



## Task 3

Each <sup>small (droplet)</sup> piece of water is moving along a parabola.

These parabolas are <sup>sets of points</sup> all equidistant from a focus (specific to each parabola) and a line (this is the <sup>horizontal</sup> height line in  $x-y$  plane that has the same potential energy as full energy of particle (kinetic + potential)). Thus this line is equal for all of the parabolas. Denote this line as  $h$ , equation  $y=h$  and in position  $P$  its

If we consider the <sup>parabola of the</sup> particle of water that just left the hose focus is clearly outside the area enclosed by the parabolas between  $(0,0)$  and  $P$  and straight line connecting both points.

If we consider the <sup>parabola of its</sup> particle that just landed at point  $Q$ , its focus is inside the shape bounded by parabola and line  $(0,0) \leq Q$ .

This is true because the hose <sup>direction</sup> was always higher than  $45^\circ$  and thus its focus of parabola is higher than line  $y=0$  (highest point  $\uparrow$  of that particle is  $x > \frac{L}{2}$ ).

Since movement was continuous ~~that over~~ the line connecting  $(0,0)$  and  $R$  <sup>and the parabola the particle at R covered</sup> is moving continuously as is the focus of that parabola. Thus by intermediate value theorem there is a point where focus was either on the parabola that particle  $\alpha$  covered (which is only possible if  $\alpha$  went straight up) or on the line  $(0,0) - R$ .

Thus there exists a point  $R$  on locus of points particles such that the inequality

$d((0,0), R) \leq d((0,0), F) + d(F, R)$  reaches equality case. (here  $F$  is the focus of parabola that particle at  $R$  covered. This means that

$$d((0,0), R) = d((0,0), F) + d(F, R) = d((0,0), L) + d(L, R) \text{ for that point } R(x_R, y_R)$$

$$= 2h - y_R$$

Thus we have to find the maximum of  $a = \sqrt{x_R^2 + y_R^2} + y_R$  <sup>among point on graph</sup> since

$$I = \max(\sqrt{x_R^2 + y_R^2} + y_R) = 2h$$

calculating the value of  $I$  we get that  $I \approx 14,66 \approx 14,7 \text{ m}$

$$h = 7,33 \text{ m} \quad m \cdot g \cdot h = m \cdot \frac{v_0^2}{2}$$

$$v_0 = \sqrt{2gh} = \underline{\underline{12 \text{ m/s}}}$$

This has been done by first eliminating all parts that have a point lower higher or more right, then calculating value at  $(10,5; 3,5)$  eliminating everything lower left from  $(10,8)$  since  $\alpha(10,8) < \alpha(10,8,8)$  then checking through remaining values.