Problem 2. | Solution page #1

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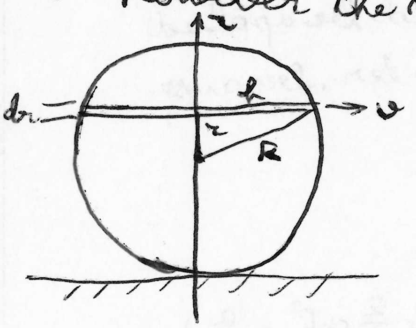
- When the ball is rolling it has kinetic energy K. Since there is no slipping $K = \text{const.}$ and the velocity of the ball is also $v_0 = \text{const.}$ ($\vec{v}_0 \neq \text{const.}$)

$$K = \frac{mv_0^2}{2} + \frac{I\omega^2}{2} = \frac{mv_0^2}{2} + \frac{\cancel{2}}{5} mR^2 \cdot \frac{1}{\cancel{2}} \cdot \frac{v_0^2}{\cancel{R^2}} = \left(\frac{1}{2}+\frac{1}{5}\right) m v_0^2 = \frac{7}{10} m v_0^2 .$$

- The average velocity of all the points of the ball is $\vec{v}_0$, the velocity of the center, so the total force on the ball created by the magnetic field is $\vec{F} = q\,\vec{v}_0 \times \vec{B}$, $|\vec{F}| = F$

$$F = q v_0 B \quad \text{(just as for a point-like particle)}$$

- However the magnetic field creates a torque τ on the ball.



all points that are a distance r above the center move with the same horizontal velocity $v = v_0 + v_0 \frac{r}{R} = v_0\left(1+\frac{r}{R}\right)$, and so have the same Ampere's force acting on them.

$$dF = vB\,dq \quad , \; dq = q\cdot\frac{dV}{V} = q\cdot\frac{\pi h^2 \cdot dr}{\frac{4}{3}\pi R^3} = \frac{3}{4} q \frac{h^2\,dr}{R^3} = \left| h^2 = R^2 - r^2 \right| =$$

$$= \frac{3}{4} q \frac{R^2 - r^2}{R^2} \cdot \frac{dr}{R}$$

Denote $\frac{r}{R} = x$: $\quad dq = q\cdot\frac{3}{4}(1-x^2)\,dx \quad , \; v = v_0(1+x)$

$$dF = v_0(1+x)B \cdot q \cdot \frac{3}{4}(1-x^2)\,dx = q v_0 B \cdot \frac{3}{4}(1+x)(1-x^2)\,dx$$

$$d\tau = dF \cdot r = q v_0 B \cdot \frac{3}{4} R\, x(1+x)(1-x^2)\,dx$$

$$\tau = \frac{3}{4} q v_0 B R \int_{-1}^{1} x(1+x)(1-x^2)\,dx \quad \text{(integrate from } r=-R \text{ to } r=R)$$

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$$\int_{-1}^{1} x(x+1)(1-x^2)\,dx = \int_{-1}^{1}(1+x-x^2-x^3)x\,dx = \int_{-1}^{1}(x+x^2-x^3-x^4)\,dx =$$

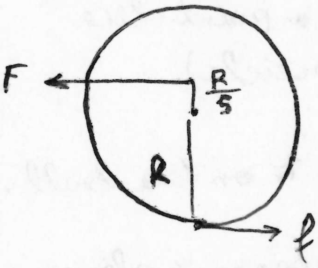
$$= \left| \frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}\right|_{-1}^{1} = \left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{5}\right)-\left(\frac{1}{2}-\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\right)=$$

$$= \cancel{\frac{1}{2}}+\frac{1}{3}-\cancel{\frac{1}{4}}-\frac{1}{5}-\cancel{\frac{1}{2}}+\frac{1}{3}+\cancel{\frac{1}{4}}-\frac{1}{5} = 2\left(\frac{1}{3}-\frac{1}{5}\right)=2\cdot\frac{5-3}{3\cdot 5}=\frac{4}{15}$$

So $\tau = \frac{\cancel{3}}{\cancel{4}} q v_0 B R \cdot \frac{\cancel{4}}{\cancel{15}} = \frac{1}{5} q v_0 B R$.

- In order for the ball to not slip there has to be a certain torque on it, which changes it's direction of rolling.

Let's loo at the ball ~~form~~ from the front:



$\odot \vec{v}_0$

the only other ~~force~~ torque on the ball is created by friction force f.
(τ is such as if F would be applied at $\frac{R}{5}$ above the center, because $\tau = \frac{1}{5} q v_0 B R = \frac{R}{5}\cdot F$)

$$\begin{cases} ma = F - f \\ \tau_{tot} = J\varepsilon = \tau + fR = \frac{R}{5}F + fR \end{cases} \qquad \left(J\varepsilon = \frac{2}{5}mR^2\cdot\frac{a}{R}\right)$$

$$\frac{2}{5}m\cancel{R}\cdot a = \left(\frac{F}{5}+f\right)\cancel{R}$$

$$\frac{2}{5}ma - \frac{F}{5} = f \Rightarrow ma = F - \left(\frac{2}{5}ma - \frac{F}{5}\right)$$

$$ma = F - \frac{2}{5}ma + \frac{F}{5}$$

$$ma\left(1+\frac{2}{5}\right) = F\left(1+\frac{1}{5}\right)$$

$$\frac{7}{\cancel{5}}ma = \frac{6}{\cancel{5}}F$$

$7ma = 6F$, where a is the acceleration of the center of the ball.

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Problem 2.

Solution page #3

Since the velocity $v_0 = \text{const}$, ~~all~~ and all forces are perpendicular to $\vec{v}_0$ and constant, we can conclude, that the ball will have a circular motion, described only by the velocity v_0 and radius R_0.

$$\begin{cases} a = \dfrac{v_0^2}{R_0} \\ 7ma = 6F = 6qv_0B \end{cases}$$

$$7m\frac{v_0^{\cancel{2}}}{R_0} = 6q\cancel{v_0}B$$

$$R_0 = \frac{7mv_0}{6qB}$$