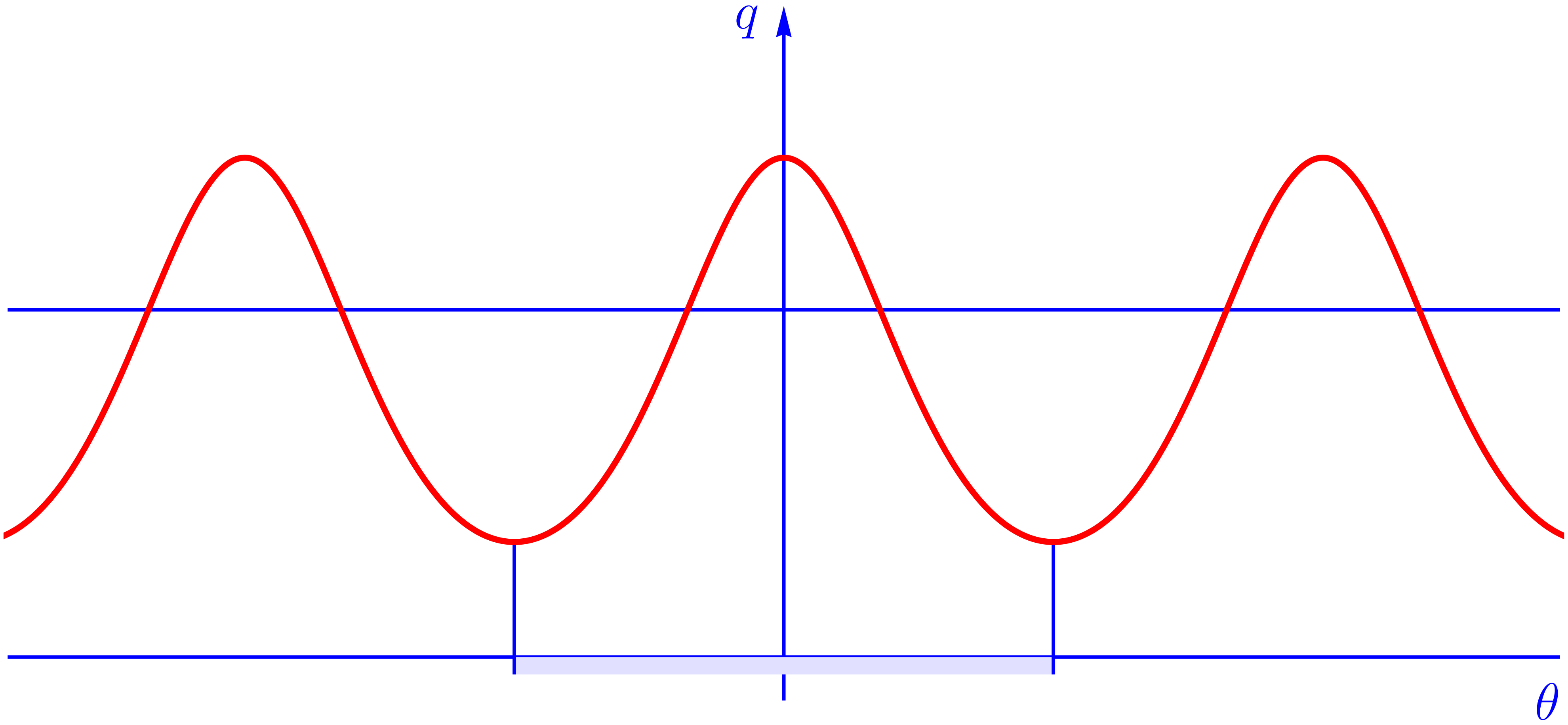

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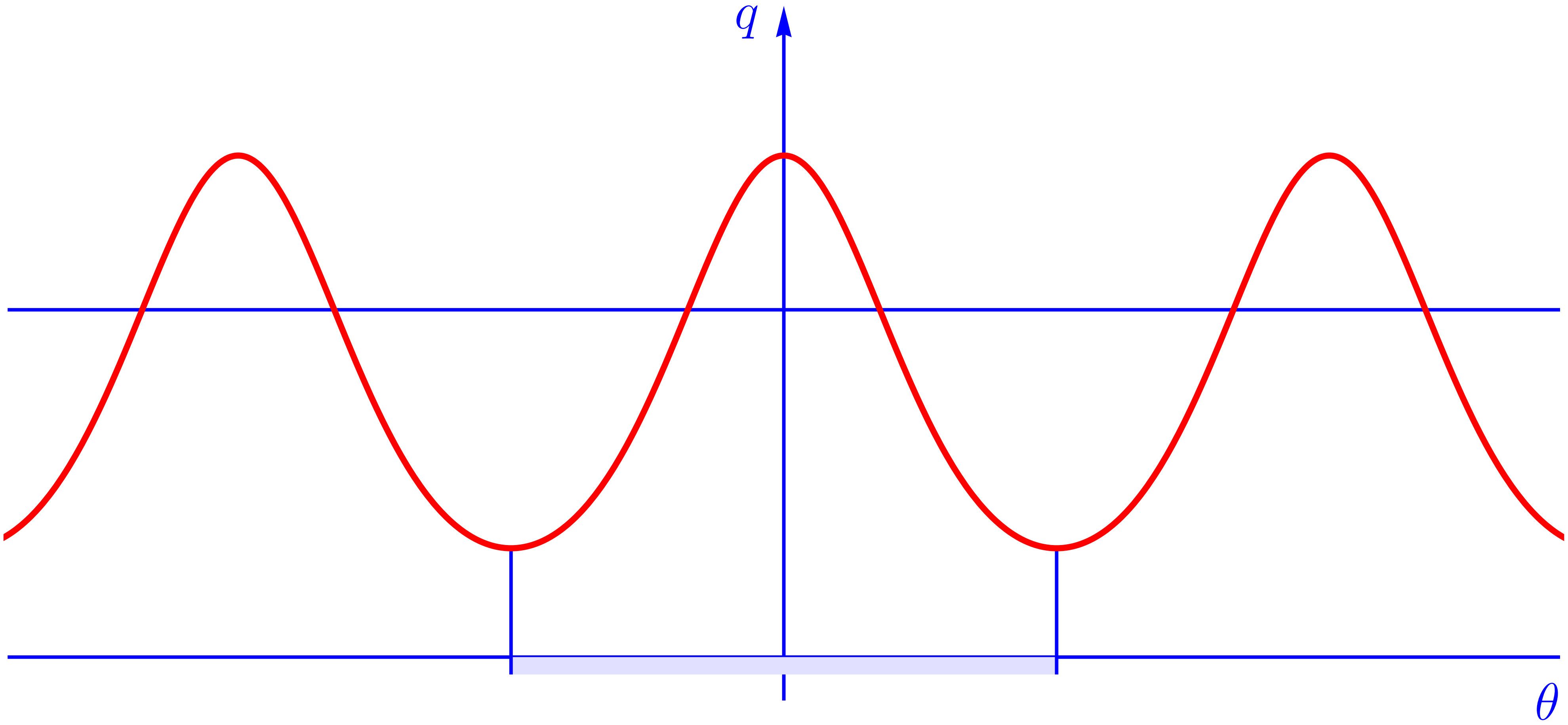

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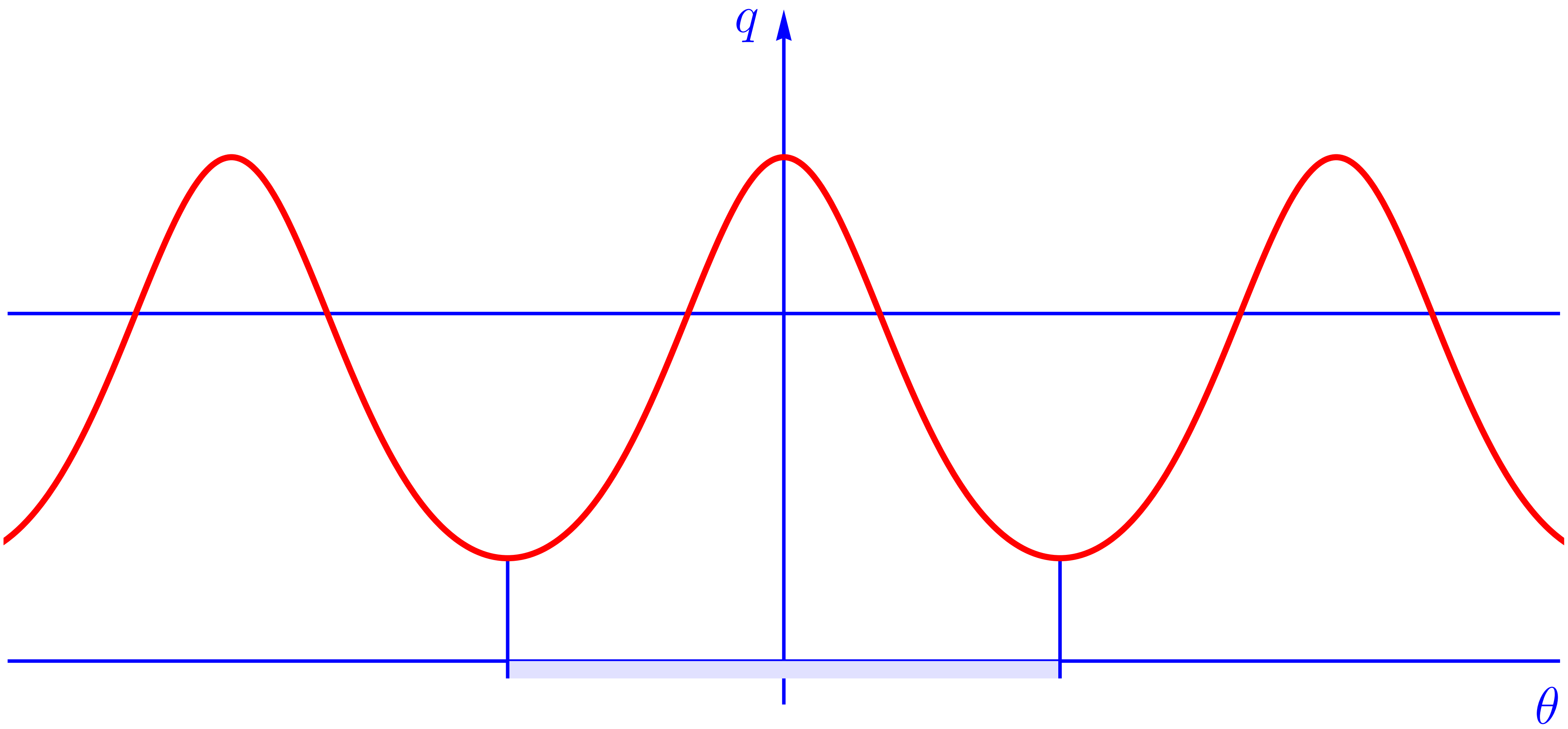

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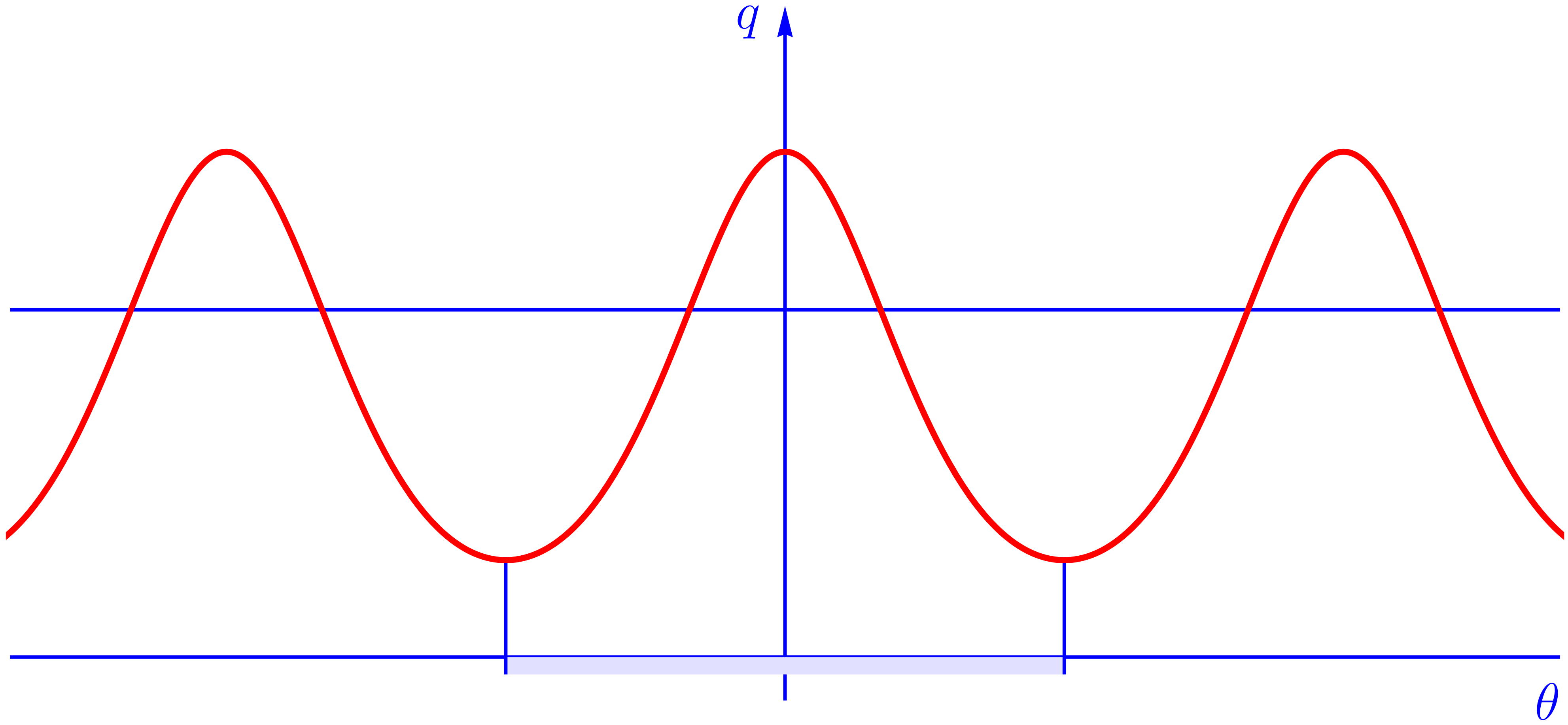

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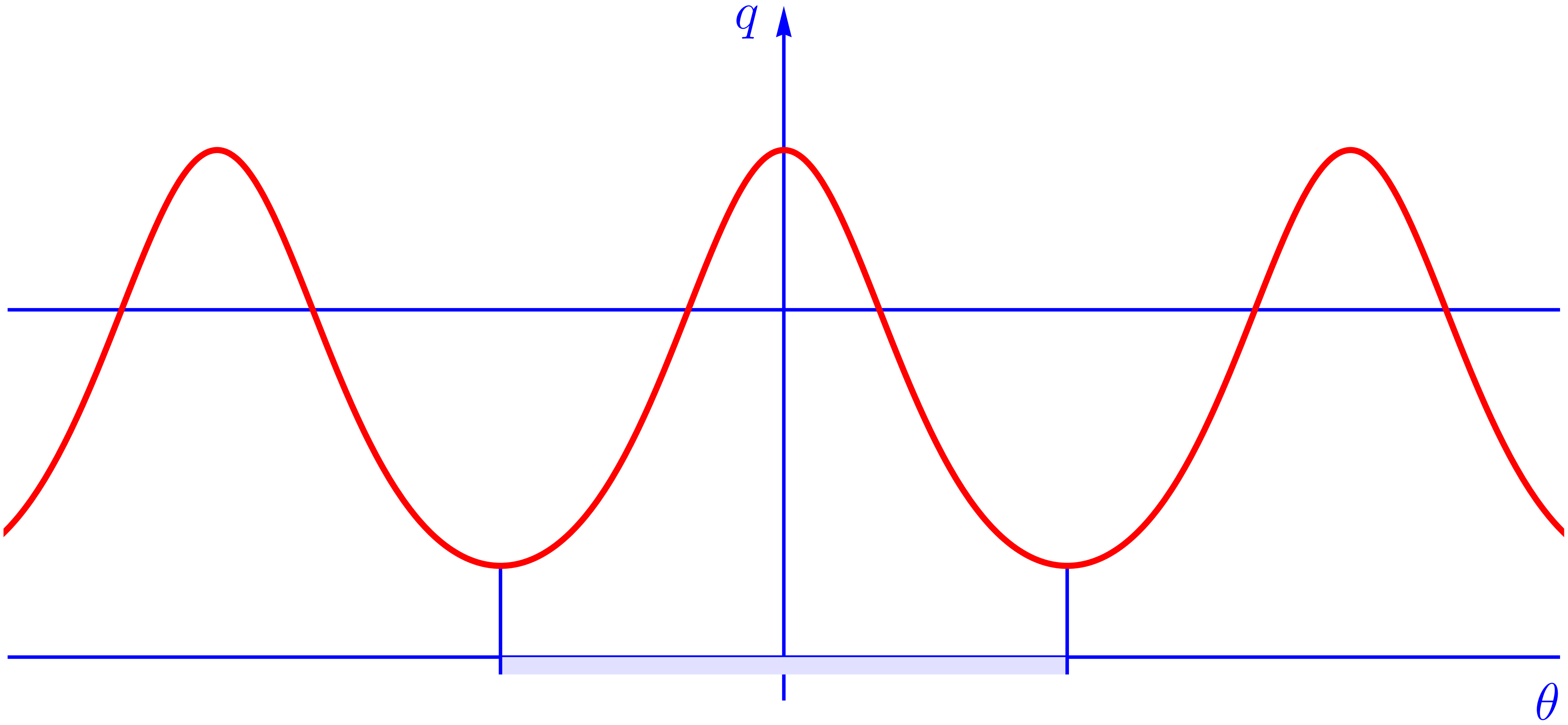

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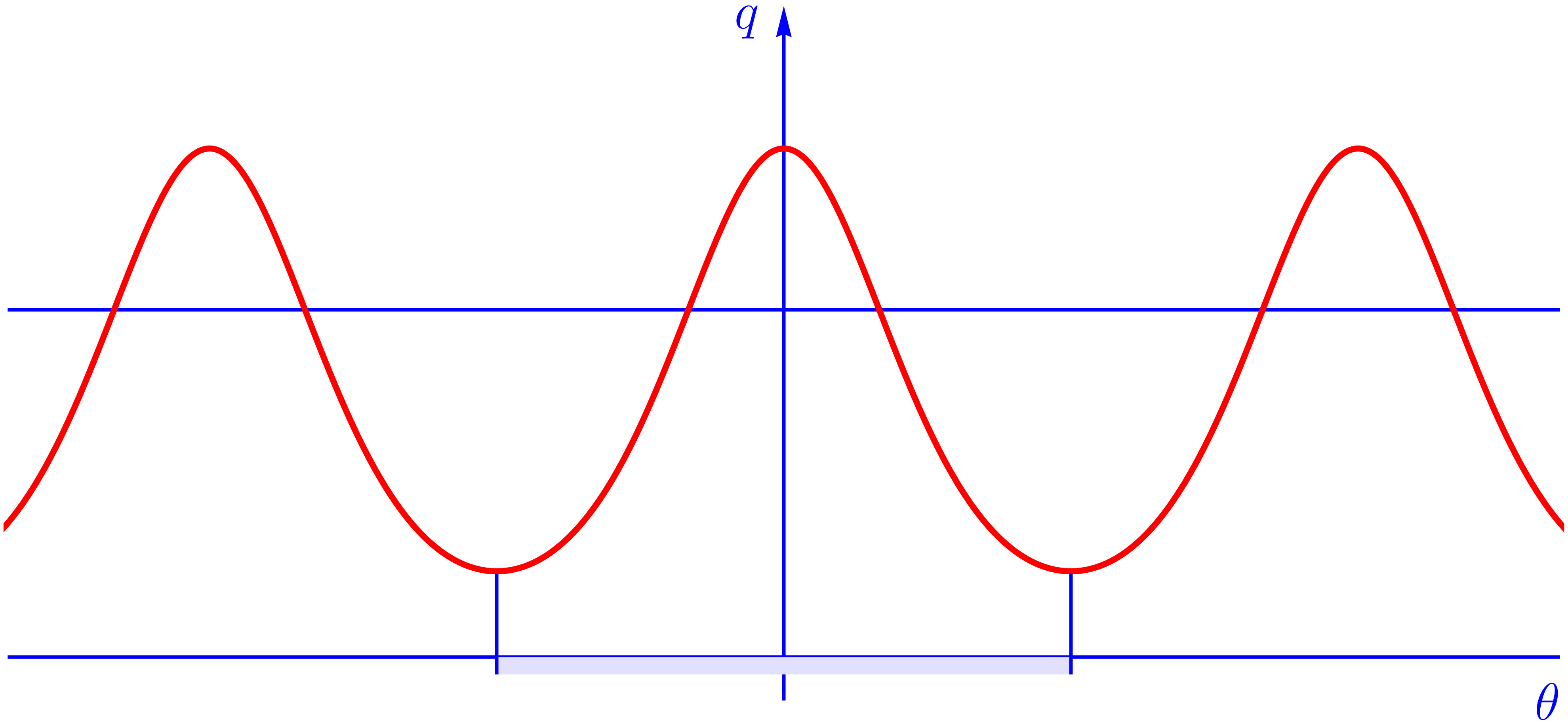

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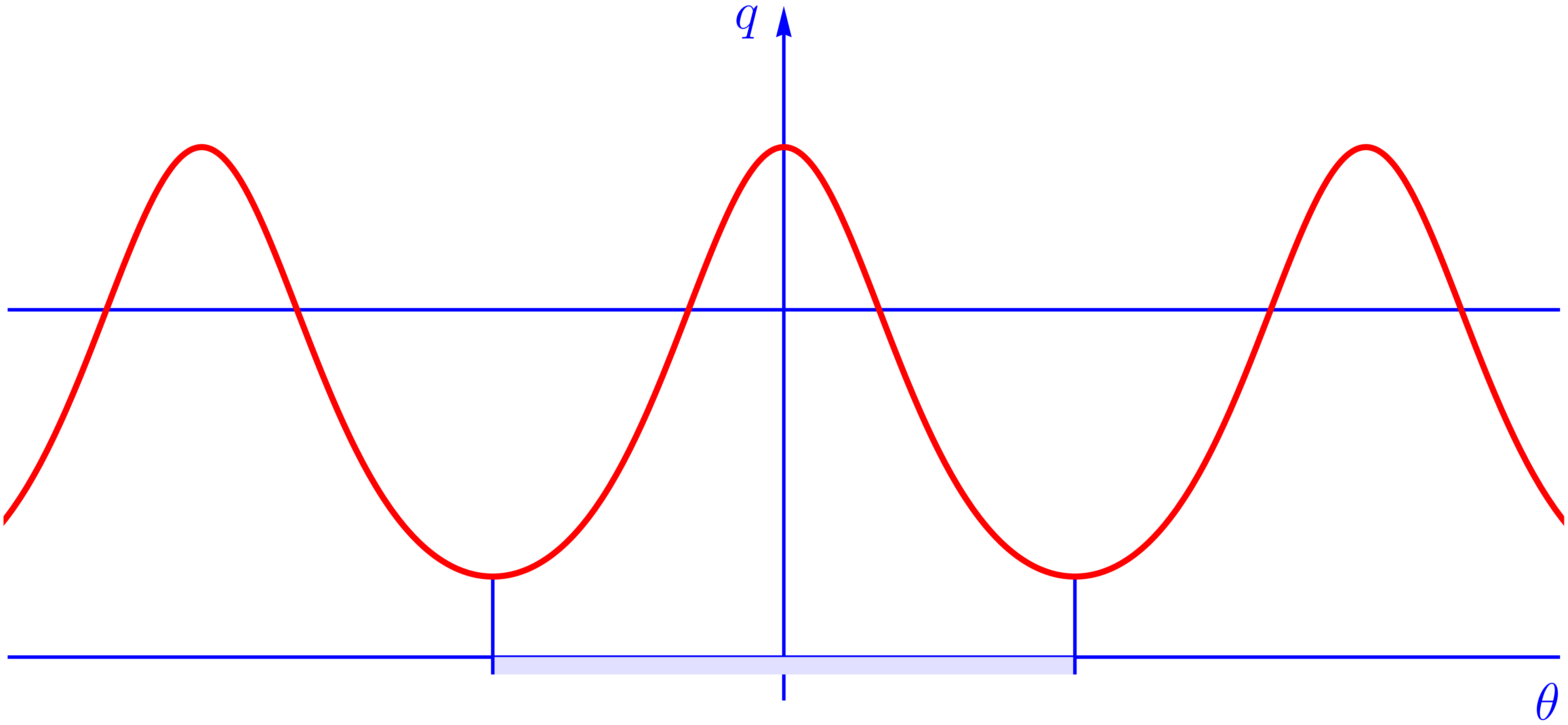

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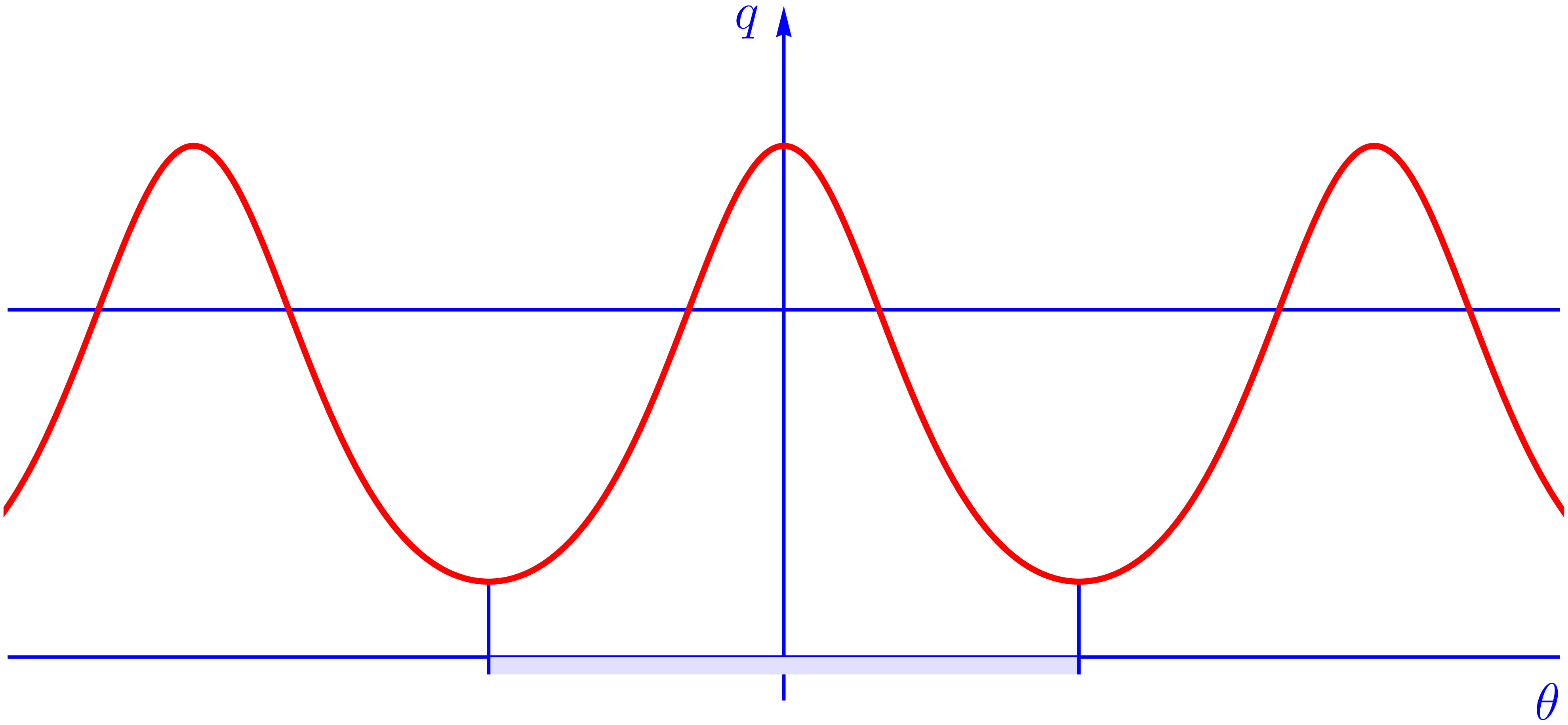

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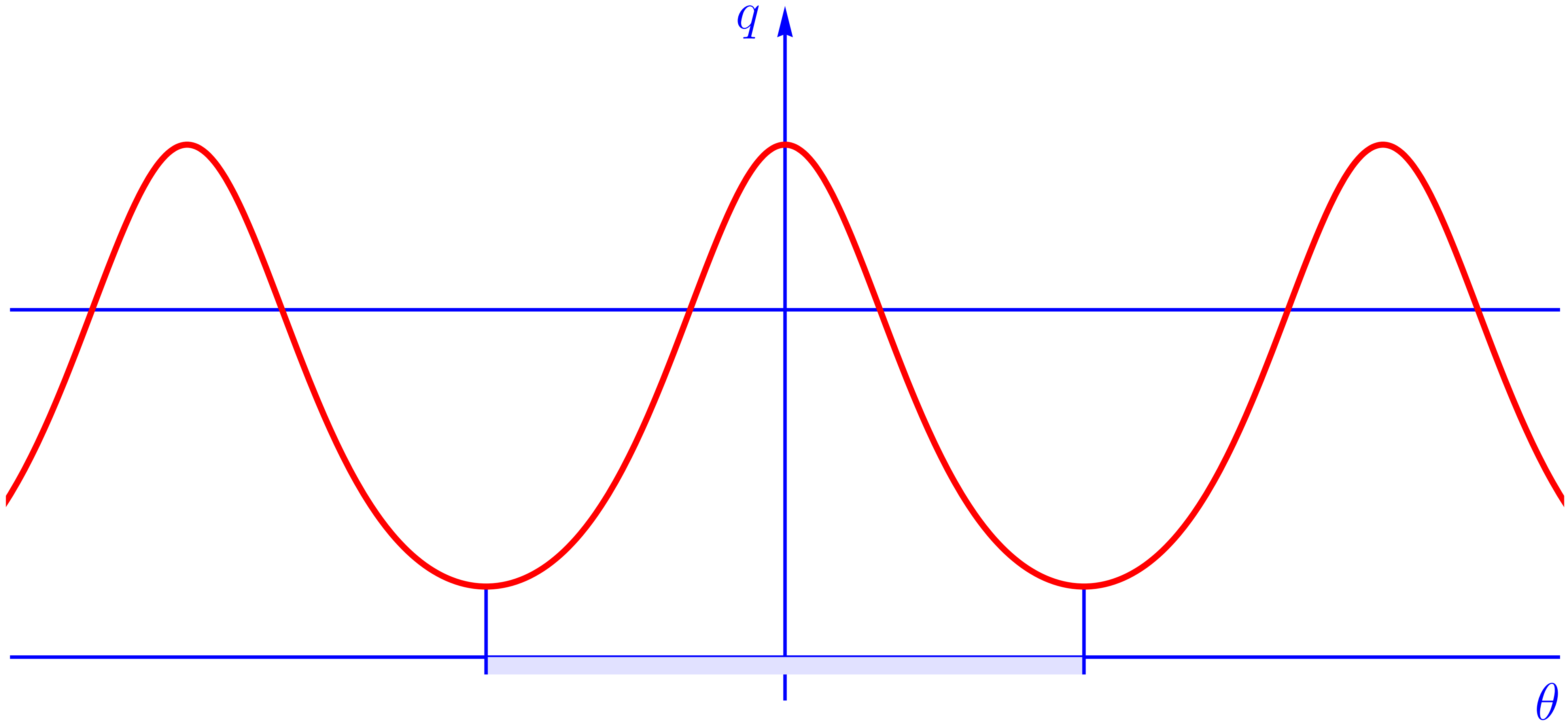

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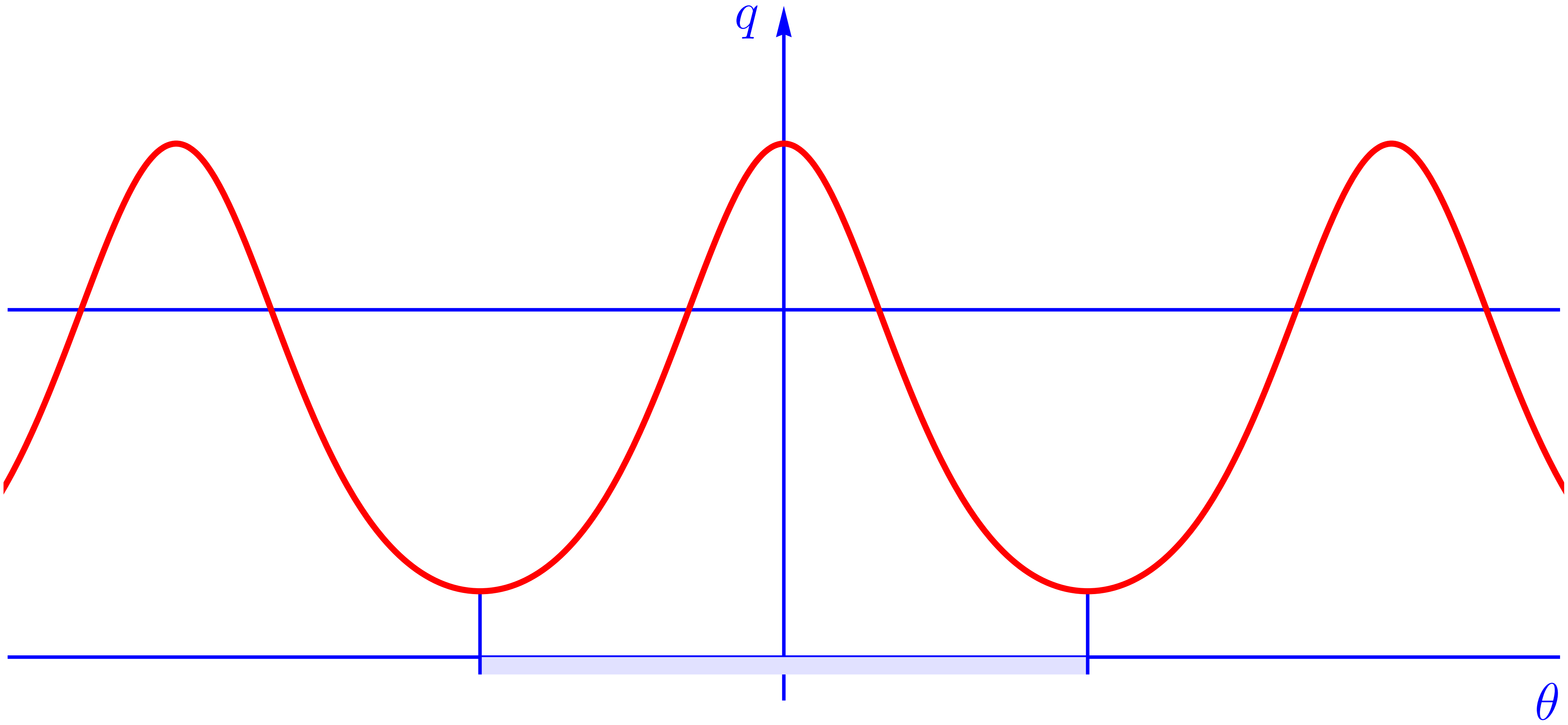

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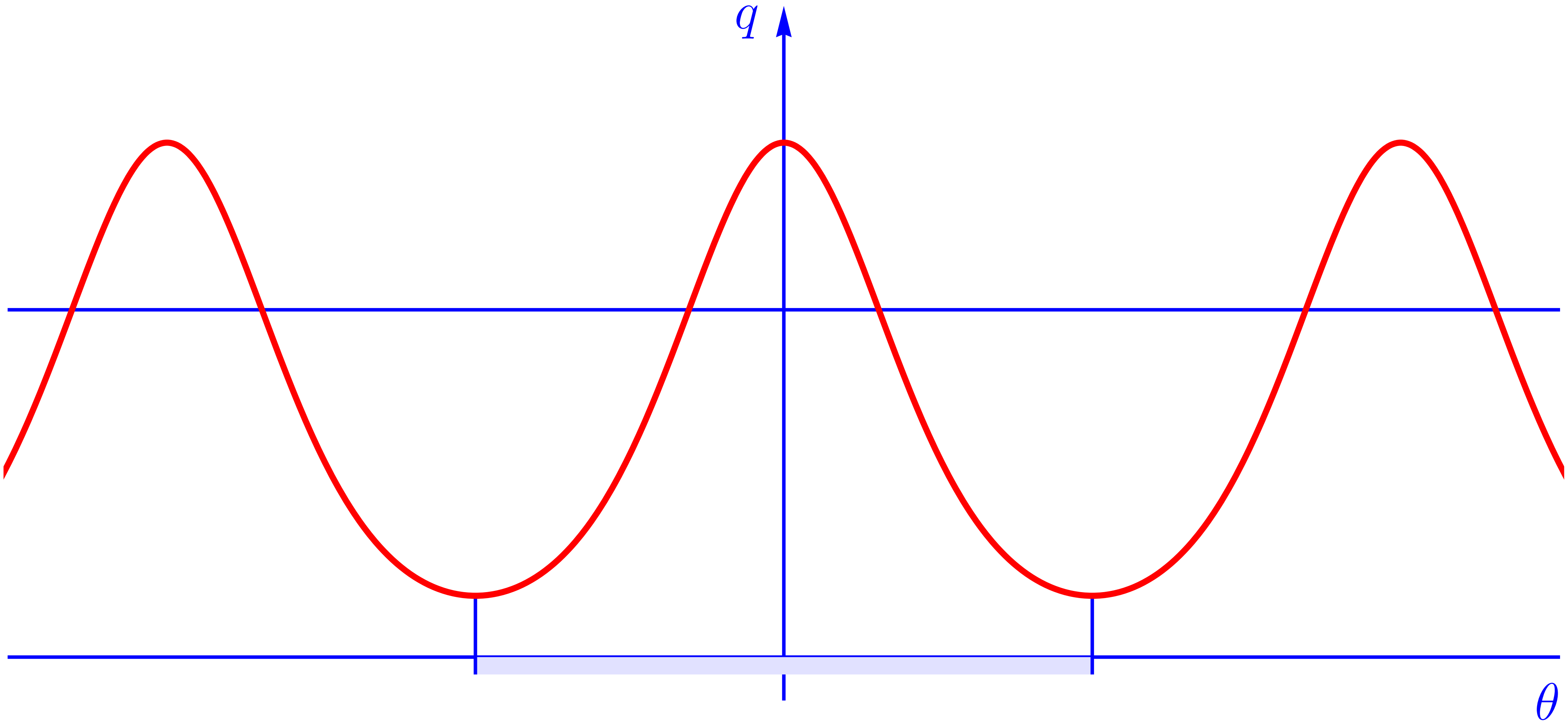

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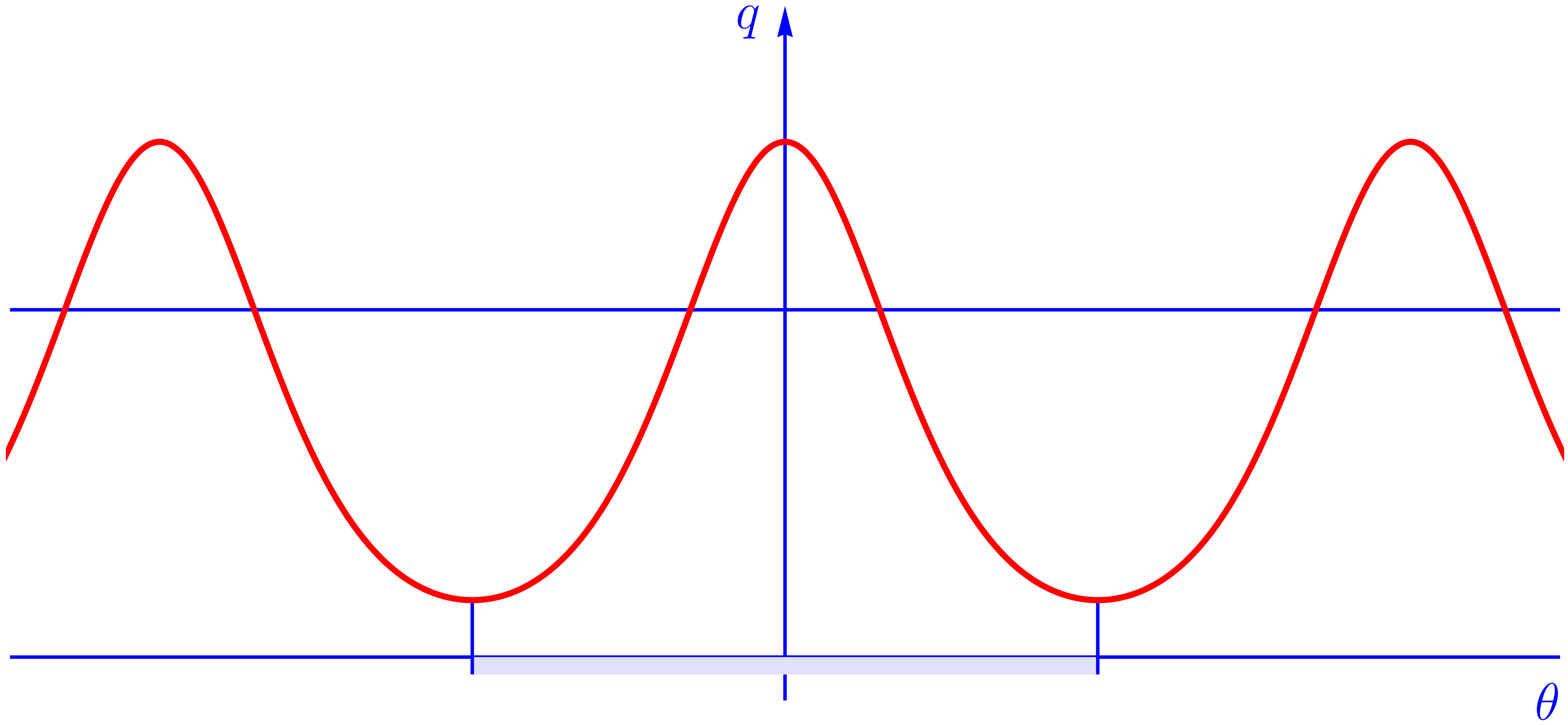

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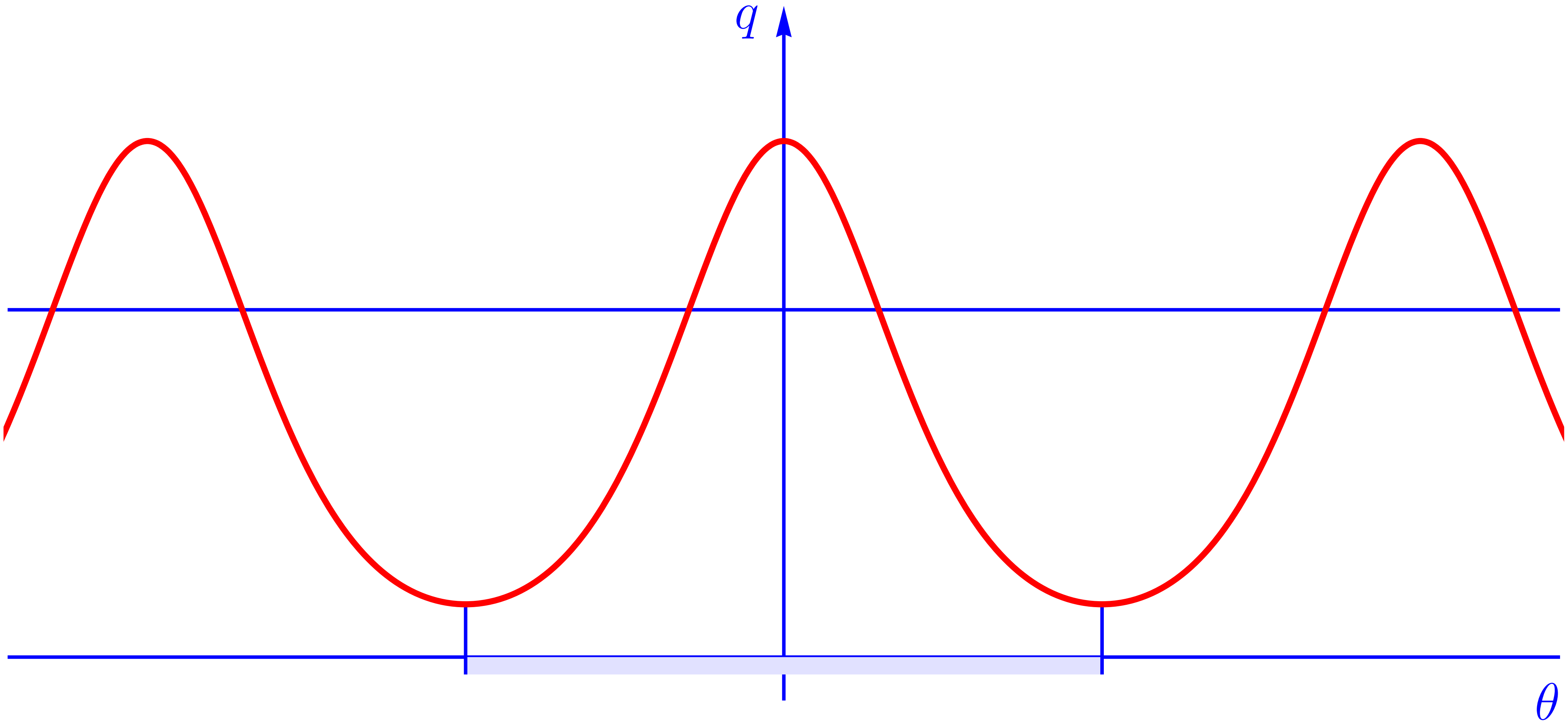

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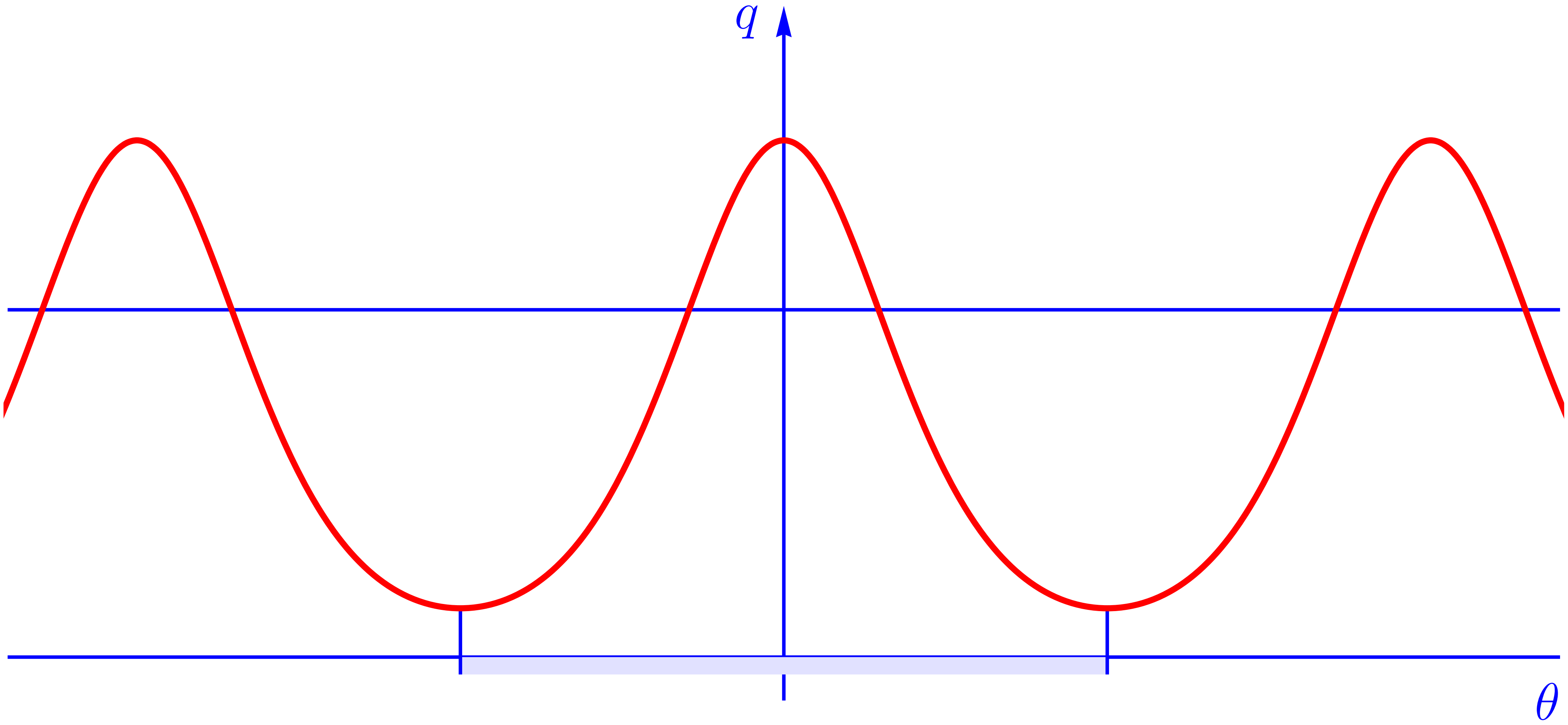

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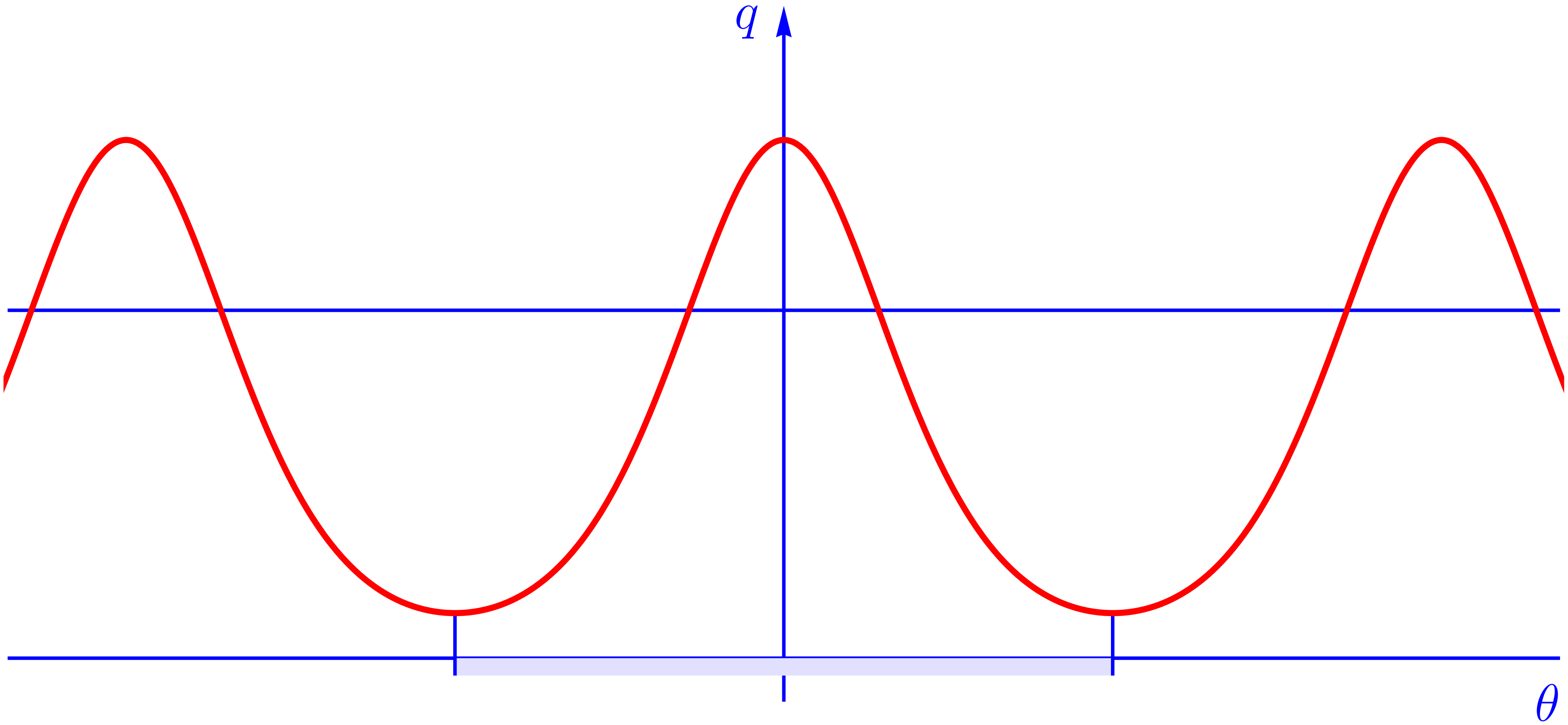

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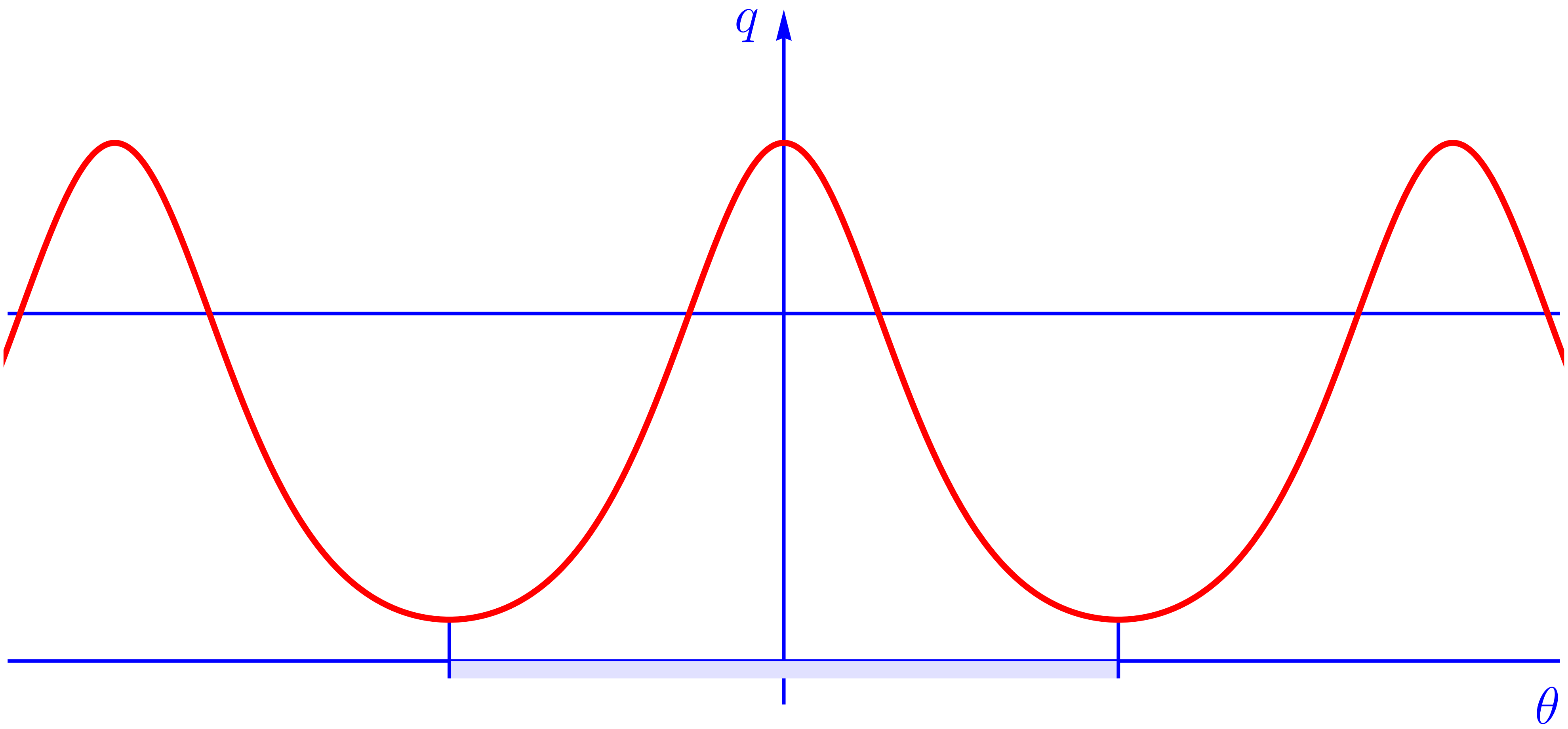

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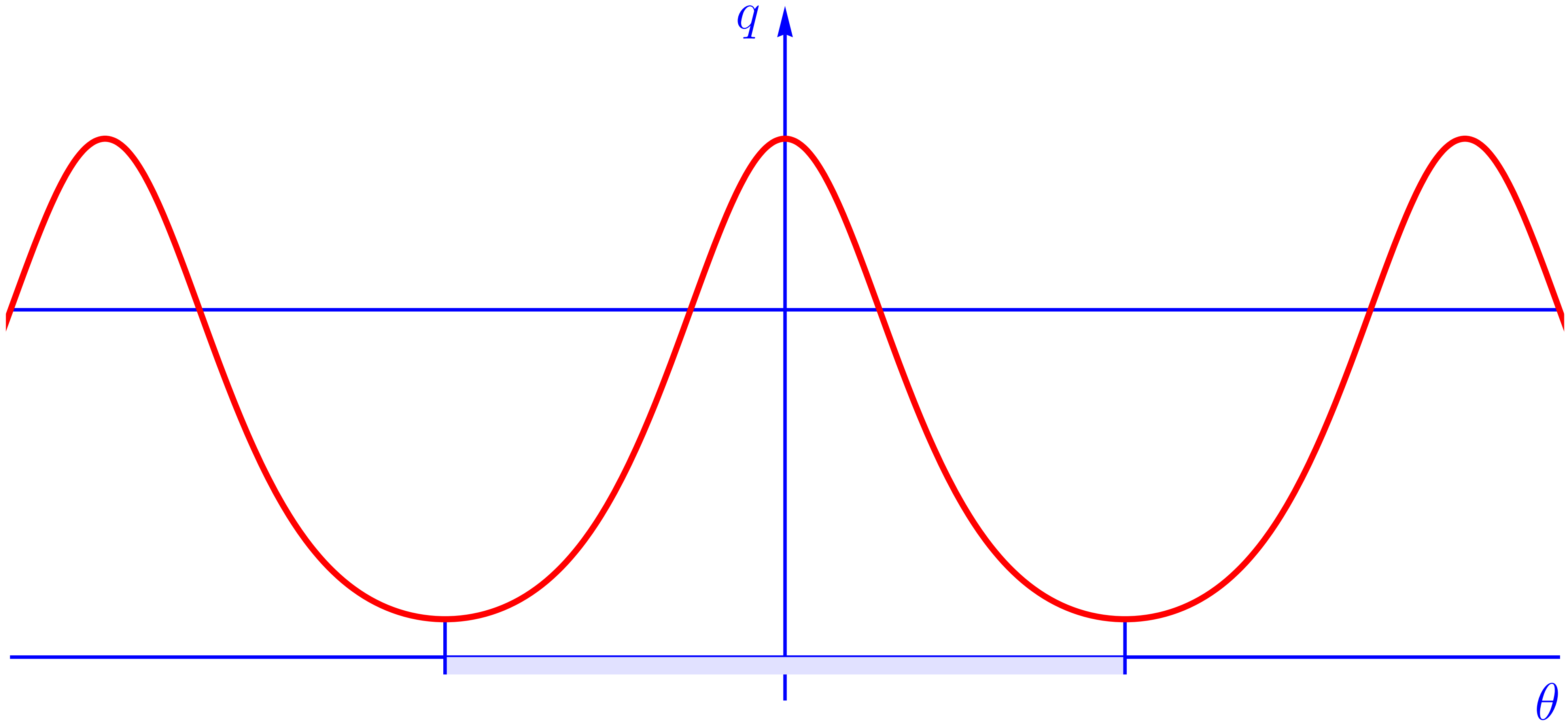

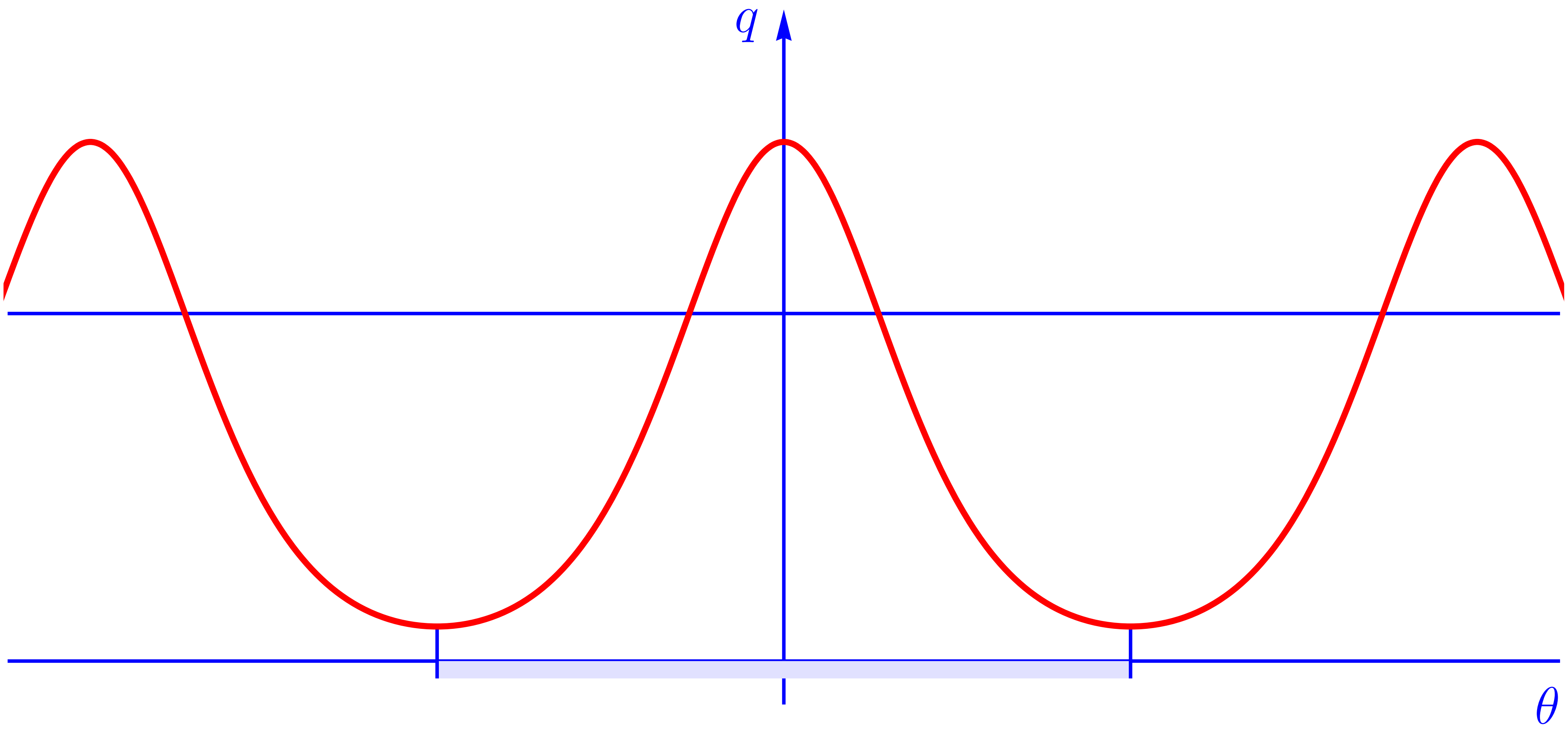
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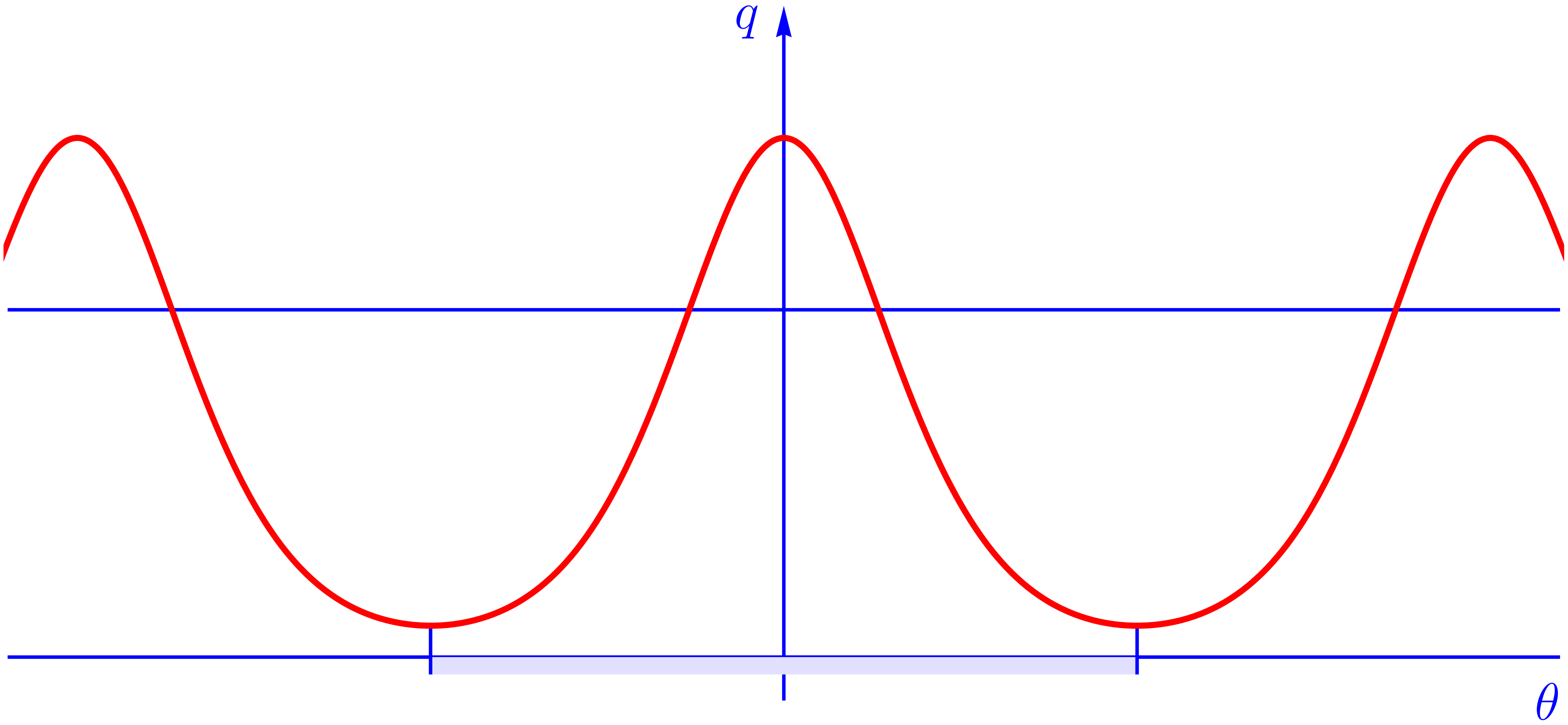
Cnoidal Waves



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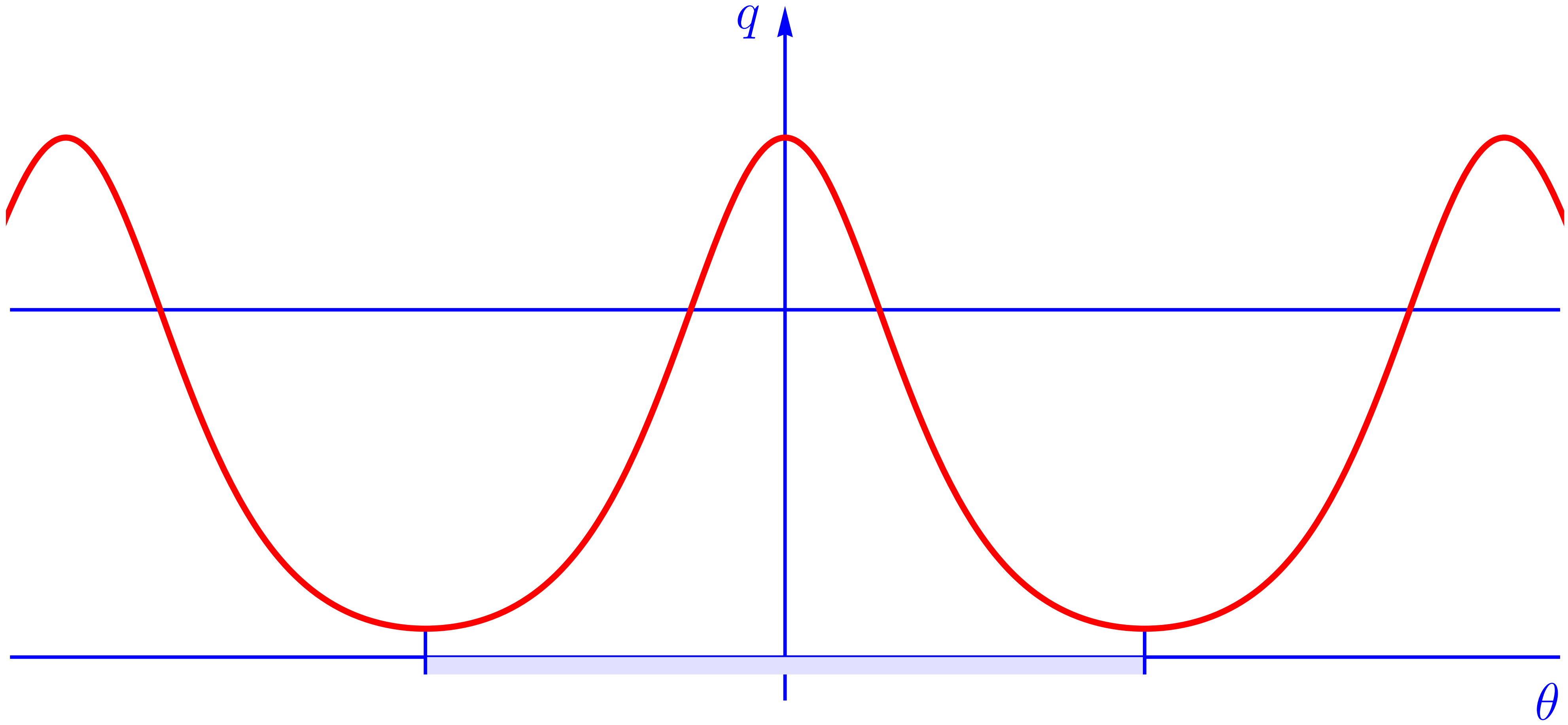

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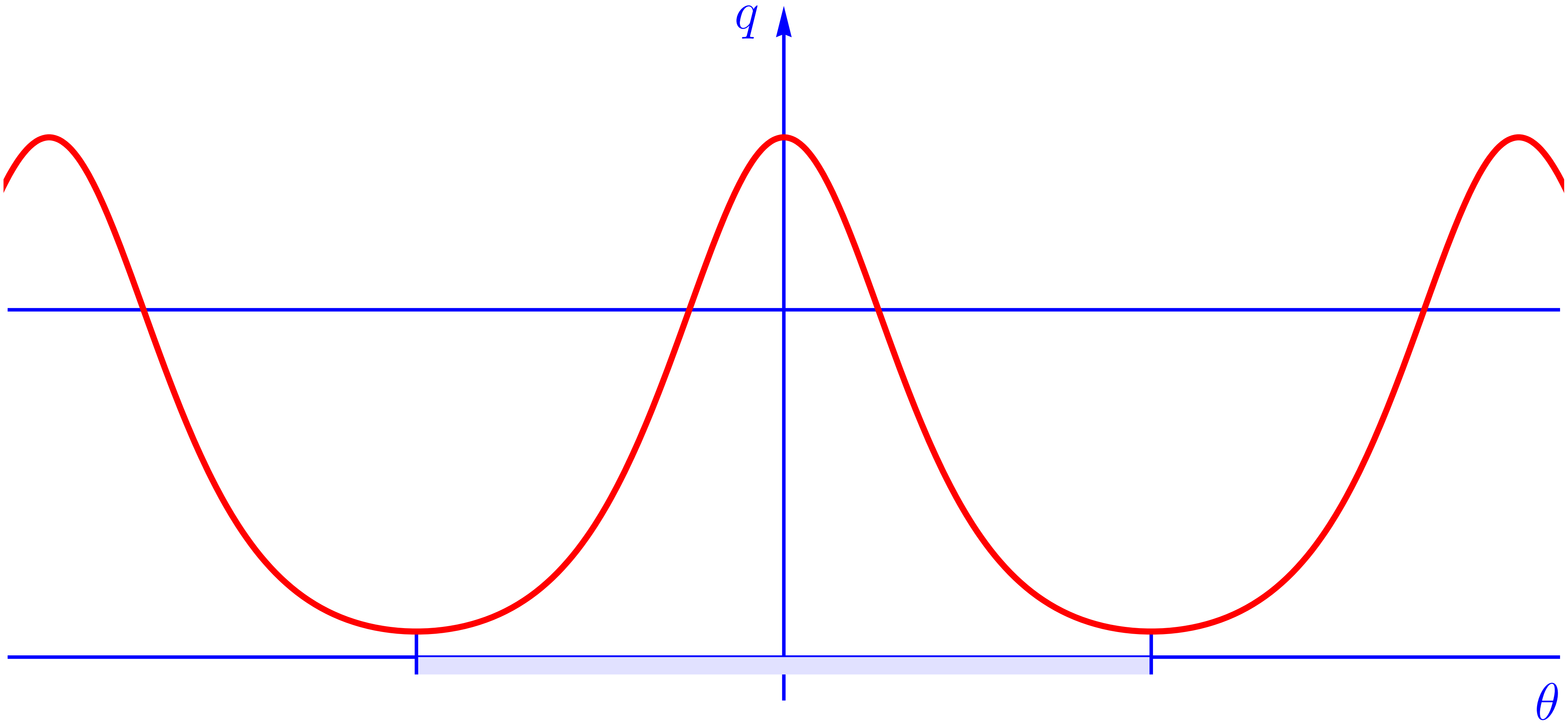

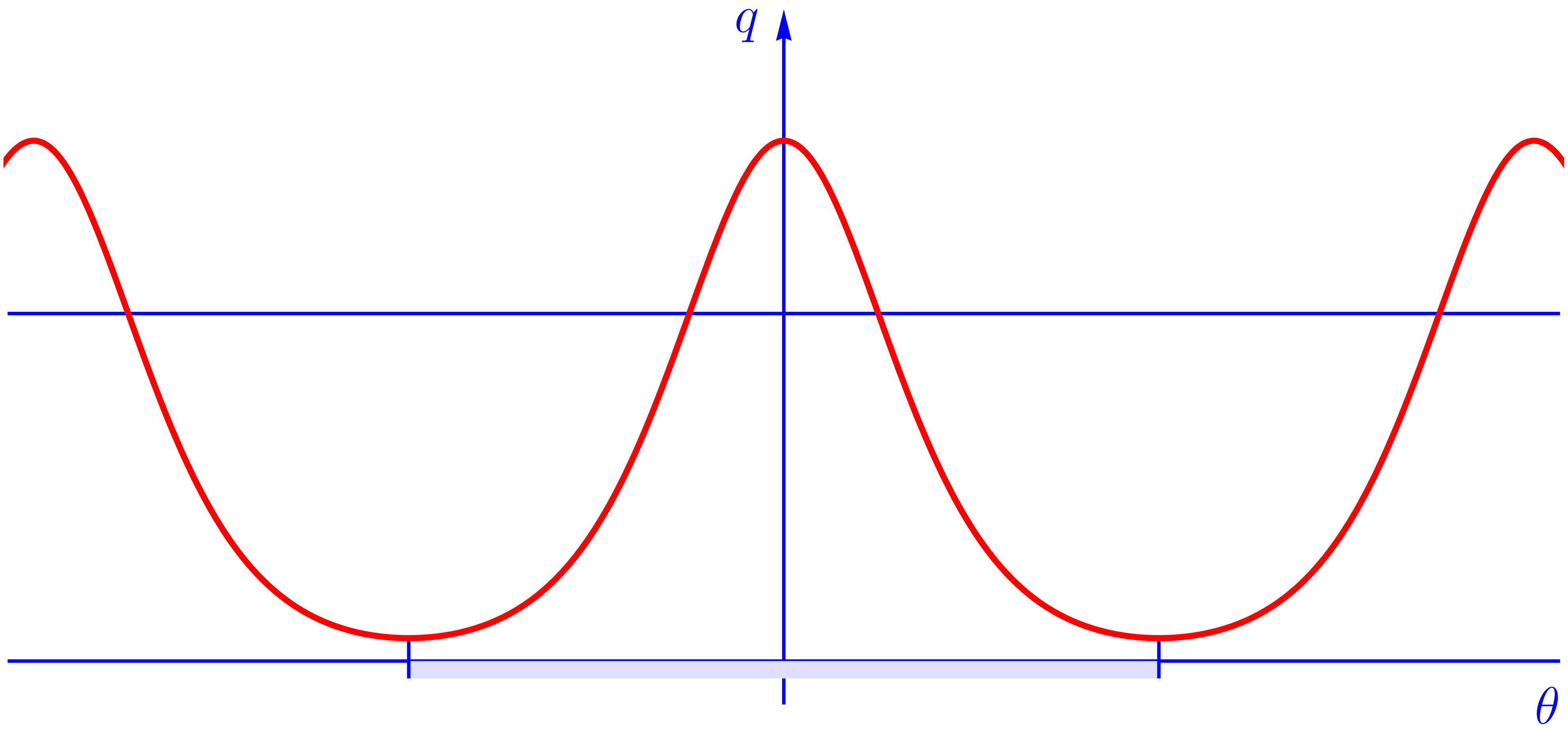
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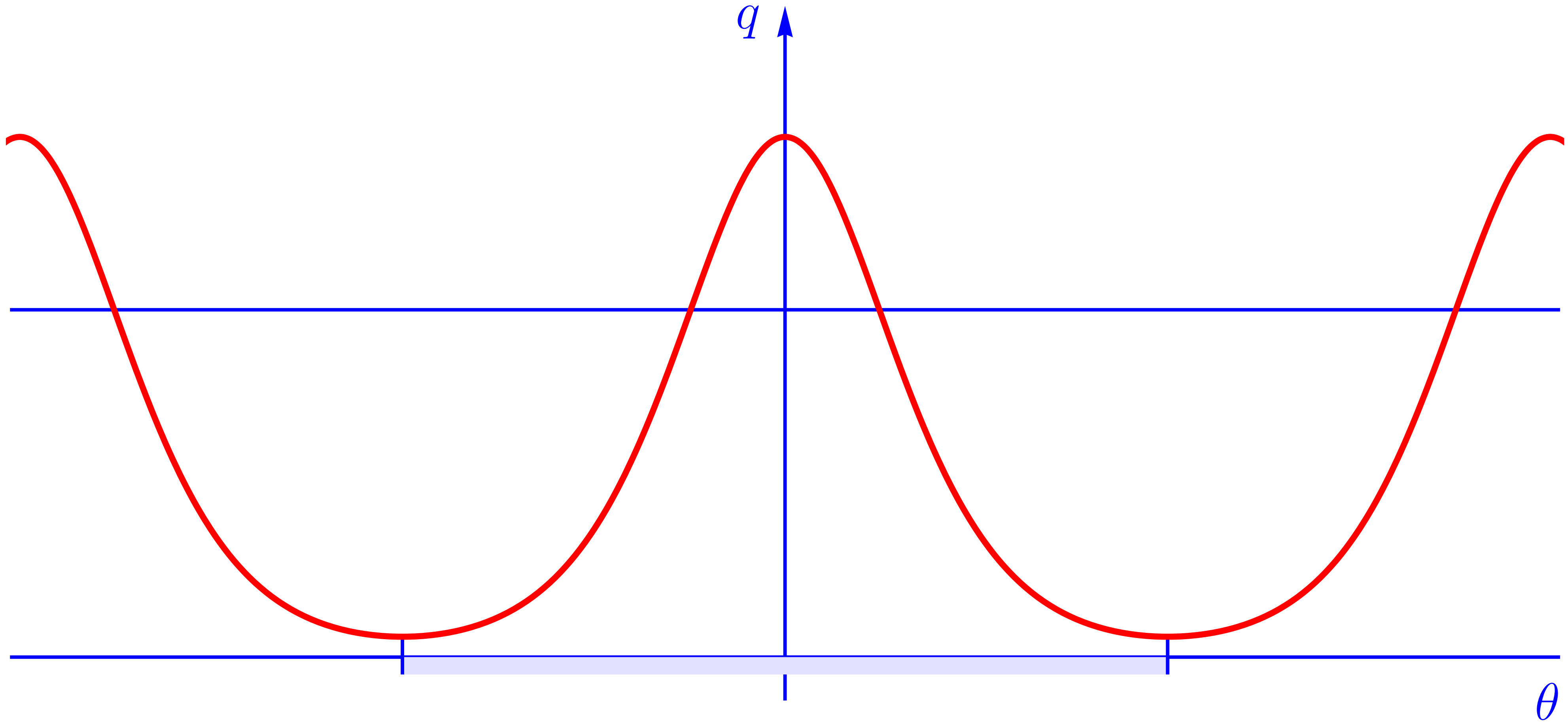
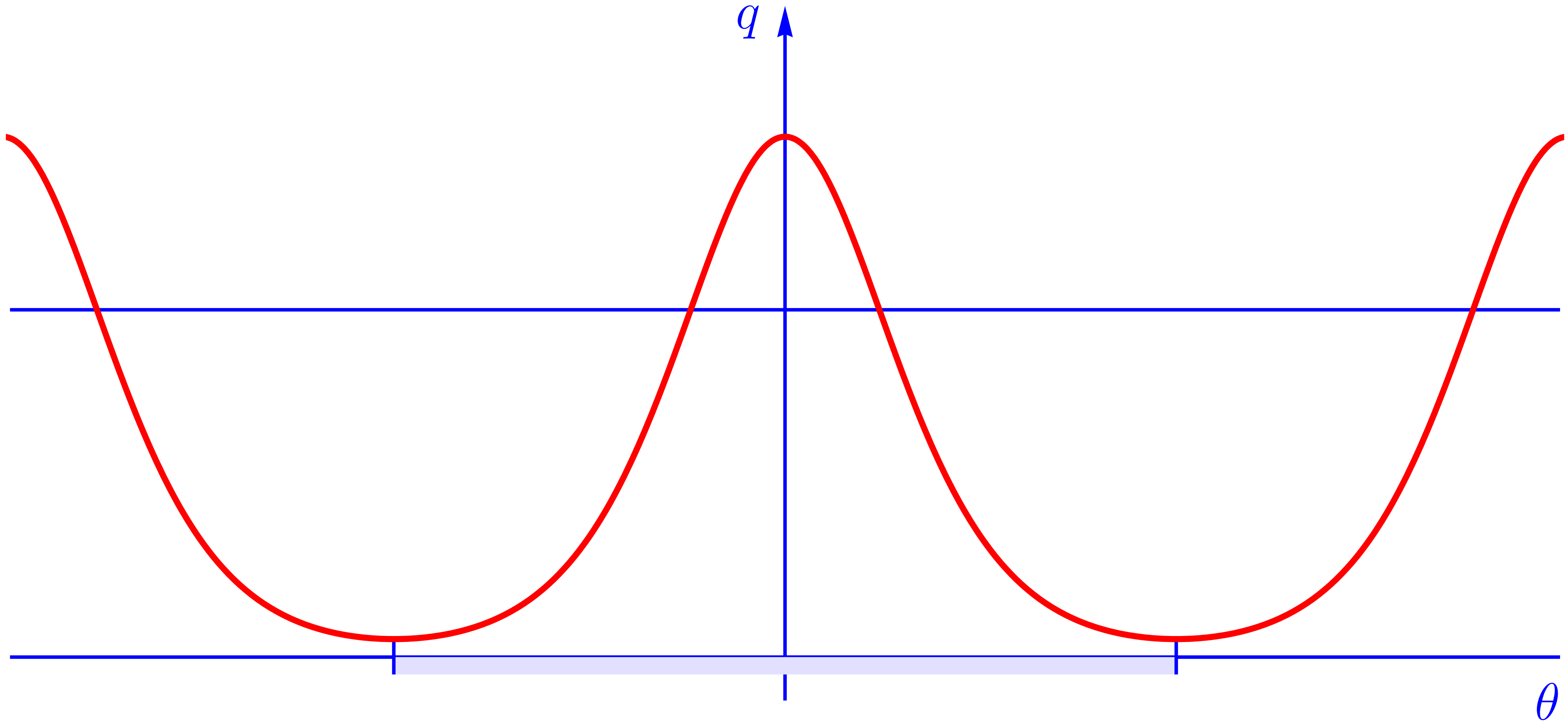
Cnoidal Waves

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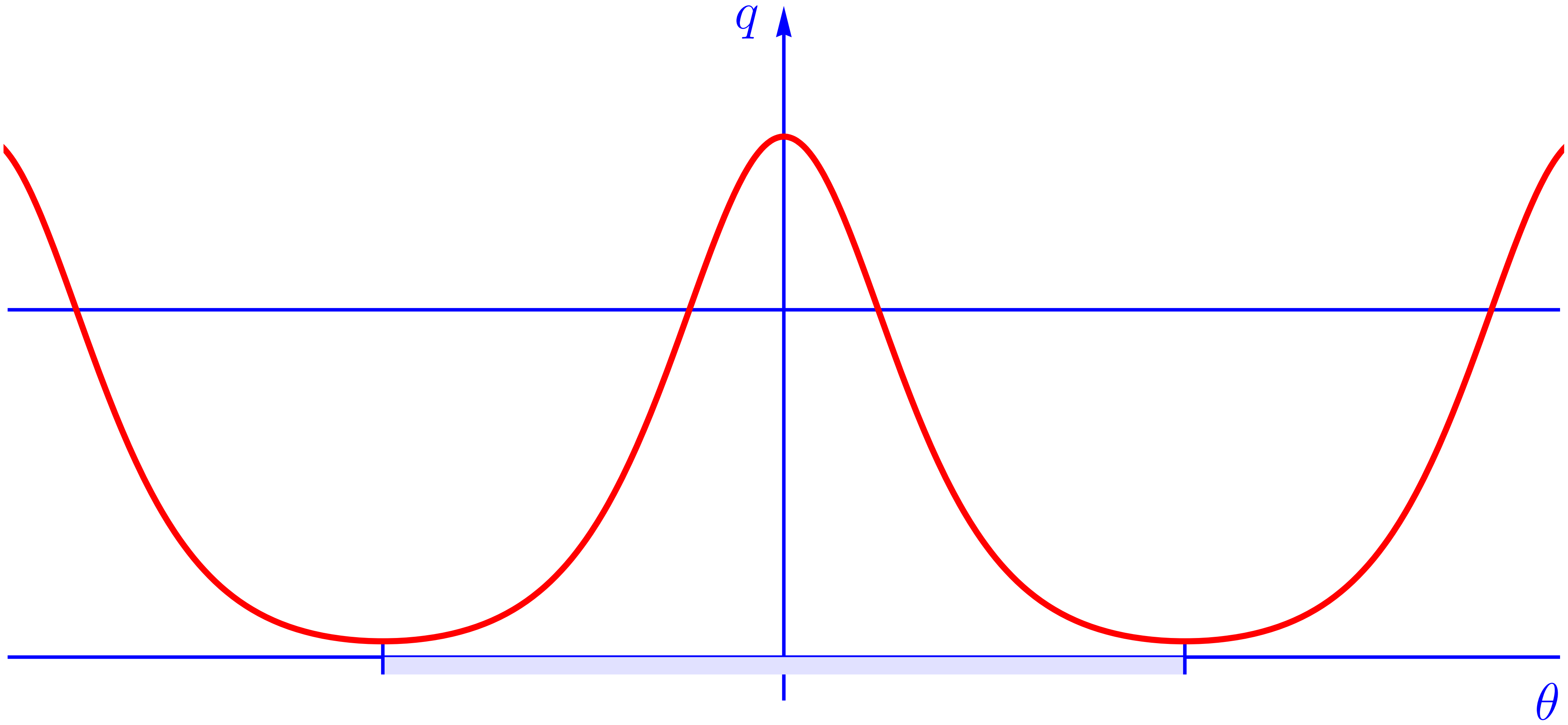
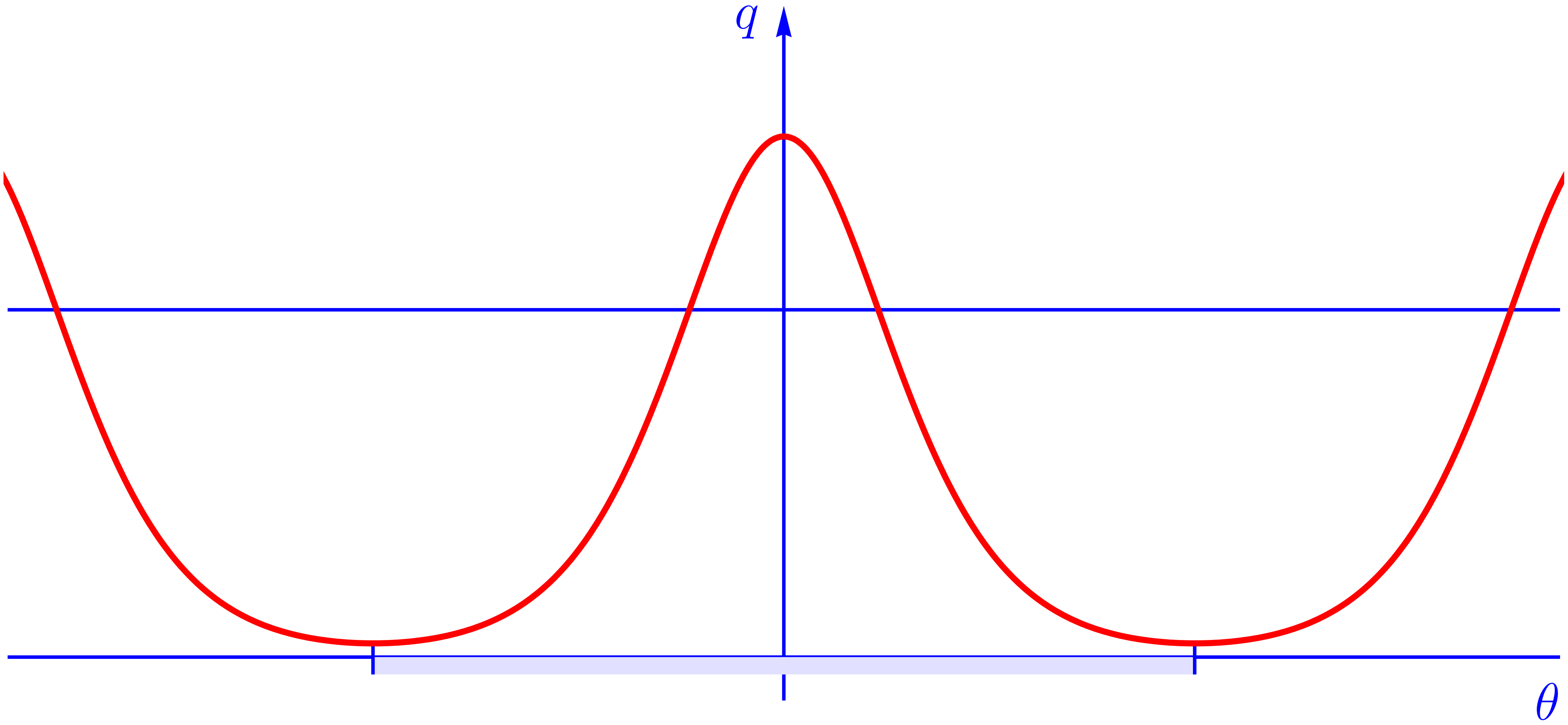
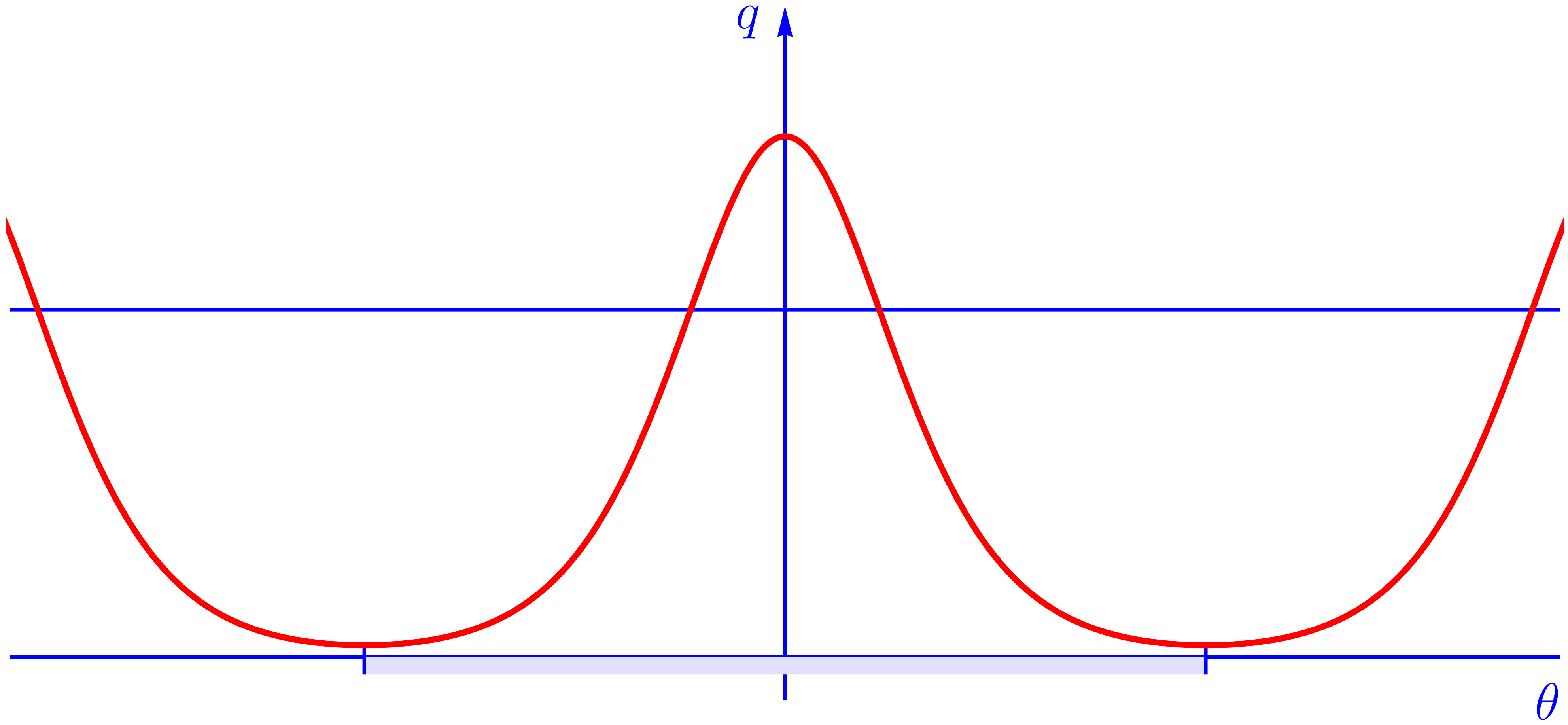
Cnoidal Waves



Cnoidal Waves

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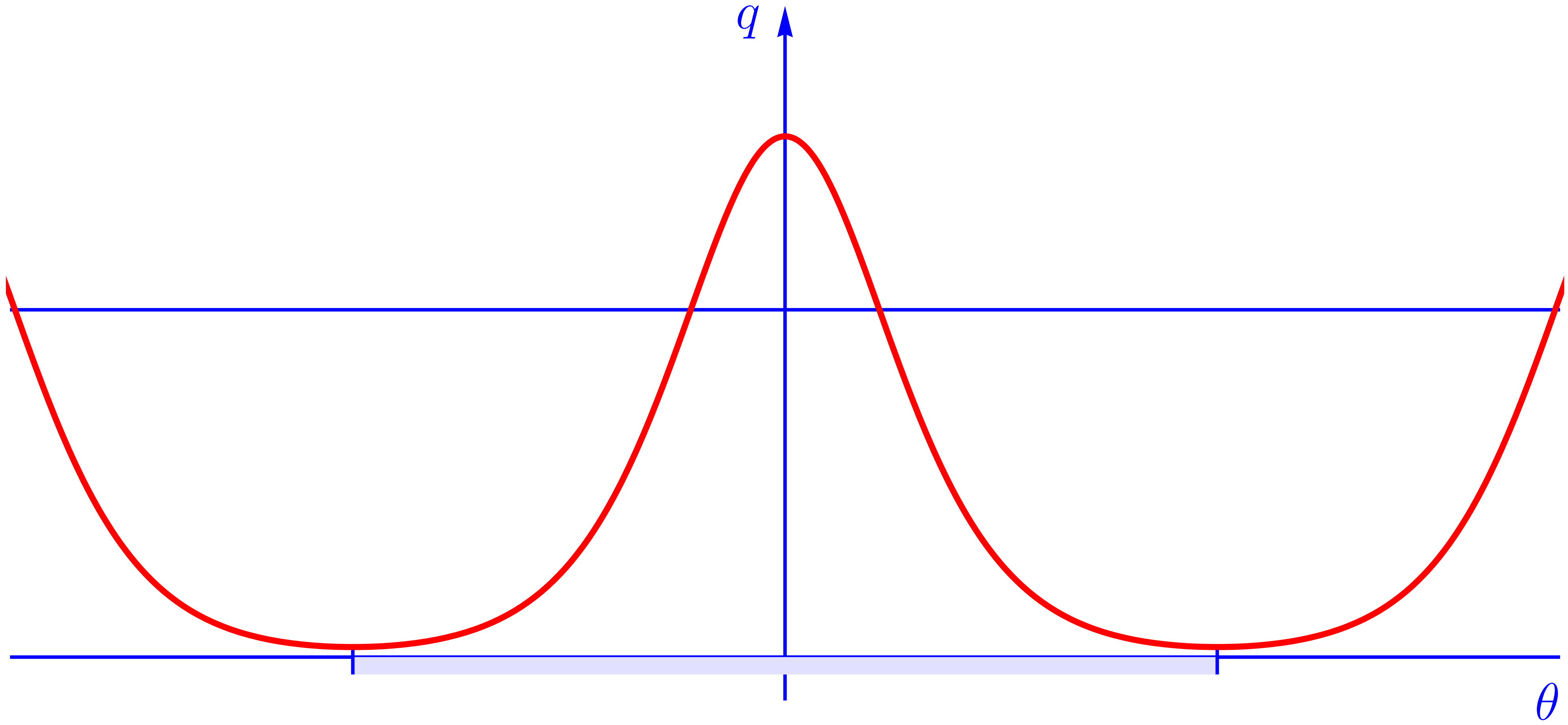
Cnoidal Waves



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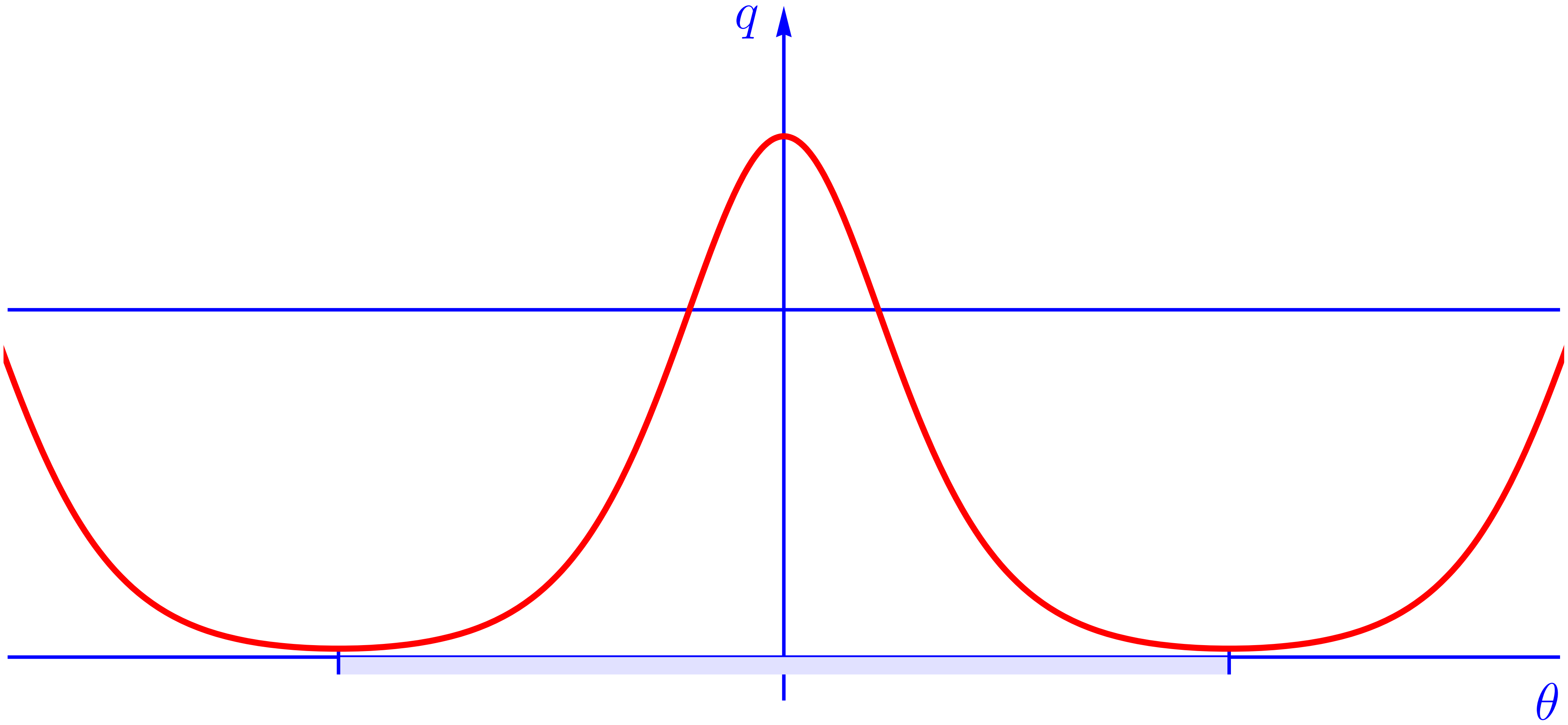

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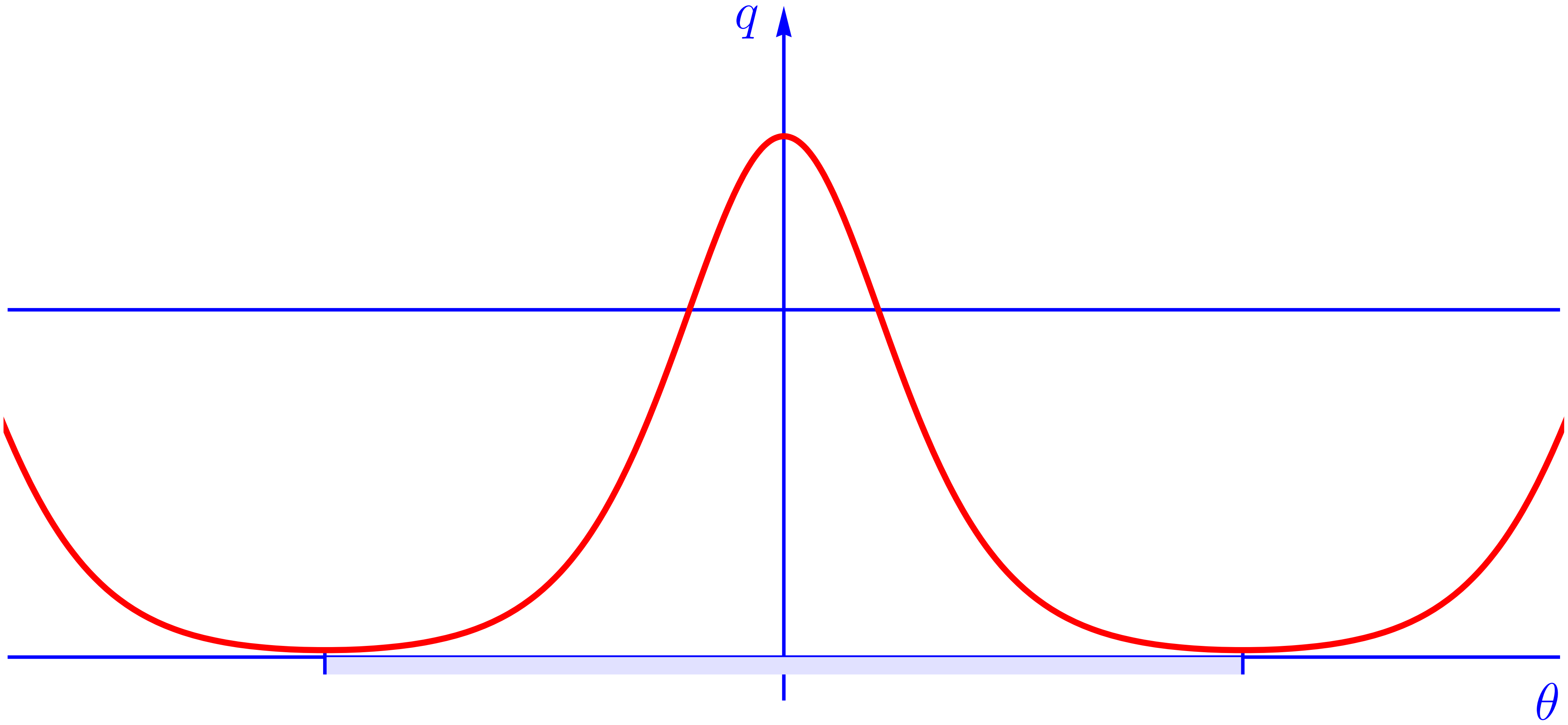

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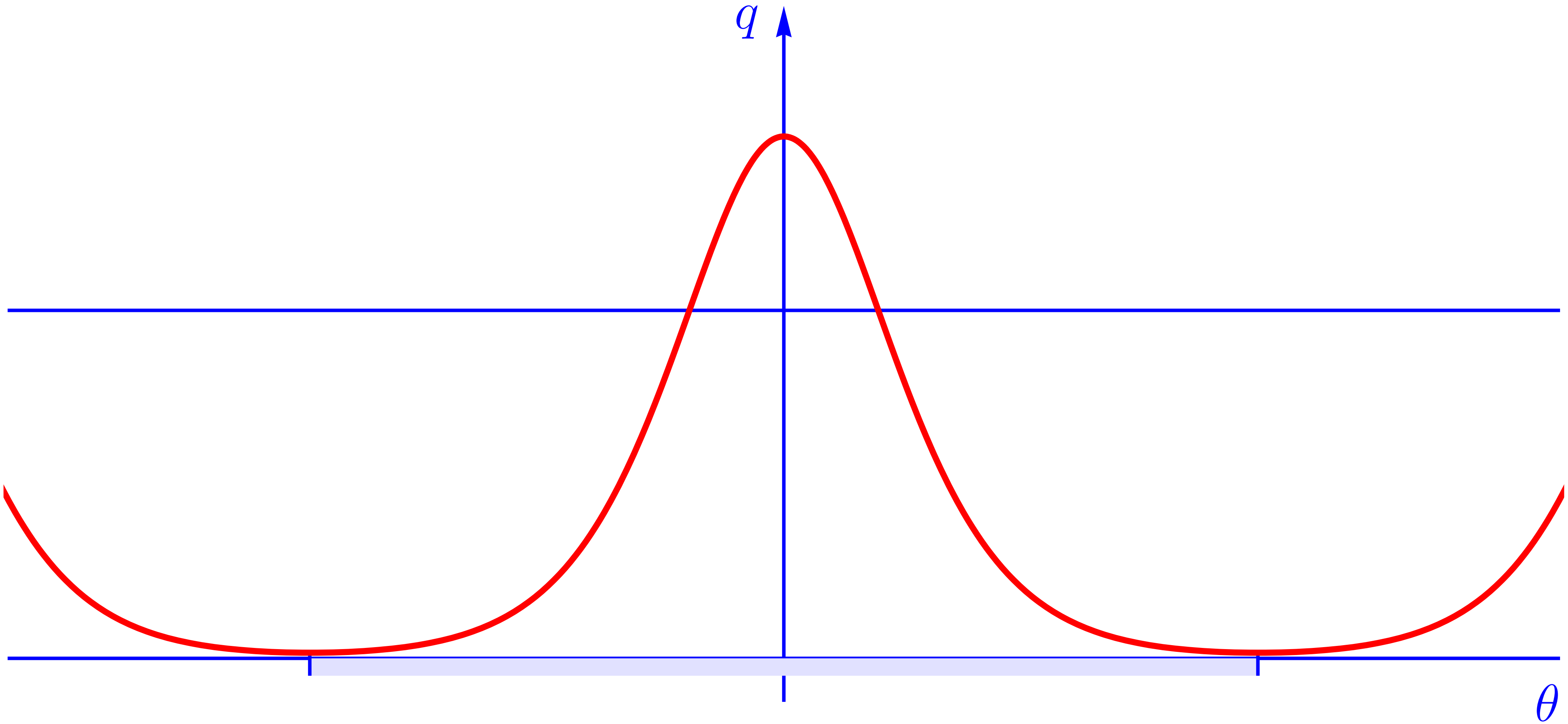

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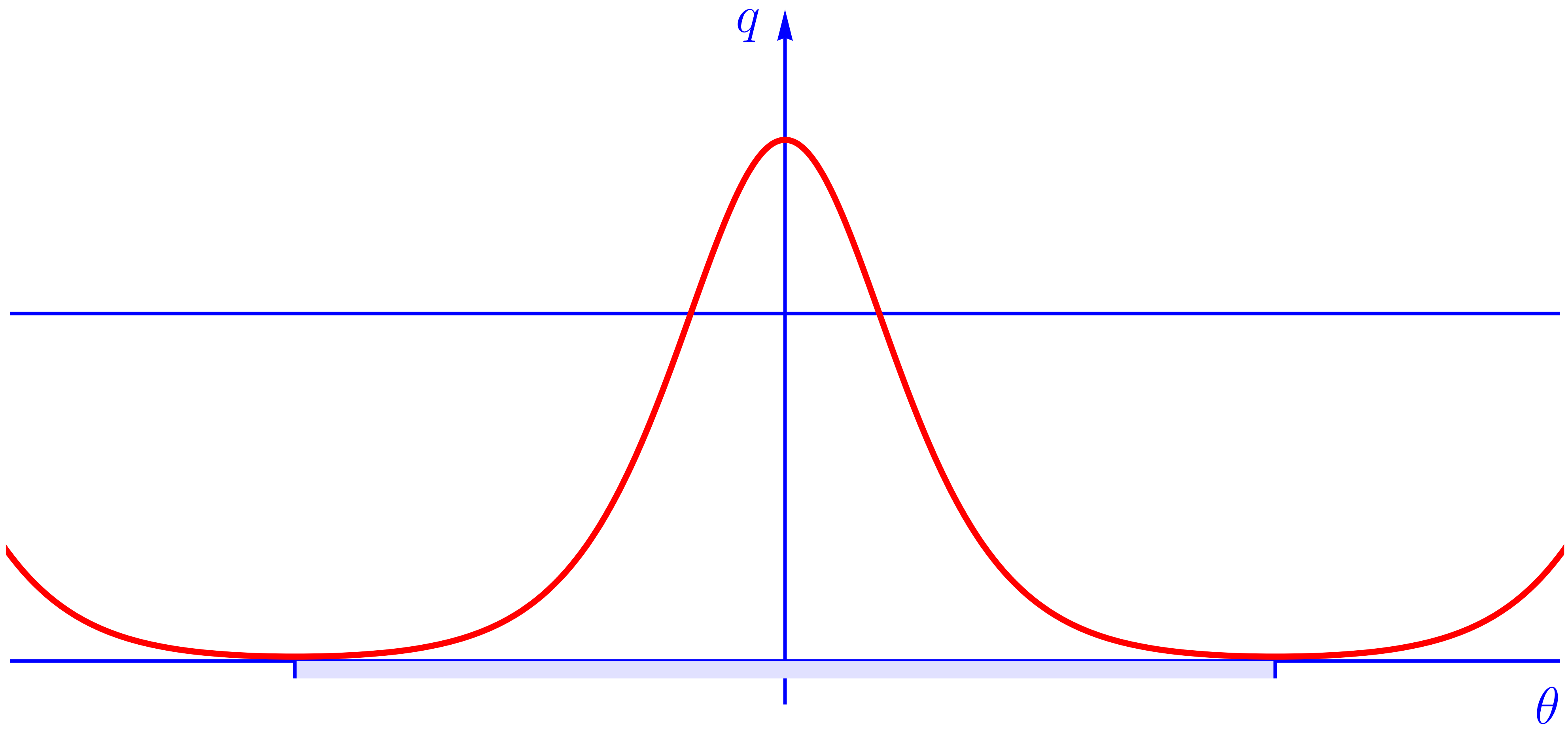

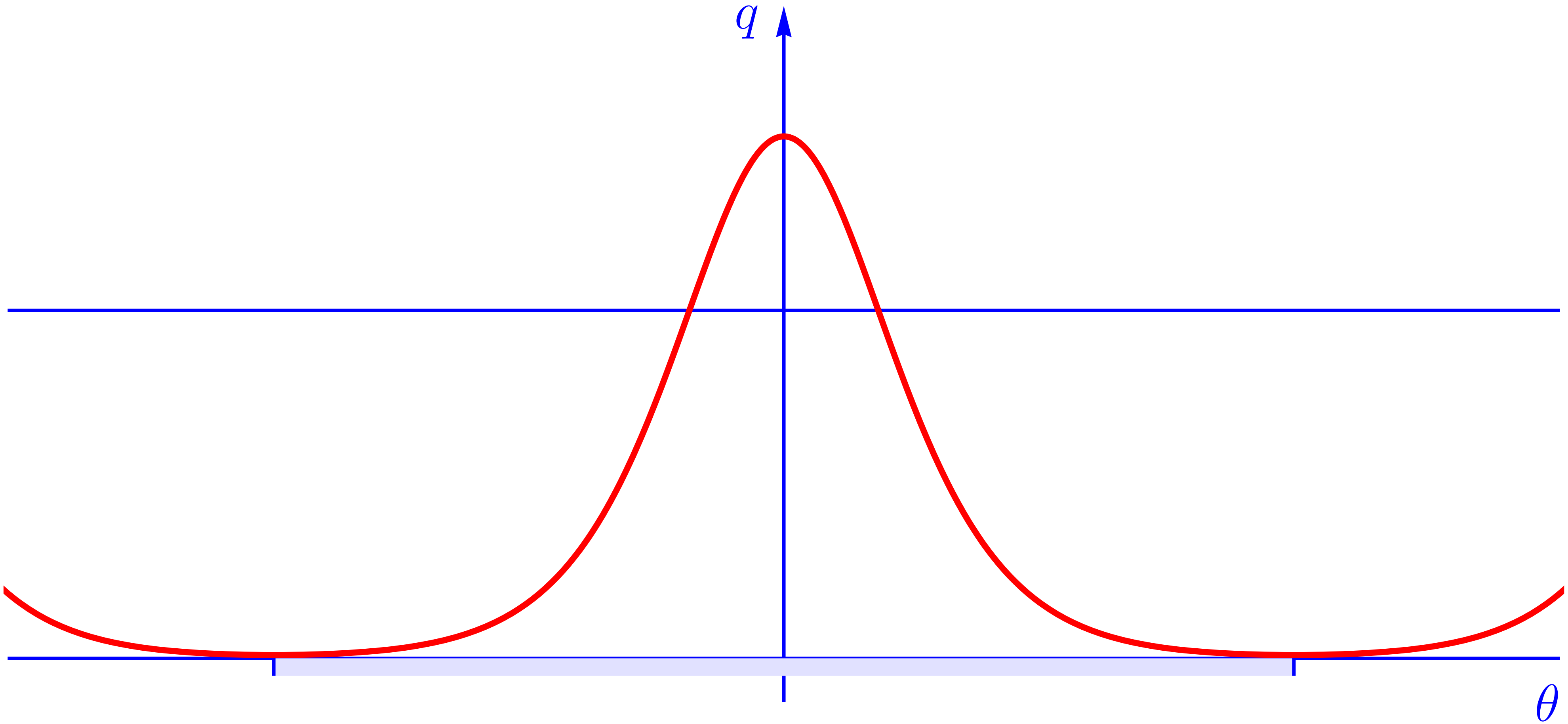
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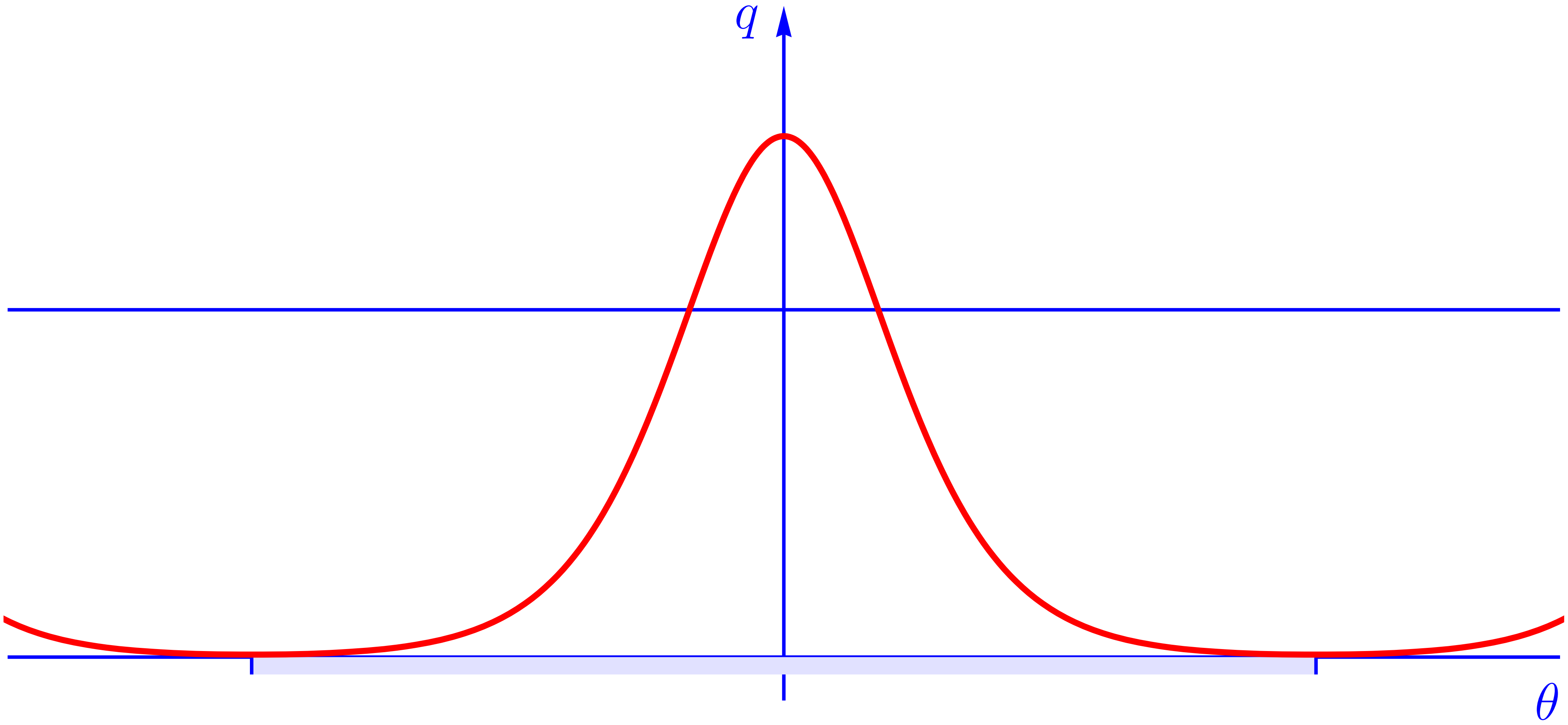
Cnoidal Waves



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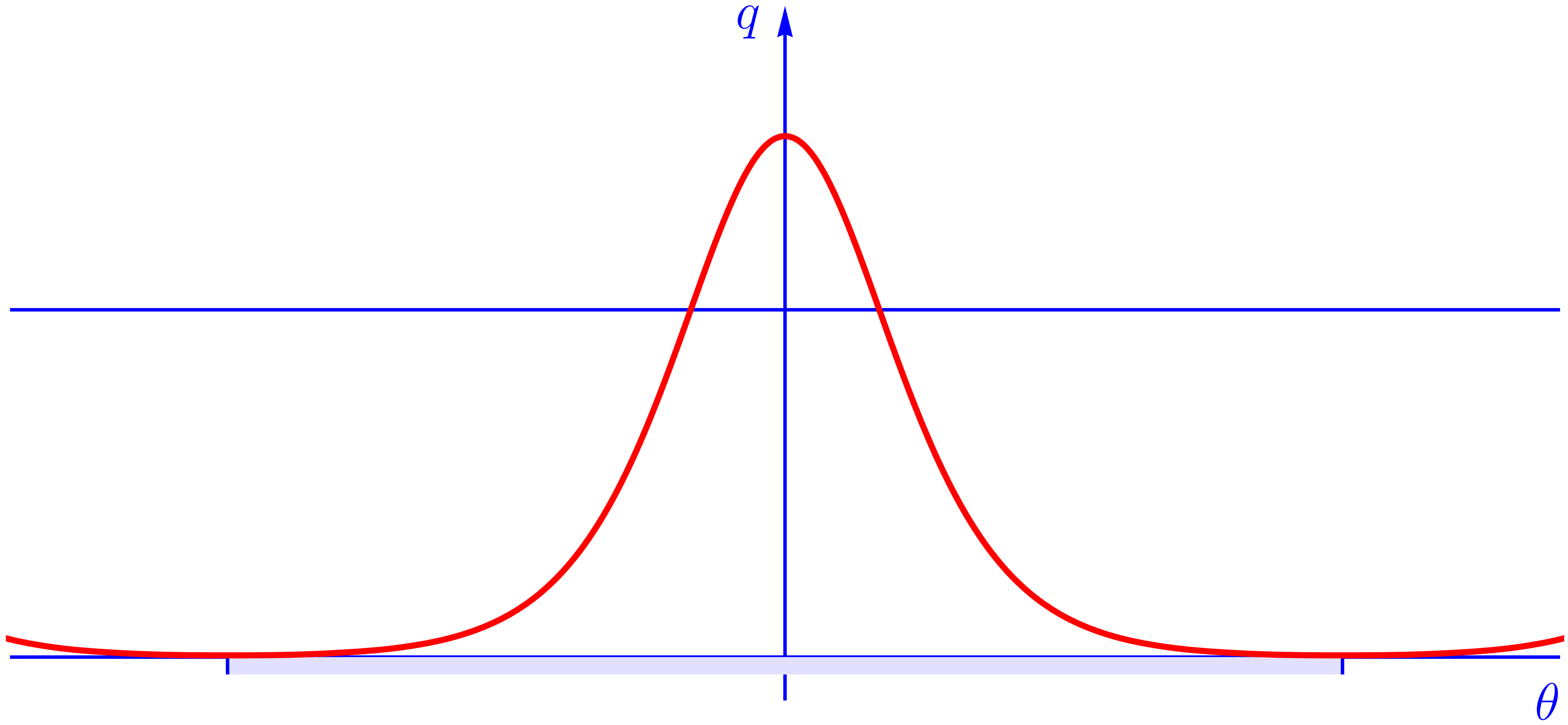

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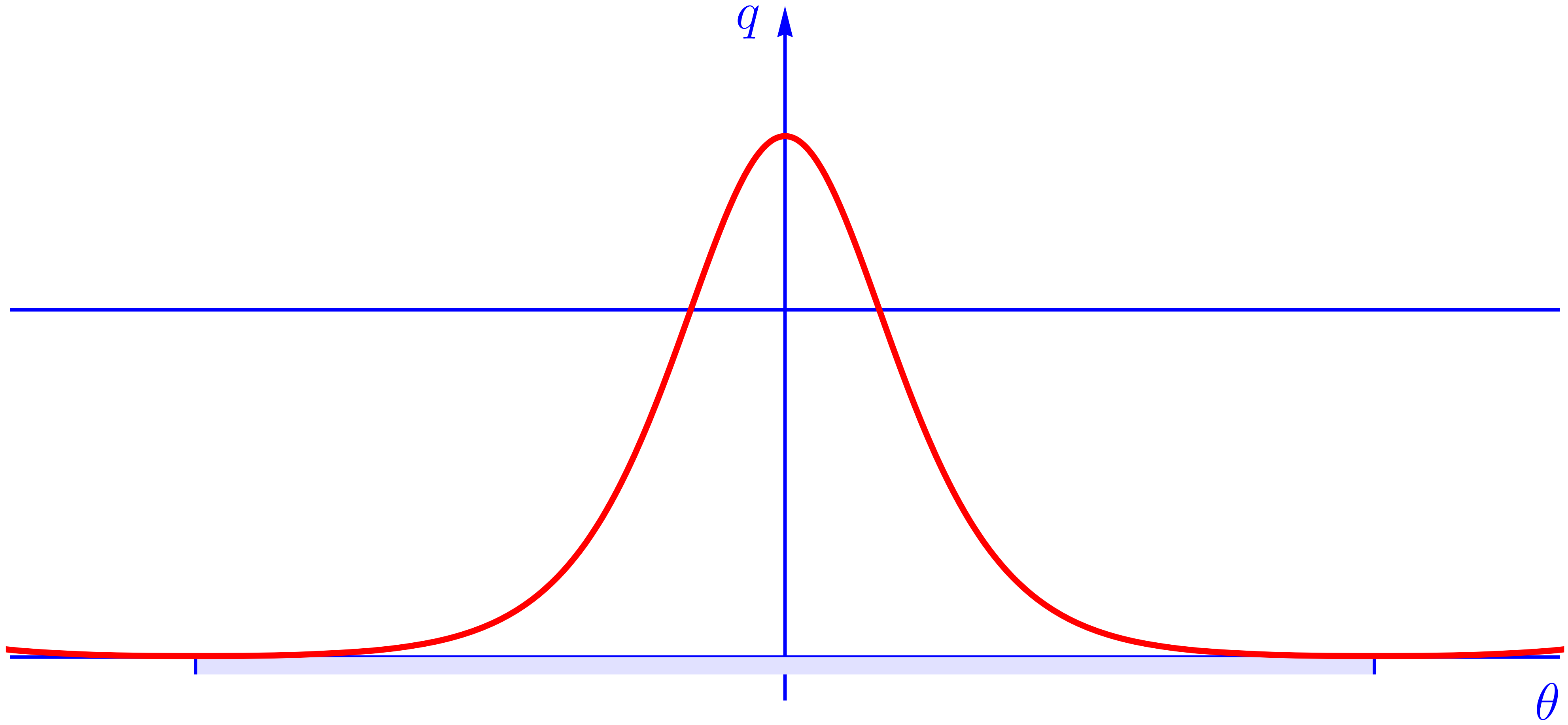
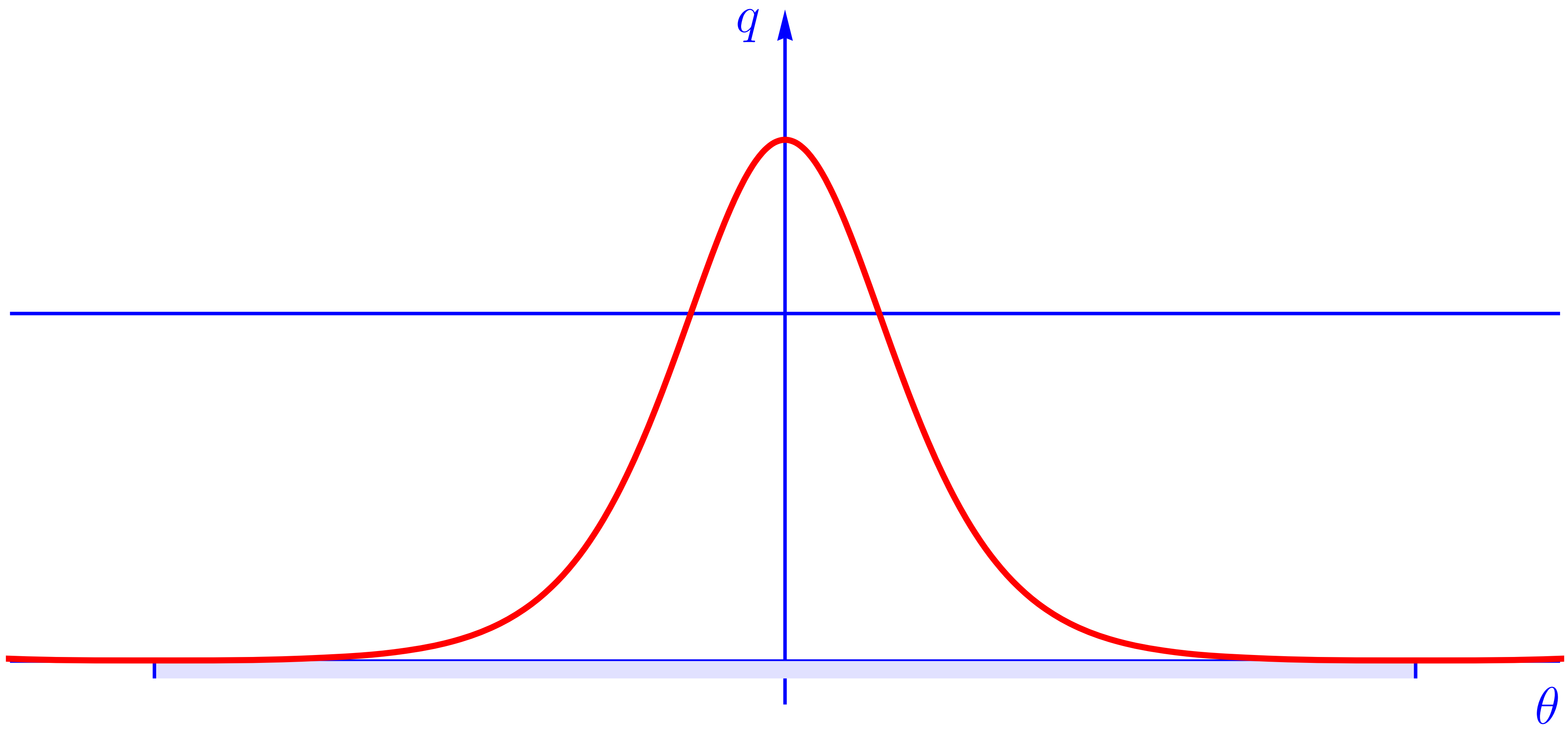
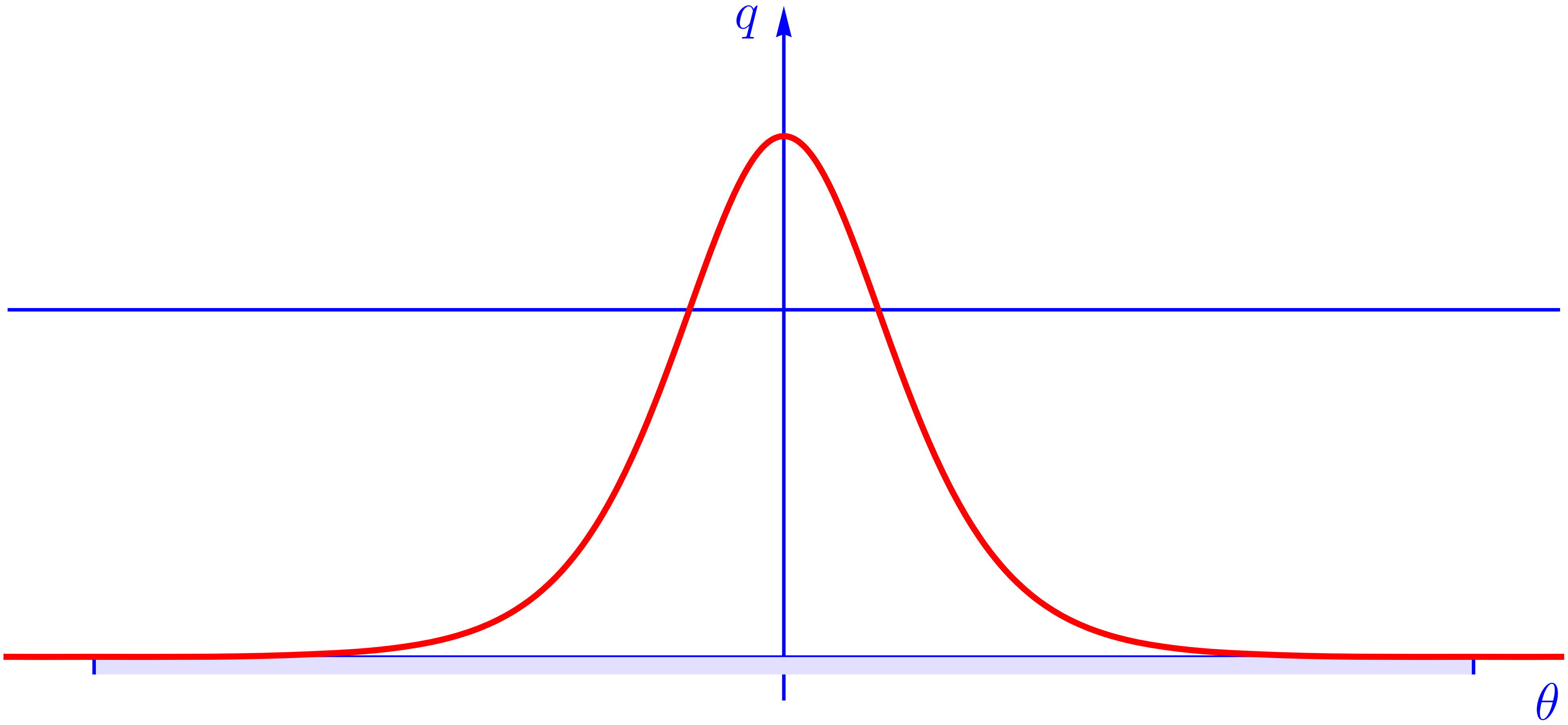
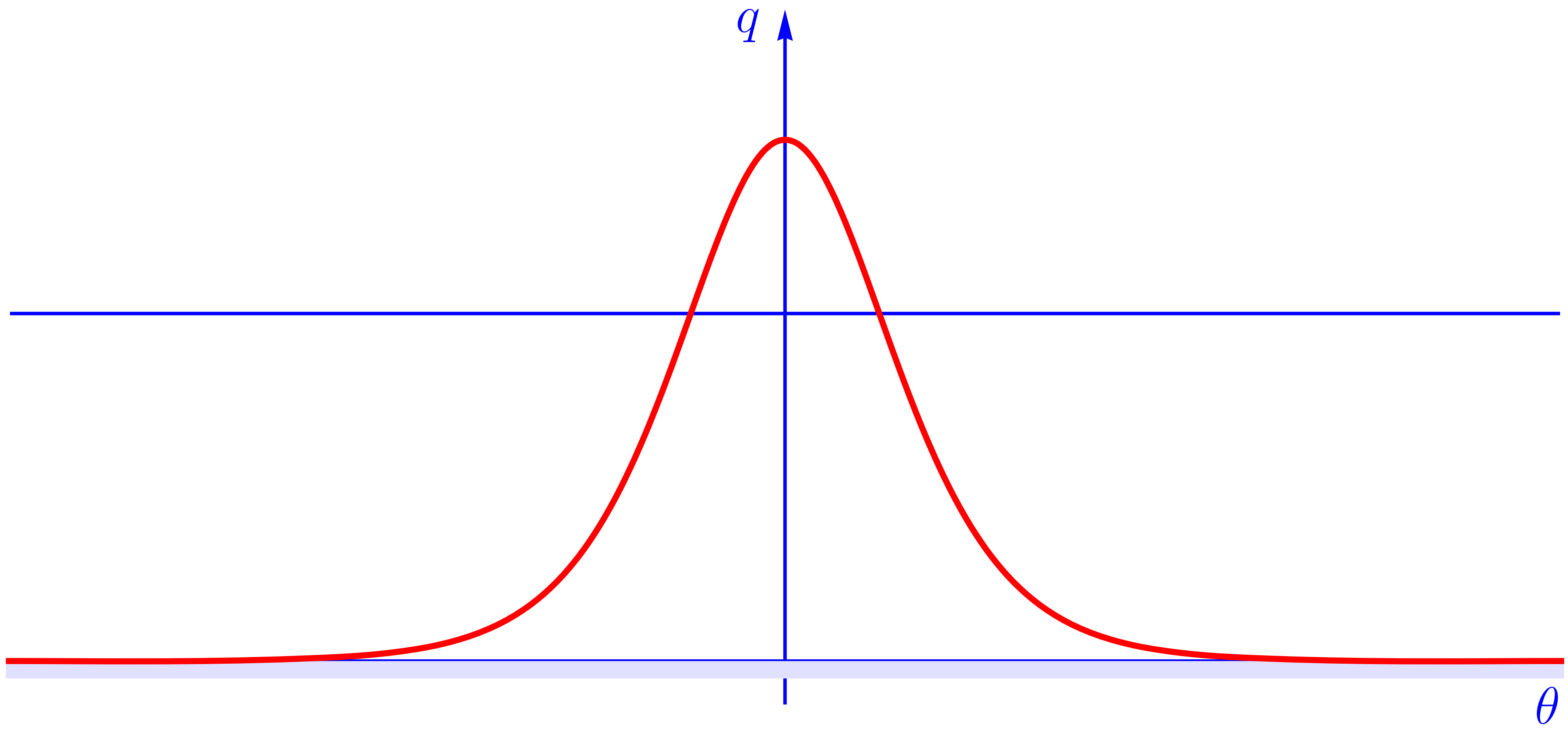


Cnoidal Waves



Cnoidal Waves




q
θ

Extended Korteweg-de Vries Equation

KdV equation extended by micro-nonlinearity term

$$y_t + 3\left(y^2\right)_x + y_{xxx} + 3\varepsilon\left(y_x^2\right)_{xx} = 0$$

Waves of constant shape

$$y(x,t) = q(\theta) \qquad \theta = x - ct$$

Ordinary differential equation

$$3\varepsilon\left(q'^2\right)'' - cq' + 3\left(q^2\right)' + q''' = 0$$

$$q' \equiv \frac{\mathrm{d}q}{\mathrm{d}\theta}$$

First integration

$$q'' + 3\varepsilon \left(q'^2\right)' = A + cq - 3q^2$$

... considering q' as a function $q'(q)$:

$$q'' = q' \frac{\mathrm{d}q'}{\mathrm{d}q} \qquad \left(q'^2\right)' = 2q'q'' = 2q'^2 \frac{\mathrm{d}q'}{\mathrm{d}q}$$

First-order differential equation

$$\left(q' + 6\varepsilon q'^2\right) \frac{\mathrm{d}q'}{\mathrm{d}q} = A + cq - 3q^2$$

Integration (Continued)

Second integration

$$4\varepsilon q'^3 + q'^2 = 2B + 2Aq + cq^2 - 2q^3 \equiv f(q)$$

Phase curves

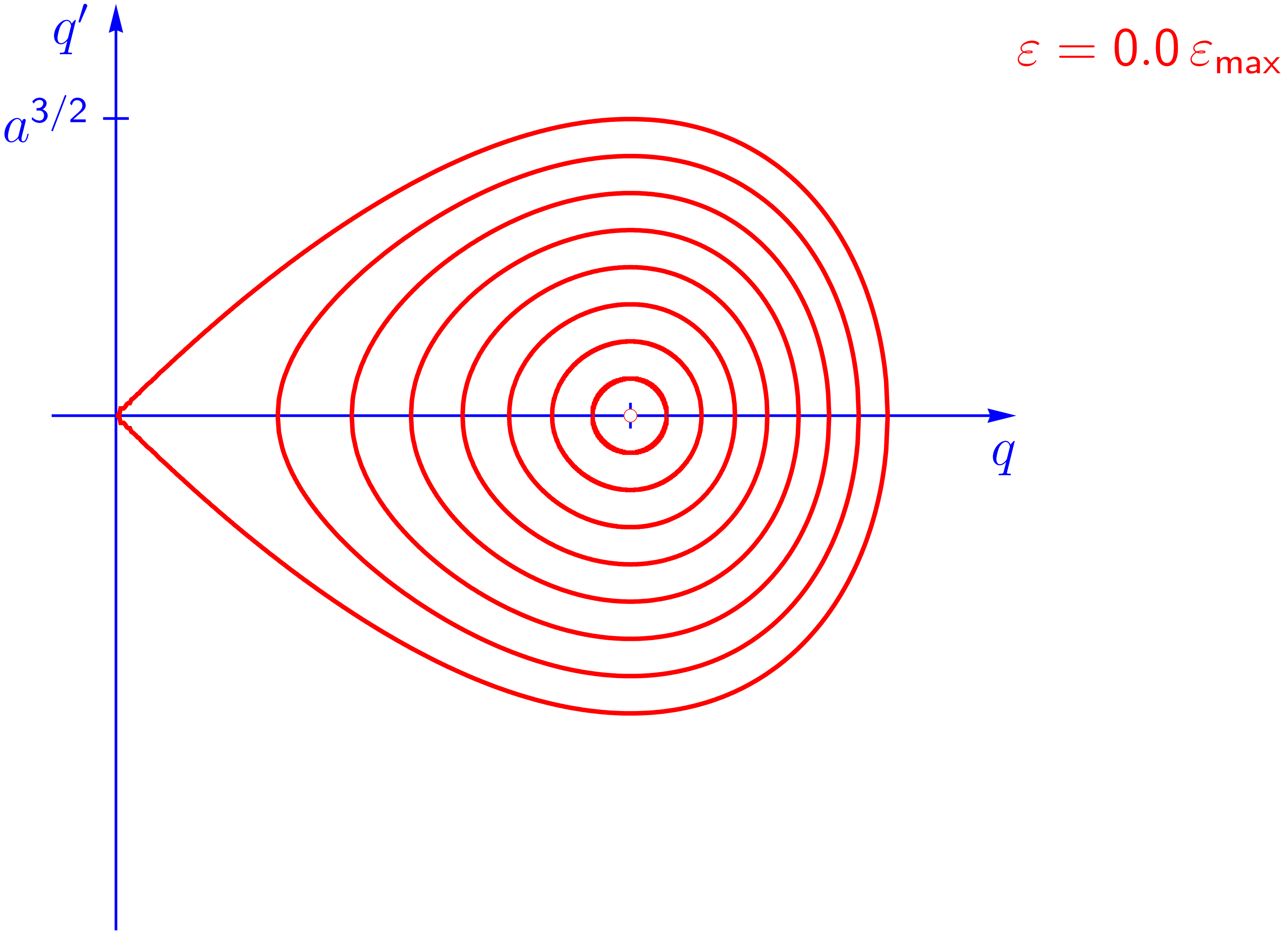
$$4\varepsilon q'^3 + q'^2 = f(q)$$

... cubic equation for $q'(q)$

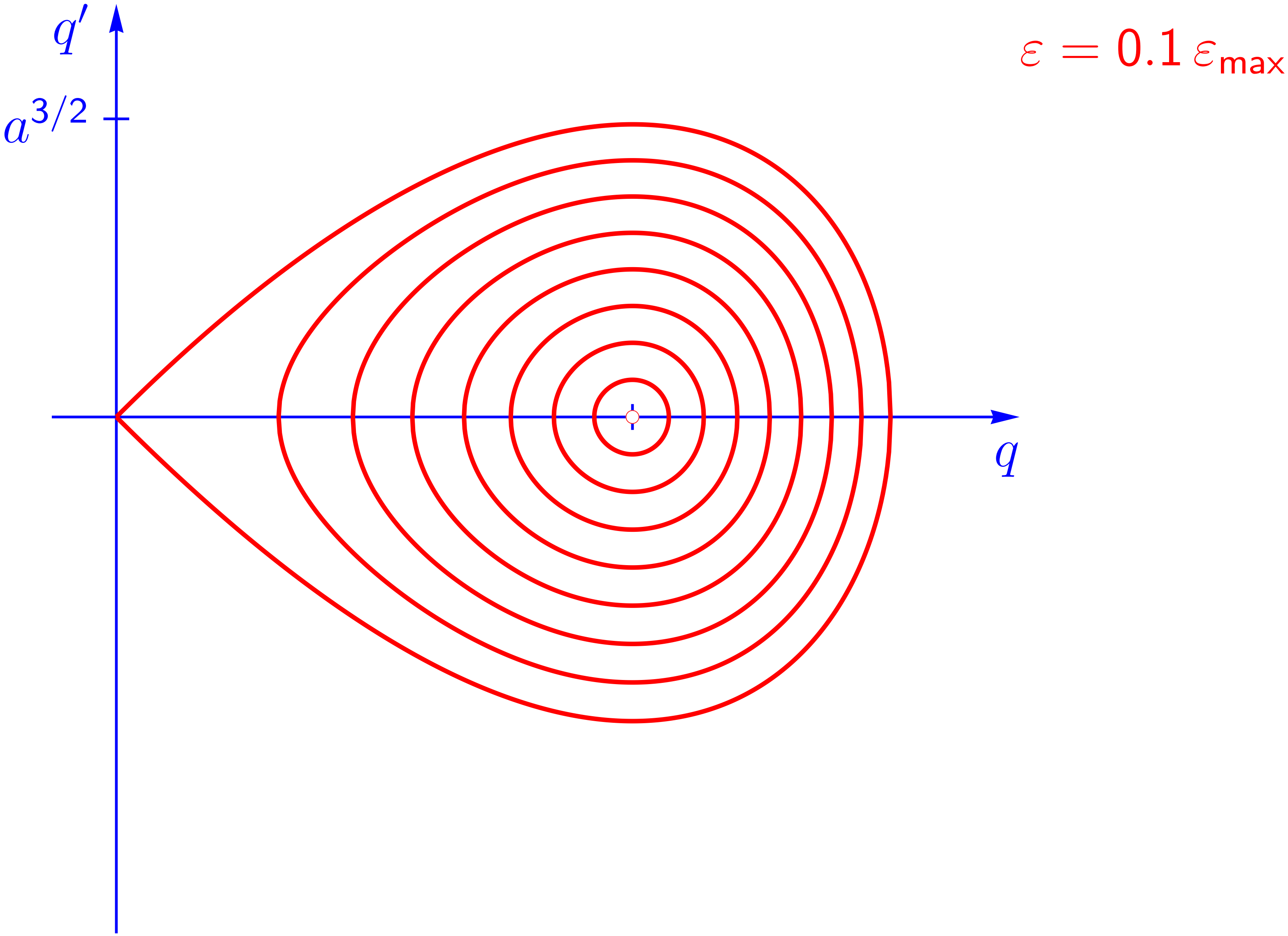
Representation of the function f by its (real) roots $q_1 \leq q_2 < q_3$

$$f(q) = 2(q - q_1)(q - q_2)(q_3 - q)$$

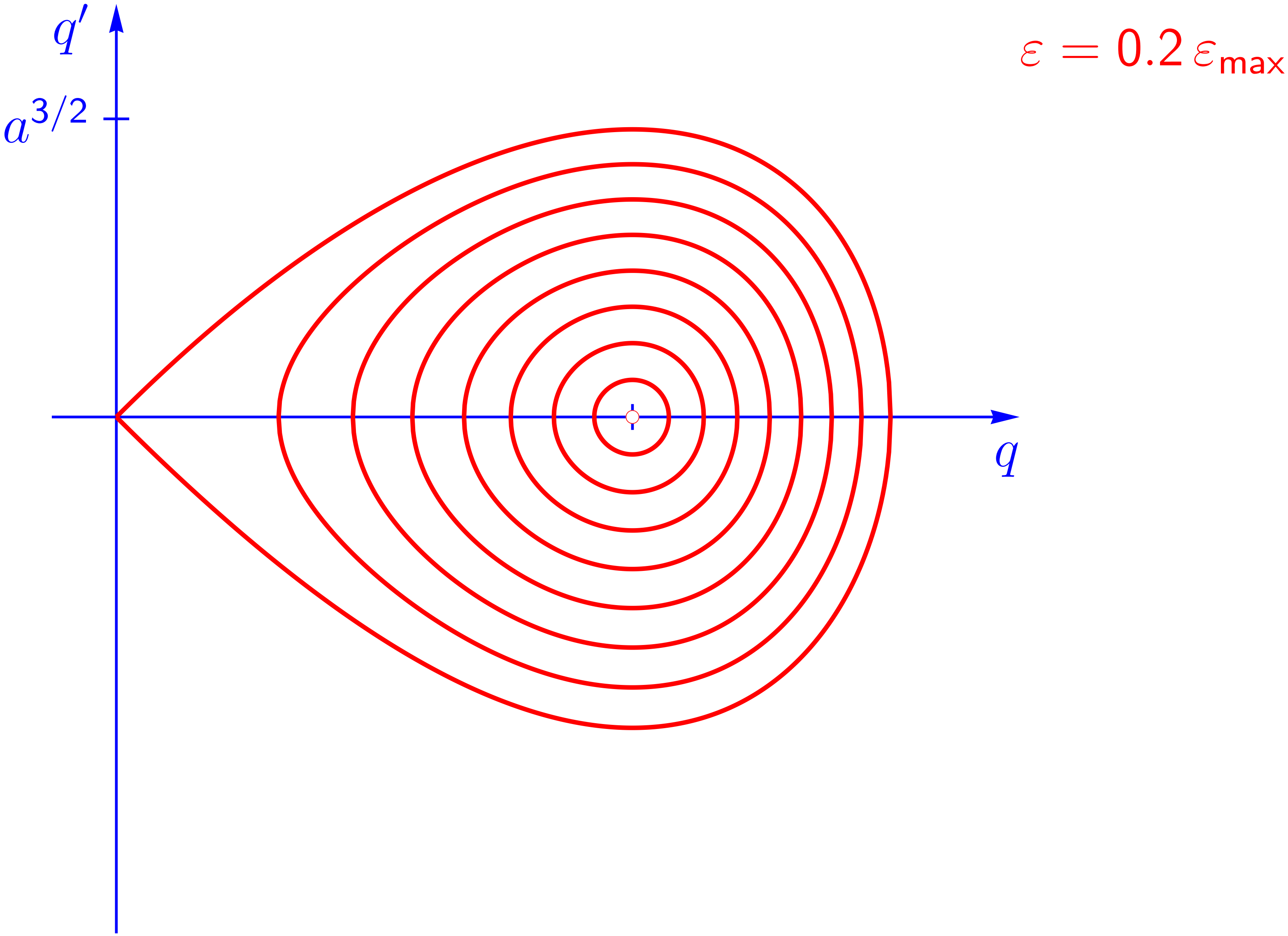
Phase Portrait



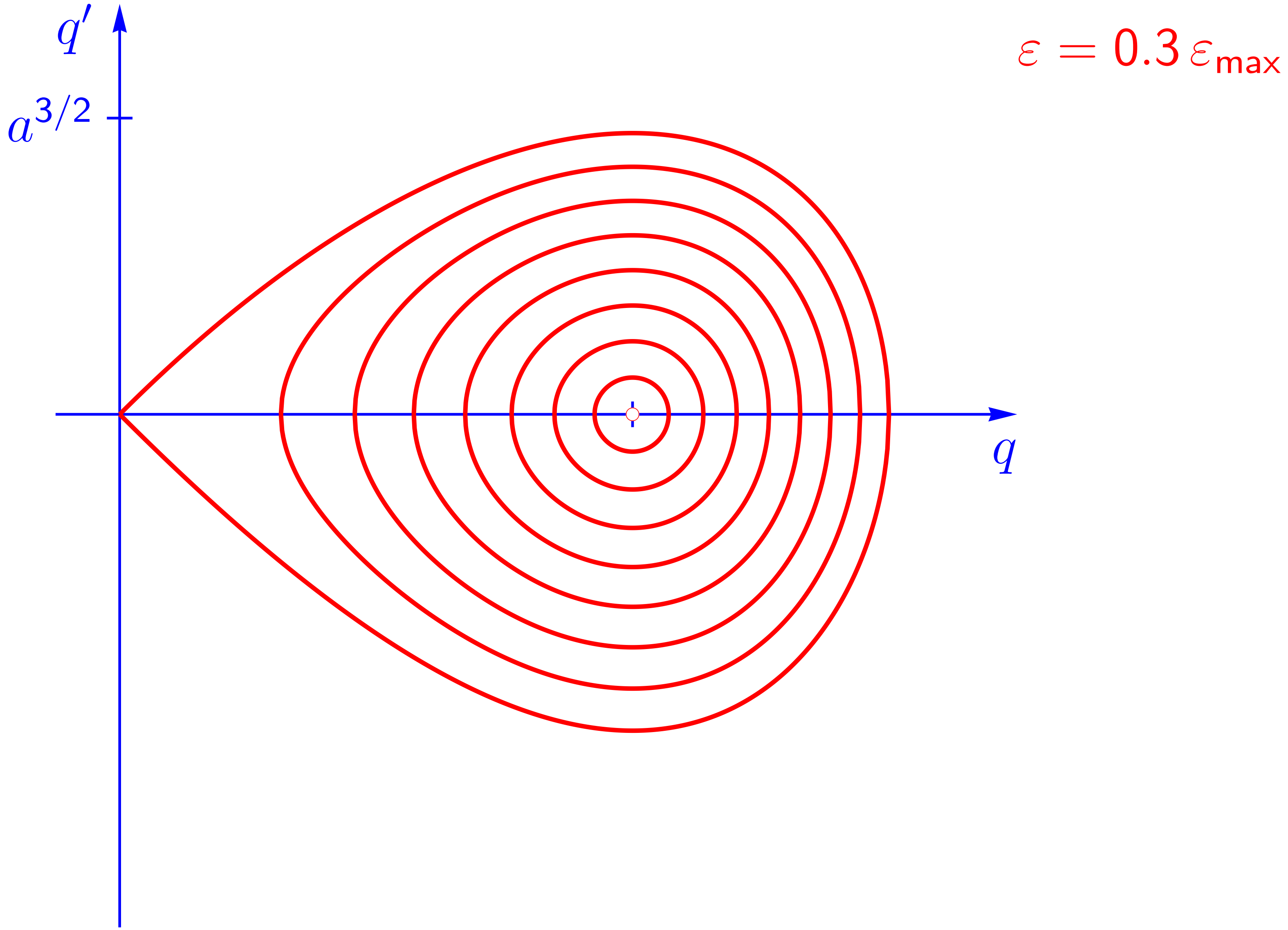
Phase Portrait



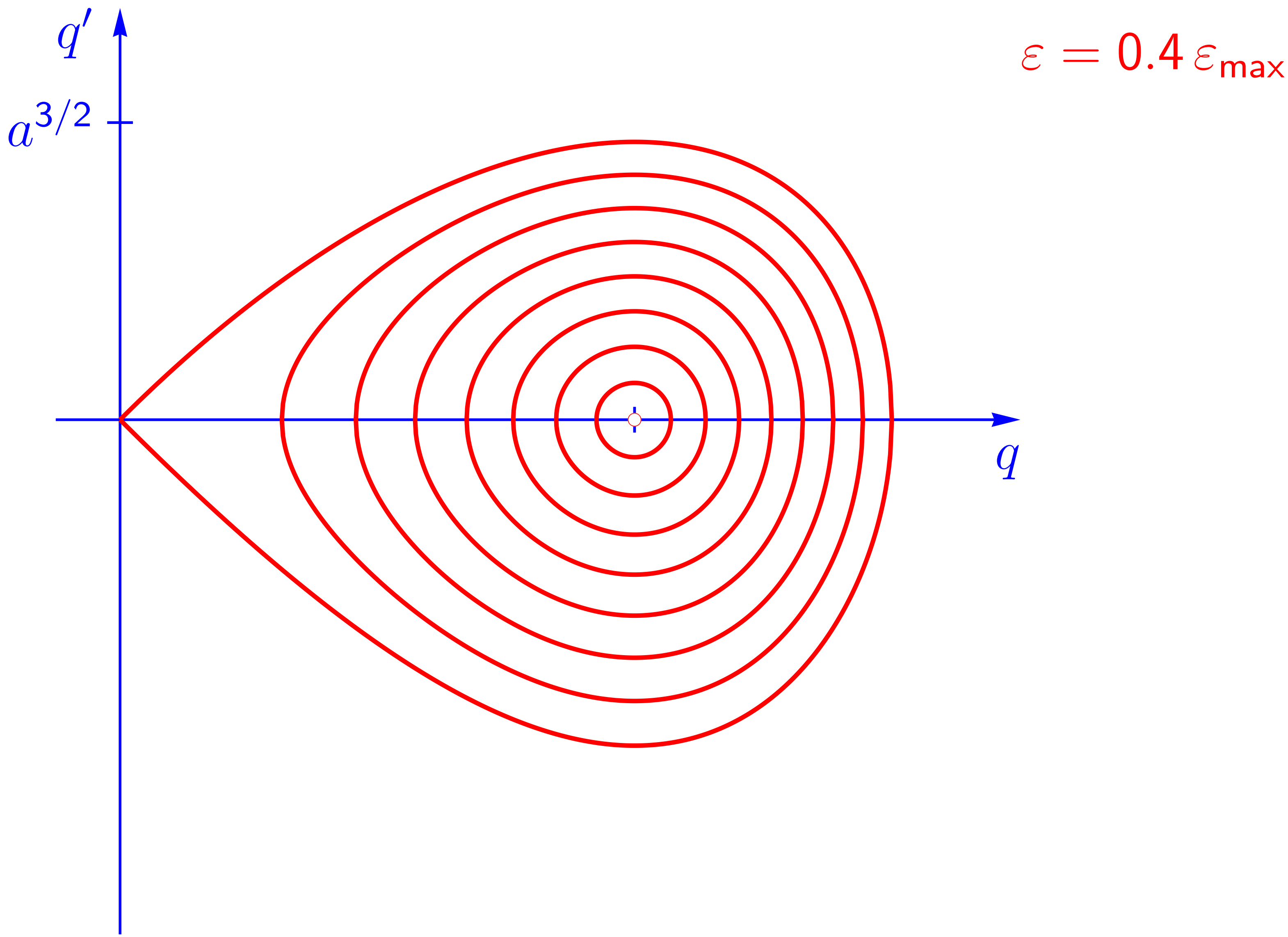
Phase Portrait



Phase Portrait

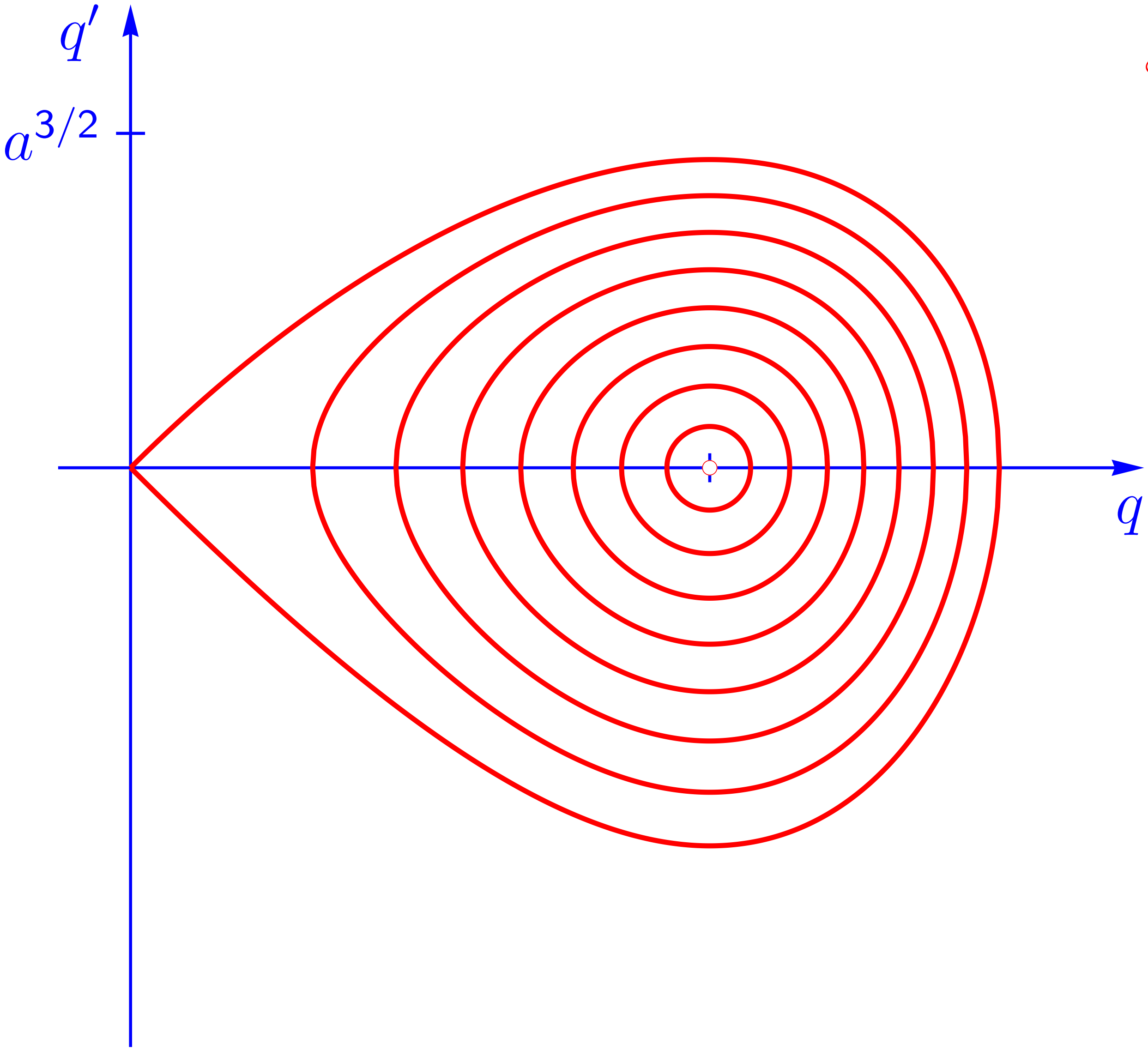


Phase Portrait

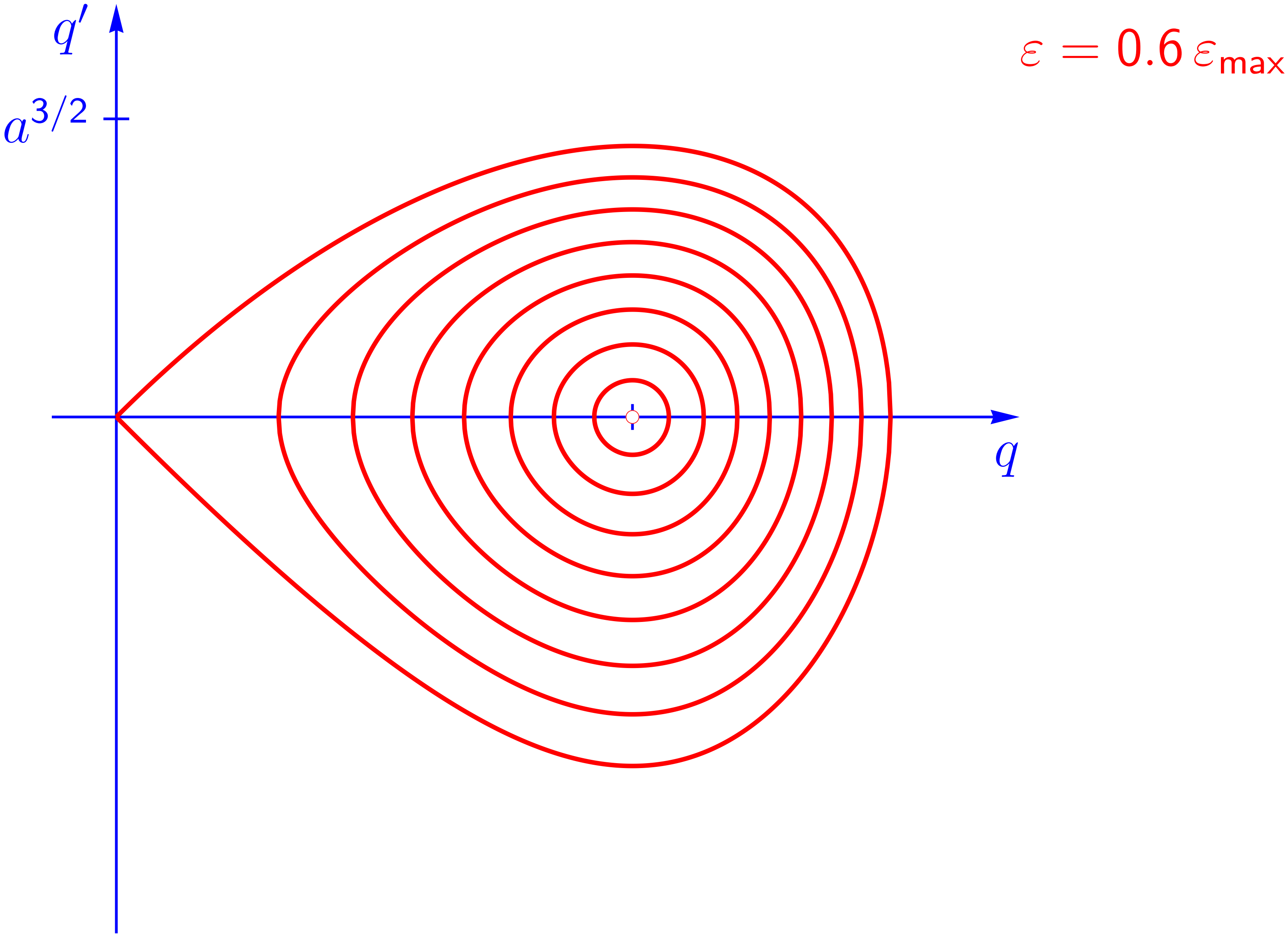


Phase Portrait

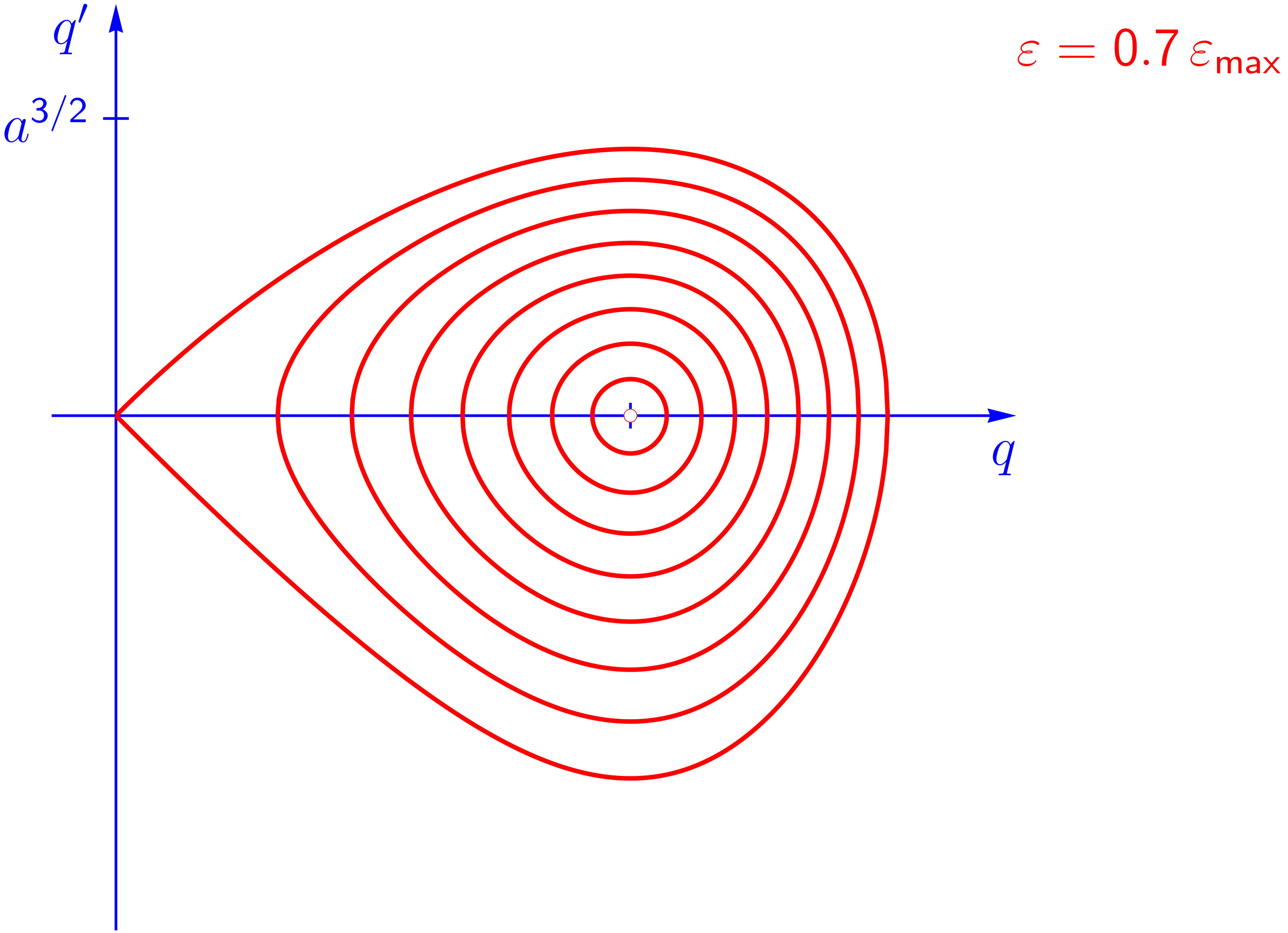
$\varepsilon = 0.5\,\varepsilon_{\text{max}}$



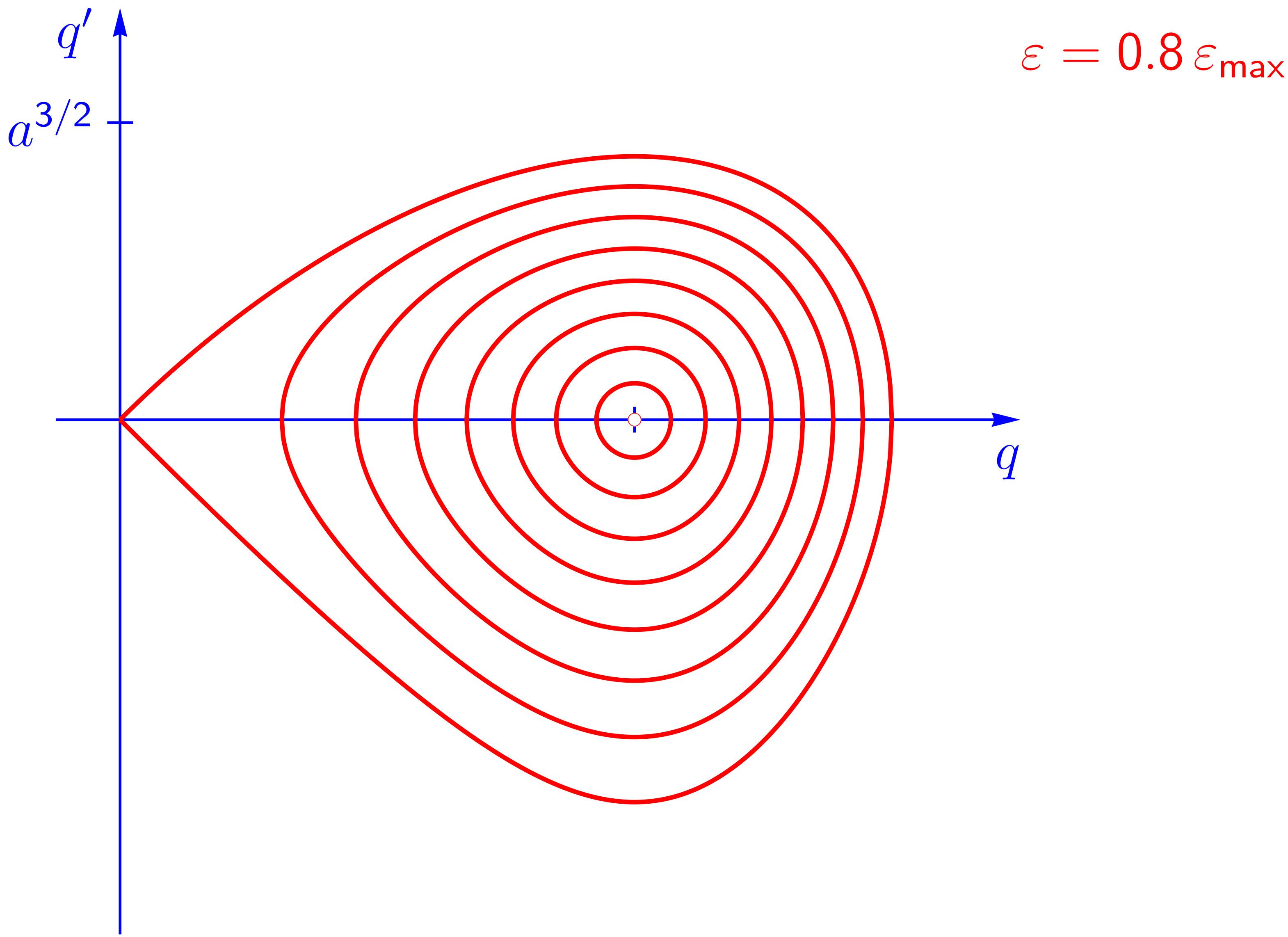
Phase Portrait



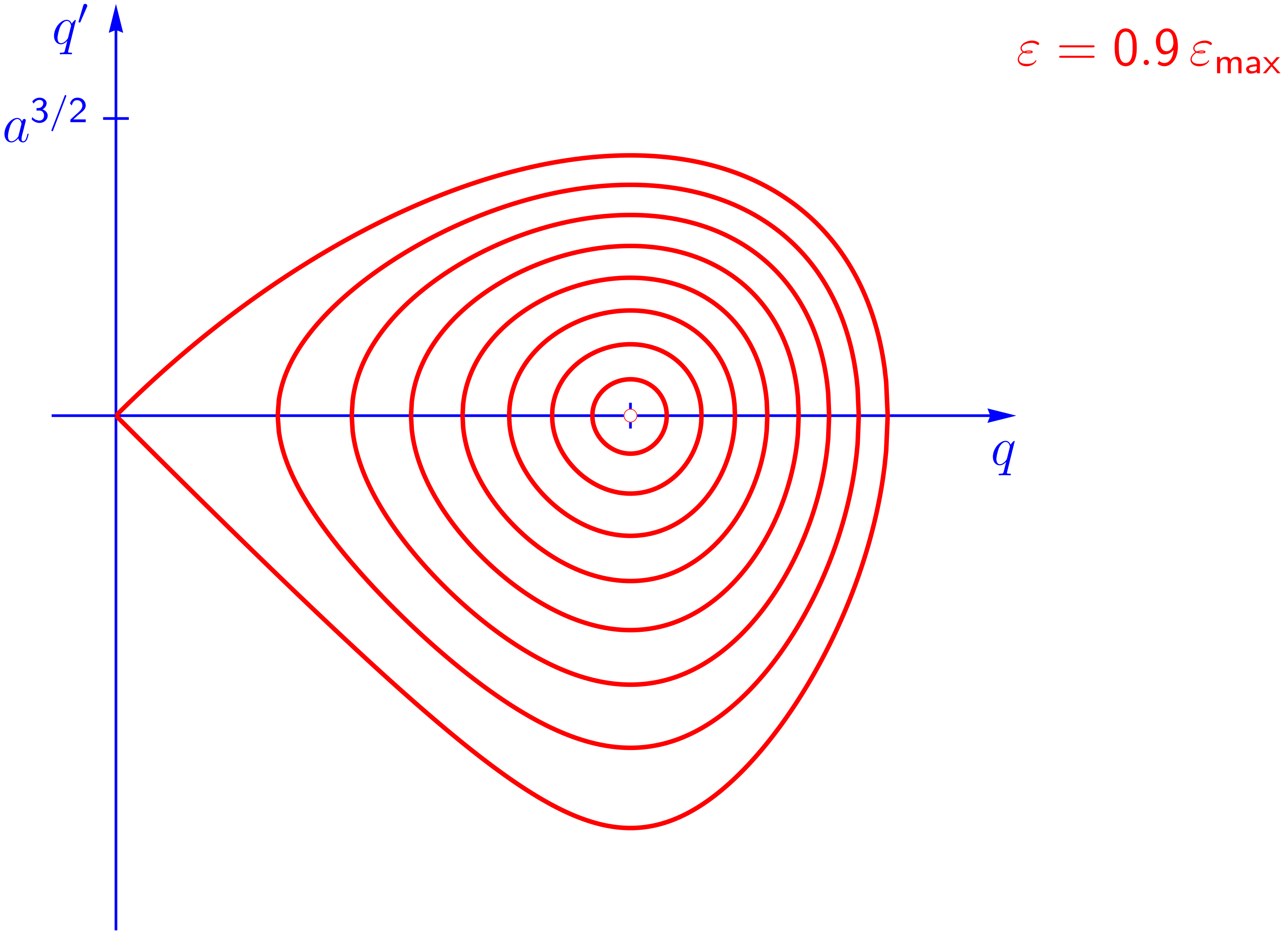
Phase Portrait



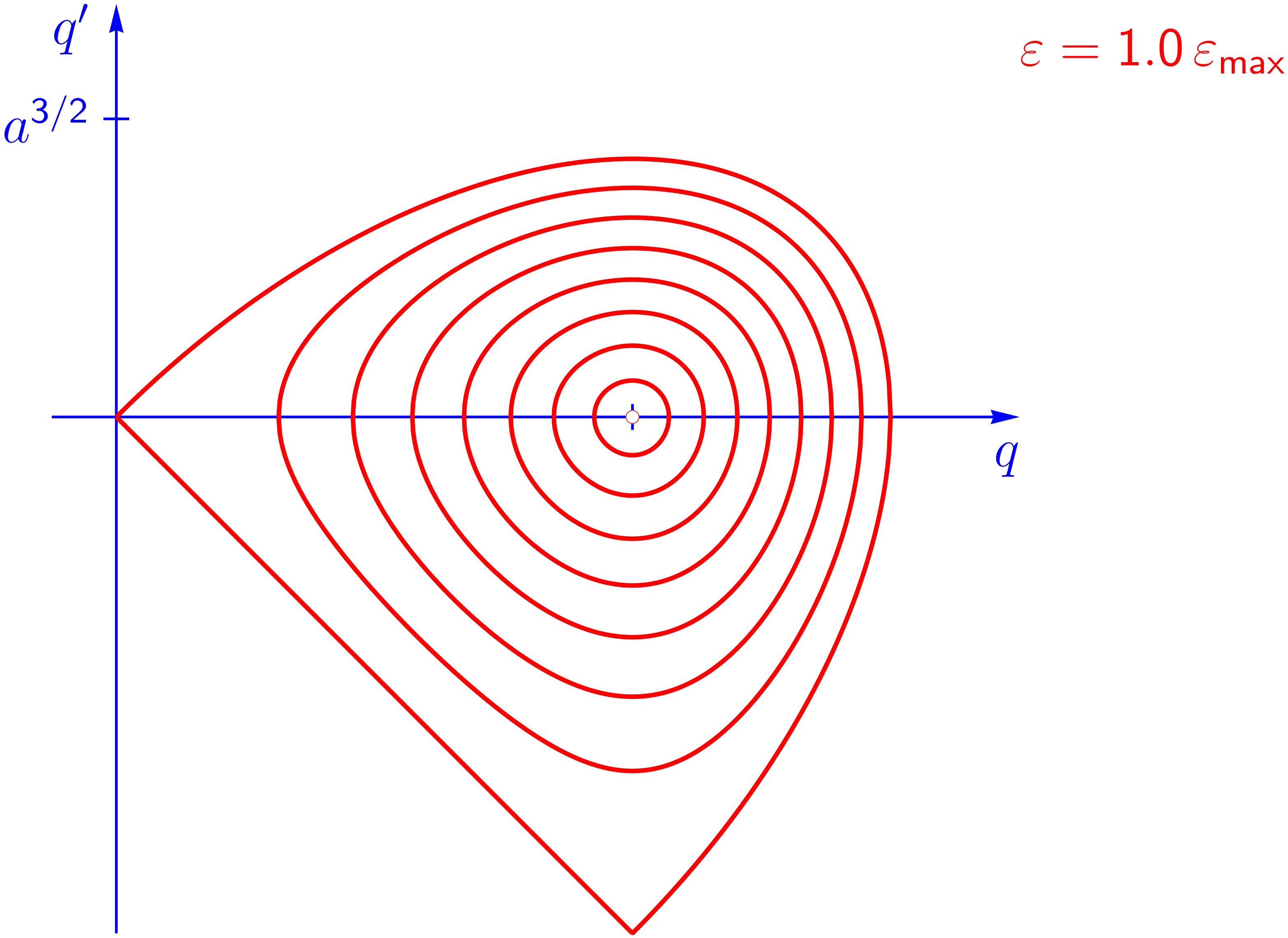
Phase Portrait



Phase Portrait



Phase Portrait



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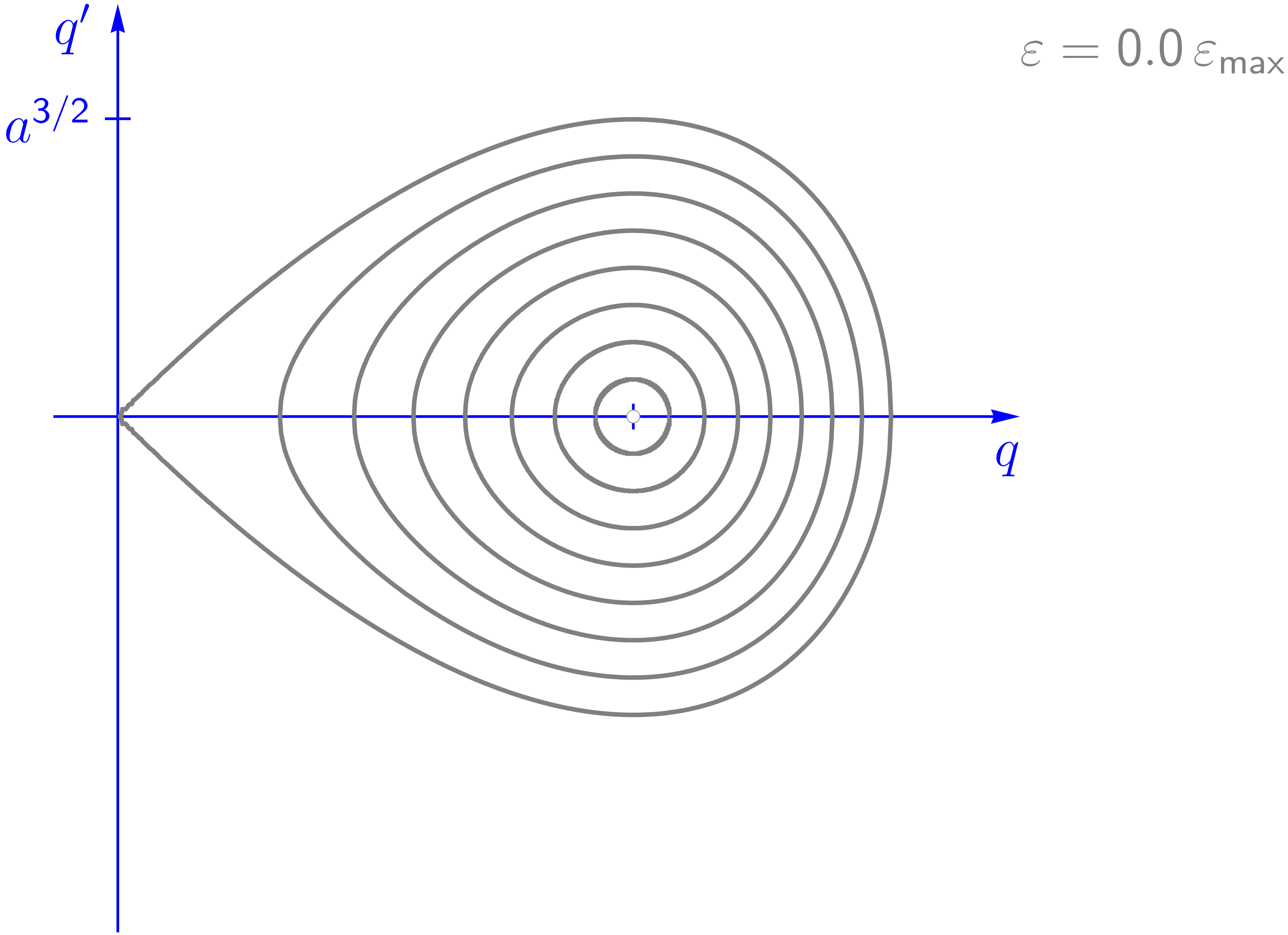

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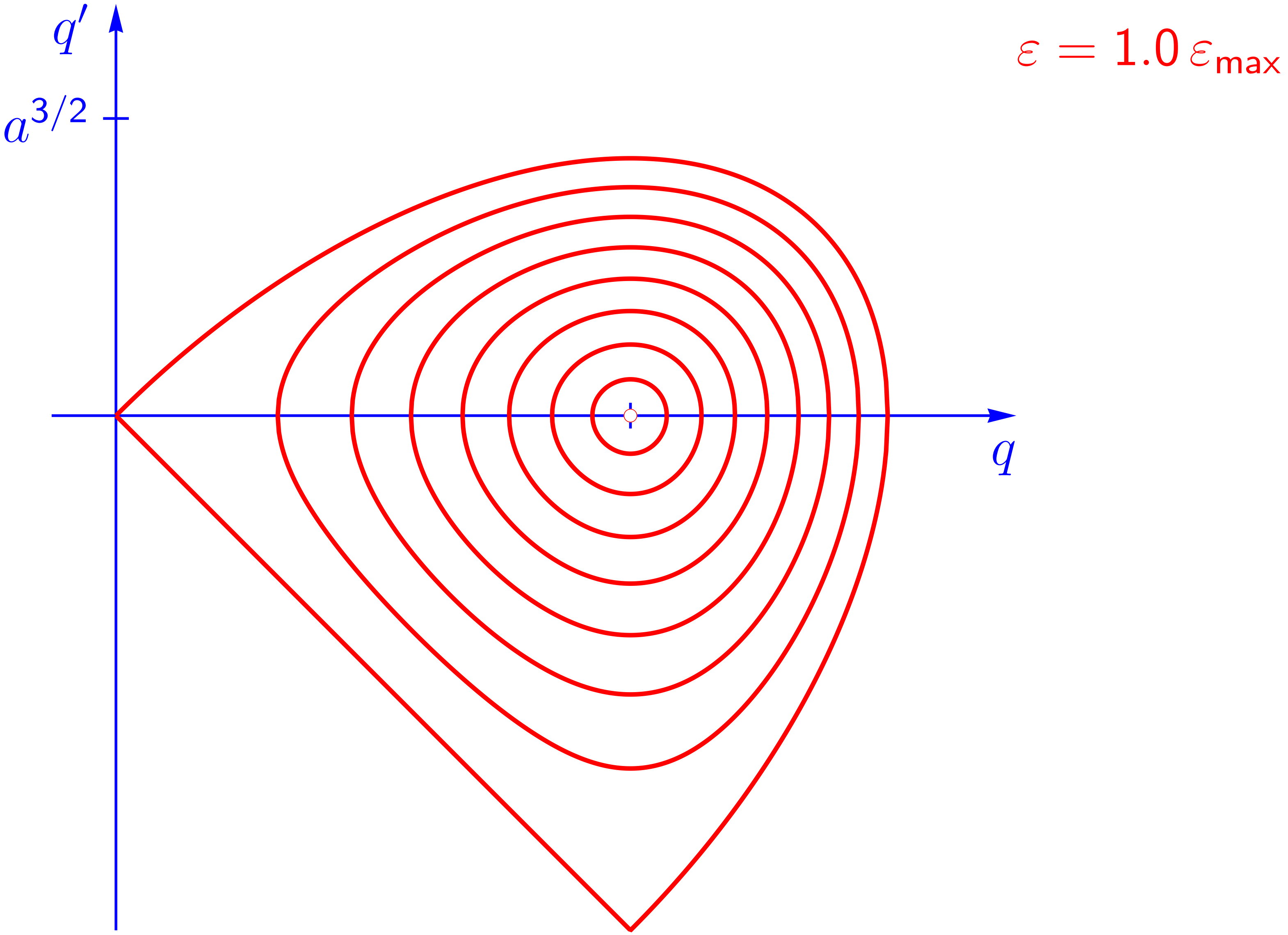


Approximate Solution of a Cubic Equation

Cubic equation

$$\varepsilon x^3 + x^2 = a^2 \qquad \varepsilon \ll 1$$

Approximate solutions emerging from $x = \pm a$

$$x_\pm = \pm a \left[1 \mp \frac{1}{2}\varepsilon a + \frac{5}{8}\varepsilon^2 a^2 \mp \varepsilon^3 a^3 + O(\varepsilon^4) \right]$$

Approximate reciprocal solution

$$\frac{1}{x_\pm} = \pm \frac{1}{a} \left[1 \pm \frac{1}{2}\varepsilon a - \frac{3}{8}\varepsilon^2 a^2 \pm \frac{1}{2}\varepsilon^3 a^3 + O(\varepsilon^4) \right]$$

(Third solution is irrelevant)

Approximation of Phase Curves

Phase curves

$$4\varepsilon q'^3 + q'^2 = f(q)$$

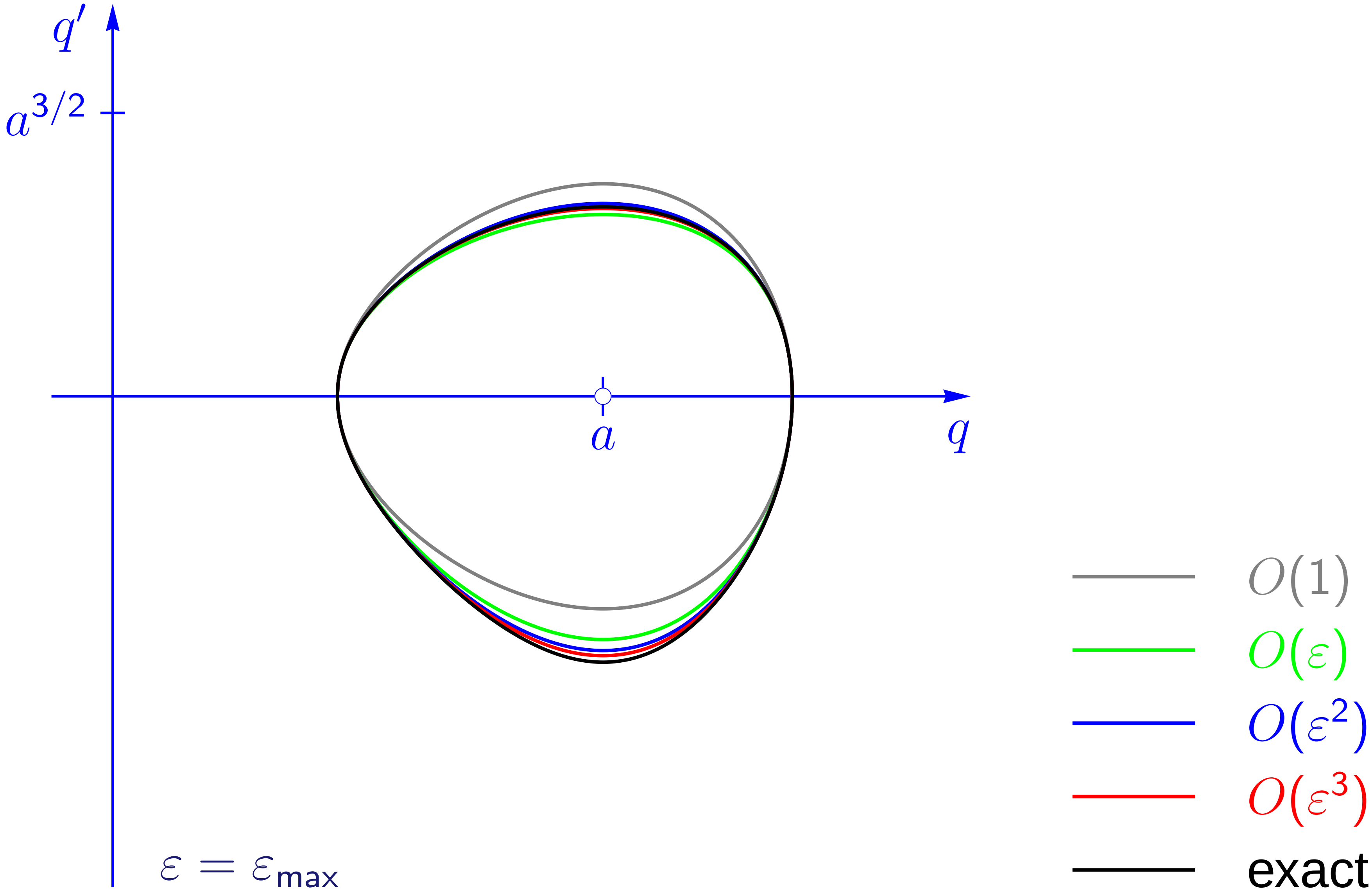
Approximation

$$q' \approx \pm\sqrt{f(q)} - 2\varepsilon f(q) \pm 10\varepsilon^2 \left[f(q)\right]^{3/2} - 64\varepsilon^3 \left[f(q)\right]^2$$

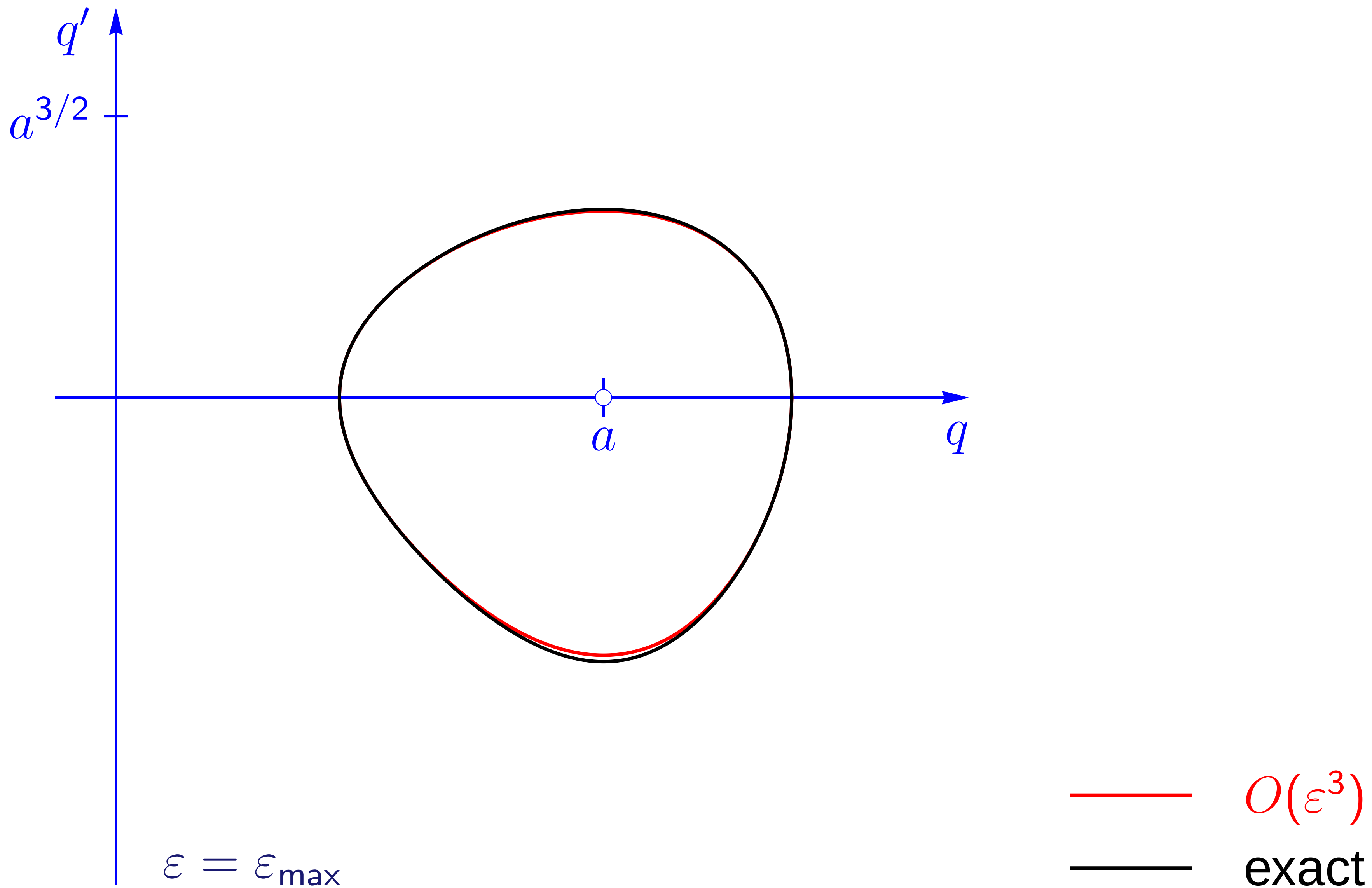
Approximation of reciprocal value

$$\frac{1}{q'} \approx \pm\frac{1}{\sqrt{f(q)}} + 2\varepsilon \mp 6\varepsilon^2\sqrt{f(q)} + 32\varepsilon^3 f(q)$$

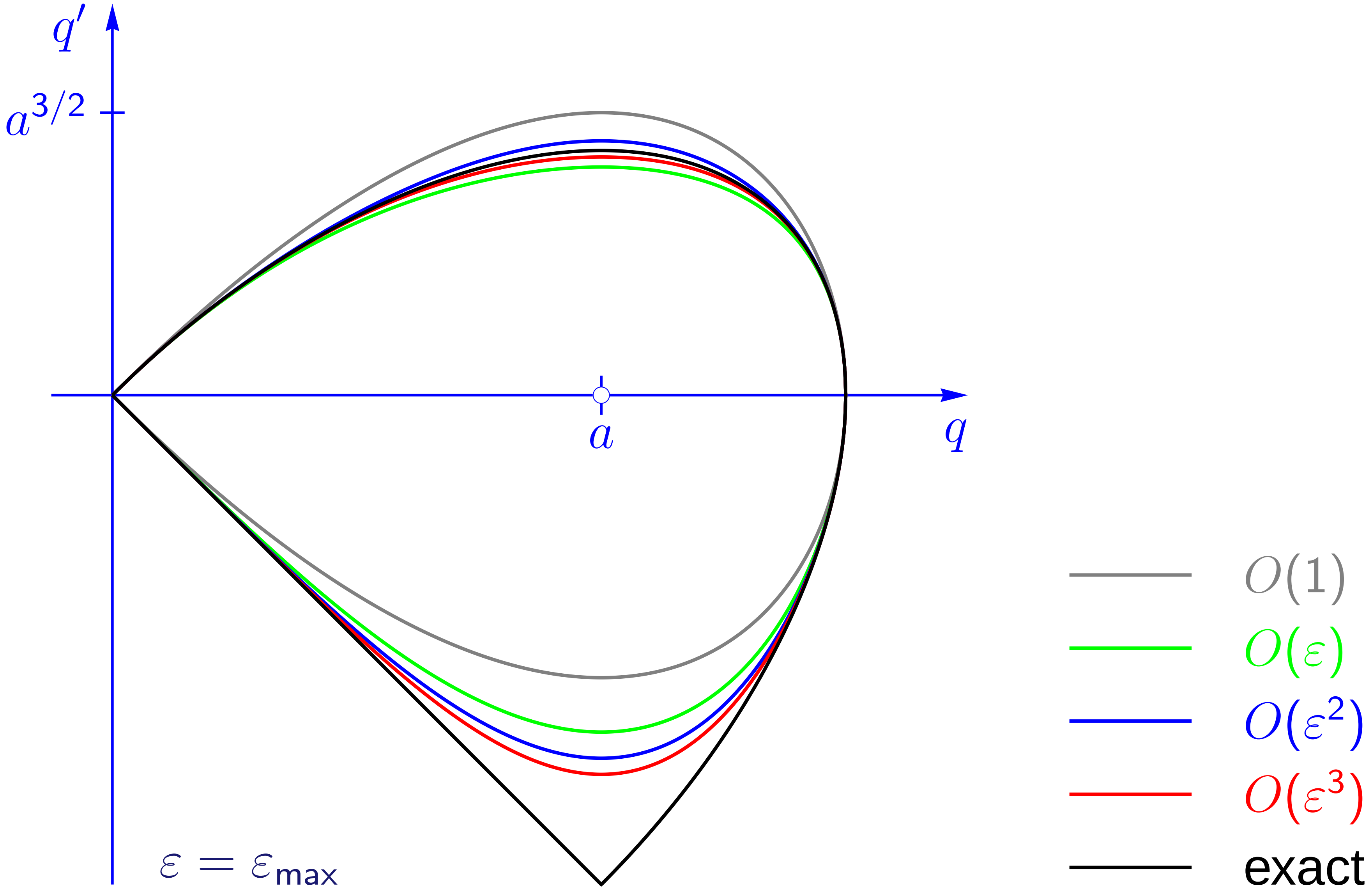
Approximation of Phase Curves



Approximation of Phase Curves



Approximation of Phase Curves



Phase curves (exact)

$$4\varepsilon q'^3 + q'^2 = f(q)$$

Approximate Final Integration

Phase curves (exact)

$$4\varepsilon q'^3 + q'^2 = f(q)$$

$O(\varepsilon)$ approximation

$$\frac{\mathrm{d}q}{\mathrm{d}\theta} = \pm\sqrt{f(q)} - 2\varepsilon f(q) \qquad \frac{\mathrm{d}\theta}{\mathrm{d}q} = \pm\frac{1}{\sqrt{f(q)}} + 2\varepsilon$$

Approximate Final Integration

Phase curves (exact)

$$4\varepsilon q'^3 + q'^2 = f(q)$$

$O(\varepsilon)$ approximation

$$\frac{\mathrm{d}q}{\mathrm{d}\theta} = \pm\sqrt{f(q)} - 2\varepsilon f(q) \qquad \frac{\mathrm{d}\theta}{\mathrm{d}q} = \pm\frac{1}{\sqrt{f(q)}} + 2\varepsilon$$

Integration, starting at $q(0) = q_3$

$$\theta = \int_{q_3}^{q} \frac{-\,\mathrm{d}q}{\sqrt{f(q)}} - 2\varepsilon(q_3 - q)$$

Approximate Final Integration (Continued)

Substitution

$$q = q_2 + (q_3 - q_2)\cos^2\varphi$$

Evaluation of the integral

$$\theta = \frac{1}{\eta}F(\varphi; k) - 2\varepsilon(q_3 - q) \qquad \Longleftrightarrow \qquad \varphi = \operatorname{am}\eta\,[\theta + 2\varepsilon(q_3 - q)]$$

Approximate Final Integration (Continued)

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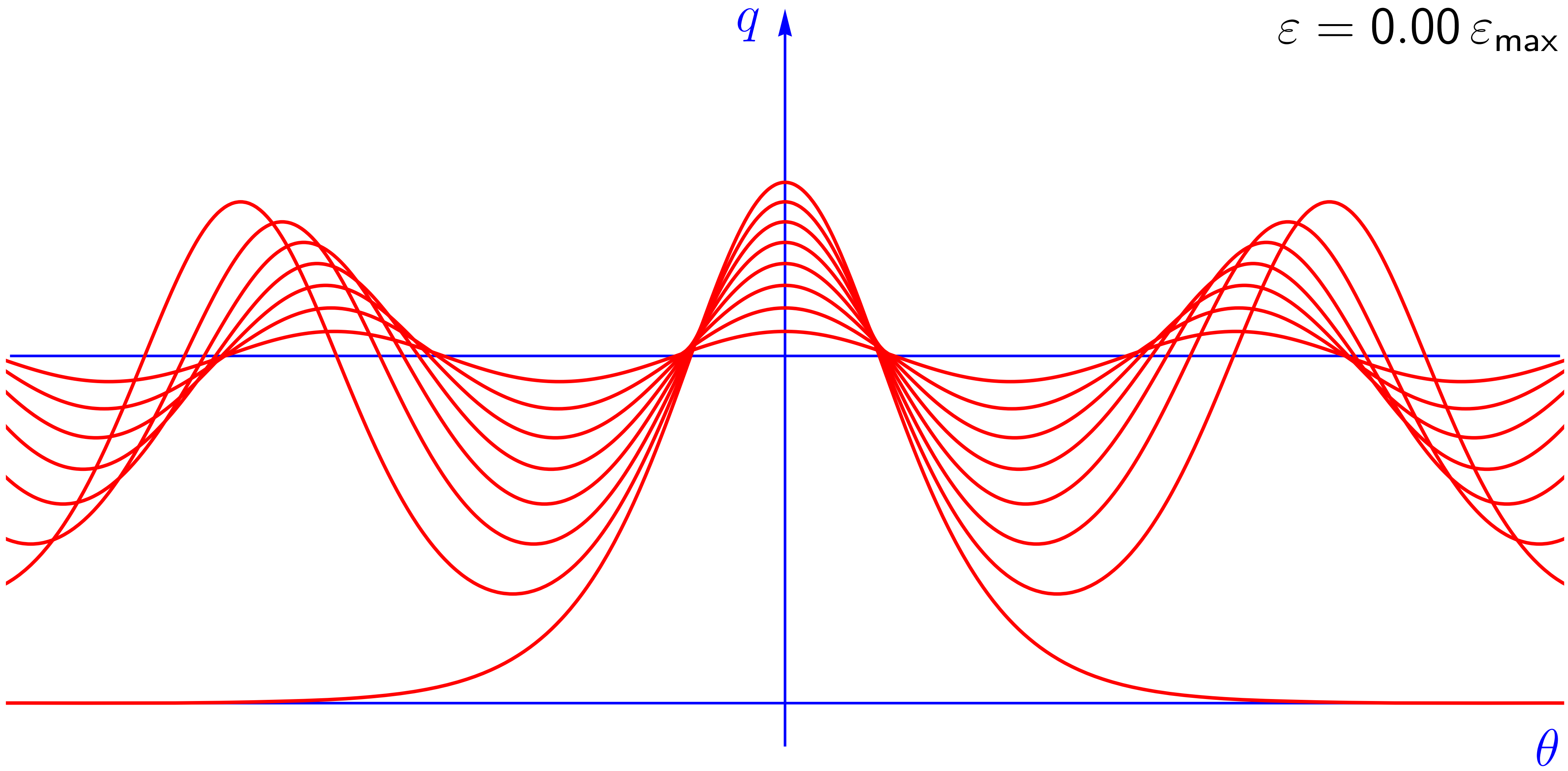
Resubstitution

$$q = q_2 + (q_3 - q_2)\,\mathrm{cn}^2\,\eta\,[\theta + 2\varepsilon(q_3 - q)]$$

implicit representation of $q = q(\theta)$

Periodic Solutions of the Extended KdV Equation



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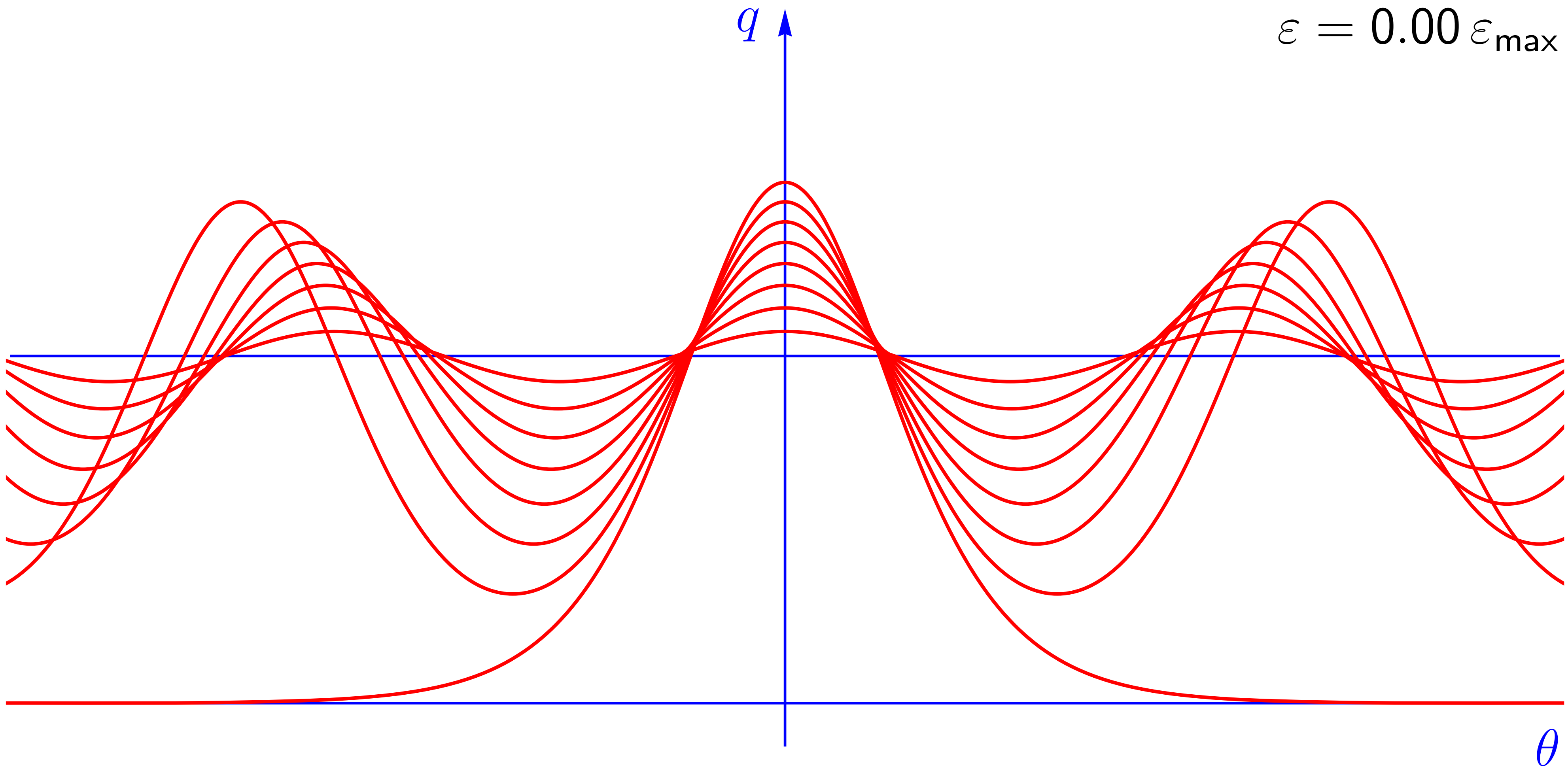

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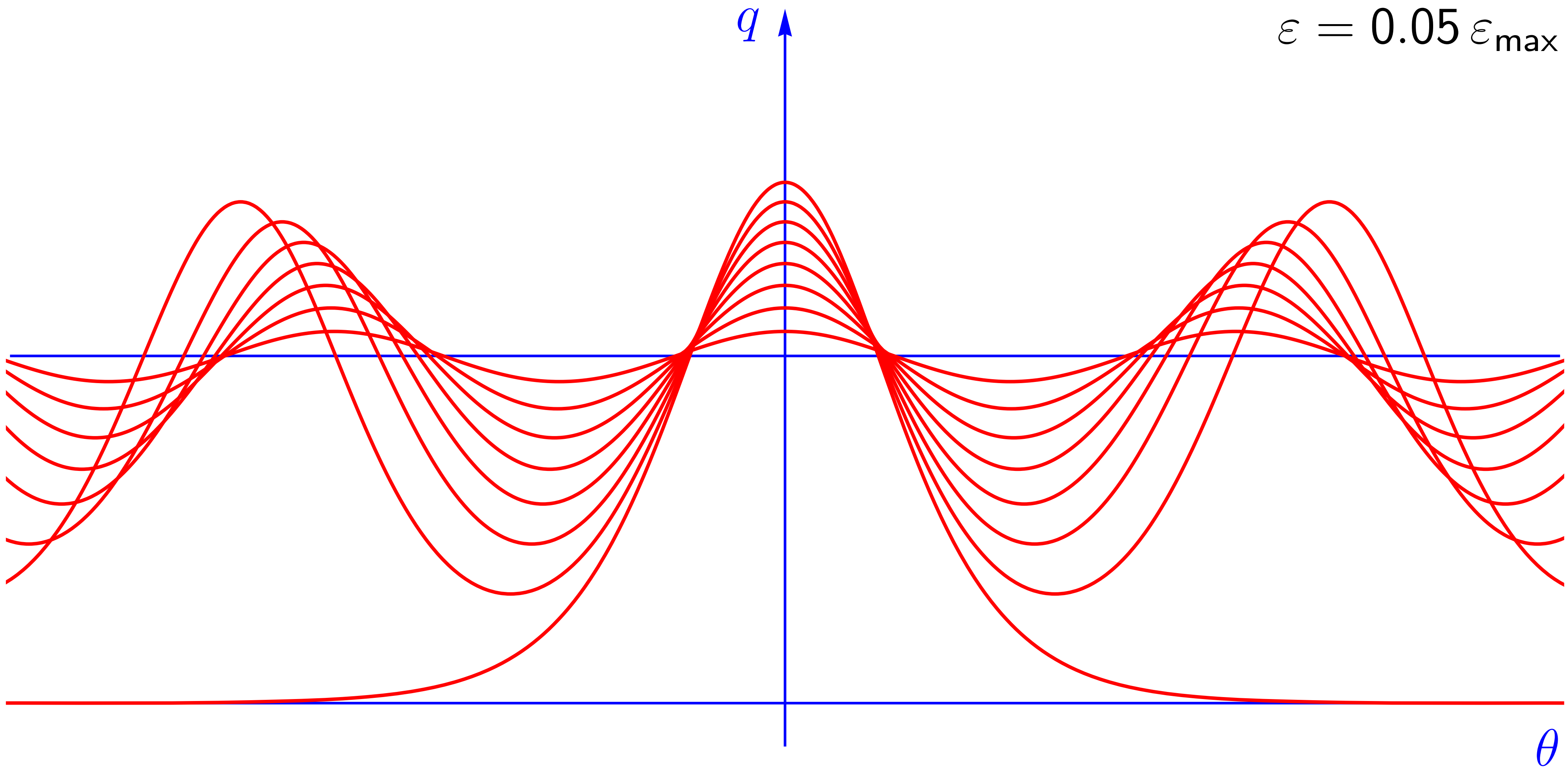

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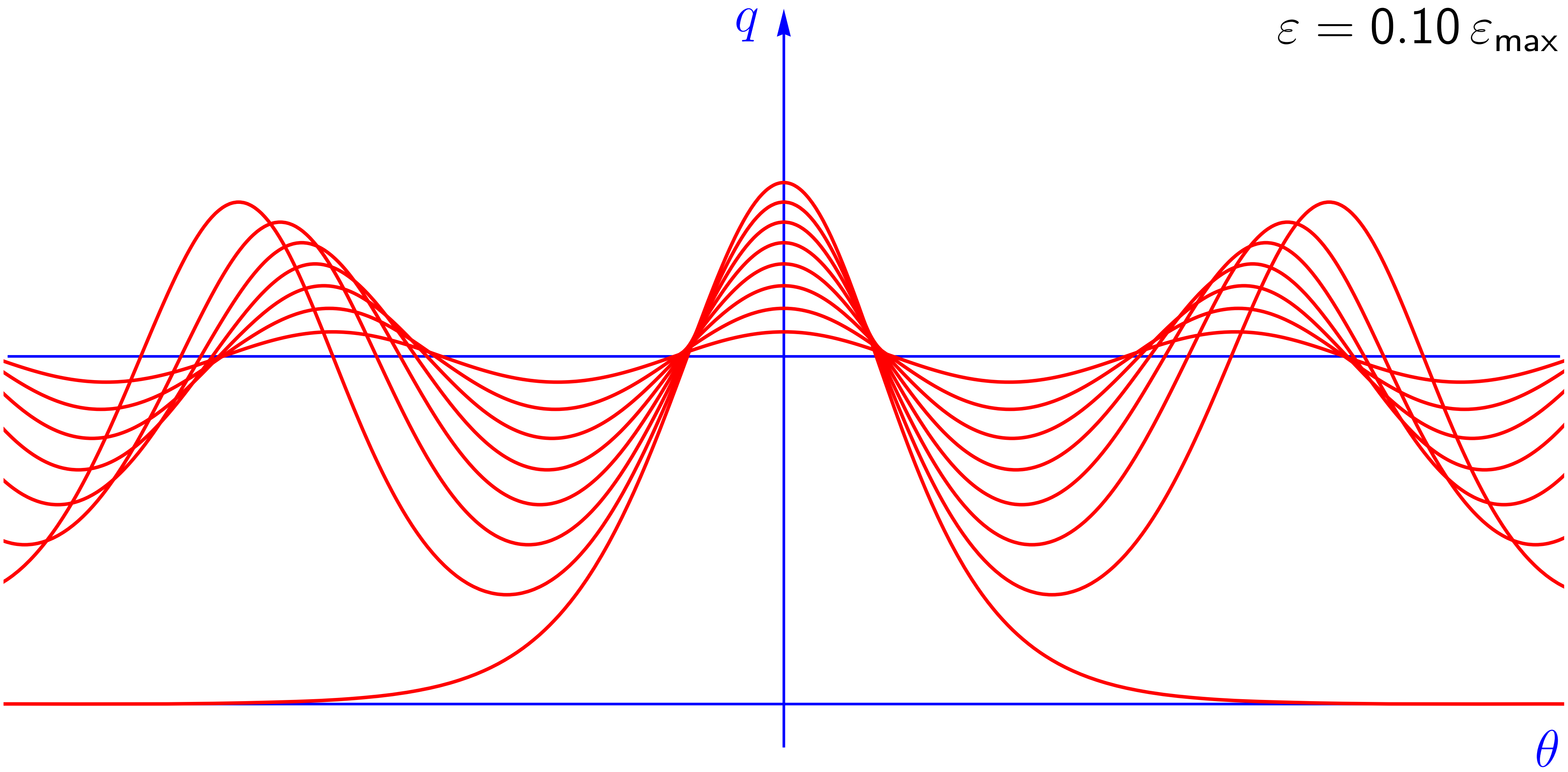

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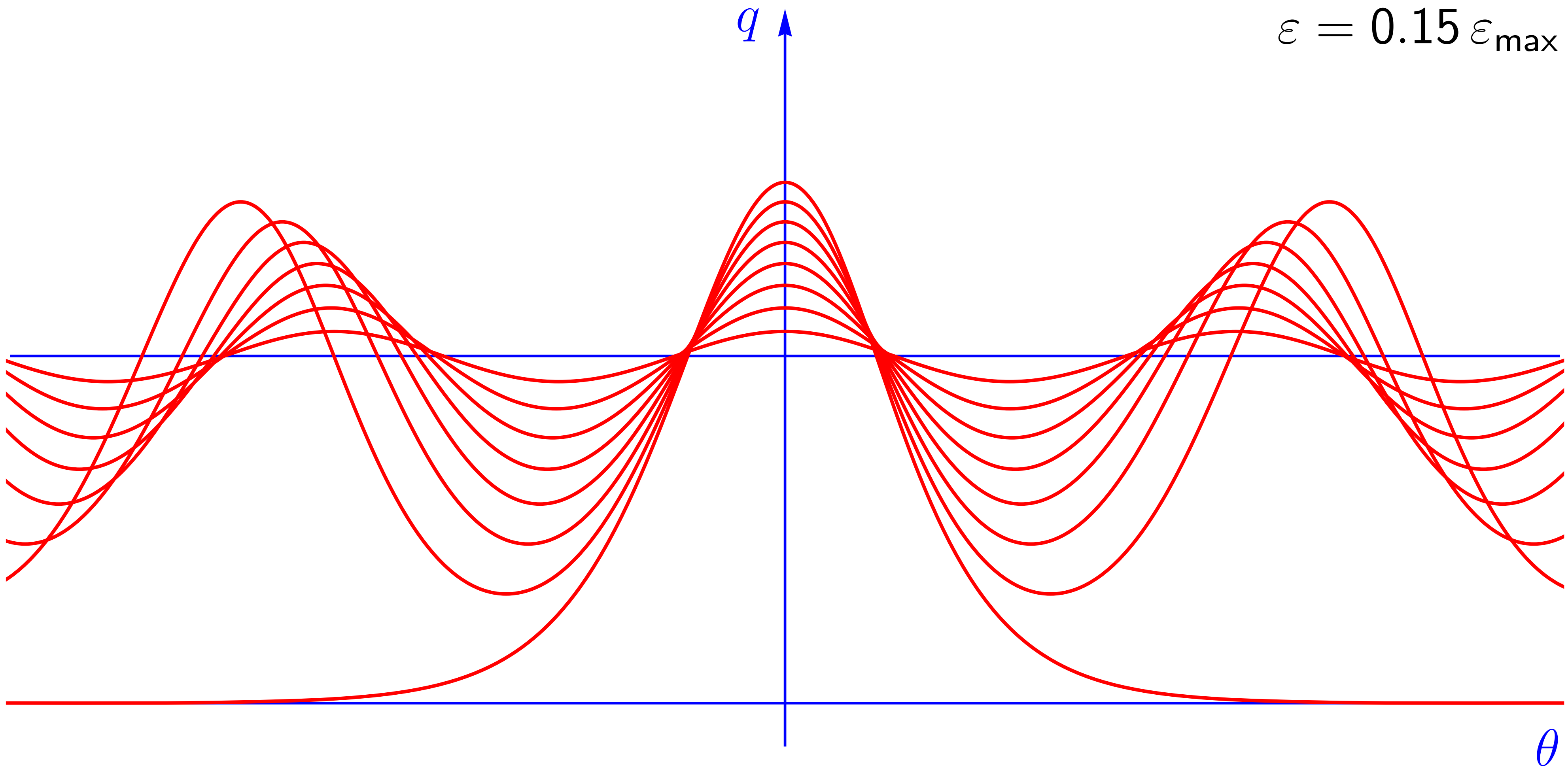

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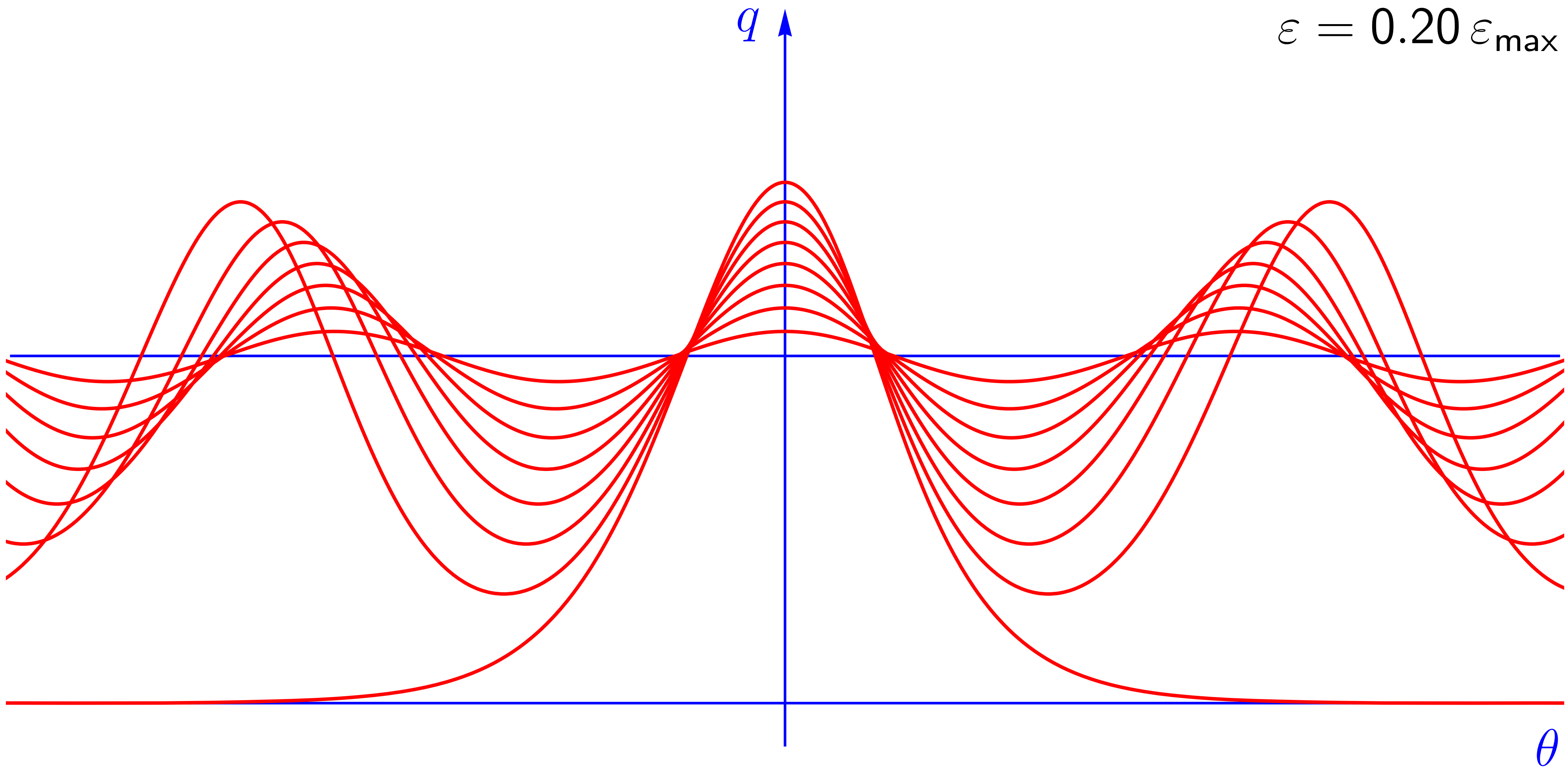

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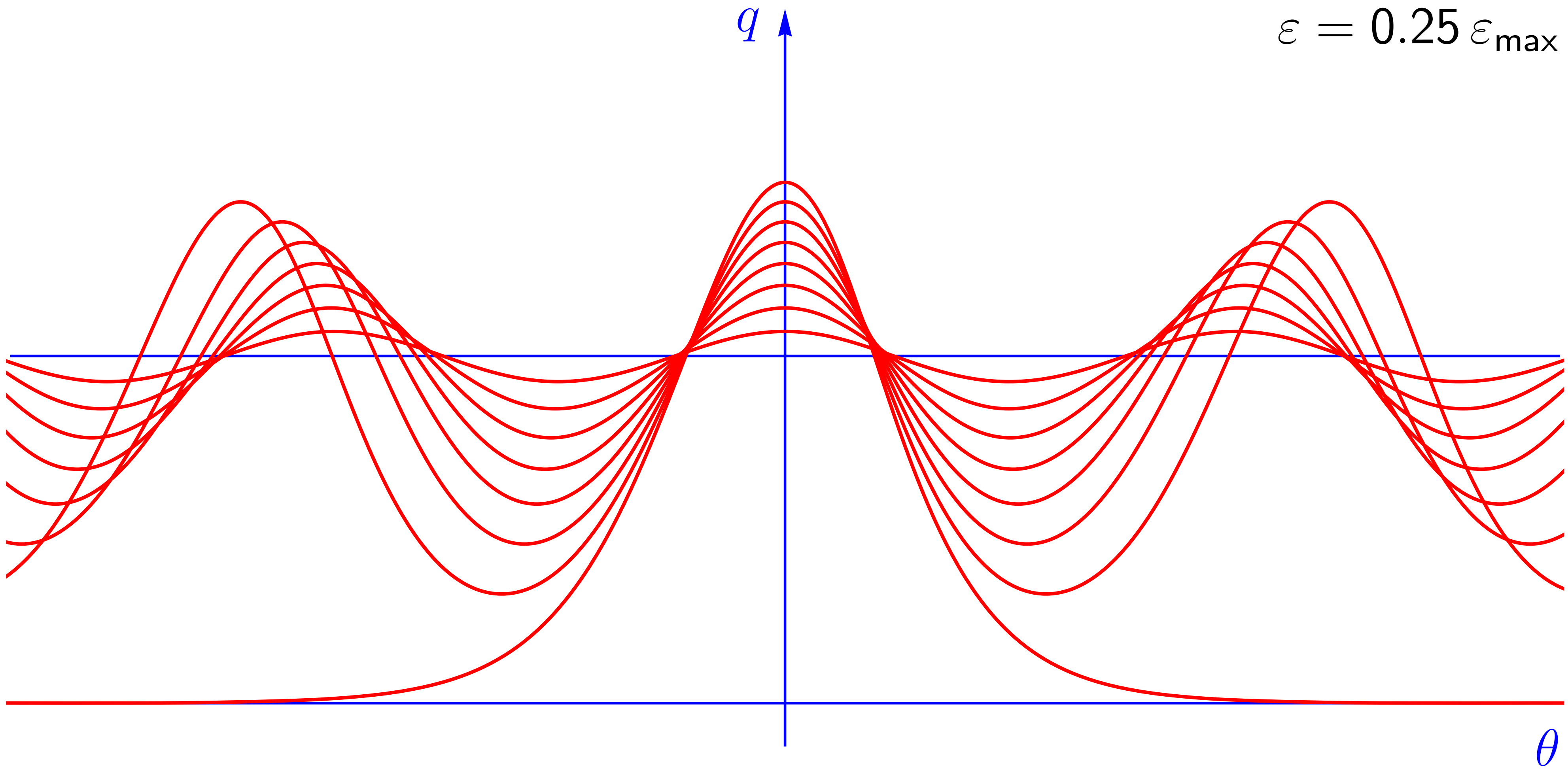

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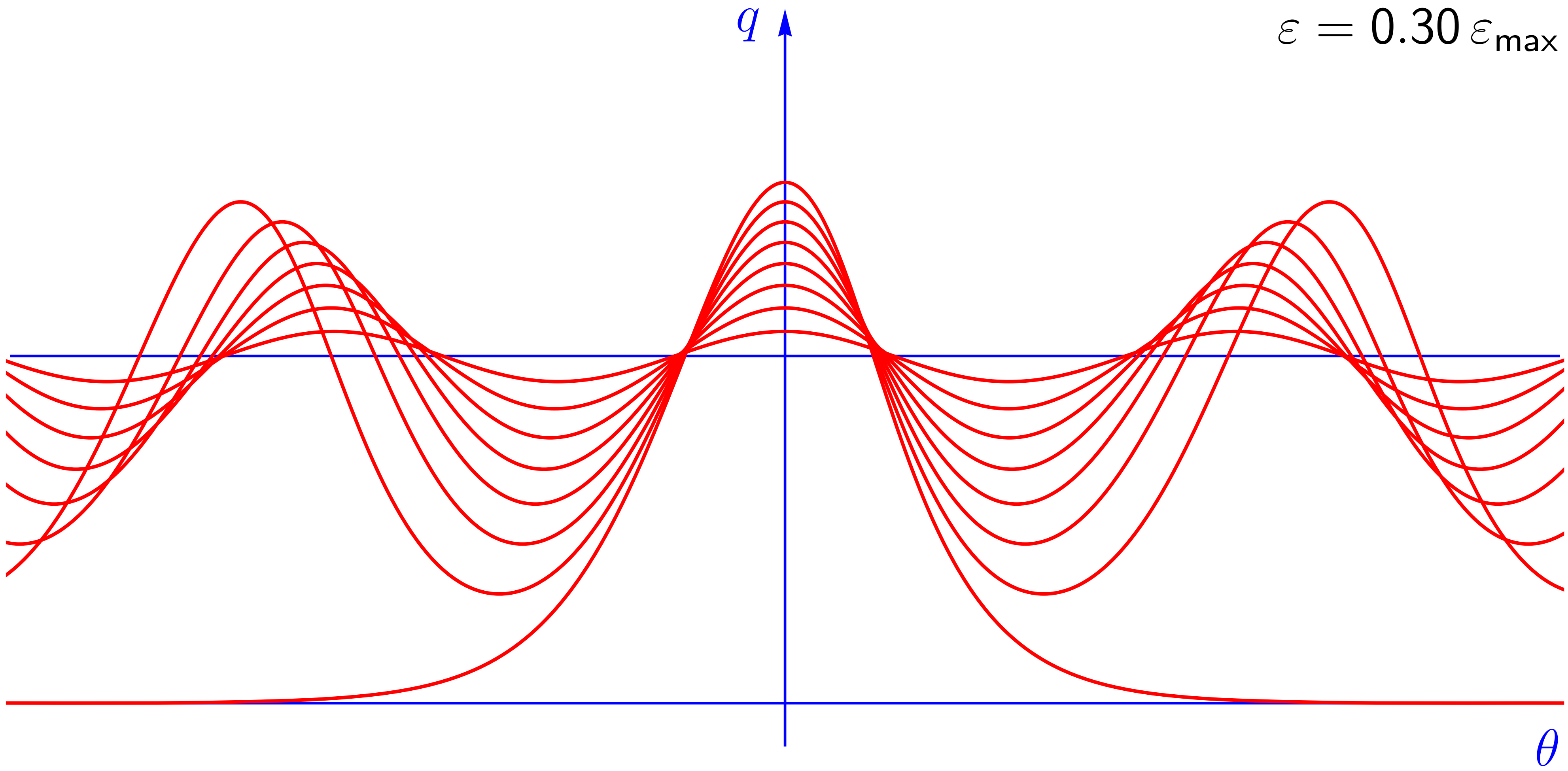

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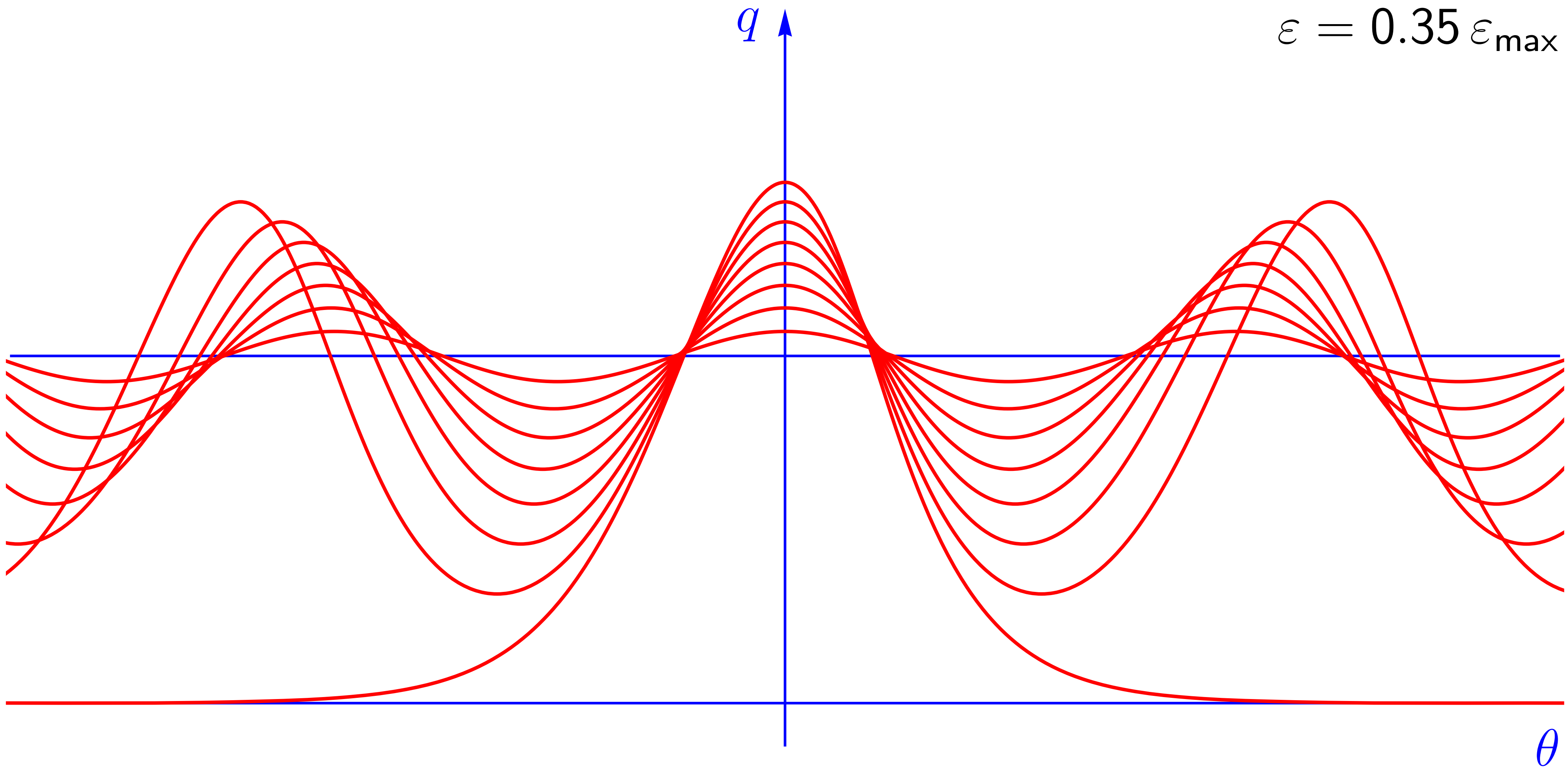

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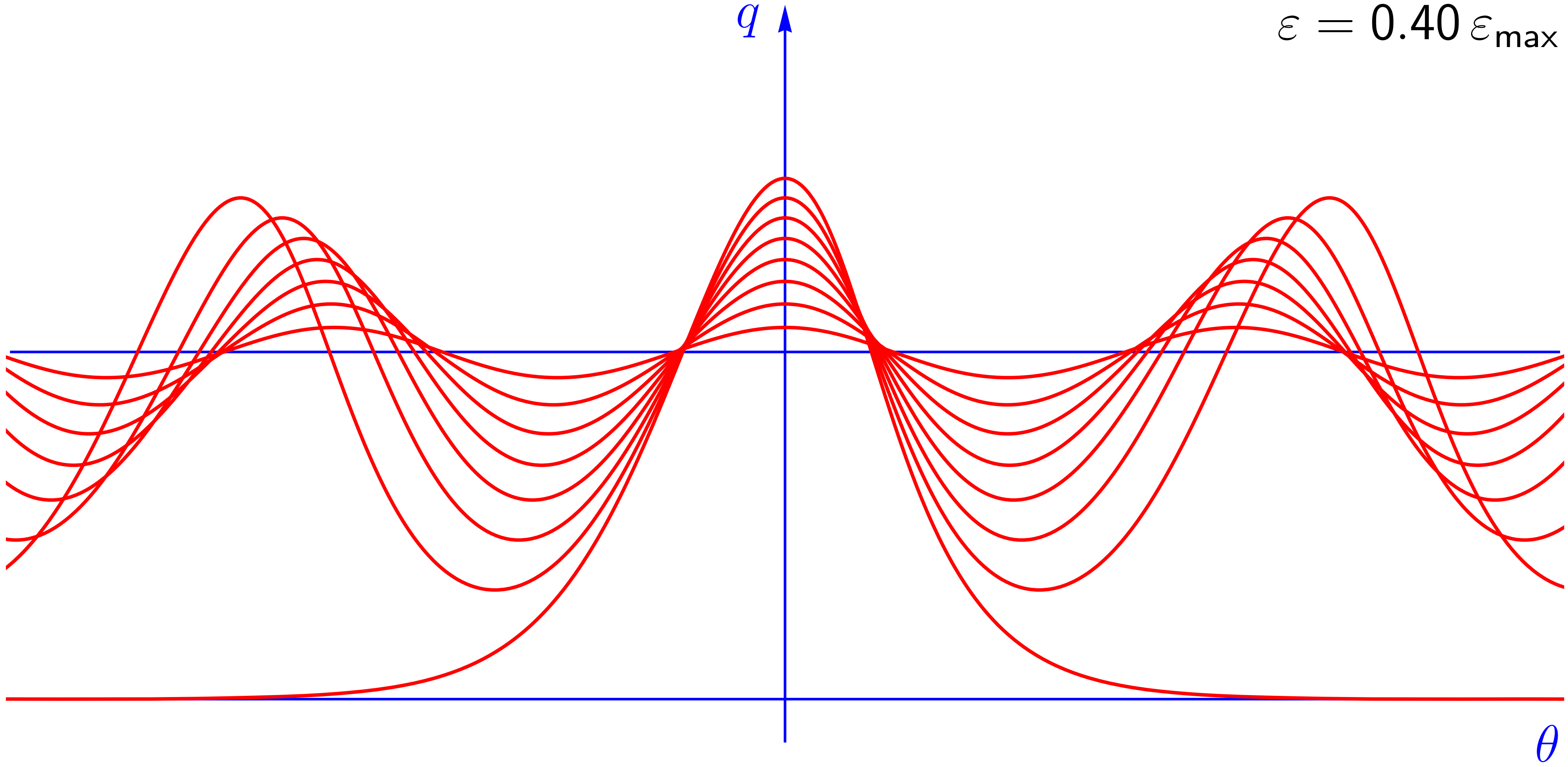

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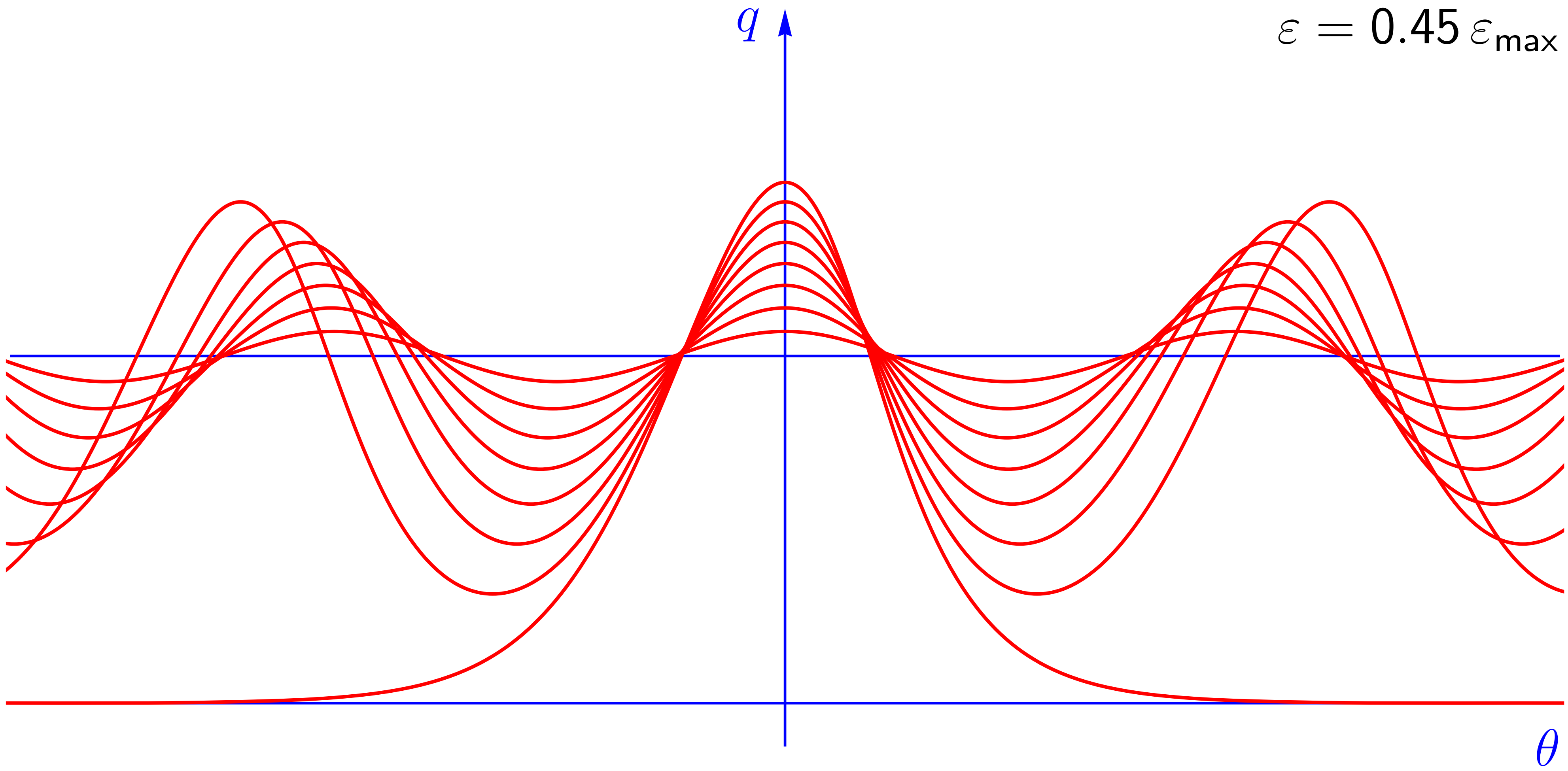

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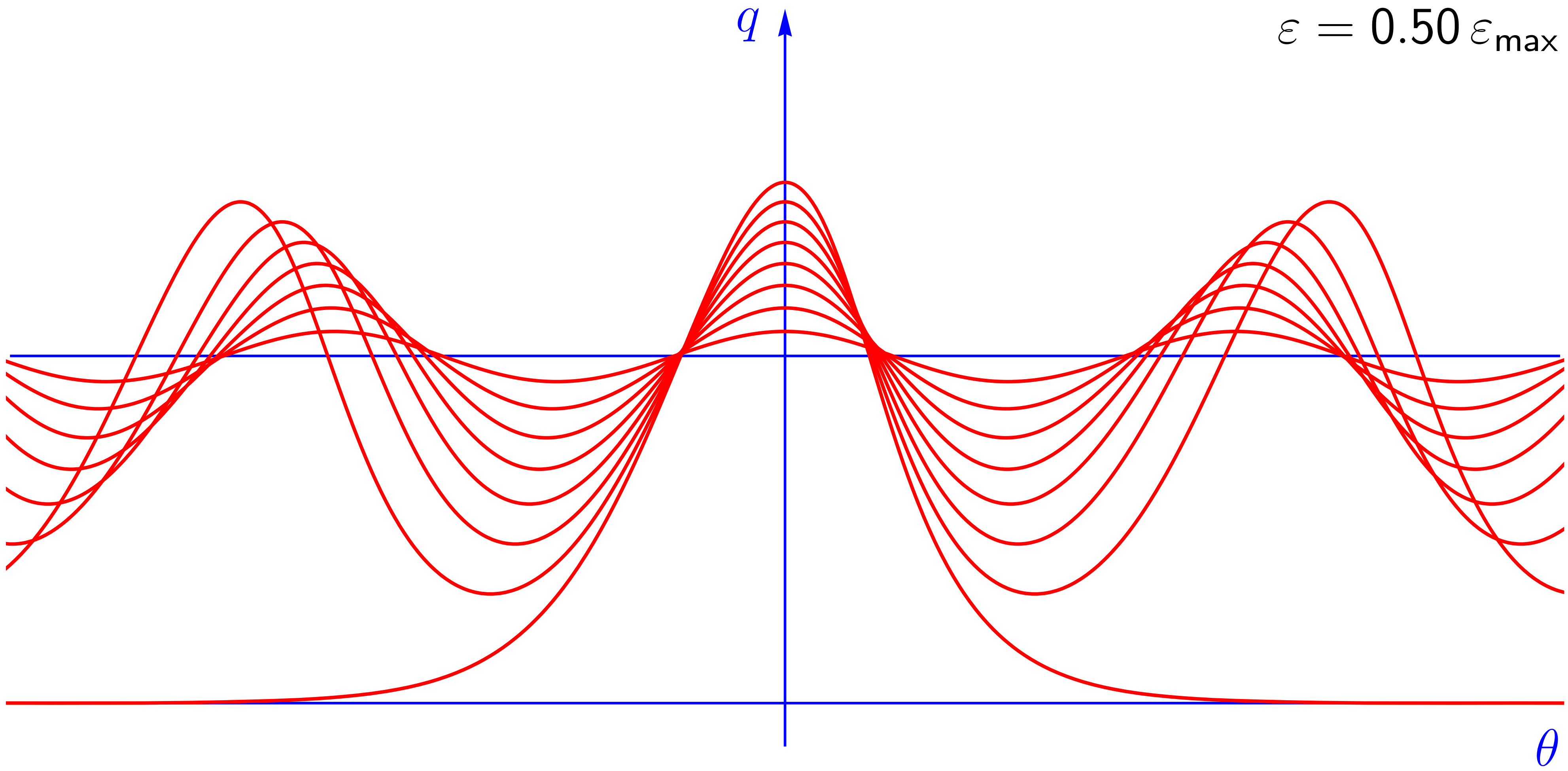

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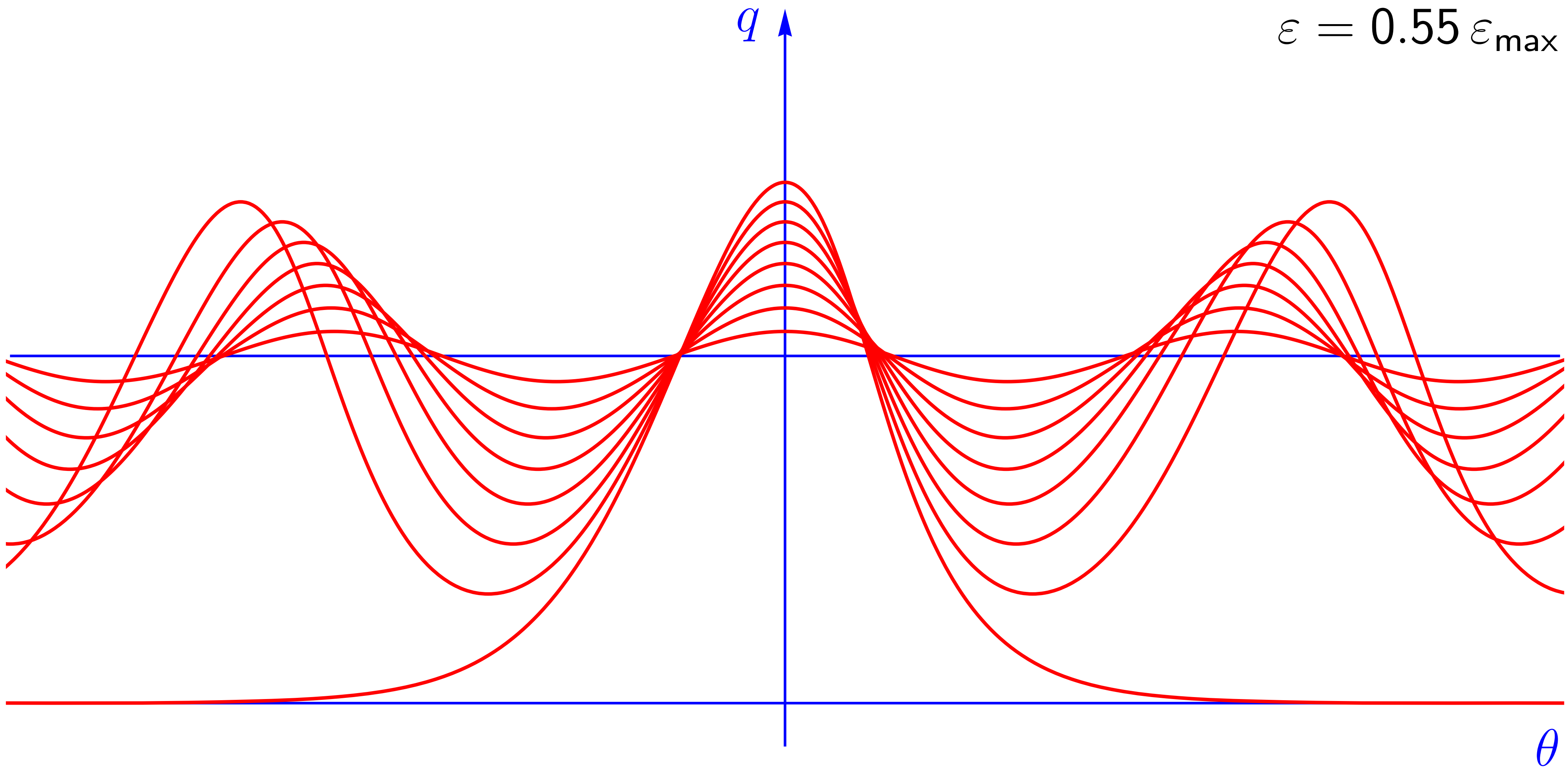

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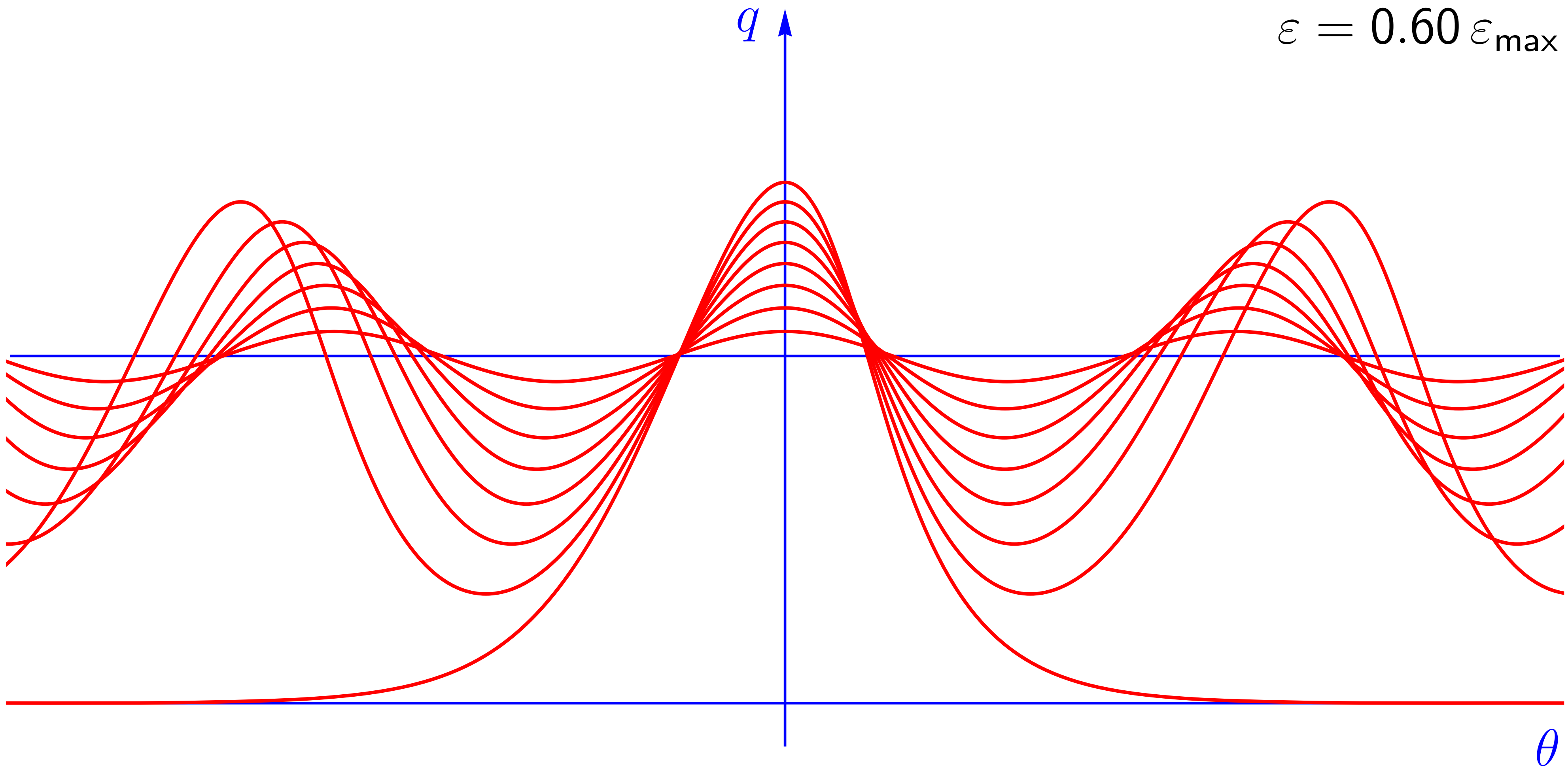

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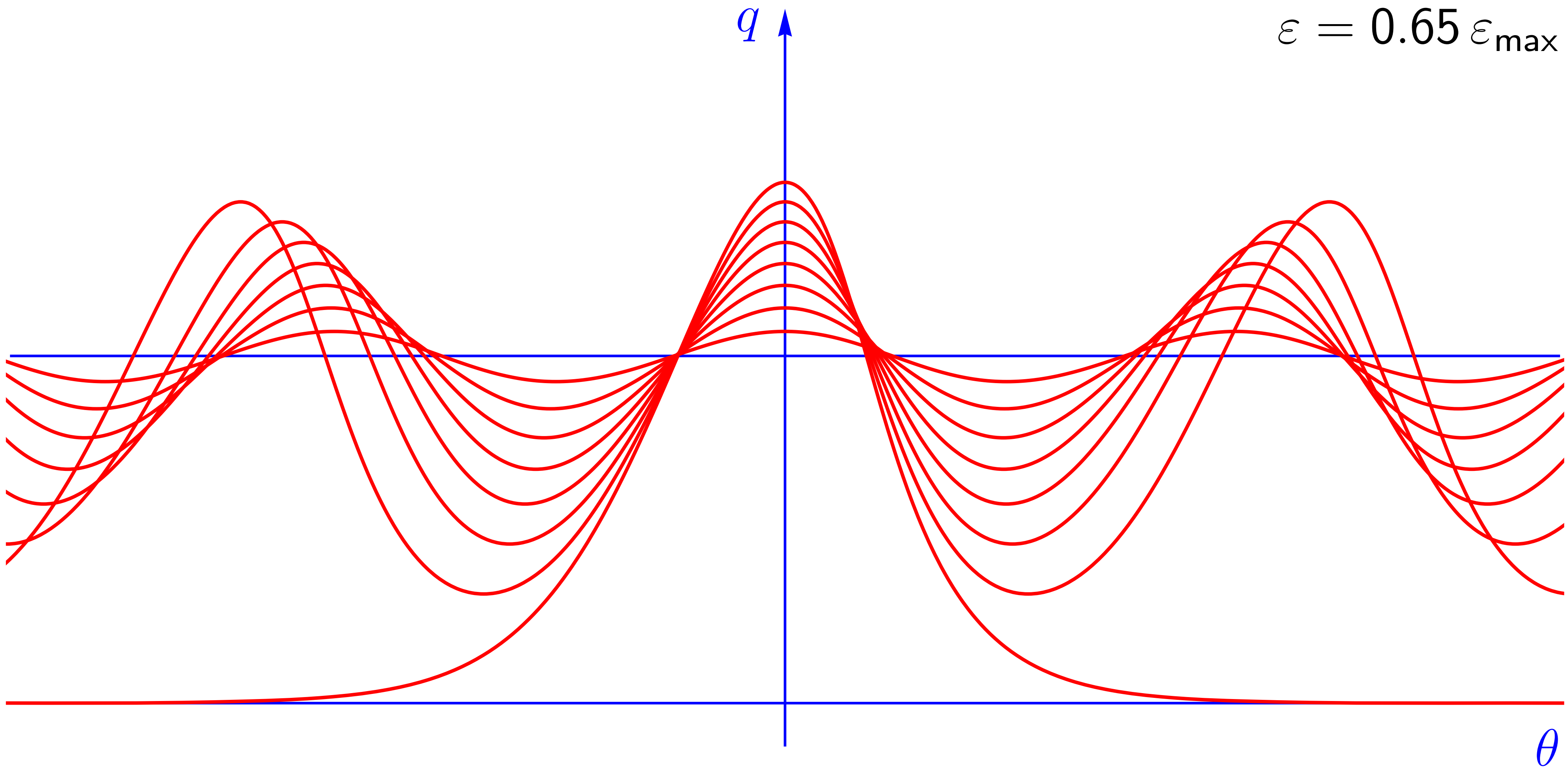

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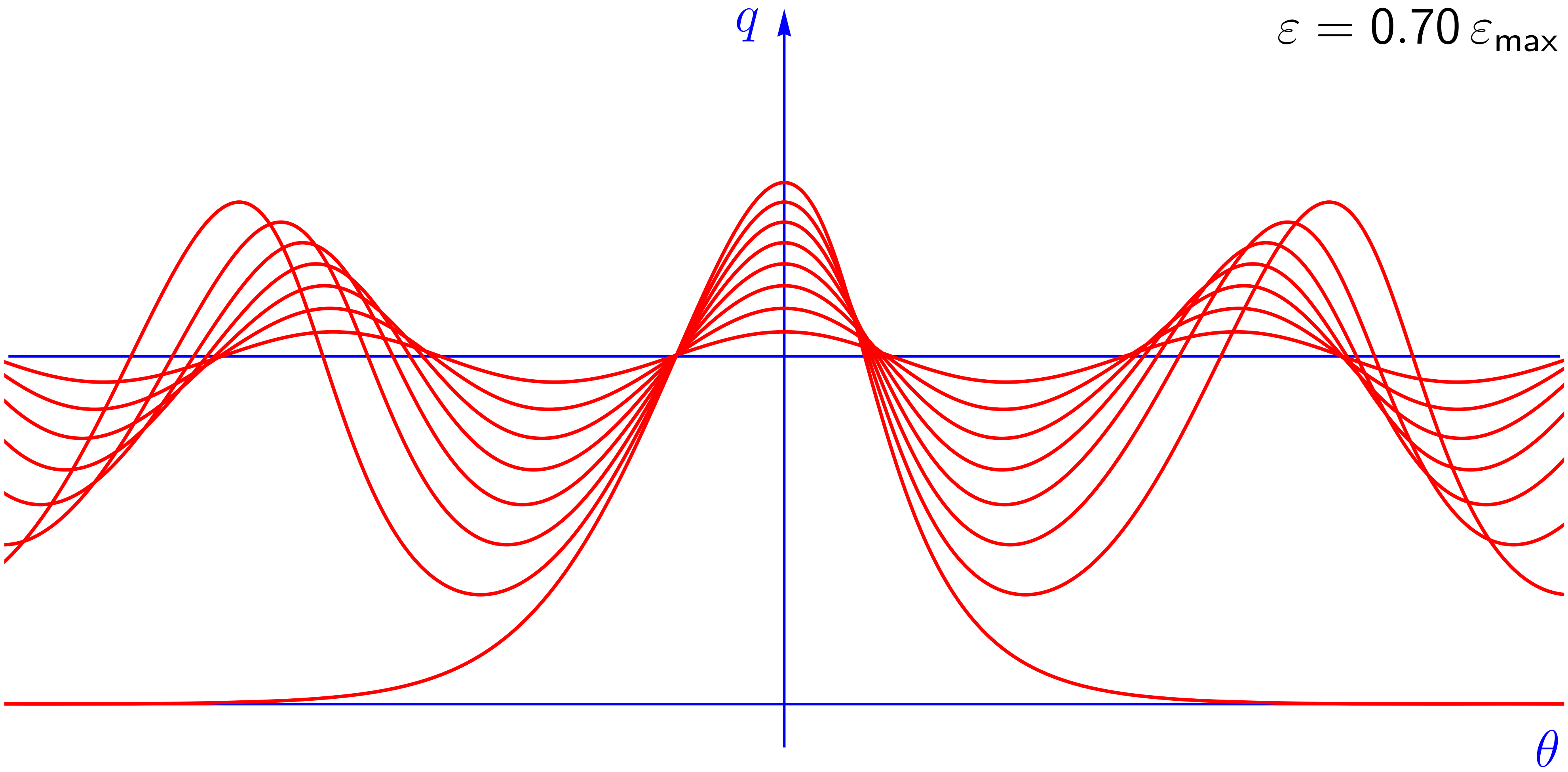

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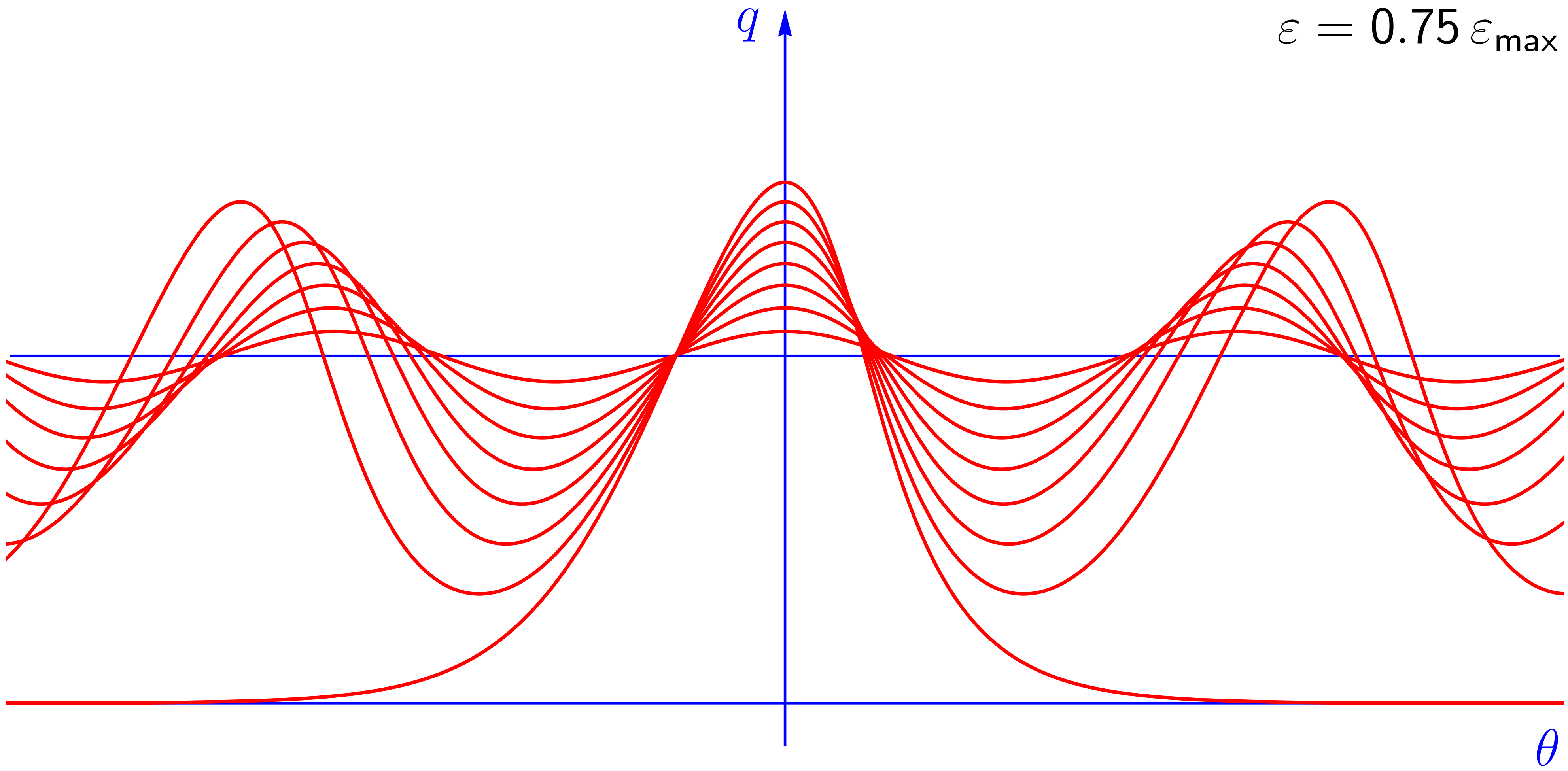

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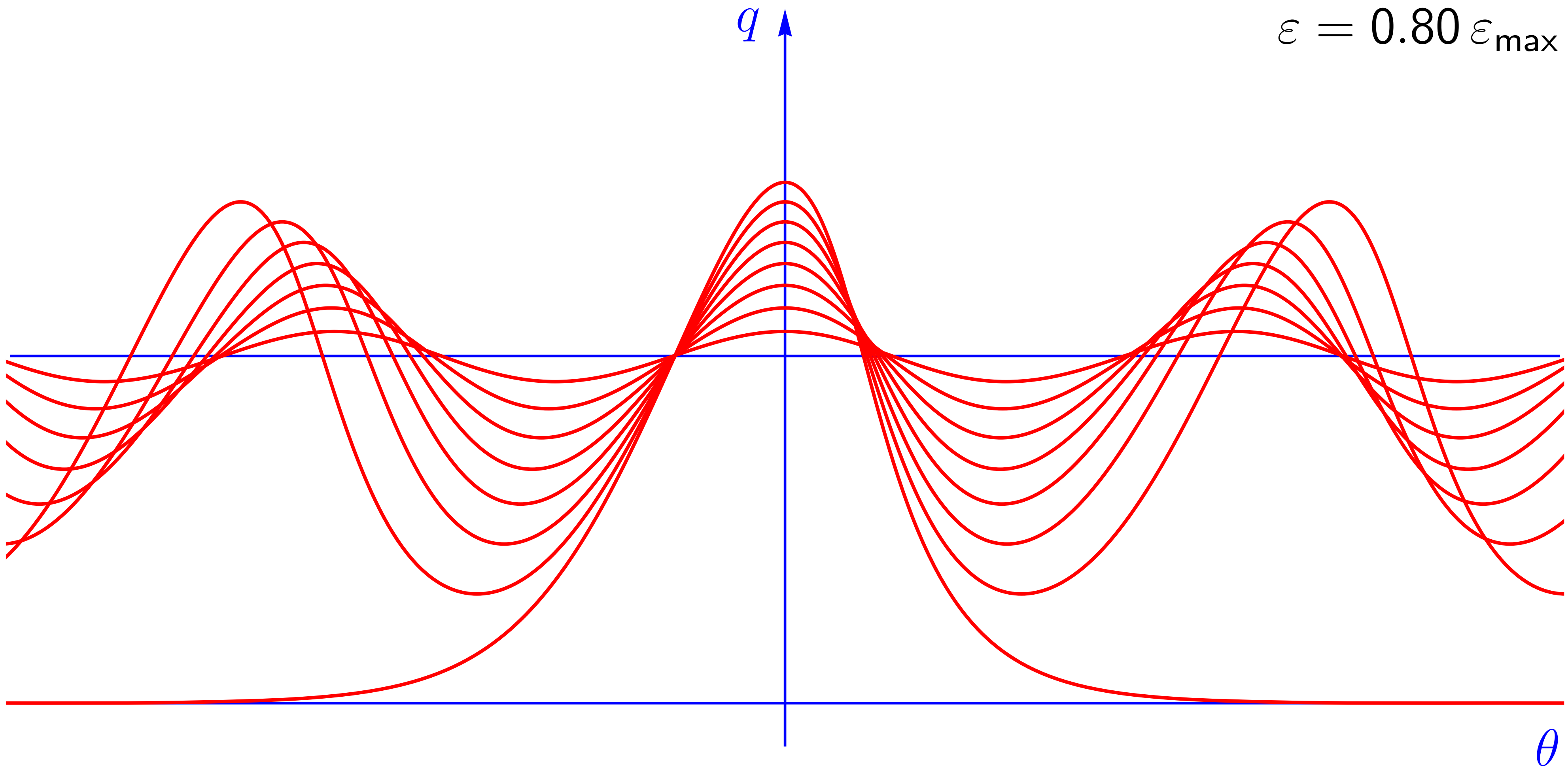

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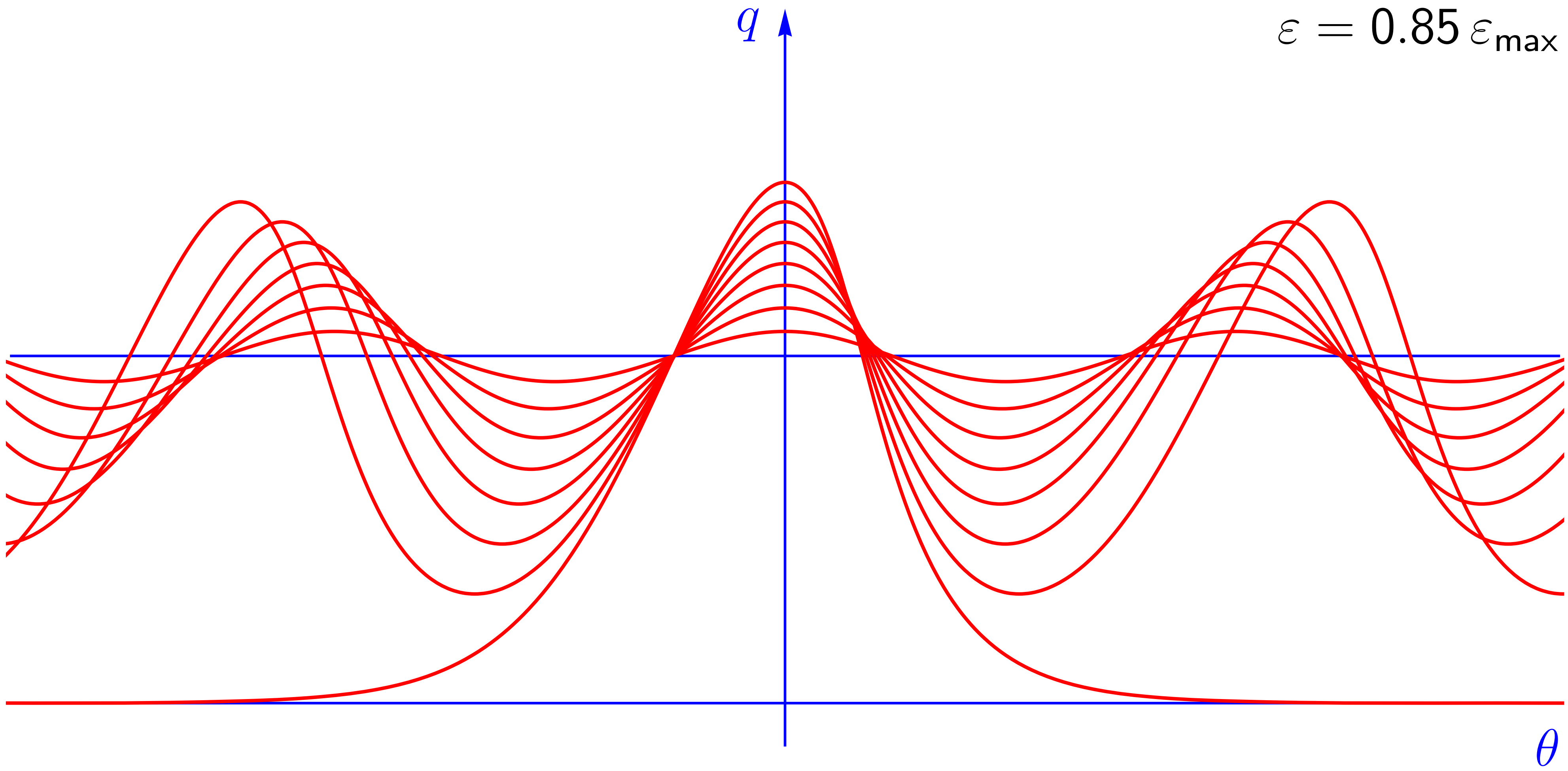

$\varepsilon = 0.85\,\varepsilon_{\mathsf{max}}$

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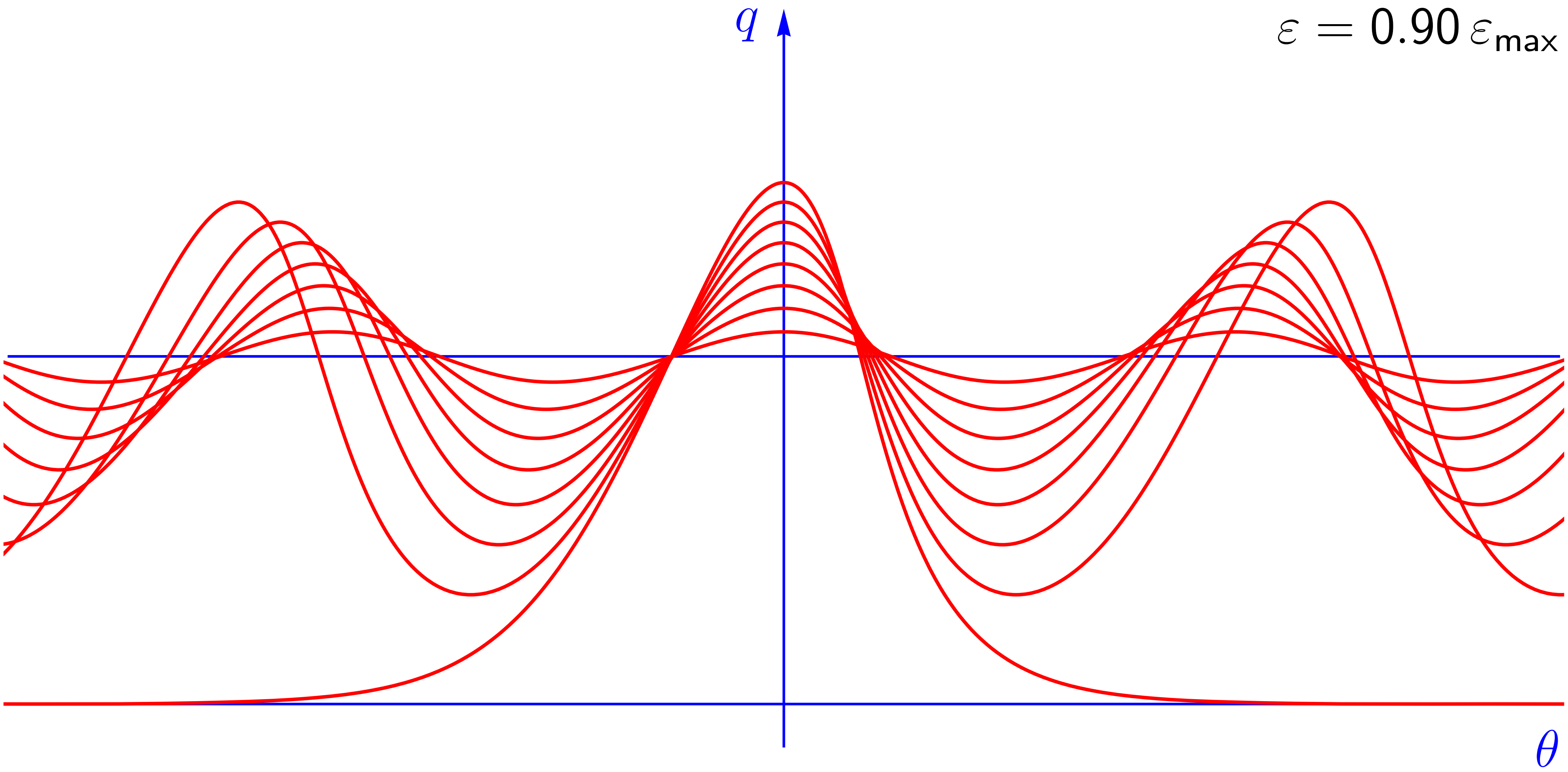

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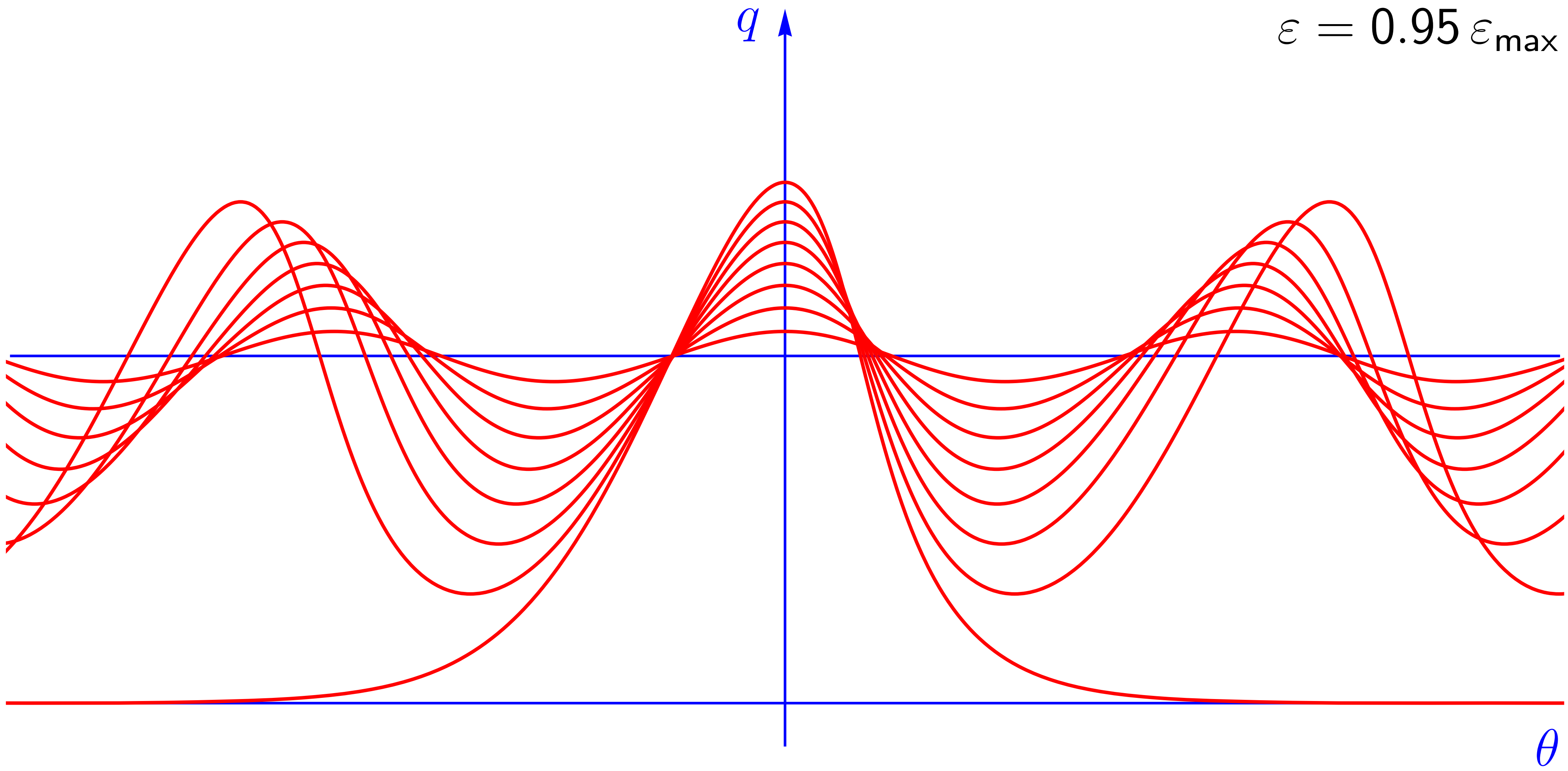

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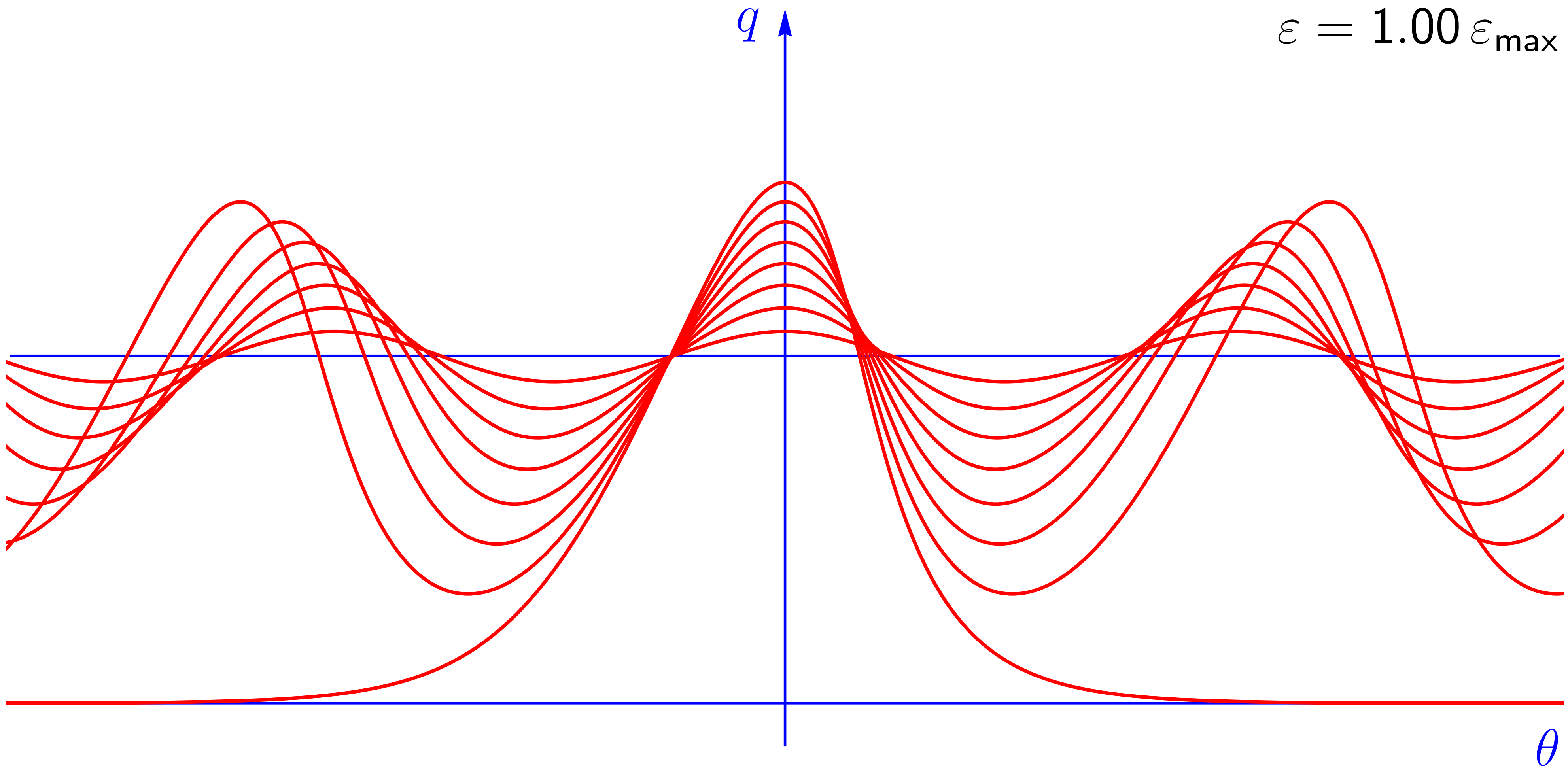
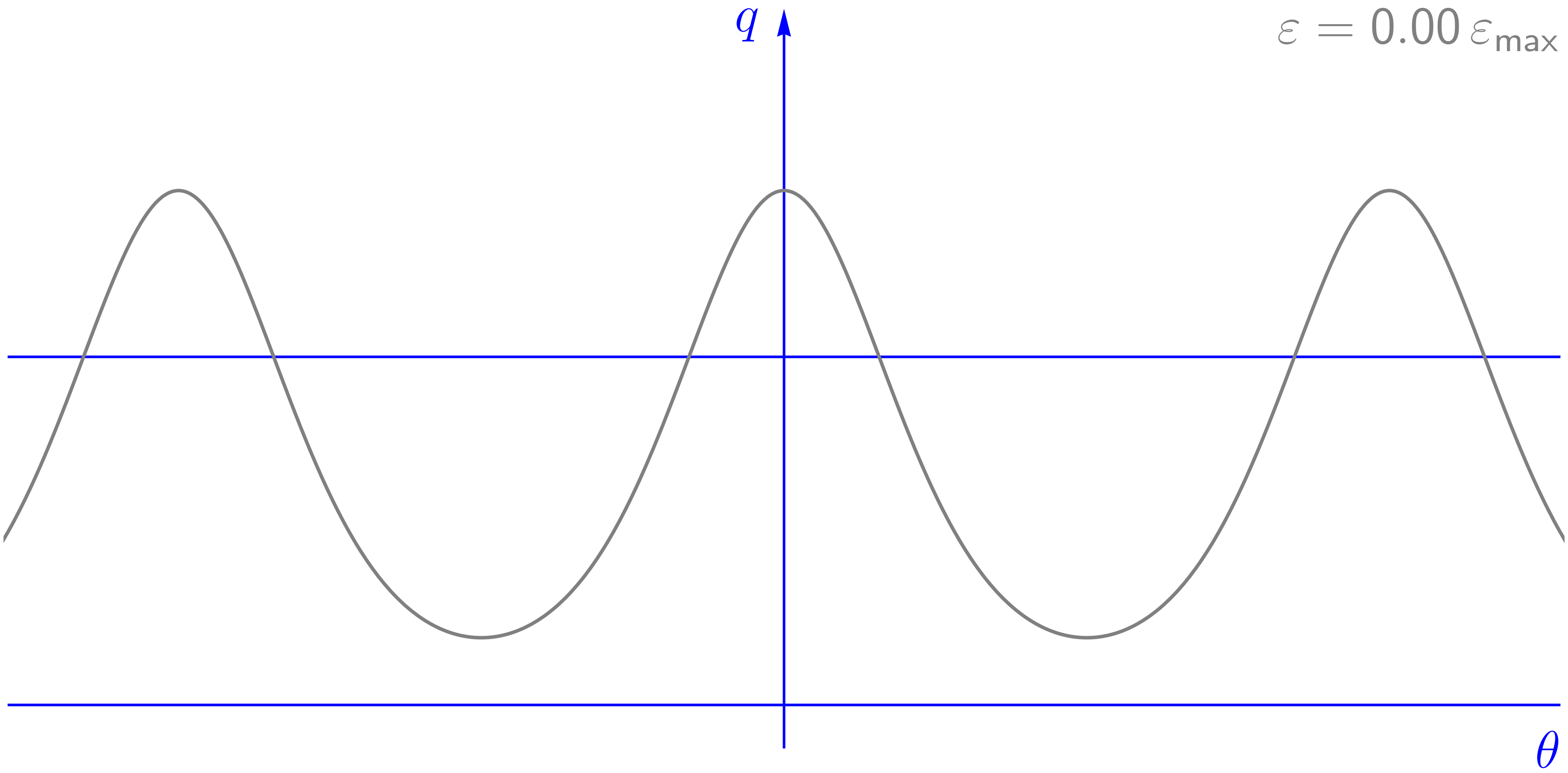
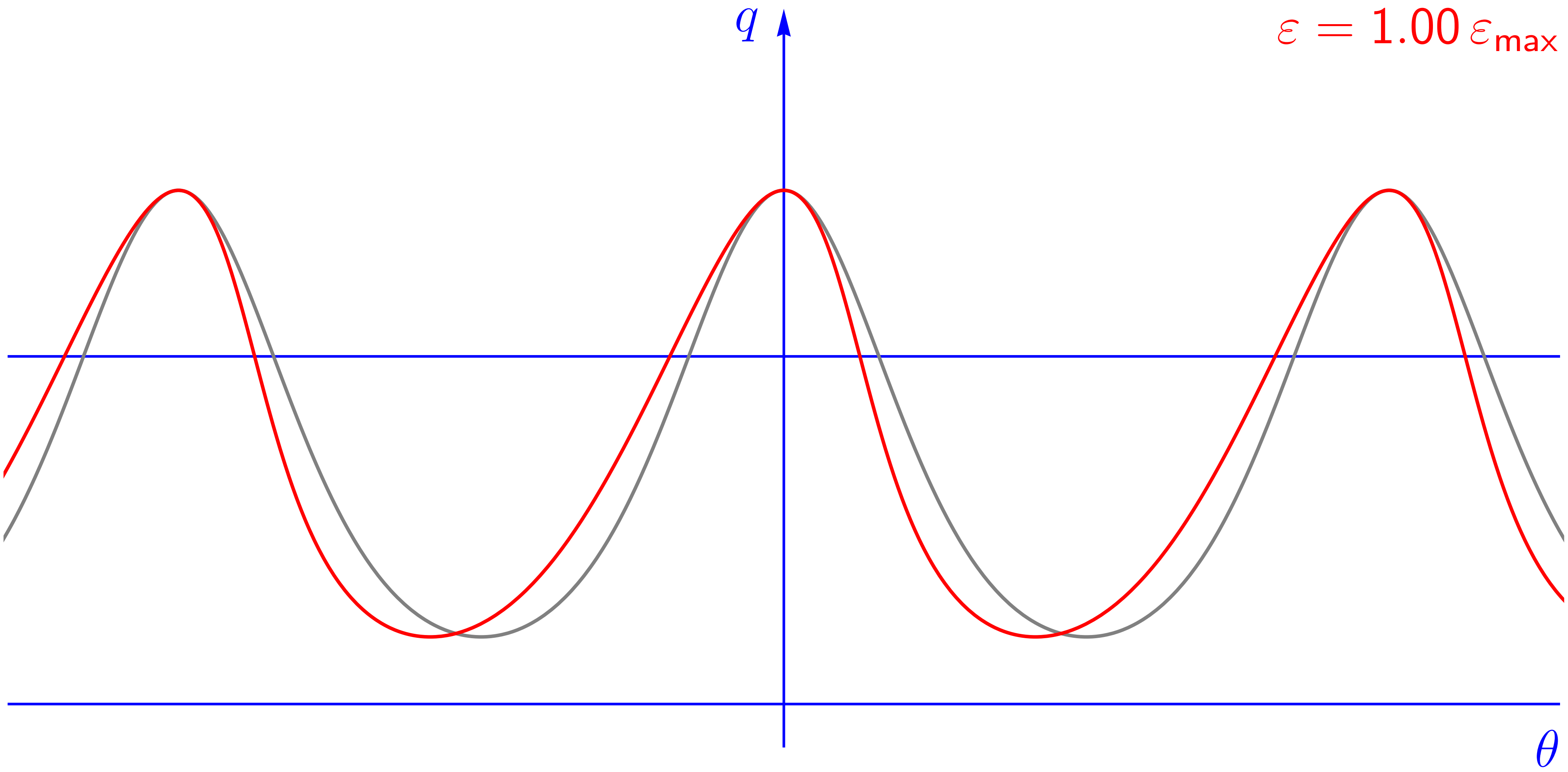


Periodic (Cnoidal) Solutions of the KdV Equation



Periodic Solutions of the Extended KdV Equation



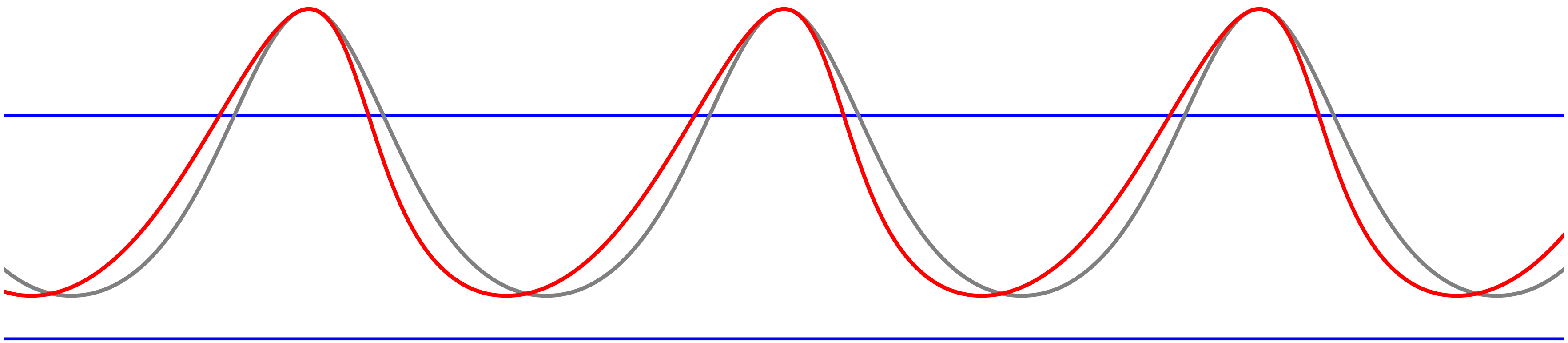
The extended Korteweg-de Vries equation

$$y_t + 3\left(y^2\right)_x + y_{xxx} + 3\varepsilon\left(y_x^2\right)_{xx} = 0$$

has solutions representing periodic waves approximated by

$$y = q_2 + (q_3 - q_2)\,\mathrm{cn}^2\,\eta\left[x - 4\eta^2 t + 2\varepsilon(q_3 - y)\right]$$

generalizing the cnoidal wave solutions of the KdV equation.



thank you

tänan

спасибо

danke