International Conference on
Complexity of Nonlinear Waves
October 5-7, 2009

Nonlinear Wave Motion and Complexity

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Outline

1. Introduction
2. Complexity
3. Waves in microstructured materials

   internal variables

   examples
4. Biophysics / biomechanics
5. Complexity around snow
6. Final remarks

One of the most highly developed skills in contemporary western civilization is dissection: the split-up of problems into their smallest possible components. We are good at it. So good, we often forget to put the pieces back together again

Armin Toffler, 1984

The whole is more than the sum of the parts

Aristotle, 384–322 BC

Prigogine I. and Stengers I.
Order out of Chaos (1984)

Nicolis G. and Nicolis C.
Foundations of Complex Systems (2007)

Complexity

- complex systems are comprised of many different parts which are connected in multiple ways;
- complex systems produce global emergent structures generated by local interactions;
- emergence occurs far from equilibrium;
- complex systems are typically nonlinear;
- emergence usually occurs at the edge of chaos.

Mechanics – classical examples

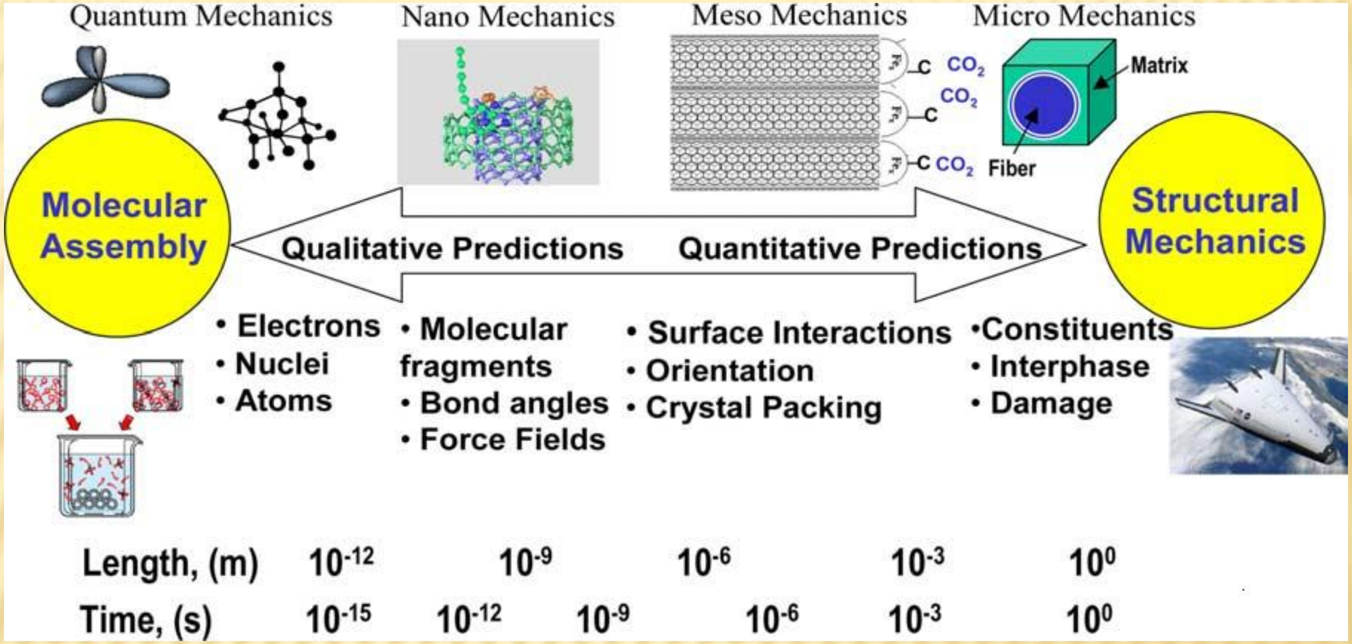
- three – body system
- double pendulum
- Lorenz attractor
- turbulence
- . . . . .
- and many more

Mechanics of microstructured solids

Problems:

- discrete vrs continuum?
- nonlinearit-y/ies?
- description of microstructure (s)?
- interaction between constituents?

Scales



Reproduced from: T.S. Gates, G.M. Odegard, S.J.V. Frankland, T.C. Clancy, 2005. Computational materials: Multi-scale modeling and simulation of nanostructured materials. Composites Science and Technology, 65, 2416-2434.

METHODS:

discretization

homogenization

advanced continuum theories

Eringen, Mindlin, ...

Maugin – pseudomomentum

internal variables

. . .

Balance laws

Canonical (material) momentum balance

$$\frac{\partial \mathbf{P}}{\partial t}\Big|_{x} - \mathrm{Div}_{R}\mathbf{b} = \mathbf{f}^{int} + \mathbf{f}^{ext} + \mathbf{f}^{inh},$$

P - material momentum, **b** – material Eshelby stress,

material inhomogeneity force – $\mathbf{f}^{inh}$,

material external (body) force – $\mathbf{f}^{ext}$,

material internal force – $\mathbf{f}^{int}$

Energy conservation

$$\frac{\partial(S\theta)}{\partial t}\Big|_x + \nabla_R \cdot \mathbf{Q} = h^{int}, \quad h^{int} := \mathbf{T} : \dot{\mathbf{F}} - \frac{\partial W}{\partial t}\Big|_x$$

the second law

$$S\dot{\theta} + \mathbf{S} \cdot \nabla_R \theta \le h^{int} + (\theta \mathbf{K})$$

S – the entropy flux, S – the entropy density per unit reference volume, θ – absolute temperature, **K** – extra entropy flux, **T** – the first Piola – Kirchhoff tensor, **F** – deformation gradient.

Internal variables (1)

observable – strains, displacements, etc.

internal – describe the internal structure of the material

see Maugin, Muschik, 1994

- damage parameter
- orientation of liquid crystals
- dislocations

etc.

In this formalism : internal variables might not be inertial.

Internal variables (2)

Microstructured solids

Single / dual variables

Berezovski, Engelbrecht, Maugin, 2008

Dual variables $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ – second order tensors

Free energy W

$$W = \overline{W}\left(\mathbf{F}, \theta, \boldsymbol{\alpha}, \nabla_R \boldsymbol{\alpha}, \boldsymbol{\beta}, \nabla_R \boldsymbol{\beta}\right)$$

Internal variables (3)

From dissipation inequality

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \mathrm{L} \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} \mathrm{L}^{11} & \mathrm{L}^{12} \\ \mathrm{L}^{21} & \mathrm{L}^{22} \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix}$$

L – depend on state variables

$\tilde{A}, \tilde{B}$ related to $\overline{\mathrm{W}}$

A simple non-dissipative process

$$\dot{\alpha} = \mathrm{L}^{12}\, \tilde{\mathcal{B}}\,, \quad \dot{\beta} = -\mathrm{L}^{12}\, \tilde{\mathcal{A}}$$

A special case $\overline{\mathrm{W}}$ independent of $\nabla_R \beta$

$$\ddot{\alpha} = \left(\mathrm{L}^{12} \cdot \mathrm{L}^{12}\right) \tilde{\mathcal{A}}$$

inertia taken into account!

Micromorphic elasticity

General theory – Mindlin (1964)

1D models – Engelbrecht and Pastrone (2003)

internal variables approach –

– Berezovski et al. (2008)

Free energy function W – quadratic → linear model

– cubic → nonlinear model

Mindlin model

Mindlin model

two balance laws

Internal variable

one balance law
dissipation inequality

$$\rho u_{tt} = \alpha u_{xx} + N u_x u_{xx} + A \psi_x$$

$$I \psi_{tt} = C \psi_{xx} + M \psi_x \psi_{xx} - A u_x - B \psi$$

Ψ – microdeformation ⟷ Ψ – internal variable

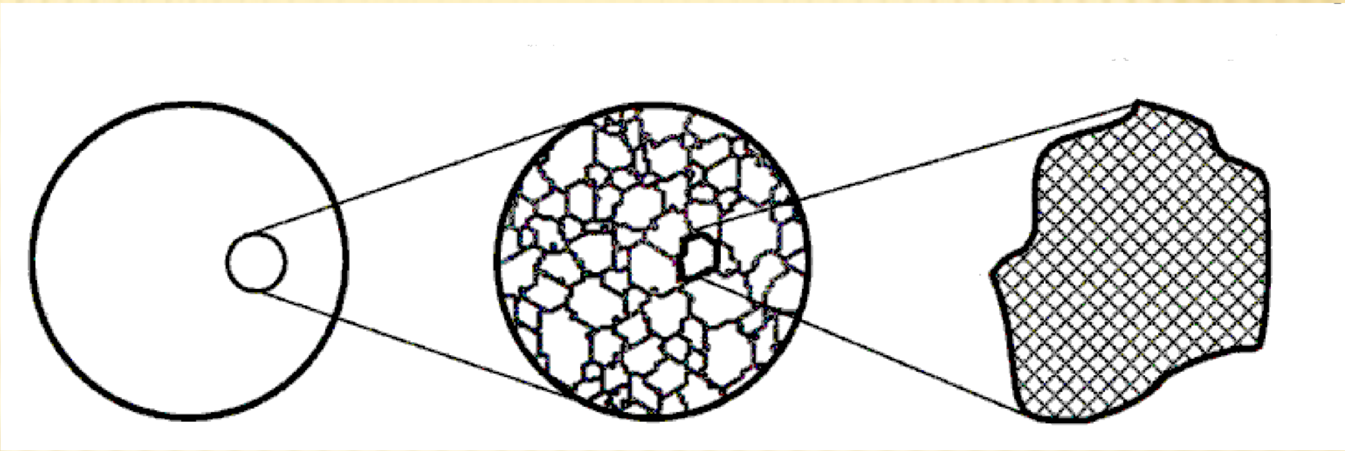
Mindlin model

exact, nonlinear, nondimensional

$$U_{TT} = \left(1 - \frac{c_A^2}{c_0^2}\right) U_{XX} + \frac{1}{2} k_N \left(U_X^2\right)_X + \frac{c_A^2}{c_B^2}\left(U_{TT} - \frac{c_1^2}{c_0^2} U_{XX}\right)_{XX} + \frac{1}{2} k_M \left(U_{XX}^2\right)_{XX}$$

Multiple scales

macrostructure microstructure 1 microstructure 2



Multiple scales

linear

$$u_{tt} = \left(c_0^2 - c_A^2\right)u_{xx} + p_1^2 c_{A1}^2 \underbrace{\left[u_{tt} - \left(c_1^2 - c_{A2}^2\right)u_{xx}\right]_{xx}} -$$

$$- p_1^2 c_{A1}^2 \;\; p_2^2 c_{A2}^2 \underbrace{\left(u_{tt} - c_2^2 u_{xx}\right)_{xxxx}}$$

Complexity of microstructured solids

hierarchy

soliton emergence

solitons asymmetric

solitonic structures

patterns of trajectories

cf. fluids

interaction of solitons

Biophysics / biomechanics

- biological systems need energy exchange with surrounding environment;
- systems are far from the thermodynamic equilibrium;
- processes operate over different time and space scales, include many hierarchies;
- in physical terms: nonlinearities, dissipation, activity/excitability.

Hierarchies in biophysics

- Structural hierarchies:

atom → molecule → cell → tissue → organ → human

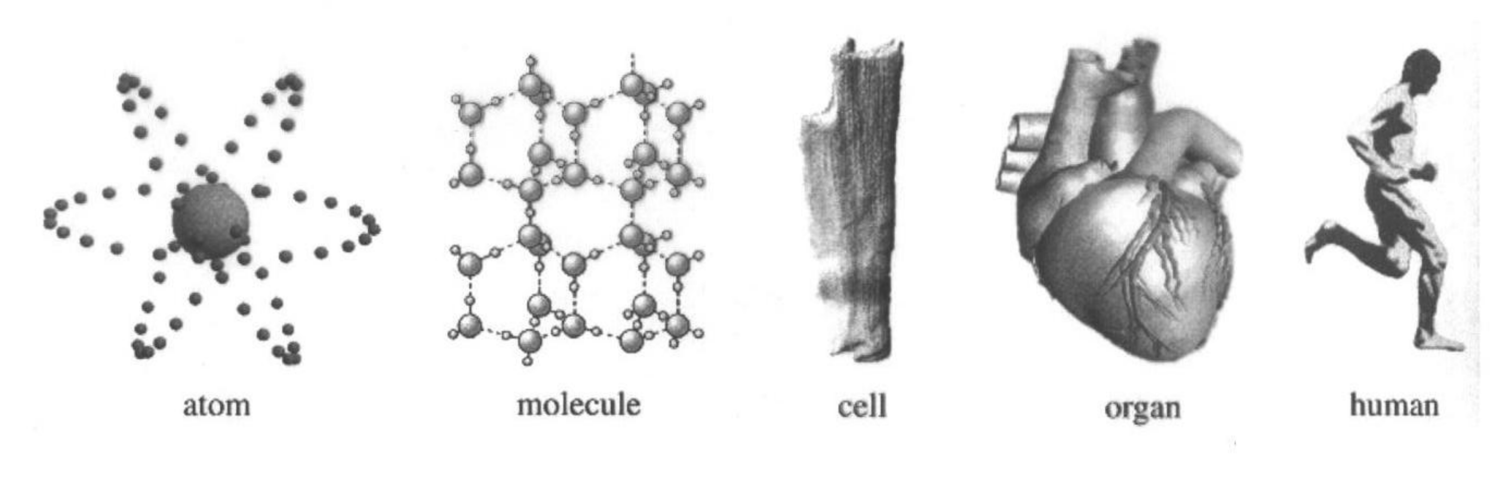
sarcomeres – myofibrils → fibres → myocardium → heart

- Process hierarchies:

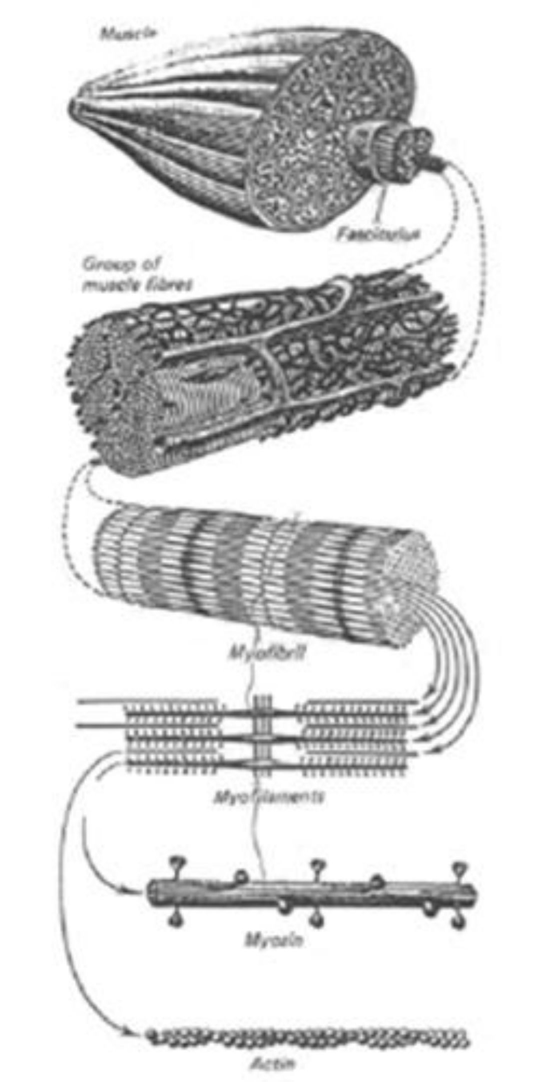
oxygen consumption → energy transfer → Ca^{2+} signals

→ cross–bridge motion → contraction

Examples



EXAMPLES


Muscle
Fasciculus
Group of
muscle fibres
Myofibril
Myofilaments
Myosin
Actin

Functional hierarchies

concept of hierarchical internal variables

. . . $\rightarrow$

$\gamma \rightarrow \beta$

$\beta \rightarrow \alpha$

$\alpha \rightarrow$ observable variables
(stress, strain)

Cardiac contraction

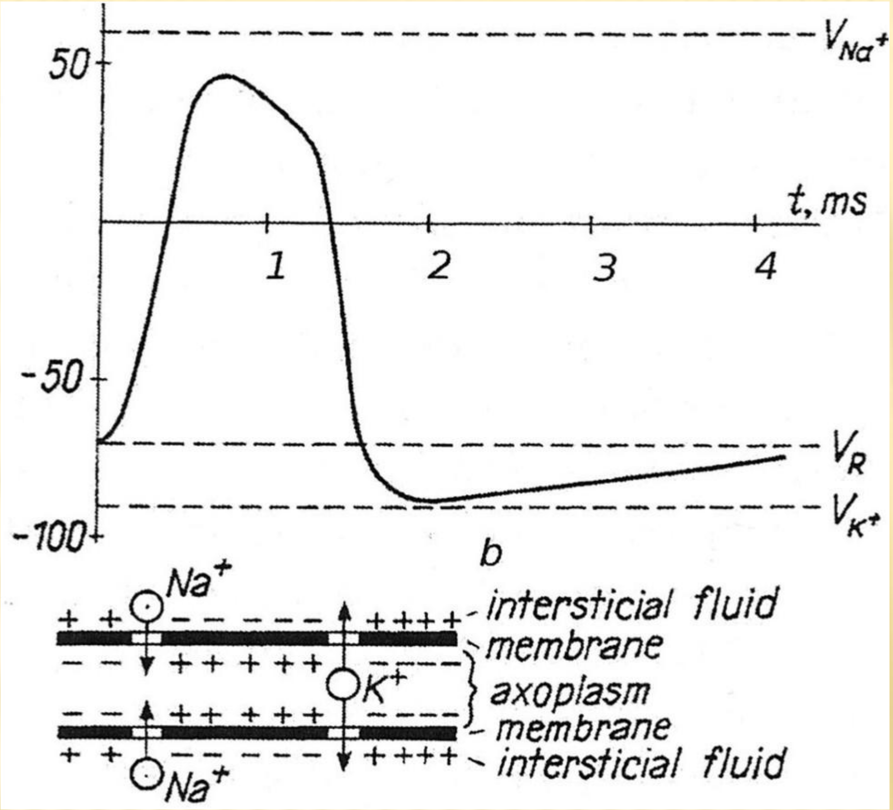
Ca^{2+} signals →

→ all activated crossbridges →

→ force producing crossbridges →

→ active stress

Nerve pulse transmission



Nerve pulse transmission

Hodgkin – Huxley model

three "phenomenological" variables

FitzHugh – Nagumo model

one "recovery" variable

In terms of continuum theory these are

internal variables

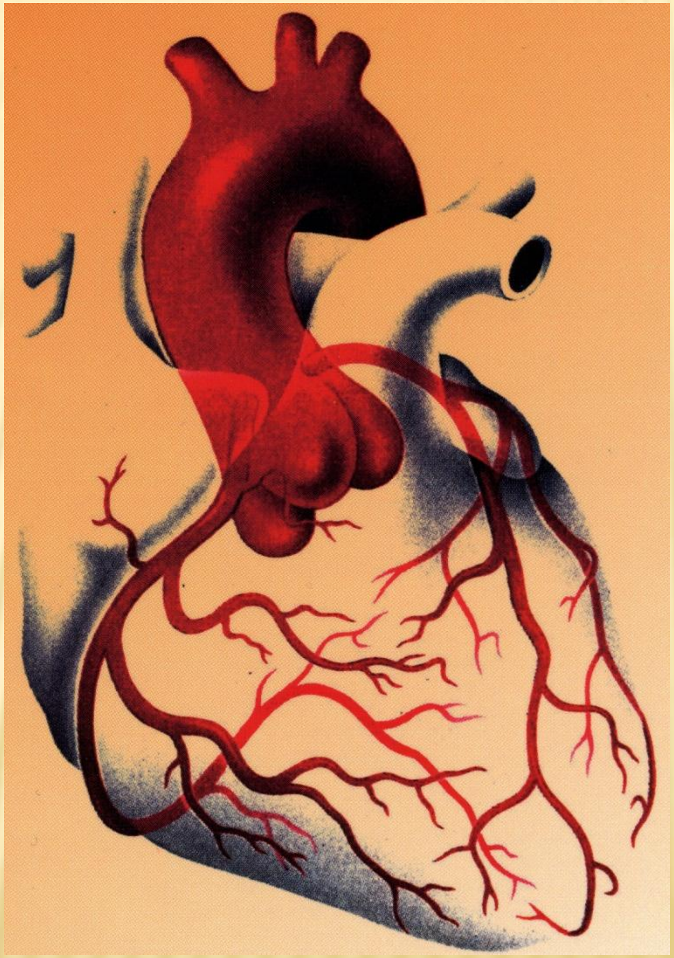
Maugin – Engelbrecht (1994)

Complexity around snow

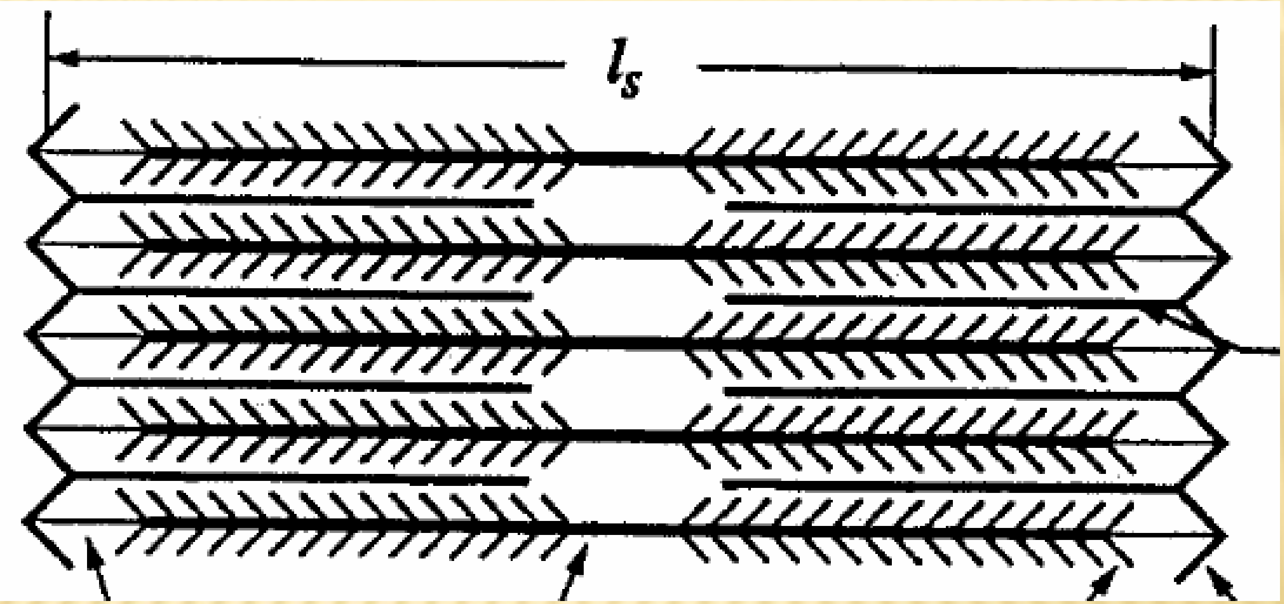
Problems

athlete	–	physiology biomechanics of muscles kinematics
equipment	–	technology mechanics chemistry
snow	–	granular (?) material phase transformation

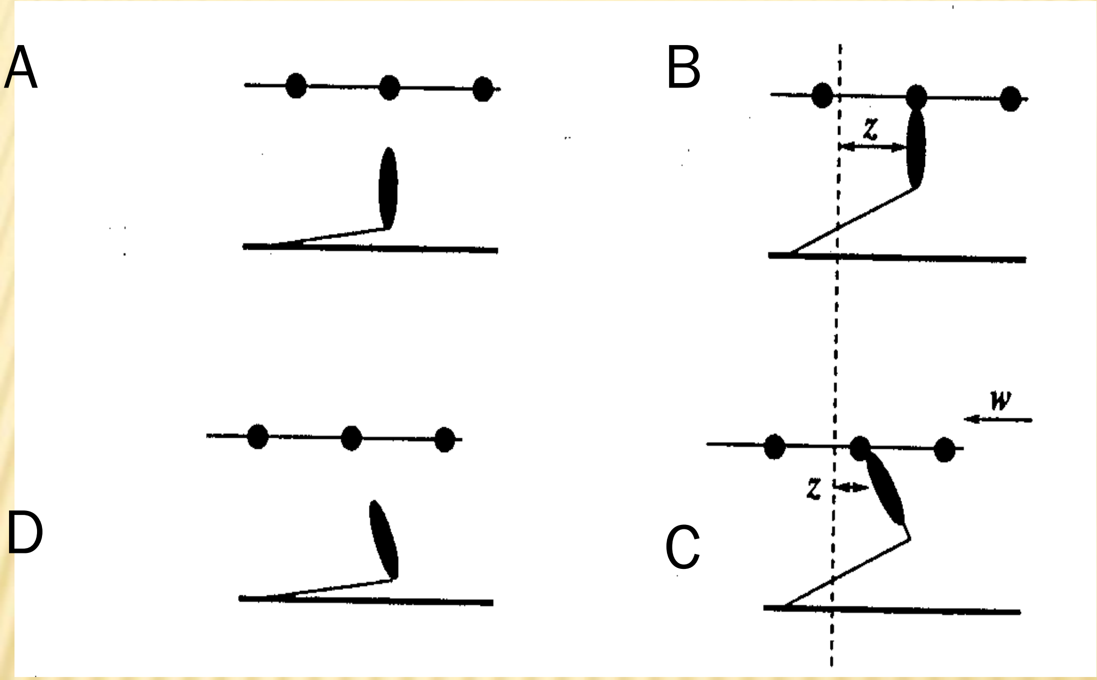
Athlete: heart



Cardiac contraction



Cardiac contraction



A to B – attaching

B to C – swivelling

C to D – detaching

D to A – resetting

Kinematics

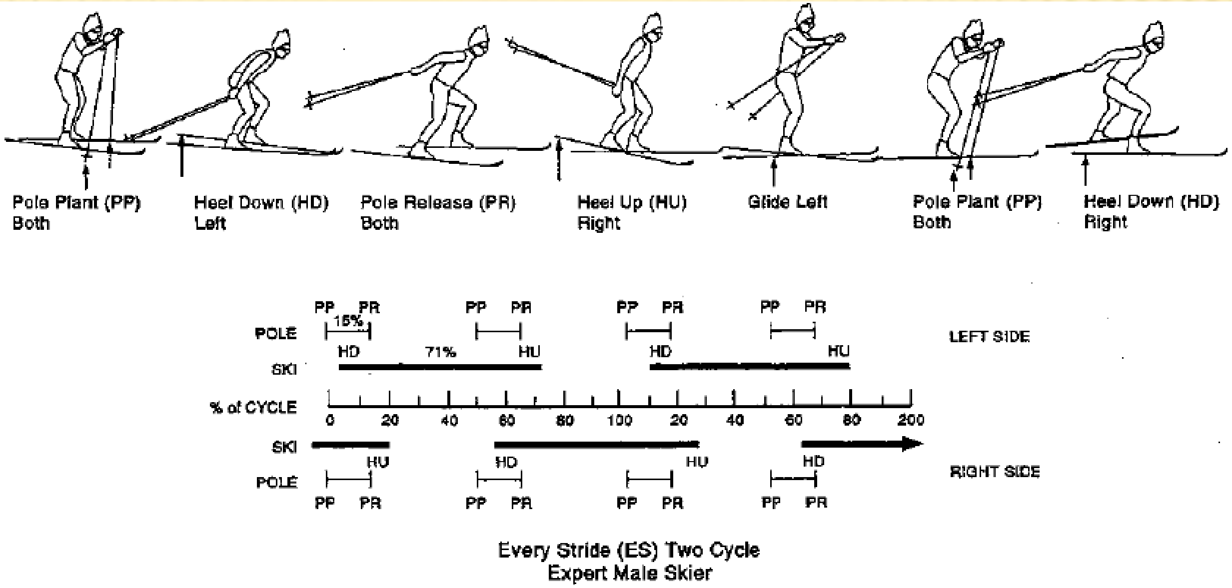


FIGURE 7.3. Every-stride (ES) ski-skating technique. The top panel shows the maneuvers for one half cycle. The lower graph shows the dwell time for each segment. Note that the top panel and the graphs come from two different sources. Although they relate well to each other, the relation is not exact. (Skier images from video of P. Peterson, PSIA Demo Team, used with permission. Data for the graph from Nelson *et al.*, 1986.)

Forces

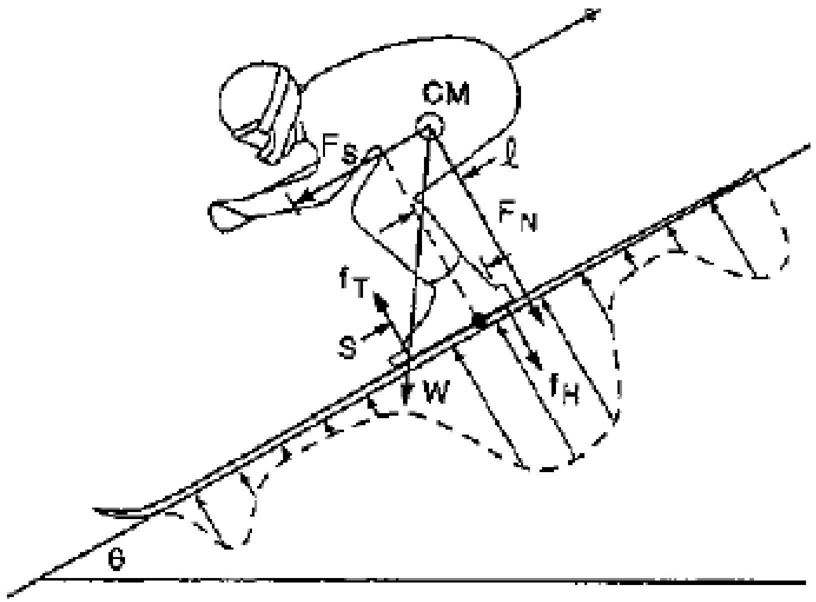


FIGURE 3.2. Forces acting on the ski through the boot.

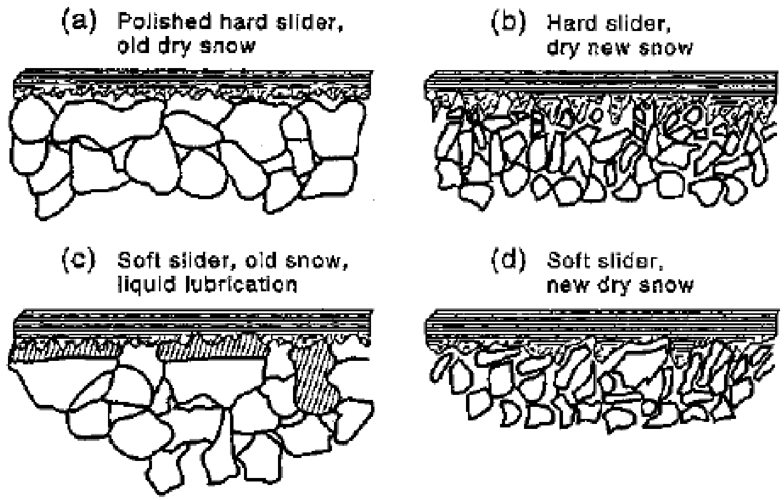


FIGURE 8.3. Four examples of the interactions that may occur at the interface of the ski bottom and the surface of the snow.

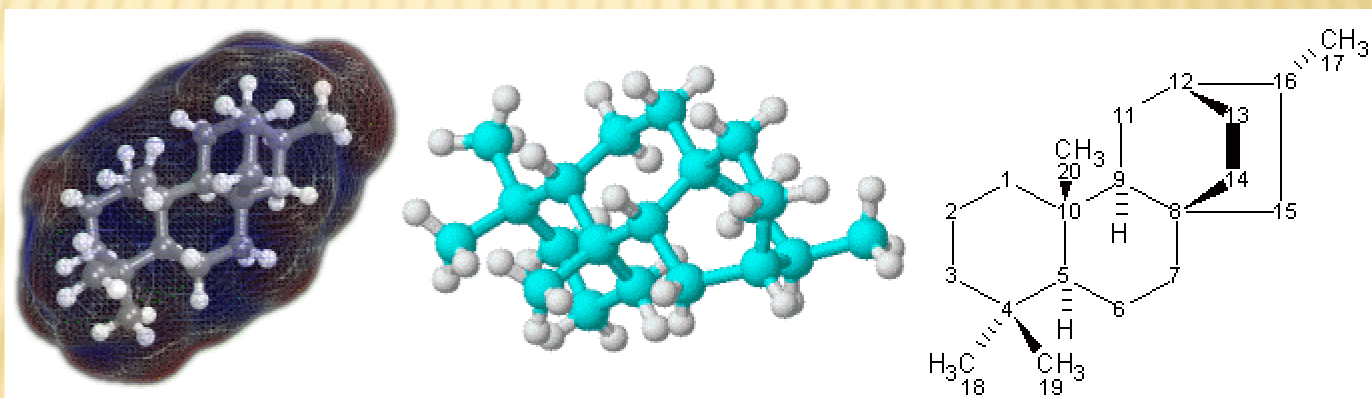
Technology:

skis, poles

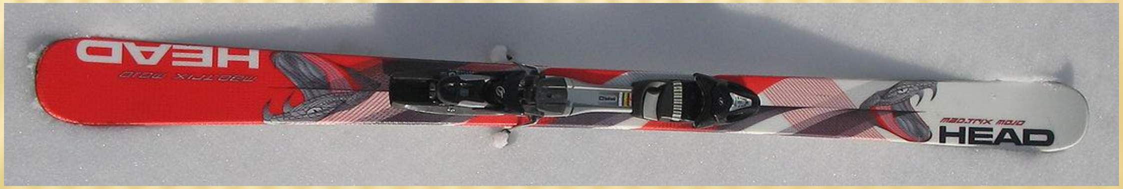
bindings, boots

clothing

waxes



Technology: examples



Snow

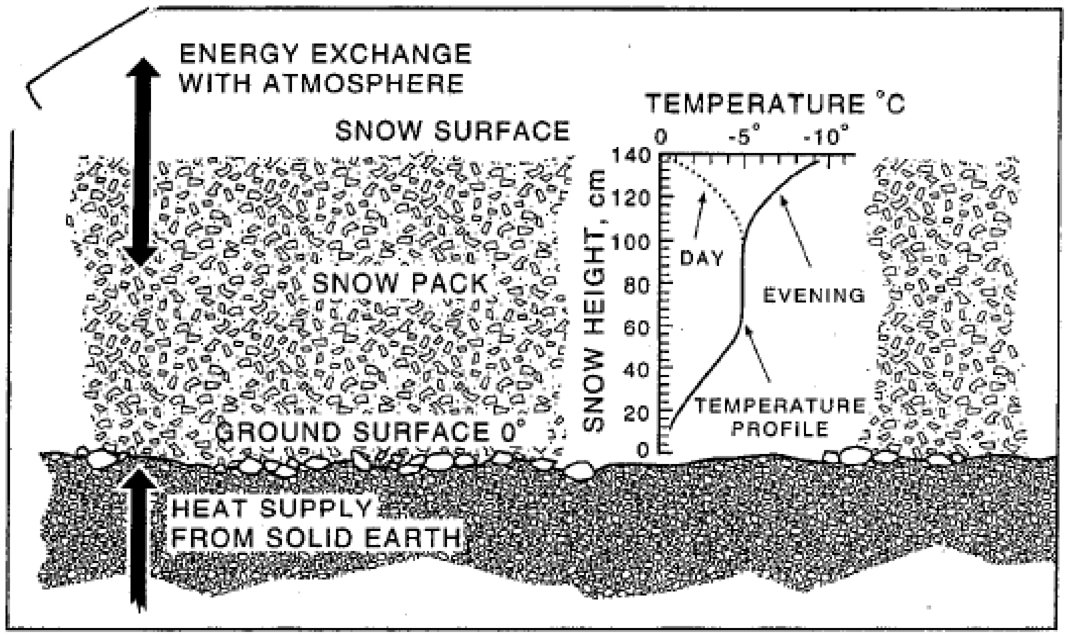
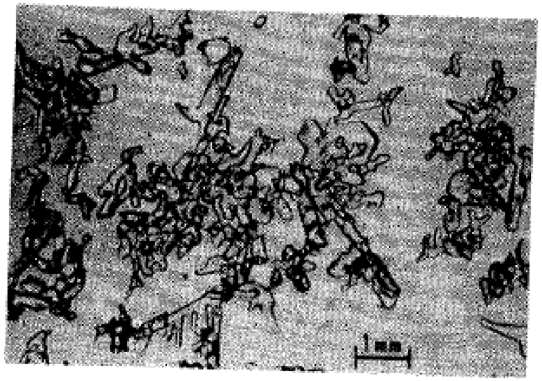


FIGURE 2.8. Temperature profile of a snowpack. Temperature gradients exist at the top and at the bottom, near the surface of the snowpack and at the ground surface. In the middle, the snowpack is at an equitemperature state (Perla and Martinelli, 1979).

Snow

(a)


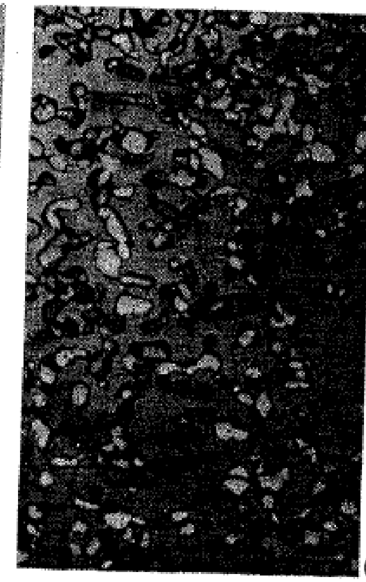
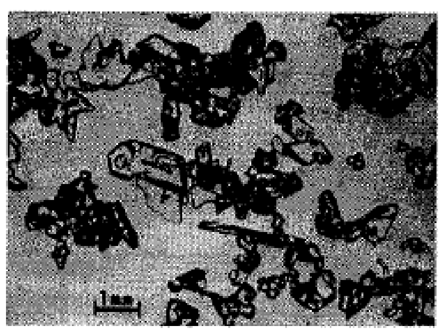

(b)

FIGURE 2.6. Equitemperature (ET) metamorphism of newly fallen snow. The faceted, dendritic arms of the new snow grains in the left frame have disappeared in the right frame, metamorphosing into rounded, sintered grains. (Reprinted with permission from Colbeck *et al.*, 1990. Photographs courtesy of E. Akitaya.)

(a)



(b)

FIGURE 2.10. Temperature-gradient (TG) snow in successive stages of growth. Notice the change in scale from (a) to (b). (Reprinted with permission from Colbeck *et al.*, 1990. Photographs courtesy of K. Izumi.)

Snow

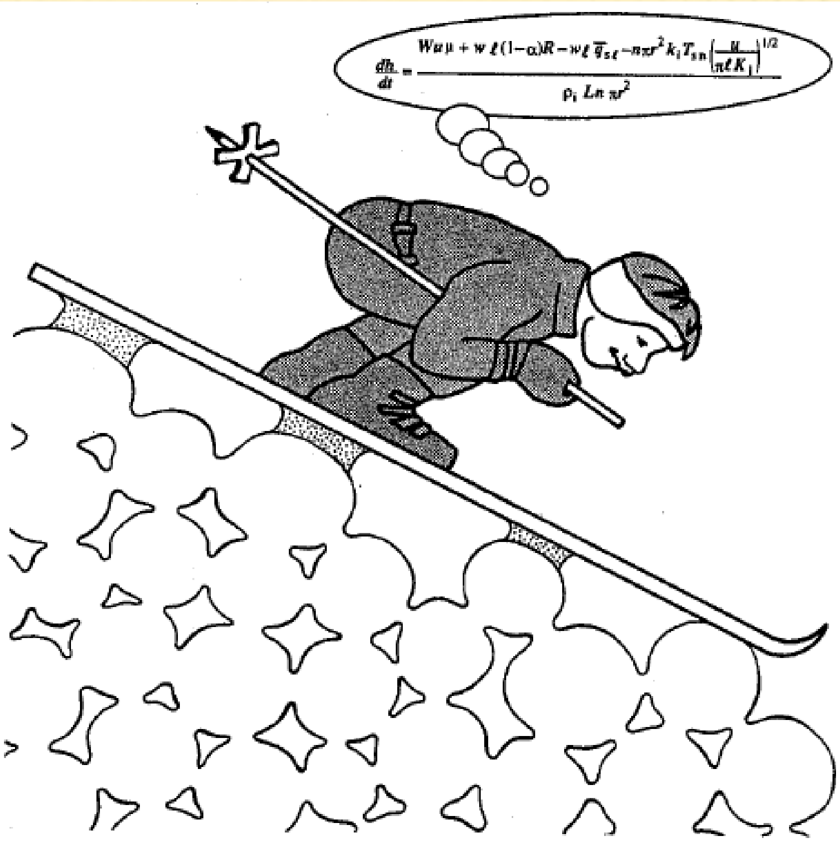
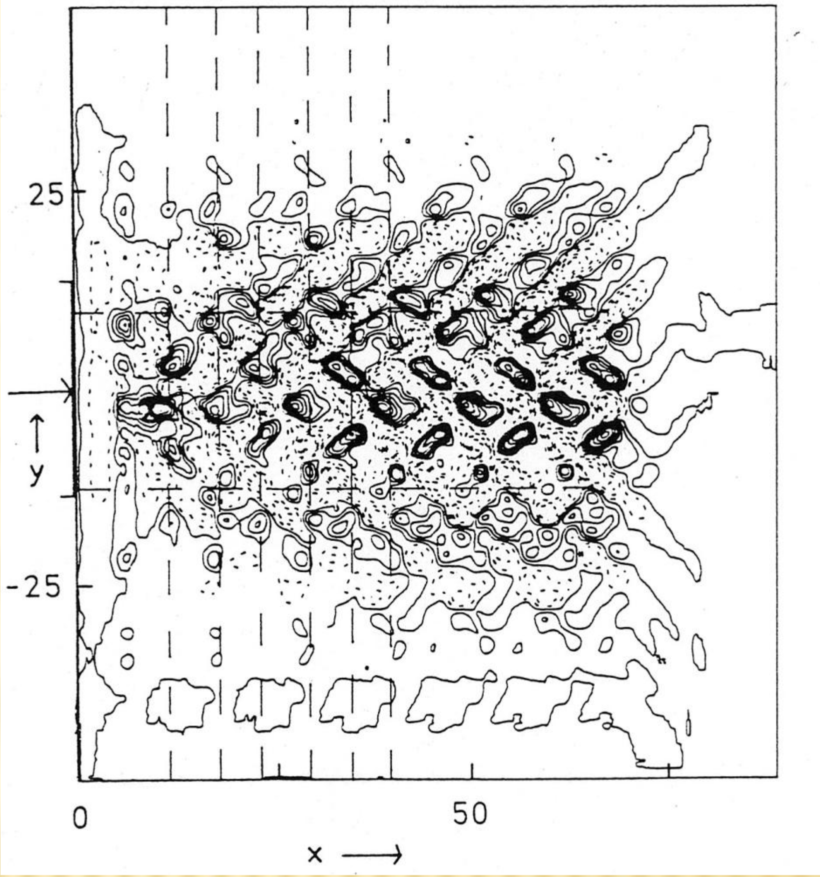


FIGURE 2.1. This skier heads down the hill, his skis lubricated by a film of water that forms under his skis. In his thoughts he mulls over a mathematical formula that we will discuss later in Chap. 8 on snow friction processes. (Colbeck, 1992. Drawn by Marilyn Aber, CRREL.)

Snow: reality

Snow: calculations

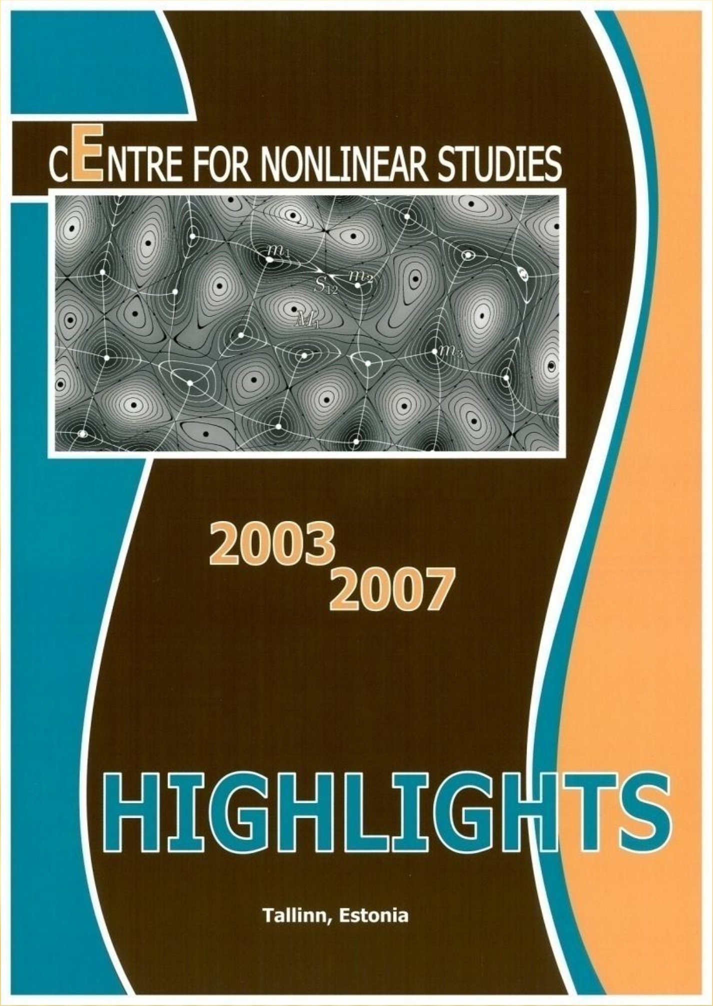


Complexity around snow

CENS: Annual reports

See also http://cens.ioc.ee/cens/about-cens/annual-reports

CENS: Books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