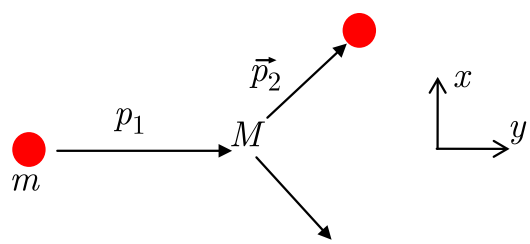
APhO-2010-1A (Taiwan)

Inelastic Scattering and Compositeness of Particles. Elementary particles can be considered as material points with no internal structure; composite particles are made of elementary particles bound together by fundamental forces. If the target particle is elementary, it has no excited states, i.e. the only form of energy it can have is the energy of translational motion. Meanwhile, composite particles can have excited states with certain internal energy (corresponding e.g. to rotations or oscillations). To determine if a particle is composite, one may set up a scattering (collision) experiment with the particle being the target and allow an elementary particle to scatter off it.

Let Q denote the total translational kinetic energy loss of the system made of (a) a scattering elementary particle of mass m, and (b) a (possibly composite) particle at rest (due to which the other particle will deviate from its trajectory) of mass M. Then $Q = K_i - K_f$, where K_i and K_f denote the total translational kinetic energies of the system before and after scattering, respectively. In what follows, all the gravitational and relativistic effects are to be neglected. Although during immediate interaction of the particles, quantum mechanical effects are important, you are asked to consider only the states before and after collision; these states are described adequately by classical mechanics.

(a) As shown in Fig., an elementary particle of mass m moves parallel to the x axis with momentum p_1. After colliding with an initially motionless target of mass M (which is either an elementary or composite particle), its momentum becomes $\vec{p}_2$. Assume that $\vec{p}_2$ lies in the $x-y$ plane and that the x and y-components of $\vec{p}_2$ (denoted by p_{2x} and p_{2y}, respectively) are known (from experiment).

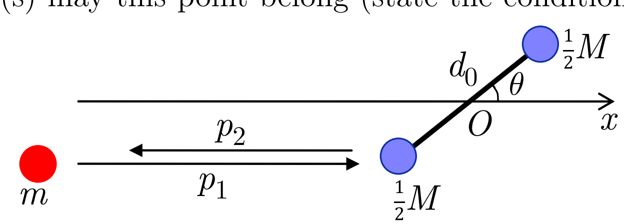


(i) Find an expression for Q in terms of m, M, p_1, p_{2x} and p_{2y}. [0.2 points]

(ii) Now assume that the target particle is elementary; then, a certain equality relating p_1, p_{2x} and p_{2y} to each other must hold. For a given p_1, plot this equality as a curve in the $p_{2x} - p_{2y}$ plane. Specify the value of p_{2x} for each intercept of the curve with the p_{2x}-axis. In the same plot, locate regions of points of $\vec{p}_2$ corresponding to $Q < 0$, $Q = 0$, $Q > 0$, and label each of them as such. [0.7 point]

(iii) Now assume that the target is a composite particle in its ground state before scattering; if we mark the outcome of a scattering as a point in the $p_{2x} - p_{2y}$ plane of the previously drawn plot, into which region(s) may this point belong (state the condition for Q)? [0.2 point]

(b) Now, consider a composite target consisting of two elementary particles each with mass $\frac{1}{2}M$. They are connected by a spring of negligible mass. See Fig.



The spring has a force constant k and does not bend sideways. Initially, the target is stationary with its center of mass at the origin O, and the spring, inclined at an angle θ to the x-axis, is at its natural length d_0. We assume that only vibrational and rotational motions can be excited in the target as a result of scattering. The incident elementary particle of mass m moves in the x-direction both before and after scattering with its momenta given, respectively, by p_1 and p_2. Note that p_2 is negative if the particle recoils and moves backward. Note that while scattering from the composite particle, due to the quantum-mechanical nature of the process, no assumptions can be made about how the energy transferred from the incident particle is distributed between the rotational and oscillatory motion of the compound particle: all the possibilities which are not prohibited by conservation laws are equally possible.

We assume that the particles did interact so that $p_2 \neq p_1$. We assume also that all three particles move in the same plane before and after scattering.

(i) If the maximum length of the spring after scattering is d_m , find an equation which relates the ratio $x = (d_m - d_0)/d_0$ to the quantities Q, θ, d_0, m, k, M, p_1, and p_2. [0.7 points]

(ii) For particle physics, an important role is played by scattering (or reaction) *cross-sections*. A certain outcome of an interaction is said to have a cross-section σ if the probability of the outcome is the same what would be for a point mass hitting a rigid ball of cross-section σ (i.e. of radius $\sqrt{\sigma/\pi}$). For this problem, we assume that the axis of the compound particle takes randomly all the possible orientations over the full solid angle of 4π with equal probabilities. Therefore, the probability that for a given scattering event, the angle θ is smaller than a certain value θ_0 is proportional to the area $A = \pi \sin^2 \theta$ of the circle drawn by the endpoint of a unit vector at origin forming angle θ_0 with the x-axis, and rotating around the x-axis. Here, we shall be studying the cross-section $\sigma = \sigma(p_2)$ of such a scattering when (for the given p_1) the momentum of the scattered particle equals to p_2. We shall assume that the translational energy loss $Q = E_r + E_o$ is distributed between oscillation energy E_o and rotation energy E_r randomly, with different values of E_o being equally probable. In that case, the probability of the scattered particle having momentum p_2 is defined just by the range of angles $\theta \in [\theta_1, \theta_2]$ for which this is not forbidden by the conservation laws. In what follows we neglect numerical prefactors (such as π or $d^2/4$), in which case the cross-section can be taken to be equal to $\sigma(p_2) = \sin^2 \theta_2 - \sin^2 \theta_1$.

It appears that while for $p_2 > p_c$, $\sigma(p_2)$ does depend on p_2, for $p_2 > p_c$ it remains constant, $\sigma(p_2) \equiv \text{const}$. In the limit of large stiffness k, estimate the value of p_c; express your answer in terms of m, M, and p_1. [1.1 points]

Assuming $M = 3m$, sketch a plot for $\sigma(p_2)$ for a given p_1 in the limit of large k. In the plot, mark the coordinates of important points. [1.1 points]