International Conference on Complexity of Nonlinear Waves, Institute of Cybernetics, Tallinn University of Technology, October 5-7, 2009

Scaling law of internal run-up duration

John Grue & J. Kristian Sveen, Mechanics Division, Department of Mathematics, University of Oslo, OSLO, NORWAY





Subsequent **ONSHORE FLOW**

MAX VEL. 0.5 m s^{-1}



Vertical excursion of pycno: 300 m (300 m / 550 m $\simeq \frac{1}{2}$)



Subsequent **ONSHORE FLOW**

MAX VEL. 0.5 m s^{-1}



Vertical excursion of pycno: 300 m (300 m / 550 m $\simeq \frac{1}{2}$)



Vertical excursion of pycno: 300 m (300 m / 550 m $\simeq rac{1}{2}$)

INTERNAL WAVE CHARACTERISTICS at Ormen Lange



Internal run-up (solibores) in Faeroe-Shetland Channel Hosegood & van Haren (2004)





Internal run-up (solibores) in Faeroe-Shetland Channel Hosegood & van Haren (2004)





Hosegood and van Haren's recordings;

- cause of run-up uncertain; not tidally driven
- leading edge velocity of 0.52 m s $^{-1}$ = $0.42c_{
 m ()}$
- c_0 internal linear long wave speed
- vertical excursions of wavy pycnocline 10-19 m
- period $\Delta t = 27 \min$
- $\Delta t c_0/h_2 \simeq 4.5$, nondimensional period
- phase speed of bolus: 0.24 m s⁻¹ = $0.19c_0$

Internal run-up (solibores) in Faeroe-Shetland Channel Hosegood & van Haren (2004)





Some studies on internal run-up

Cacchione and Southard (1974)

Kao, Pan and Renouard (1985)

Wallace and Wilkinson (1988)

Helfrich (1992)

Michallet and Ivey (1999)

Sveen, Guo, Davies and Grue (2002)

Klymak and Moum (2003)

Hosegood and van Haren (2004)

Study internal run-up experimentally



- short $5h_2$ and long $21.9h_2$ initial elevation/depression
- $a = \pm 0.5 h_2$
- small slope s=0.3/6.8=0.044
- FOV1, FOV2, FOV3
- scale 1:5000



1. quantify internal wave induced velocities

2. duration of onshore/offshore flow

3. internal run-up/gravity currents/boluses





Observations, short initial elevation

- FOV1: leading elevation
- dispersive wave train
- $u_{1,2}/c_0$ up to ± 0.5
- $\Delta t c_0 / h_2 = 20$



Observations, short initial elevation

- FOV1: leading elevation
- dispersive wave train
- $u_{1,2}/c_0$ up to ± 0.5
- $\Delta t c_0 / h_2 = 20$



- FOV2: (intersection) leading elevation
- duration of event: $\Delta t c_0 / h_2 = 20$
- u_1/c_0 up to 0.45 and down to -0.35 (lower layer)
- a bolus forms behind the bore; $\Delta t c_0/h_2 = 4.5$; about the same duration as measured by Hosegood & van Haren







- FOV1: leading undular bore
- duration of bore $\Delta t c_0/h_2 \sim 60$
- for the bore: u_1/c_0 up to 0.25, u_2/c_0 down to -0.4
- FOV2: onshore flow of dense water; velocity u_1/c_0 of leading front slightly less than 0.5
- duration of onshore flow $\Delta t c_0 / h_2 = 40$

Run-up, long initial elevation, $\Delta t c_0/h_2 = 7.8$



Position of front; $l/h_2 = 21.9$ (o), $l/h_2 = 5$ (x)



Bottom currents, internal waves - p.21/4

Position of front; $l/h_2 = 21.9$ (o), $l/h_2 = 5$ (x)



Bottom currents, internal waves - p.22/4



Bottom currents, internal waves - p.23/4

Position of front; $l/h_2 = 21.9$ (o), $l/h_2 = 5$ (x)











Bottom currents, internal waves - p.28/4

FOV2; gravity current



Gravity current; plot of $c_{front}/\sqrt{g'H}$; H thickness of tail



Gravity current; plot of $c_{front}/\sqrt{g'H}$; H thickness of tail



Gravity current; plot of $c_{front}/\sqrt{g'H}$; H thickness of tail







Long initial depression, leading waves



Onshore flow duration.

- $\Delta t c_0 / h_2 = 20$ for $l/h_2 = 5$ (short ini. elevation)
- $\Delta t c_0/h_2 = 40$ for $l/h_2 = 21.9$ (long ini. elevation)
- $\Delta t c_0 / h_2 = 212$ field observation at ORMEN LANGE

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.9$$
 for $l/h_2 = 5$ (short ini. elevation)

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.5$$
 for $l/h_2 = 21.9$ (long ini. elevation

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.9$$
 for $l/h_2 = 5$ (short ini. elevation)

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.5$$
 for $l/h_2 = 21.9$ (long ini. elevation)

How long should l be at Ormen Lange in order that $\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}}\sim 9?$

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.9$$
 for $l/h_2 = 5$ (short ini. elevation)

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.5$$
 for $l/h_2 = 21.9$ (long ini. elevation)

How long should l be at Ormen Lange in order that $\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} \sim 9$? Answer: \sim 300 km!

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.9$$
 for $l/h_2 = 5$ (short ini. elevation)

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 8.5$$
 for $l/h_2 = 21.9$ (long ini. elevation)

$$\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} = 9.1$$
 for $l/h_2 = 300$ km/550 m (OL)

Width of Norwegian Atlantic Current?



Bottom currents, internal waves - p.41/40

Width of Norwegian Atlantic Current?



Bottom currents, internal waves - p.42/40

Conclusions I

Experiments show nondimensional velocity up to $0.5c_0$

Current event in the field at Ormen Lange, velocity up to $0.37c_0$

Duration of event in laboratory and field: $\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} \simeq 9$

The onshore flow has a leading front velocity of $0.43c_0$

Hosegood & van Haren found leading front velocity $0.42c_0$

Onshore surge moves initially like a gravity current $c_{front}/\sqrt{g'H}$ =0.7 (here), 0.707 (Benjamin, 1968), 0.75 (Simpson & Britter, 1979) (*H* thickness of the tail) Maximal run-up: $s/h_2 = 15$ and 12 ($y/h_2 = 0.68$ and 0.54) for long/short initial elevation

Conclusion II

Initial depression results in a train of boluses moving up slope

Number of boluses in agrees with present investigations

- Period of bolus: $\Delta t c_0/h_2 \sim 4.5$; similar time as in Hosegood & van Haren's measurements in the Faeroe-Shetland Channel
- Bolus induced velocity, up to $0.35c_0$. Phase velocity of bolus observed by Hosegood & van Haren is $0.19c_0$; by Klymak & Moum is $0.25c_0$ (but this depends much on the phase of motion along the slope)

Initial depression gives small run-up ($s/h_2 = 6.2$; $y/h_2 = 0.28$)



MAX VEL. $0.37c_0$ DURATION $\frac{\Delta t c_0/h_2}{\sqrt{l/h_2}} \simeq 9$



Bottom currents, internal waves - p.45/4

