English version

1 Oscillating rope

A heavy uniform rope of length L is suspended vertically from the ceiling. The rope may oscillate around its equilibrium position with different natural frequencies, which will be denoted by f_i $(i = 1, 2, \ldots)$ in an ascending order. The figure below presents the shape of the rope in the first three natural vibrations, as obtained by a computer simulation. Note that the horizontal and vertical scales in the figures are not equal. You may assume that the actual lateral displacement of the rope is much smaller than its length (small-amplitude approximation).

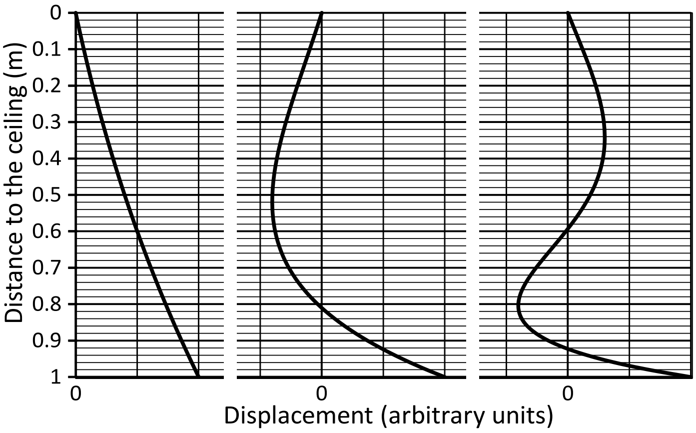

Figure: Shapes of the oscillating rope for the first three natural vibrations ($i = 1, 2, 3$ from left to right)

A) Develop a simplified model, which will allow you to estimate the frequency f_1 of the first (fundamental) vibration of the rope. Hence, calculate approximately f_1 for a rope of length $L = 1.0$ m. Assume that $g = 9.8\,\mathrm{m/s^2}$.

B) Take necessary readings from the figure to estimate the frequency ratio $f_1 : f_2 : f_3$.

2 Disk in gas

Consider a thin flat disk of mass M and face area S at initial temperature T_1 resting initially in weightlessness in a gas of mass density ρ at temperature T_0 $(T_1 = 1000T_0)$. One of the faces of the disk is covered with a thermally insulating layer, the other face has a very good thermal contact with the surrounding gas: gas molecules of mass m obtain the temperature of the disk during a single collision with the surface.

Estimate the initial acceleration a_0 and maximal speed $v_{\max}$ of the disk during its subsequent motion.

Assume the heat capacity of the disk to be on the of order Nk_B, where N is the number of atoms in it, and k_B is the Boltzmann constant, and molar masses of the gas and the disk's material to be of the same order. The mean free path length of molecules is much larger than the size of the disk. Neglect any edge effects occuring at the edge of the disk.

3 Superconducting mesh

Consider a mesh made from a flat superconducting sheet by drilling a dense grid of small holes into it. Initially the sheet is in a non-superconducting state, and a magnetic dipole of dipole moment m is at a distance a from the mesh pointing perpendicularly towards the mesh. Now the mesh is cooled so that it becomes superconducting. Next, the dipole is displaced perpendicularly to the surface of the mesh so that its new distance from the mesh is b. Find the force between the mesh and the dipole. The pitch of the grid of holes is much smaller than both a and b, and the linear size of the sheet is much larger than both a and b.