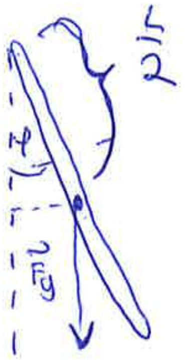
EuPhO 2017
Theory Solution Sheet   Problem Number: 1.   Student Code: CRO3

Considering that the lateral displacement of the rope is much smaller than the rope itself, its curvature while suspended will be ( ) miniscule so we can approximate this motion as a rigid body pendulum.



As the rope is uniform, its center of mass is placed in the middle, at distance $\frac{L}{2}$ from the ends of the rope. If it is placed under the angle φ from the central axis of the pendulum, the gravitational force creates a moment which starts the motion

I - moment of inertia $\rightarrow I = \frac{mL^2}{3}$ (m - mass of the rope).

$$I\ddot{\varphi} = -\frac{F_g L}{2}\sin\varphi \approx -\frac{F_g L}{2}\varphi \quad (\text{for small } \varphi)$$

$$F_g = mg \rightarrow \quad \frac{mL^2}{3}\ddot{\varphi} = -\frac{mgL}{2}\varphi$$

$$\frac{L}{3}\ddot{\varphi} = -\frac{g}{2}\varphi$$

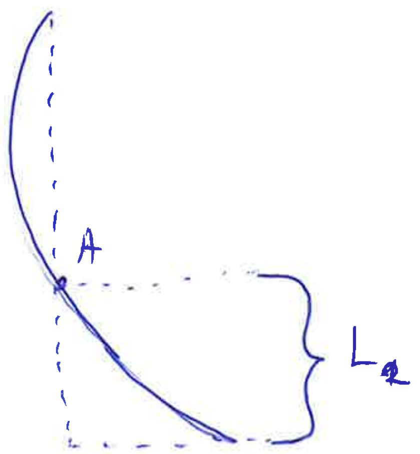
$$\ddot{\varphi} = -\frac{3g}{2L}\varphi$$

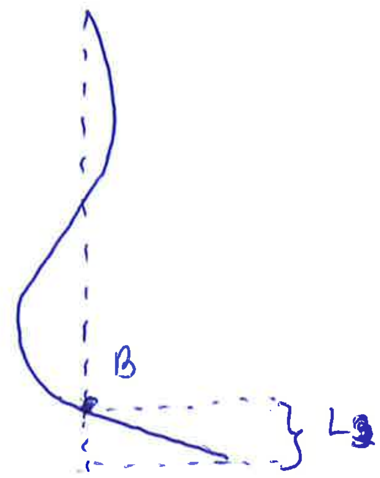
$$\ddot{\varphi} + \frac{3g}{2L}\varphi = 0$$

This is a differential equation which gives us a solution

$$\varphi = \varphi_0 \sin(\omega t), \text{ where } \omega_1 = \sqrt{\frac{3g}{2L}}$$

$$f = \frac{\omega}{2\pi} \rightarrow f_1 = \frac{\omega_1}{2\pi} \rightarrow \boxed{f_1 = \frac{1}{2\pi}\sqrt{\frac{3g}{2L}} = 0.61\,\text{Hz}}$$





To calculate f_2 and f_3 we have to notice that every part of the rope has to move at the same frequency, including the parts shown in the upper graphs. These parts can also be considered as rigid body pendulums just like the whole rope in the case for f_1. This is because the points A and B don't change their positions during the motions. So the same results are valid for them:

$$\omega_2 = \sqrt{\frac{3g}{2L_2}}, \quad \omega_3 = \sqrt{\frac{3g}{2L_3}}$$

We can read the lengths L_2 and L_3 from the given graphs.

$L_2 \approx 0.19\,m$, $L_3 = 0.08\,m$

$$f_2 = \sqrt{\frac{3g}{2L_2}} \cdot \frac{1}{2\pi} = f_1\sqrt{\frac{1}{0.19}} \qquad f_3 = \frac{1}{2\pi}\sqrt{\frac{3g}{2L_3}} = f_1\sqrt{\frac{1}{0.08}}$$

$\boxed{f_2 = 1.4\,Hz}$ $\boxed{f_3 = 2.158\,Hz}$

$$\boxed{f_1 : f_2 : f_3 = 0.61 : 1.4 : 2.158}$$