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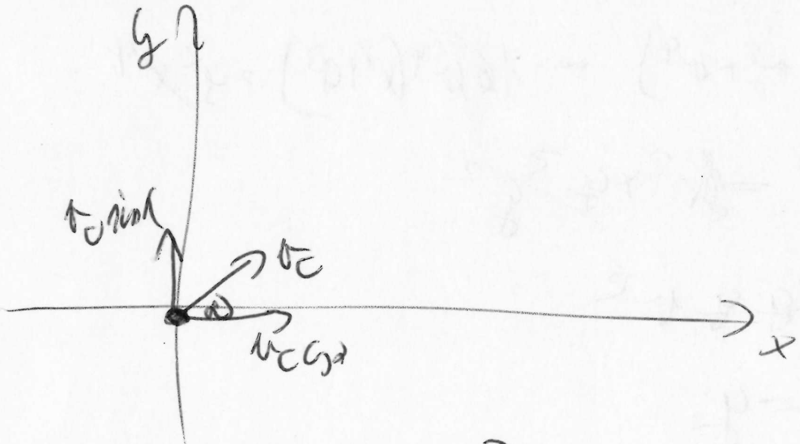
First part: The ballistic problem

What region of space can be hit by a particle which has velocity v?

I'll show that this is a parabola which's equation is: $y = \frac{v^2}{2g} - \frac{g}{2v^2}x^2$

Let's start with the equations of motion



$$\left.\begin{aligned} y(t) &= v_0 \sin\alpha\, t - \frac{g}{2}t^2 \\ x(t) &= v_0 \cos\alpha\, t \end{aligned}\right\} y = \frac{x \sin\alpha}{\cos\alpha} - \frac{g}{2v^2\cos^2\alpha}x^2$$

$$y = \frac{2v^2 x \sin\alpha \cos\alpha - g x^2}{2v^2\cos^2\alpha}$$

$$2v^2\cos^2\alpha\, y = 2v^2 x \cos\alpha \sqrt{1-\cos^2\alpha} - gx^2$$

$$\left(2v^2\cos^2\alpha\, y + gx^2\right)^2 = 4v^4x_0^2\cos^2\alpha - 4v^4x_0^2\cos^4\alpha$$

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$$4v^4 y^2 \cos^4\alpha + 4gx^2v^2 y \cos^2\alpha + g^2x^4 = 4v^4x^2\cos^2\alpha - 4v^4x^2\cos^4\alpha$$

$$4v^4(x^2+g^2)\cos^4\alpha + 4v^2(gx^2y - v^2x^2)\cos^2\alpha + g^2x^4 = 0$$

$$4v^4(x^2+y^2)\cos^4\alpha + 4v^2x^2(gy - v^2)\cos^2\alpha + g^2x^4 = 0$$

$P(x;y)$ is a point in the boundary if and only if only 1 projectile can reach it $\Longleftrightarrow$ Discriminant vanishes

$$D = b^2 - 4ac = 0$$

$$16v^4\cancel{x^4}(g^2y^2 - 2gyv^2 + v^4) - \cancel{16v^4}(x^2+y^2)\cdot g^2\cancel{x^4}$$

$$\cancel{g^2y^2} - 2gy\,v^2 + v^4 = g^2x^2 + \cancel{y^2g^2}$$

$$v^4 - g^2x^2 = 2gyv^2$$

$$\frac{v^2}{2g} - \frac{g}{2v^2}x^2 = y$$

So the boundary is a parabole whith ~~focus~~

$$y_{(x)} = \frac{v^2}{2g} - \frac{g}{2v^2}x^2$$

$$y - \frac{v^2}{2g} = -\frac{1}{2\cdot\frac{v^2}{g}}x^2$$

From the equation of a parabol one could see that it's parameter is $\frac{v^2}{g}$

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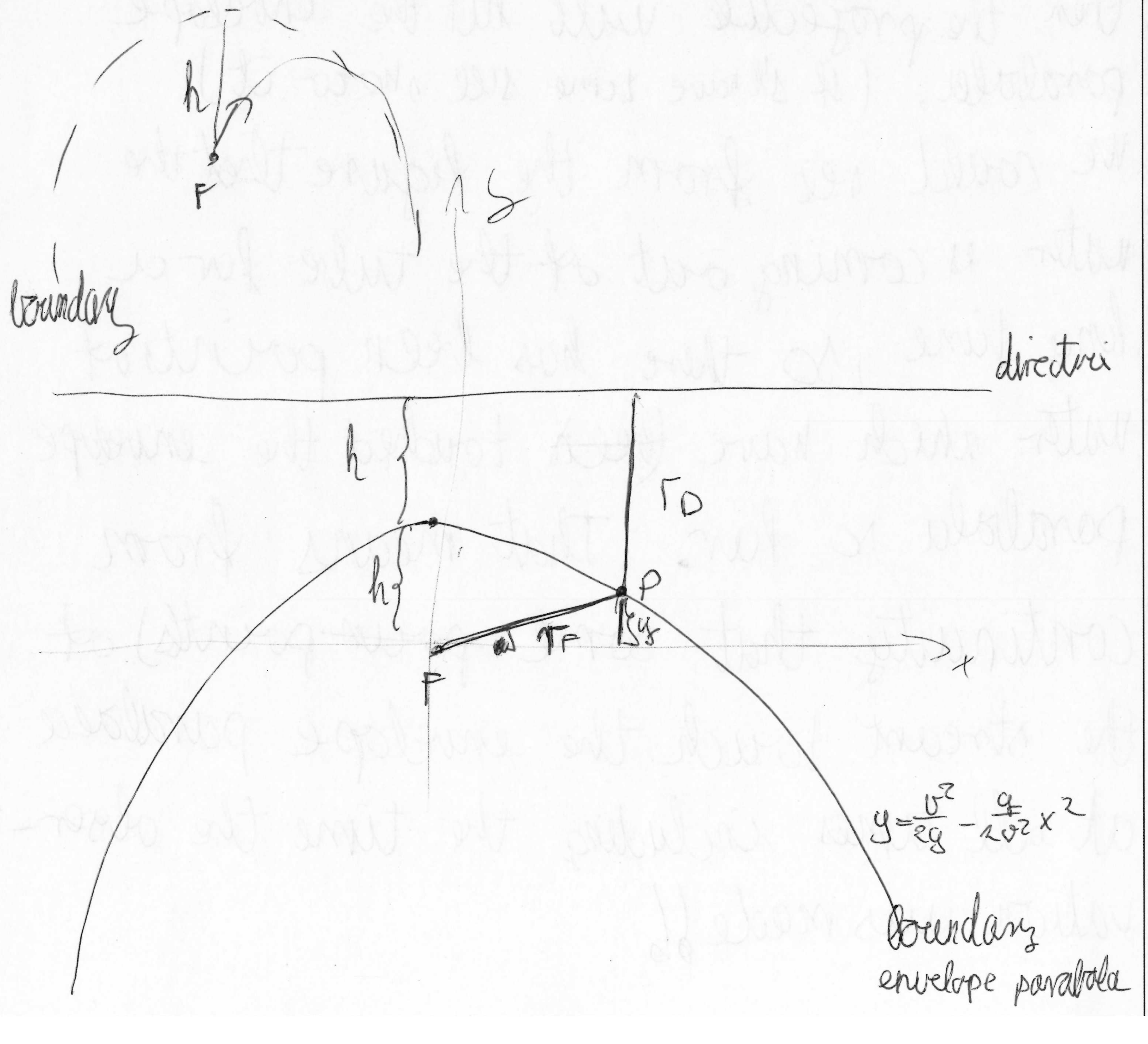
By symmetry the directix („vezéregyenes") of the parabol is horizontal $\Rightarrow$ From this one could see that the focus of the parabola is the launching point, since $h = \frac{v^2}{2g}$ and $p = 2h = \frac{v^2}{g}$



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If we know one point in the boundary using the definition: $r_F = r_D$ we could find „h" hence „v"!!

Part 2.

One could show that if $45° \leq \alpha \leq 90°$ then the projectile will hit the envelope parabola. (If I'll have time I'll show it)

We could see from the figure that the water is coming out of the tube for a long time, so there has been points of water which have ~~been~~ touched the envelope parabola so far. That means from continuity that some ~~poun~~ point(s) of the stream touch the envelope parabola at all times including the time the observation was made!!

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This is because if one particle touches the envelope parabola then at the next instant another one does too since the water stream is continous. / The time difference to reach the envelope parabola between two very close value of „h" is also very close – infinitely close indeed. /

That means we only have to find the point /points that touches the envelope parabola.

~~For that reason we want to find „r_F" max, the maximal distance of the tracetory from the launching point. / Strictly speaking we want to find „$2h$" max = „$r_F + y$" max see figure but it doesn't really makes much of a difference)~~

For that reason we want to find

$(2h)_{max} = (r_D + y)_{max} = (r_F + y)_{max}$ see figure. Let $(r_F + y)_P = S(P)$

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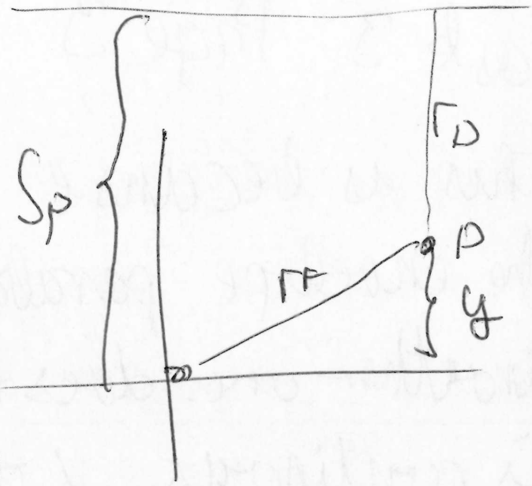

I marked some points on the diagram with they $S_{(P)}$ value. $/S_{(P)} = r_F(P) + y_{(P)}/$

S_{max} will correspond to the touching point

The biggest "S" I could find was for the point "P" $(10{,}75 \,;\, 3{,}4) \Rightarrow S_P = 14{,}07\,m$

~~$(= r_D + y = r_D + 3{,}4 \Rightarrow r_D =)$~~

$S_P = 2h \Rightarrow h = 7{,}337\ m$

$h = \frac{v^2}{2g} \Rightarrow \sqrt{2gh} = v$

$\underline{v} = \sqrt{2 \cdot 9{,}8\frac{m}{s^2} \cdot 7{,}337\,m} = \underline{\underline{11{,}99 \approx 12\,m/s}}$