Propagation of Deformation Waves in the Piano Hammer Felt

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Outline

- Introduction
- Piano hammer properties
- Contact time and compression model of strike
- Wave model of hammer strike
- Summary

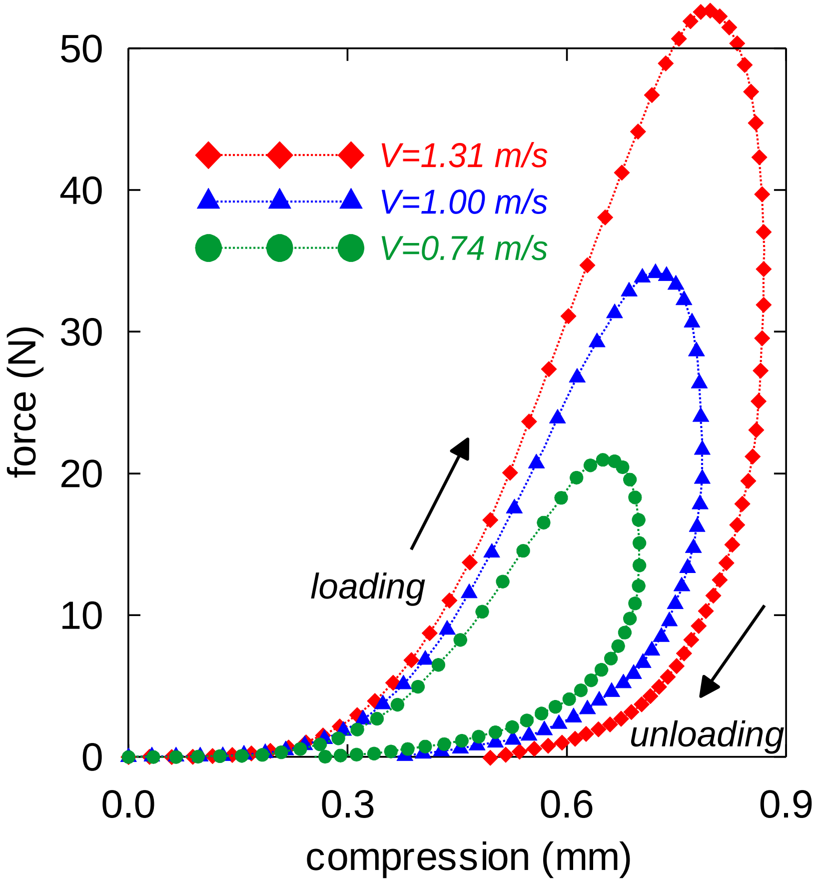
Piano Hammers

Piano hammer testing device

Experimental results

Piano hammer features

- The nonlinearity of the force-compression characteristics of the hammer
- The strong dependence of the slope of the loading curve on the hammer velocity
- The significant influence of hysteresis, i.e. the loading and unloading of the hammer felt are not alike



Hysteretic model of piano hammer

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right]$$

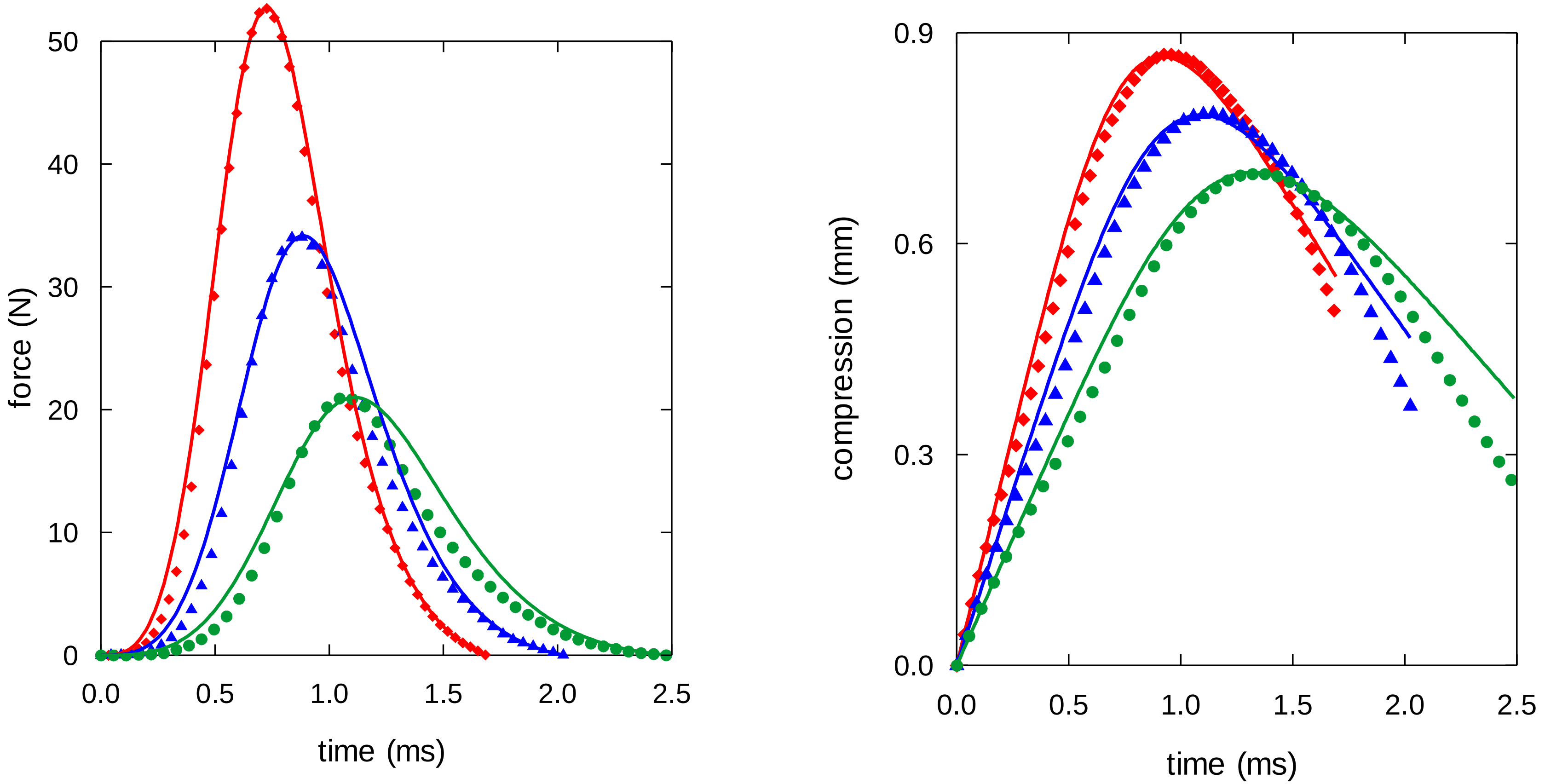
Equation of motion : $$m\frac{d^2u}{dt^2} + F(u) = 0$$

Initial conditions : $u(0) = 0\,;\ \left.\frac{du}{dt}\right|_{t=0} = V$

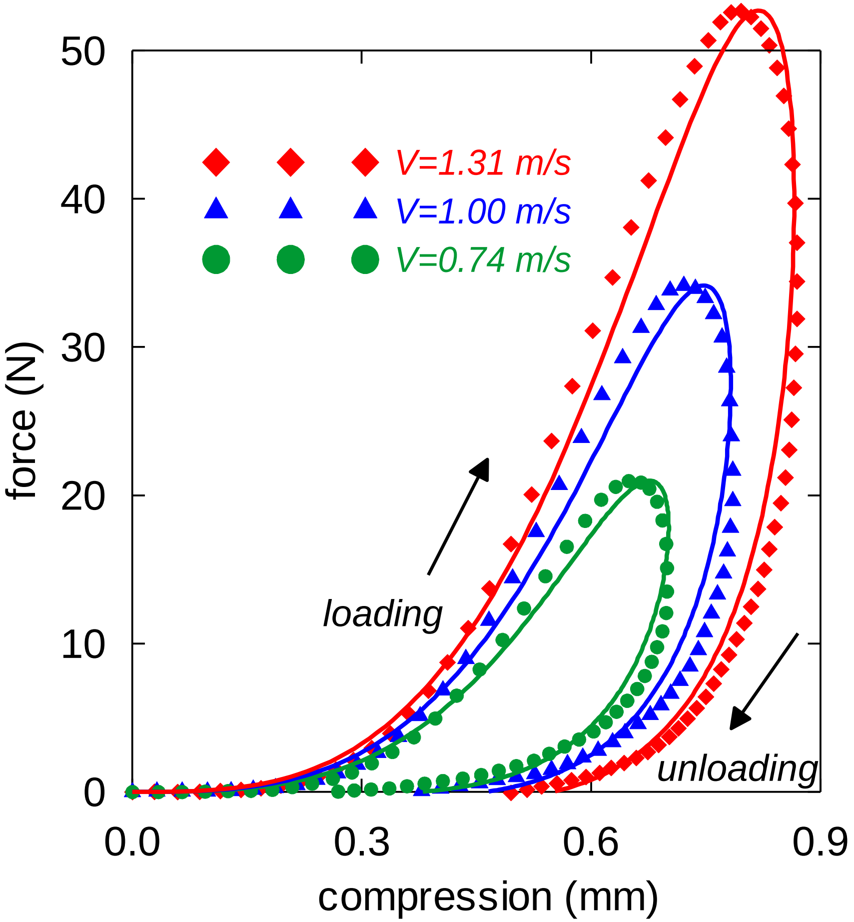
F_0 - instantaneous hammer stiffness; ε - hereditary amplitude

p - compliance nonlinearity exponent; τ - relaxation time

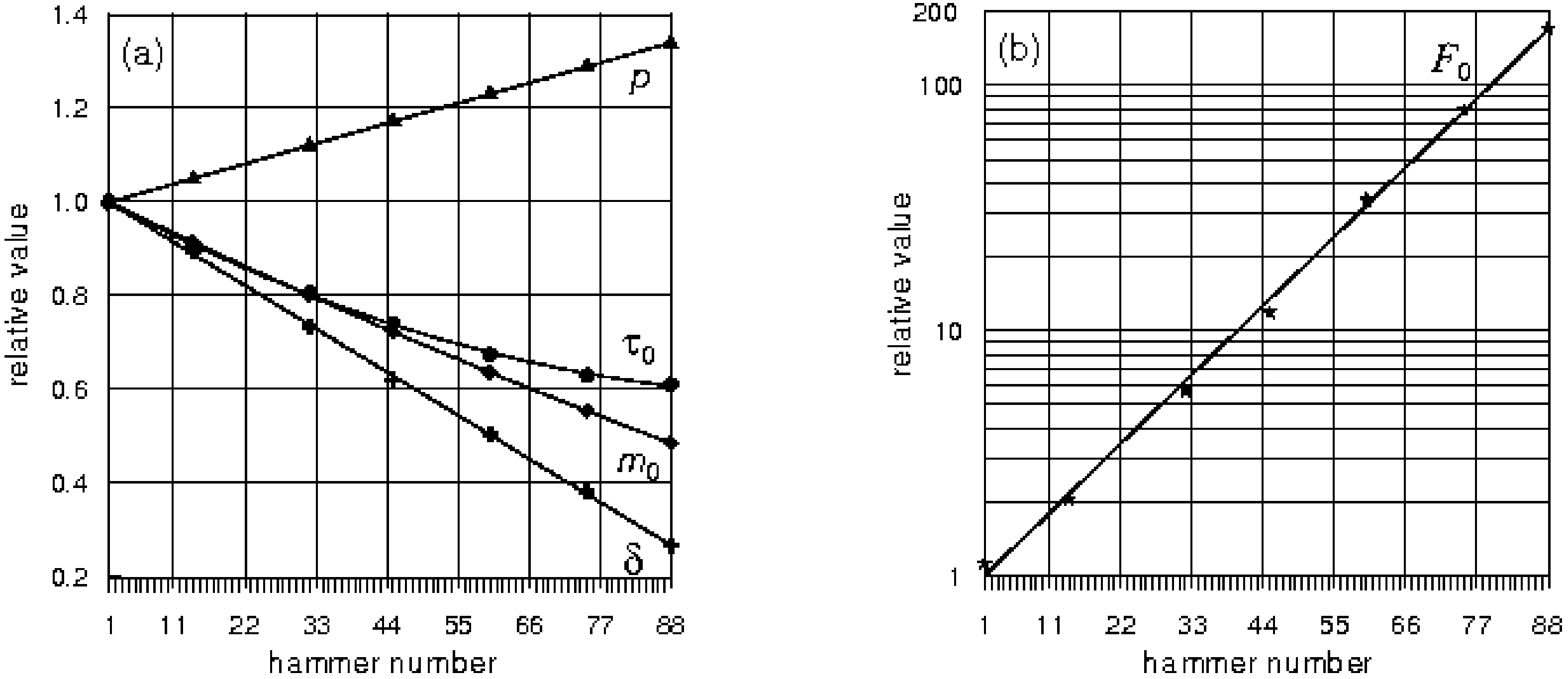
Force and compression histories for various hammer velocities



Force-compression characteristics for various hammer velocities



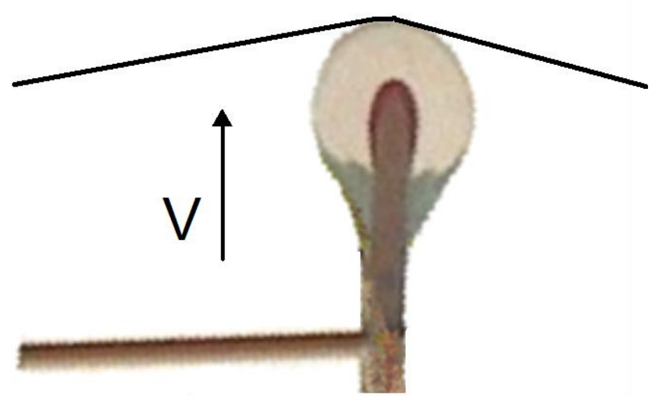
Relative variation in hammer parameters across the compass of the piano



Parameters of hammer n=1

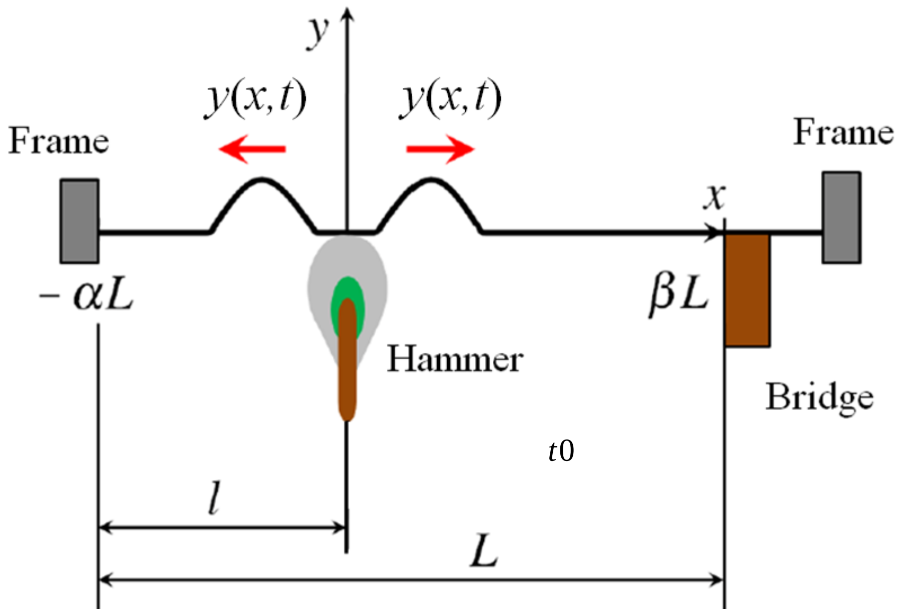
p = 3.72; τ_0 = 2.7 μs; m_0 = 11 g; $\delta = 1 - \varepsilon$ = 0.011; F_0 = 16440 N/mm^p

The contact duration t_0 between the hammer and the string, and what can cause the hammer to rebound?



$$A_n + jB_n = j\left(\frac{2\sin n\pi\alpha}{n\pi c\mu}\right)\int_{-\infty}^{t_0} F(t)e^{j\omega_n t}\,dt$$

Hammer-string interaction

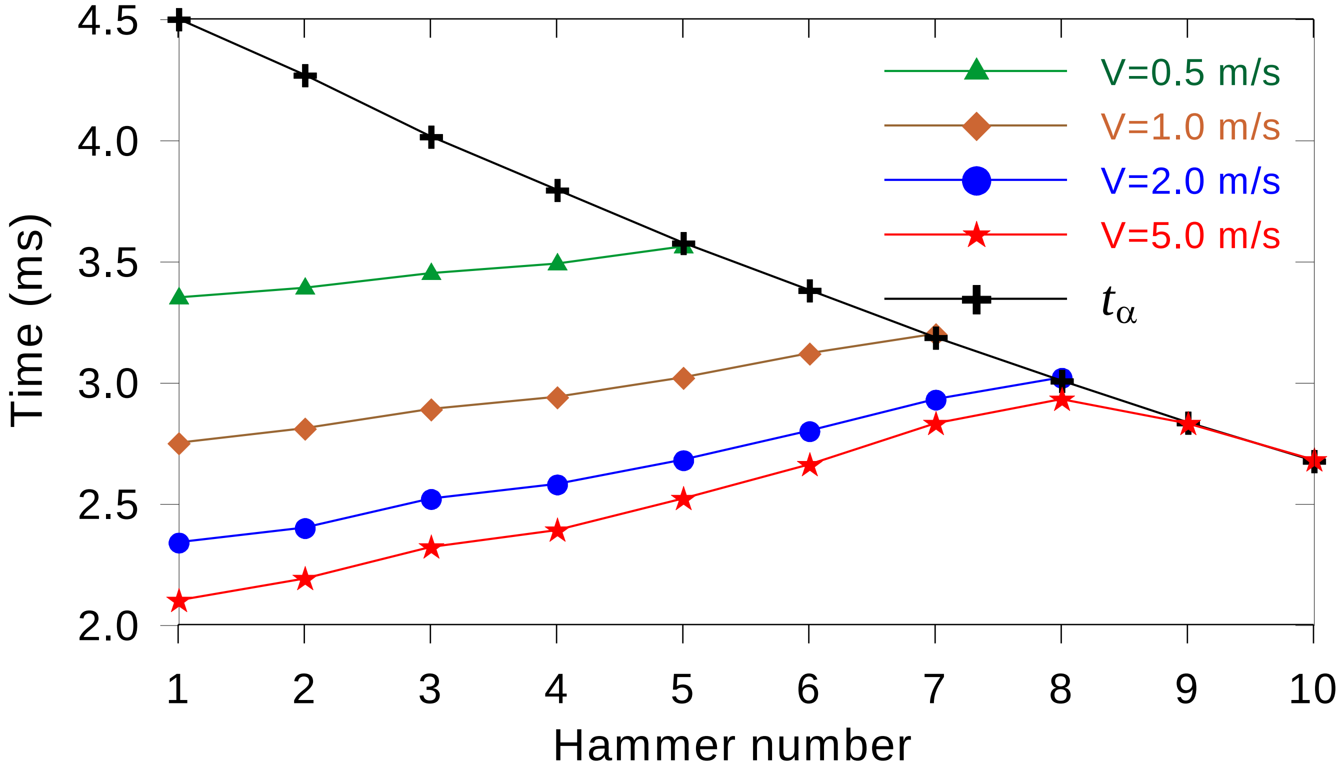


"When the hammer has less mass than the string, it will most likely be thrown clear of the string by the first reflected pulse."

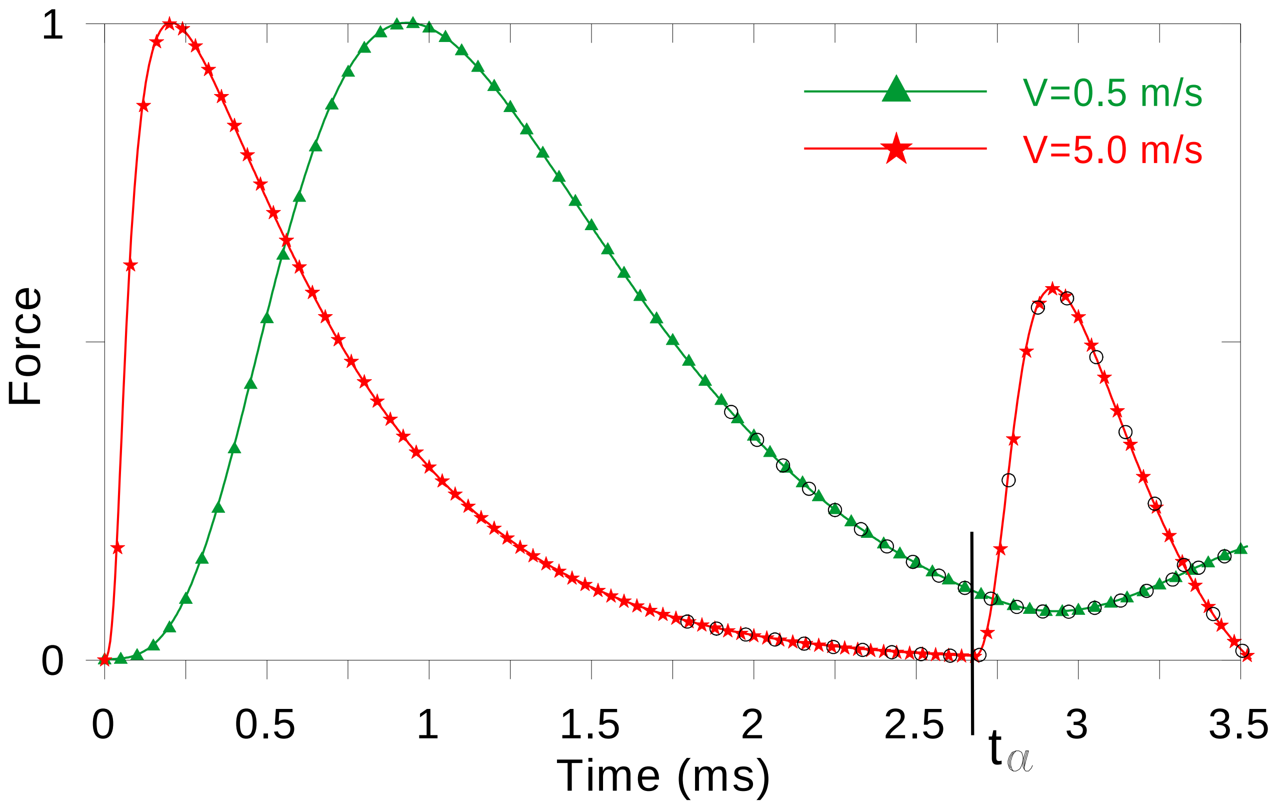
Rossing T. D. *The Science of Sound*

$$t_0 \geq t_\alpha = \frac{2\alpha L}{c}$$

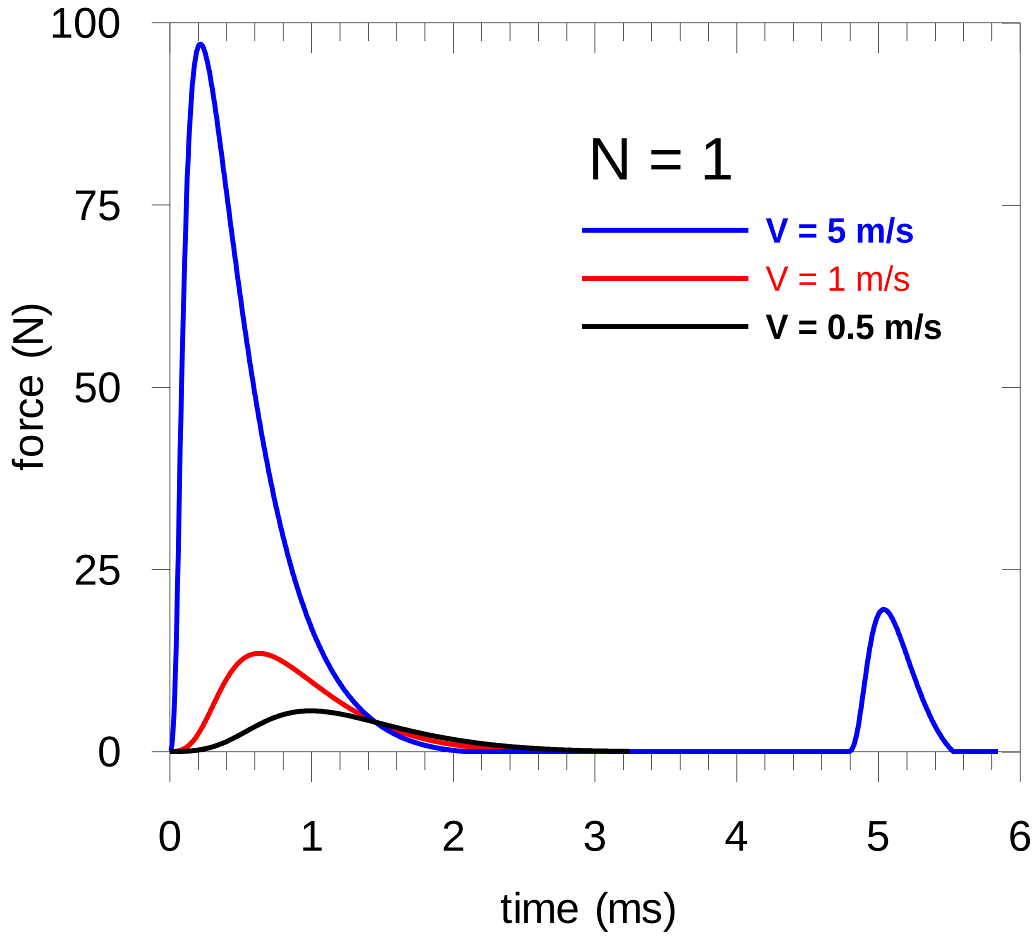
Contact times as functions of hammer number



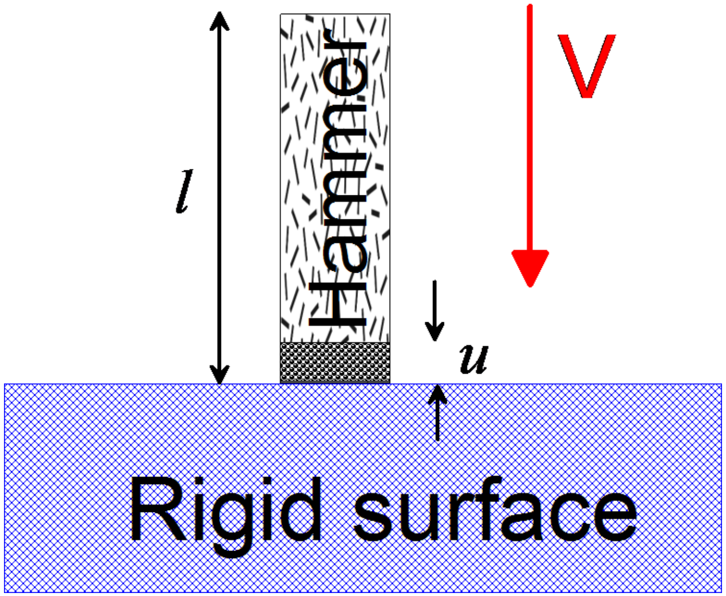
Normalized force histories computed for hammer N=10



Force histories computed for hammer N=1



Compression model of hammer strike



$$t_0 = \frac{\pi l}{c}\,; \qquad c = \sqrt{E/\rho}$$

Compression model of hammer strike

- In case of slow loading the nonlinear force is given by

$$F(u(t)) = F_0(1-\varepsilon)\, u^p(t) = Q_0\, u^p(t)$$

- During the strike the initial energy of the hammer is

$$\frac{mV^2}{2} = \frac{m}{2}\left(\frac{du}{dt}\right)^2 + \frac{Q_0}{p+1} u^{p+1}$$

- The maximum compression corresponds to the moment, when $du/dt = 0$

$$u_{\max} = \left(\frac{p+1}{2}\frac{m}{Q_0}V^2\right)^{\frac{1}{p+1}}$$

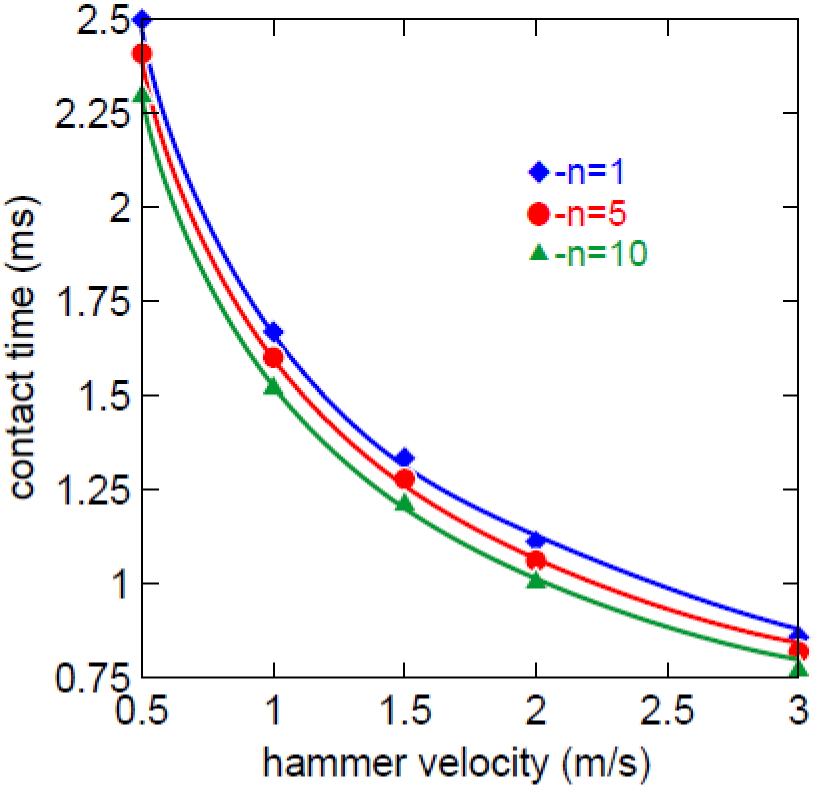
- The contact time

$$t_0 = \frac{2u_{\max}}{V}\int_0^1 \frac{dx}{\sqrt{1-x^{p+1}}} = \frac{2b\sqrt{\pi}}{V}\frac{\Gamma(1+a)}{\Gamma(1+b)}u_{\max}$$

- here $a = 1/(p+1)$, $b = a + ½$

$$u_{\max} = \left(\frac{p+1}{2}\frac{m}{Q_0}V^2\right)^{\frac{1}{p+1}}$$

Hysteretic and compression models comparison



Wave model of hammer strike

Hysteretic model of piano hammer

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right]$$

We shall consider the propagation of the one-dimensional transient longitudinal waves.
Equation of motion

$$\rho \ddot{U}(x,t) - \sigma'(x,t) = 0$$

Initial conditions

$$U(x,0) = 0,\ \dot{U}(x,0) = 0,\ \sigma(x,0) = 0$$

Constitutive equation

$$\sigma(x,t) = E\left\{[U'(x,t)]^p - \frac{\varepsilon}{\tau}\int_0^t [U'(x,\xi)]^p \exp\left(\frac{\xi - t}{\tau}\right)d\xi\right\}$$

$$U \Rightarrow U/l;\quad x \Rightarrow x/l;\quad t \Rightarrow t/\alpha$$

$$\alpha = \frac{\tau}{1-\varepsilon};\quad \delta = 1-\varepsilon = \frac{\tau}{\alpha};\quad l = \alpha\sqrt{\frac{\delta E}{\rho}} = \alpha c\sqrt{\delta}$$

$$[(U')^p]' + [(U')^p]'^{\bullet} - \mathrm{U}^{\bullet\bullet} - \delta \mathrm{U}^{\bullet\bullet\bullet} = 0$$

δ = 0.01 ; ρ = 0.42 g/cm^3 ; c = 25 m/c

α = 0.25 ms ; l = 6 mm;

E = E_s = 0.2 MPa ; E_d = E_s / δ = 20 MPa

Linear case: $p = 1$

$$U'' - \ddot{U} + \dot{U}'' - \delta \dddot{U} = 0$$

$$\omega^2 - k^2 + i\omega k^2 - i\delta\omega^3 = 0$$

$$k = k_1 + ik_2 = \omega\sqrt{\frac{1 - i\delta\omega}{1 - i\omega}}$$

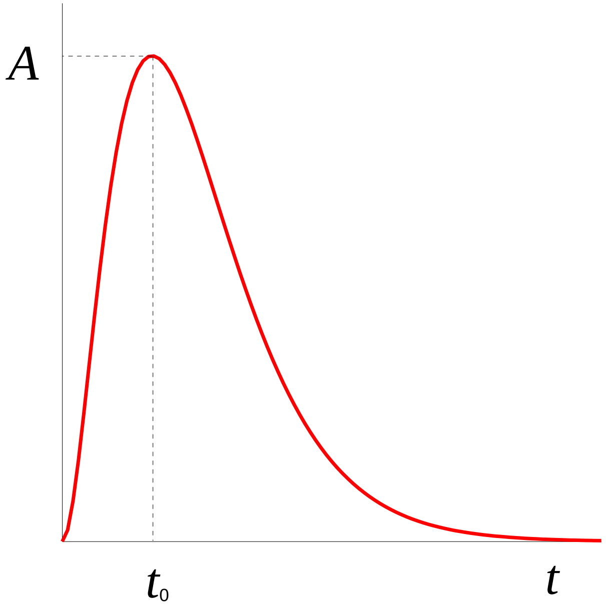
$$[(U')^p]' + [(U')^p]'^{\bullet} - \mathrm{U}^{\bullet\bullet} - \delta \mathrm{U}^{\bullet\bullet\bullet} = 0$$

Initial conditions

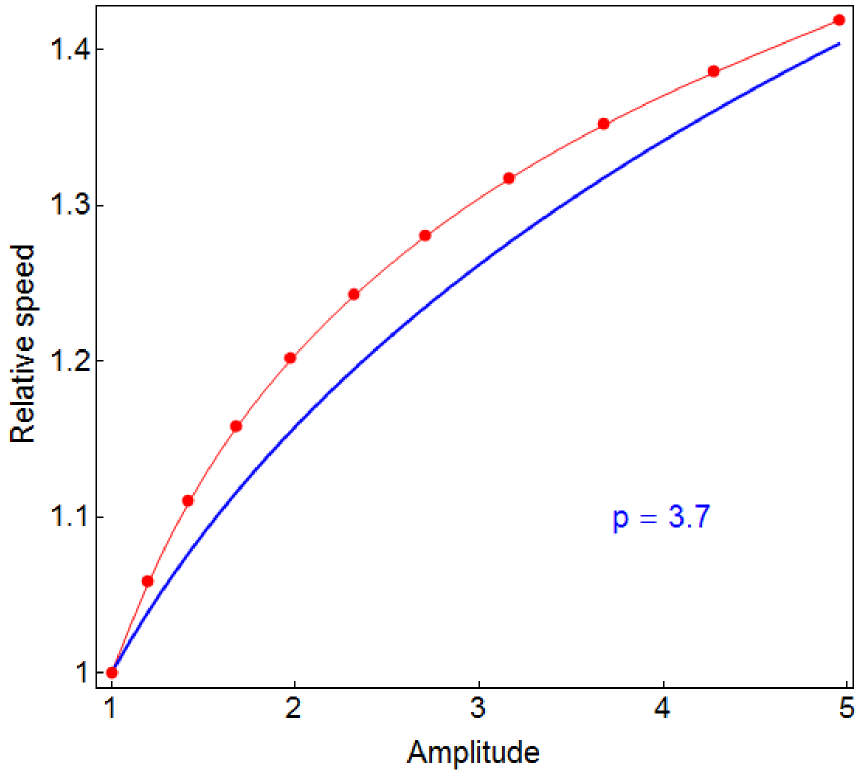
$$U(x,0) = 0,\ U^{\bullet}(x,0) = 0$$

Boundary condition

$$U(0,t) = A\left(\frac{t}{t_0}\right)^2 e^{2(1-t/t_0)}$$



Compression and wave models comparison



Summary

- Three models of the hammer strike are compared
- The contact time for bass hammers is correlated with the speed of compression wave generated by the impact
- The speed of a compression wave increases with the growth of its amplitude
- The compression wave speed is in interval 25 - 75 m/s