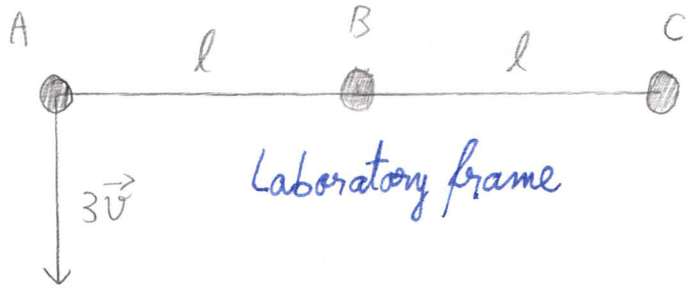
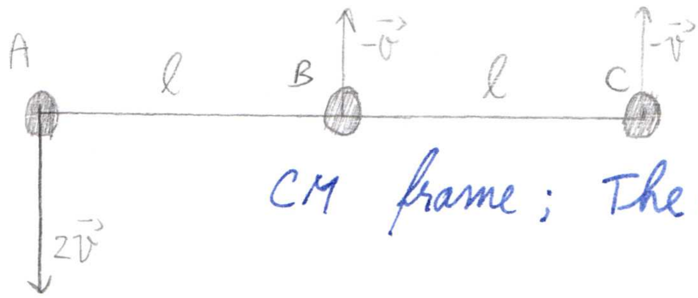
After the ball A is given its velocity, no external forces act on the system. Therefore, we will ~~work~~ work in the center-of-mass reference frame to simplify calculations.



Let the initial velocity of ball A be $3\vec{v}$. The total momentum is $3mv = m_{tot} \cdot v_{CM}$ where $m_{tot} = 3m$ is the total mass of the system and v_{CM} is the center of mass' velocity.

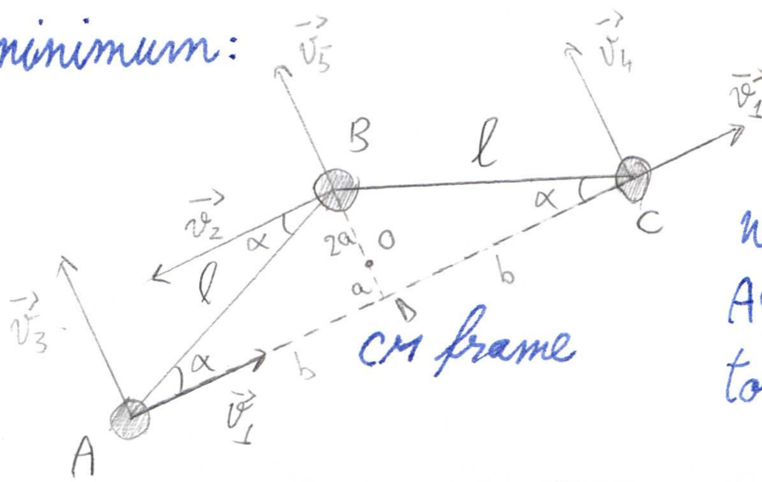
$3mv = 3mv_{CM} \Rightarrow v_{CM} = v$

The initial situation in the CM frame is:



CM frame ; The center of mass is exactly at ball B.

Let us consider the situation when the distance between A and C is minimum:



We have written, for each ball, the components of the velocity which are parallel to the line AC, and respectively perpendicular to the line AC.

If the distance AC is minimum, the relative velocity of A and C along the line joining them must be zero; let them both have velocities v_1 along that line.

Because we are in the CM frame, the total momentum along any axis must be zero:

$$m v_1 + m v_1 - \cancel{2}m v_2 = 0 \Rightarrow v_2 = 2 v_1$$

$$m v_3 + m v_4 + m v_5 = 0 \Rightarrow v_3 + v_4 + v_5 = 0$$

As the rods have fixed length, the distances AB and BC are constant; therefore A and B must have identical velocities along line AB: $v_1 \cos\alpha + v_3 \sin\alpha = v_5 \sin\alpha - v_2 \cos\alpha$

$$(v_1 + v_2)\cos\alpha + v_3 \sin\alpha = v_5 \sin\alpha \qquad (1)$$

$$3 v_1 \cos\alpha + v_3 \sin\alpha = v_5 \sin\alpha$$

Analog for BC:

$$v_4 \sin\alpha - v_1 \cos\alpha = v_2 \cos\alpha + v_5 \sin\alpha$$

$$-(v_1 + v_2)\cos\alpha + v_4 \sin\alpha = v_5 \sin\alpha \qquad (2)$$

Adding (1) and (2):

$$(v_3 + v_4)\sin\alpha = 2 v_5 \sin\alpha$$

As $\alpha \in (0, \frac{\pi}{2})$, $\sin\alpha \neq 0$ and we can divide by it:

$$v_3 + v_4 = 2 v_5$$

But $v_3 + v_4 + v_5 = 3 v_5 = 0 \Rightarrow v_5 = 0$ and $v_3 = -v_4$

We have $3v_1 \cos\alpha - v_4 \sin\alpha = 0$

$$3v_1 \cos\alpha = v_4 \sin\alpha \qquad (3)$$

As no energy is dissipated by friction or turned to potential energy, kinetic energy is conserved:

$$\frac{m(2v)^2}{2} + \frac{mv^2}{2} + \frac{mv^2}{2} = \frac{m(v_3^2+v_1^2)}{2} + \frac{m(v_2^2+v_5^2)}{2} + \frac{m(v_1^2+v_4^2)}{2} \Big/ \cdot \frac{2}{m}$$

$$4v^2+v^2+v^2 = v_3^2+v_1^2+v_2^2+v_5^2+v_1^2+v_4^2$$

As $v_5=0$, $v_2=2v_1$ and $v_3=-v_4$:

$$6v^2 = v_4^2+2v_1^2+4v_1^2+v_4^2$$

$$6v^2 = 6v_1^2+2v_4^2$$

$$3v_1^2+v_4^2 = 3v^2 \qquad (4)$$

As no external forces act upon the system, the total angular momentum around the center of mass is conserved; in the sketch, the CM is point O, situated on the line BΔ ($B\Delta \perp AC$), with $BO = 2O\Delta$ ($\equiv 2a$) by simple geometry arguments. Let us make the notation $A\Delta = C\Delta \equiv b$.

The initial angular momentum:

$$L = ml\cdot 2v + mlv = 3mlv$$

In the discussed situation:

$$L = mv_1a - mv_3b + mv_1a + mv_4b + mv_2\cdot 2a$$

$L = 6 m v_1 a + 2 m v_4 b$

We see that $3a = l \sin\alpha$ and $b = l \cos\alpha$

$L = 2 m v_1 l \sin\alpha + 2 m v_4 l \cos\alpha$

$2 m l v_1 \sin\alpha + 2 m l v_4 \cos\alpha = 3 m l v$

$2 v_1 \sin\alpha + 2 v_4 \cos\alpha = 3 v \qquad (5)$

Therefore, equations (3), (4) and (5) form the system:

$$\begin{cases} 3 v_1 \cos\alpha = v_4 \sin\alpha \\ 3 v_1^2 + v_4^2 = 3 v^2 \\ 2 v_1 \sin\alpha + 2 v_4 \cos\alpha = 3 v \end{cases}$$

We will try to find α or a trigonometric function of it.

$$v_1 = \frac{1}{3} v_4 \frac{\sin\alpha}{\cos\alpha}$$

$$2 \cdot \frac{1}{3} v_4 \frac{\sin\alpha}{\cos\alpha} \cdot \sin\alpha + 2 v_4 \cos\alpha = 3 v$$

$$\left(\frac{2}{3} \cdot \frac{\sin^2\alpha}{\cos\alpha} + 2\cos\alpha\right) v_4 = 3 v \Rightarrow \frac{v}{v_4} = \frac{1}{3}\left(\frac{2}{3} \frac{\sin^2\alpha}{\cos\alpha} + 2\cos\alpha\right)$$

$$3 \cdot \left(\frac{1}{3} v_4 \frac{\sin\alpha}{\cos\alpha}\right)^2 + v_4^2 = 3 v^2$$

$$\frac{1}{3} v_4^2 \frac{\sin^2\alpha}{\cos^2\alpha} + v_4^2 = 3 v^2 \Rightarrow \left(\frac{v}{v_4}\right)^2 = \frac{1}{3}\left(\frac{1}{3} \frac{\sin^2\alpha}{\cos^2\alpha} + 1\right)$$

But $\left(\frac{v}{v_4}\right)^2 = \frac{1}{9}\left(\frac{4}{9} \frac{\sin^4\alpha}{\cos^2\alpha} + 4\cos^2\alpha + \frac{8}{3}\sin^2\alpha\right)$

$$\frac{4}{9}\cdot\frac{\sin^4\alpha}{\cos^2\alpha}+4\cos^2\alpha+\frac{8}{3}\sin^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}+3$$

We note $\cos^2\alpha \equiv t$ and therefore $\sin^2\alpha = 1-\cos^2\alpha = 1-t$

$$\frac{4}{9}\cdot\frac{(1-t)^2}{t}+4t+\frac{8}{3}(1-t)=\frac{1-t}{t}+3 \quad /\cdot t$$

$$\frac{4}{9}\left(t^2-2t+1\right)+4t^2+\frac{8}{3}\left(t-t^2\right)=1-t+3t \quad /\cdot 9$$

$$4\left(t^2-2t+1\right)+36t^2+24\left(t-t^2\right)=9+18t$$

$$4t^2-8t+4+36t^2+24t-24t^2=18t+9$$

$$16t^2-2t-5=0 \qquad \Delta=4+320=324=18^2$$

$$t_{1/2}=\frac{2\pm18}{32}$$

As $t=\cos^2\alpha$, we need $t>0$

$$t=\frac{20}{32}=\frac{5}{8}$$

$$\cos^2\alpha=\frac{5}{8} \Rightarrow \cos\alpha=\frac{1}{2}\sqrt{\frac{5}{2}}$$

The distance $d=2b=2l\cos\alpha=l\sqrt{\frac{5}{2}}$

Observation: We imposed the condition that the time derivative of the distance AC is zero; however, this does not guarantee that the distance is minimum (it could be a maximum, theoretically). The reason why it is a minimum is because we started at the maximum possible value under the given constraints ($2l$) and as the function is bounded and continuous, we must have a minimum. We only found one value of α where the derivative is zero, so that must be our desired solution.