EuPhO 2019 Theory 3 Marking Scheme Timestamp: June 2nd, 02:20

Criteria	Max	Points
Realizing that the envelope is parabola	1.0	
Realizing that focus point of parabola is origin (or giving correct algebraic equation for the envelope)	1.0	
Realizing that each trajectory must touch envelope	1.0	
Realizing that touching point is at $y \geq 0$	1.0	
Realizing that at least one point of curve must lie on envelope	1.0	
Option A:		
Realizing that at this point water curve is tangent to envelope	1.0	
Understanding geometric property at this point from reflective behaviour	1.0	
Option B:		
Realizing that the point at the envelope has the largest value of $y + \sqrt{x^2 + y^2}$	2.0	
Finding the position of the point P at the envelope with reasonable accuracy	1.0	
Finding height of topmost point of parabola	1.0	
Determining velocity as $v = y_P + \sqrt{x_P^2 + y_P^2} \approx 12\,\mathrm{m\,s^{-1}}$	1.0	
Total	**10.0**	

Alternative approach	Max	Points
Find lower bound $v > \sqrt{g \cdot 11.5\,\mathrm{m}}$	2.0	
Alternative lower bound $v > \sqrt{2g \cdot 4.9\,\mathrm{m}}$	1.0	
Find upper bound $v < \sqrt{4g \cdot 4.9\,\mathrm{m}}$	2.0	
Improving lower bound correctly by using $y = 2.6\,\mathrm{m}$ at $x = 11.5\,\mathrm{m}$	1.0	
Improving upper bound in a similar way	1.0	
Considering different points to show that $v \geq 11.8\,\mathrm{m/s}$	1.0	
Considering different points to show that $v \geq 11.9\,\mathrm{m/s}$	1.0	
Considering different points to show that $v \leq 12.2\,\mathrm{m/s}$	1.0	
Considering different points to show that $v \leq 12.1\,\mathrm{m/s}$	1.0	
Total	**10.0**	

If contestant considers envelope of trajectories, the first scheme is applied. If he/she obtains a formula for the minimal speed to reach the given point as $v = \sqrt{g(y + \sqrt{x^2 + y^2})}$ without mentioning the envelope, and finds a maximal value of this expression (maximized over the water curve), the second alternative scheme is applied. In that case, this expression provides only a lower bound for v and upper bound would be also needed. If the upper bound is not found, but the obtained result (v around $12.0\,\mathrm{m/s}$) is presented as if an exact answer, the contestant is penalized with 0.5 pts for not realizing that only a lower bound has been found.