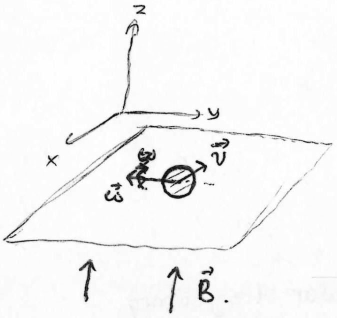
Student: IDN-S4

Sheet: T-002

Side: A

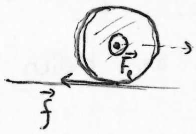
Problem 2



* No slip condition :

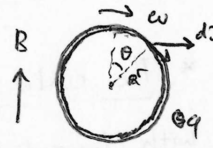
$$\vec{v} + \vec{\omega} \times R\hat{z} = 0 \quad \dots (1)$$

* Force diagram :



$$\vec{F}_e = Q \cdot \vec{v} \times B\hat{z}$$

* Determination of torque ~~from~~ to the ball from the magnetic field.

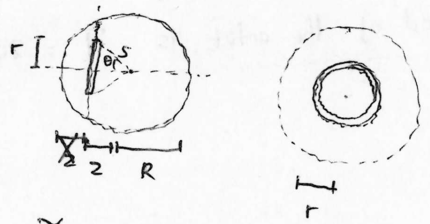


$$d\tau = dq \cdot \omega r B \cos\theta \cdot r\cos\theta$$

$$dq = \frac{q}{2\pi} d\theta \qquad \Rightarrow \tau = \frac{1}{2\pi} \cdot q\omega r^2 B \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$\tau = \frac{q\omega r^2 B}{2} \quad \text{(for ring charge)}$$

⇒ for spherical ball :



$$z = s\cos\theta$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$\tau \to d\tau$; ~~r → s sin θ~~ ; $q \to \rho \cdot dV$

$$dV = 2\pi r \cdot dz$$

$$\Rightarrow \tau = \frac{1}{2}\rho\omega B \int_{z=-R}^{z=+R}\int_{r=0}^{r=\sqrt{R^2-z^2}} 2\pi r^3 \, dr \, dz$$

$$\tau = \frac{\pi}{4}\rho\omega B\left(2R^5 + \frac{2}{5}R^5 - \frac{4}{3}R^5\right)$$

$$= \frac{1}{5}Q\omega R^2 B. \qquad \Rightarrow \vec{\tau} = \frac{1}{5}QR^2 \cdot \vec{\omega} \times B\hat{z}$$

⇒ Equations of motion of the ball :

$$m \cdot \frac{d\vec{v}}{dt} = Q \cdot (\vec{v} \times B\hat{z}) + \vec{f} \quad \dots (2)$$

$$\frac{2}{5}mR^2 \frac{d\vec{\omega}}{dt} = -R\hat{z} \times \vec{f} + \frac{1}{5}QR^2 \cdot \vec{\omega} \times B\hat{z} \quad \dots (3)$$

From eq. (1) :
$$\hat{z} \times \vec{v} = R\left(\hat{z} \times (\vec{\omega} \times \hat{z})\right)$$
$$= \vec{\omega} R$$
$$\vec{\omega} = \frac{1}{R}(\hat{z} \times \vec{v})$$

From eq. (2) & (3) :

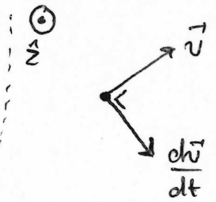
$$\frac{2}{5}mR^2 \frac{d\vec{\omega}}{dt} = -mR\hat{z} \times \frac{d\vec{v}}{dt} + QBR\vec{v} + \frac{1}{5}QR^2 B \cdot \vec{\omega} \times \hat{z}$$

$$\Rightarrow \frac{7}{5}mR\left(\hat{z} \times \frac{d\vec{v}}{dt}\right) = \frac{6}{5}QBR\vec{v} \quad \dots (4)$$

* Acceleration of the ball is perpendicular to the velocity of the ball.

* Simplify eq. (4) :

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{6QB}{7m} \cdot (\vec{v} \times \hat{z})$$



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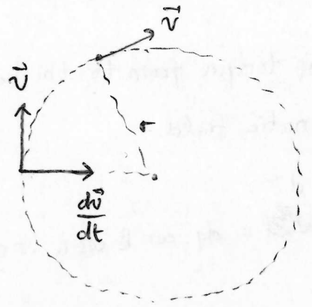
Problem 2 (cont.)



* Magnitude of $\vec{v}$: $|\vec{v}| = v_0$.

~~* The ball will~~

* The center of the ball will follow a circular trajectory with tangential velocity v_0 and radius r, where r is :

$$\frac{v_0^2}{r} = \frac{6QBv_0}{7m}$$

$$r = \frac{7mv_0}{6QB}.$$

* The period of the orbit is $T = 2\pi \cdot \frac{7m}{6QB}$.