COVER

The 3rd Gulf Physics Olympiad — Experimental Competition Solutions

Muscat, Oman — October 8th 2019

Error analysis

In what follows, any time errors of the mean of tabulated data are calculated, standard deviation is used. Assuming there are N data points of the form x_i, $i \in \{1, \ldots, N\}$, the mean is

$$x_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} x_i,$$

and the standard deviation of the mean

$$\Delta x_{\text{avg}} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - x_{\text{avg}})^2}{N(N-1)}}.$$

For error propagation through equations, Pythagoran rule for adding errors in quadrature is used (alternatively, one could use min-max but for lower accuracy). In general, when you have a variable y be a function of variables x_i, $i \in \{1, \ldots N\}$ with errors Δx_i, then the error of y is given by

$$\Delta y = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial y}{\partial x_i}\right)^2 \Delta x_i^2}.$$

Any time the methods used in finding the errors is not specified, 50 % of the marks for error analysis are to be deducted.

Problem E1. Magnetic properties of matter *(20 points)*

Part A. Diameter of the syringe needle *(3 points)*

In order to maximize the accuracy, the path length of the diffracted laser needs to be maximized since that increases the separation of the maxima on the screen. **(0.2 pts)**
A two-fold increase in the path length can be achieved by using a mirror to reflect the laser from one end of the table to the other and then back again. **(0.3 pts)**

For both needles, the distance from needle to the mirror and from the mirror to the screen were both $L_0 = 119\,\text{cm} \pm 0.5\,\text{cm}$. The optical path length is then $L = 2L_0 = 238\,\text{cm} \pm 0.5\,\text{cm}$. **(0.3 pts)**
On the screen, the optical path difference between two neighbouring maxima is λ, which corresponds to a distance of $l_1 = \frac{\lambda}{d}L$ on the screen, where d is the diameter of the syringe needle. In order to increase the accuracy, the measurements should cover as many maxima, N as possible. Then the separation is $l_N = NL\frac{\lambda}{d}$ and $d = \frac{NL\lambda}{l_n}$. **(0.4 pts)**

For the white needle, following measurements were made:

i	N	l_N(mm)	l_N/N(mm)	d(mm)
1	22	89	4.05	0.313
2	18	74	4.11	0.308
3	20	79	3.95	0.321

1 measurement **(0.4/0.6 pts)**
2 measurements **(0.5/0.6 pts)**
3 or more measurements **(0.6/0.6 pts)**

The average diameter is found to be $d = 0.31\,\text{mm}$**(0.1 pts)** with an error of $\Delta d = 0.004\,\text{mm}$. **(0.2 pts)**

For the green needle, following measurements were made:

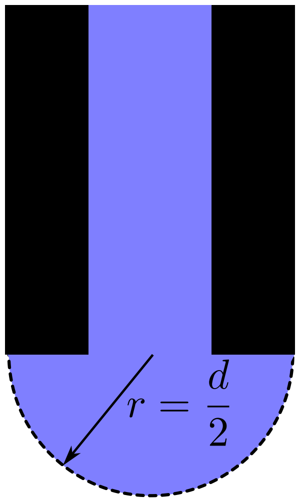
i	N	l_N(mm)	l_N/N(mm)	d(mm)
1	18	98	5.44	0.233
2	15	76	5.07	0.250
3	16	85	5.31	0.238

1 measurement **(0.4/0.6 pts)**
2 measurements **(0.5/0.6 pts)**
3 or more measurements **(0.6/0.6 pts)**

The average diameter is found to be $d = 0.24\,\text{mm}$ **(0.1 pts)** with an error of $\Delta d = 0.005\,\text{mm}$. **(0.2 pts)**

Part B. Surface tension of water *(4 points)*

After correctly setting up the equipment and pulling the middle part downwards, the needle should start dripping at one point. This happens because the surface tension isn't enough to hold back the additional pressure pushing the water down. Note that the critical case is when the radius of curvature of the water drop is the smallest, because then the extra pressure, $2\sigma/r$, is the biggest. This happens when $r = d/2$, shown in the figure below. **(0.6 pts)**



The pressure balance reads

$$2\sigma/r = \rho g h,$$

where $\rho g h$ is the pressure from the water column of height h with respect to the needle. If we measure h, we can thus express the surface tension as

$$\sigma = \frac{\rho g h r}{2} = \frac{\rho g h d}{4}. \tag{1}$$

(0.6 pts)

The following repeated measurements were made for the white needle:

i	h(mm)
1	84
2	80
3	83
4	82
5	83

1 measurement **(0.2/0.6 pts)**
2-4 measurements **(0.4/0.6 pts)**
5 or more measurements **(0.6/0.6 pts)**

The average height is found to be $h_{\text{avg}} = 82.4\,\text{mm}$ with an error of $\Delta h_{\text{avg}} = 0.7\,\text{mm}$, and using (1), we get $\sigma = 63.4\,\text{mN/m}$ with an error of $\Delta\sigma = \sigma\sqrt{(\Delta h_{\text{avg}}/h_{\text{avg}})^2 + (\Delta d/d)^2} = 1\,\text{mN/m}$.

values **(0.3 pts)**
errors **(0.4 pts)**

The measurements for the green needle were:

i	h(mm)
1	114
2	113
3	116
4	112
5	113

1 measurement **(0.2/0.6 pts)**
2-4 measurements **(0.4/0.6 pts)**
5 or more measurements **(0.6/0.6 pts)**

The average height is $h_{\text{avg}} = 113.6\,\text{mm}$ with an error $\Delta h_{\text{max}} = 0.7\,\text{mm}$. That gives $\sigma = 67.0\,\text{mN/m}$, and an error of $\Delta\sigma = 1.5\,\text{mN/m}$.

values **(0.3 pts)**
errors **(0.4 pts)**

The final expression for the surface tension is found by averaging the results of the white and green needle. This yields
$\sigma = (\sigma_{\text{white}} + \sigma_{\text{green}})/2 = 65.2\,\text{mN/m}$, **(0.1 pts)**
$\Delta\sigma = \sqrt{\Delta\sigma^2_{\text{white}}/2 + \Delta\sigma^2_{\text{green}}/2} = 1.3\,\text{mN/m}$. **(0.1 pts)**

Part C. Susceptibility of graphite *(4 points)*

Taking only 1 measurement of the diameter of the magnet is enough since the measurement results are all virtually the same. The error of the result comes from the accuracy of a ruler, taken to be 0.3 mm. The diameter of the magnet is measured to be $d = 10.0\,\text{mm}$.

value **(0.3 pts)**
error **(0.2 pts)**

In both configurations, the graphite will experience three forces: gravity, normal force and magnetic force, all of them being in balance, with the magnetic force being at an angle $45\,°$ with respect to the horizon. Purely under the force of gravity, the graphite would start accelerating with acceleration $a_g \approx g\alpha$ (where we have used small angle approximations) along the water surface (normal force does not contribute to this as it's perpendicular to the surface). Since the angles are small, this translates to horizontal acceleration of the same magnitude. This is counteracted by the acceleration from the magnet, given by the formula

explaining the force balance **(0.5 pts)**

$$a_m = \frac{F}{m} = (|\chi_g - \chi_w|)\frac{1}{2\mu_0}\frac{\mathrm{d}B^2}{\mathrm{d}z} = (|\chi_g - \chi_w|)\frac{\partial w}{\partial z}.$$

Since the magnet is pushing the graphite at a $45\,°$ angle, the force balance is $a_m/\sqrt{2} = a_g$. This let us express χ_g in terms of the measured quantities α and $\frac{\partial w}{\partial z}$:

$$\chi_g = \chi_w - \left|\frac{\sqrt{2}g\alpha}{\frac{\partial w}{\partial z}}\right|. \tag{2}$$

(0.6 pts)

Here we've been careful with the signs, since graphite must have a larger susceptibility in magnitude than water but it's still negative. This concludes the necessary theoretical calculations.

In the first configuration with the equilateral triangle, following measurements for the distance from the wall were made:

i	l(mm)
1	12
2	10
3	11
4	10
5	11

1-2 measurement **(0.3/0.5 pts)**
3-4 measurements **(0.4/0.5 pts)**
5 or more measurements **(0.5/0.5 pts)**

this gives an average of $l = 10.8\,\text{mm}$ with an error of $\Delta l = 0.4\,\text{mm} + 0.5\,\text{mm} = 0.9\,\text{mm}$, where we have added the measurement error from the ruler and the standard deviation from the tabulated data (we usually omit the ruler's accuracy in the case of repeat measurements, but that's usually because the ruler's accuracy is much smaller than standard deviation).

The corresponding surface slope can be read from the graph, but first we need to convert l to units of $\lambda = \sqrt{\sigma/(\rho g)} = 2.58\,\text{mm}$, with $\Delta\lambda = 0.03\,\text{mm}$. Thus, $l = (l/2.58\,\text{mm})\lambda = 4.2\lambda$ with $\Delta l = 0.4\lambda$. From figure 7, we measure

$$\alpha = 2.4 \times 10^{-3}\,\text{rad}.$$
$$\Delta\alpha = 1.8 \times 10^{-3}\,\text{rad}$$

where the error was measured from the figure by looking at $\alpha(\lambda \pm \Delta\lambda)$. The error is big but this is to be expected due to the exponential nature of the graphs.

To get a reading of the magnetic pressure gradient, we first calculate the distance of the graphite from the surface of the magnet. We see that the distance is that of the height of an equilateral triangle with side length $d = 10\,\text{mm} \pm 0.3\,\text{mm}$.

The distance is then simply $l_1 = \sqrt{3}/2d = 8.7\,\text{mm} \pm 0.3\,\text{mm}$. From figure 8, we read the magnetic pressure gradient to be $\frac{\partial w}{\partial z} = 6.1 \times 10^5\,\text{J/m}^4 \pm 0.9 \times 10^5\,\text{J/m}^4$.

Finally, using equation (2), we find

$$\chi_g = 6.4 \times 10^{-8}\,\text{m}^3/\text{kg} \pm 4 \times 10^{-8}\,\text{m}^3/\text{kg}.$$

reading the data from the graphs properly **(0.2 pts)**
final value for χ_g **(0.1 pts)**
error propagation **(0.3 pts)**

We go through the same calculations in the 2nd configuration to get a different estimate for χ_g. The measurements for 2nd configuration are tabulated below

i	l(mm)
1	7
2	8
3	8
4	6
5	7

1-2 measurement **(0.3/0.5 pts)**
3-4 measurements **(0.4/0.5 pts)**
5 or more measurements **(0.5/0.5 pts)**

We find $l = 7.2\,\text{mm}$, $\Delta l = 0.4\,\text{mm} + 0.5\,\text{mm} = 0.9\,\text{mm}$. This corresponds to $l = 2.8\lambda \pm 0.4\,\lambda$ From figure 7, we read $\alpha = 1.7 \times 10^{-2}\,\text{rad} \pm 0.9 \times 10^{-2}\,\text{rad}$.

In this case, a right-angled isosceles is formed. It's easy to see that the graphite is distance $d/2 = 5\,\text{mm} \pm 0.15\,\text{mm}$ from the surface of the magnet. This gives the magnetic pressure gradient to be $\frac{\partial w}{\partial z} = 4.6 \times 10^6\,\text{J/m}^4 \pm 0.4 \times 10^6\,\text{J/m}^4$. This gives the susceptibility to be $\chi_g = 6.0 \times 10^{-8}\,\text{kg/m}^3 \pm 3\,\text{kg/m}^3$.

reading the data from the graphs properly **(0.2 pts)**
final value for χ_g **(0.1 pts)**
error propagation **(0.3 pts)**

Finally, we average the two results to get

$$\chi_g = 6.2 \times 10^{-8}\,\text{kg/m}^3 \pm 4 \times 10^{-8}\,\text{kg/m}^3.$$

final answer **(0.1 pts)**
error **(0.1 pts)**

Part D. Relative permeability of ferromagnetic strip *(9 points)*

i. (1 pt) The voltage on the output leads of the battery holder can be measured to be $\mathcal{E} = 3.17\,\text{V}$, no uncertainty is needed. **(0.5 pts)**

After correctly setting up the experimental equipment, the zero off-set is measured to be $V_0 = 2\,\text{mV}$. **(0.5 pts)**

ii. (4 pts) Rearranging the expression for the magnetic field strength between the strips, $B = B_0\cosh(z/\lambda)$, we get

$$\cosh^{-1}(B/B_0) = z/\lambda. \qquad \textbf{(0.5 pts)}$$

We can measure z and V, and V can be converted into magnetic field strength B by noting that the maximal field strength $B_{\text{max}} = 3\,\text{V} \cdot \frac{10\,\mu\text{T}}{1\,\text{mV}} = 30\,\text{mT}$ is measured at $V = \mathcal{E}$, and thus, we must have

$$B = B_{\text{max}}\frac{V}{\mathcal{E}} = 30\,\text{mT} \cdot \frac{V}{\mathcal{E}}. \tag{3}$$

(0.3 pts)

This relies on the fact that the magnetic field strength scales linearly with voltage. To linearize the measured data, we need to know B_0. This can be found by noting that $\cosh(0) = 1$ and so $B_0 = B(z = 0)$. This we can read from the measured data to be $B_0 = 0.227\,\text{T}$. **(0.2 pts)**

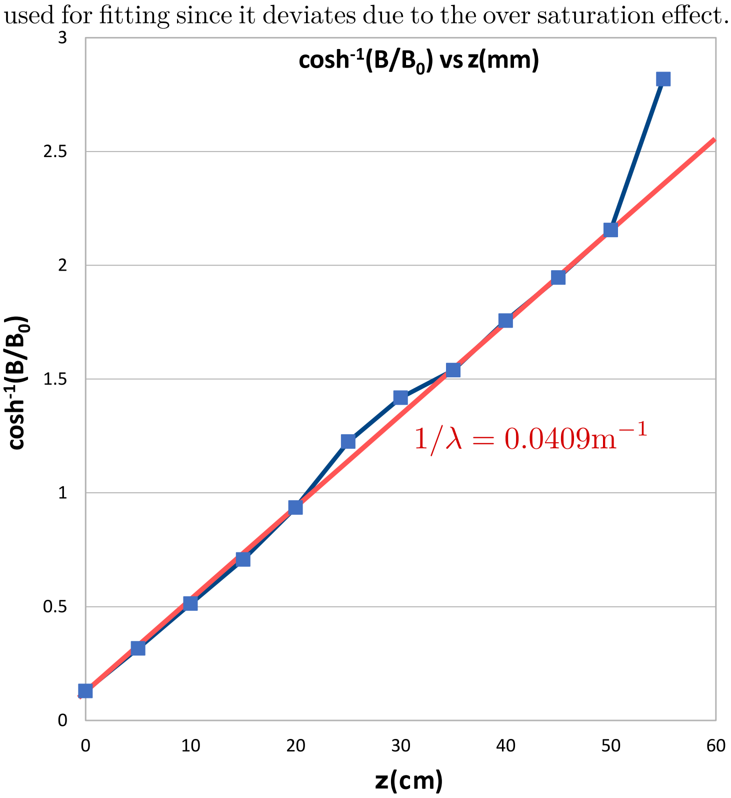
Now note that we can plot $\cosh^{-1}(B/B_0)$ against z to get a linear graph with slope $1/\lambda$. The tabulated data is given below.

z(cm)	V(mV)	B(mT)	$\cosh^{-1}(B/B_0)$
0	24	0.227	0.130
5	25	0.237	0.316
10	27	0.256	0.513
15	30	0.284	0.707
20	35	0.331	0.936
25	44	0.416	1.225
30	52	0.492	1.418
35	58	0.549	1.539
40	71	0.672	1.757
45	85	0.804	1.946
50	104	0.984	2.154
55	200	1.893	2.818

less than 3 measurements **(0.0/0.4 pts)**
3 - 11 measurements **(0.2/0.4 pts)**
correct number of measurements **(0.4/0.4 pts)**

calculations **(0.4 pts)**

It is important to note that on the graph, the line doesn't need to pass through origin due to systematic errors affecting all the points equally. For example, the measured $z = 0$ doesn't coincide with the actual origin due to the physical dimensions of the magnet. Furthermore, the final point in the graph is not used for fitting since it deviates due to the over saturation effect.



plotting **(1.0 pts)**

From the graph, we read the slope to be $1/\lambda = 0.0409\,\mathrm{m}^{-1}$.

(0.2 pts)

The uncertainty can be estimated by looking at the spread of lines that can be reasonably expected to pass through the points. This yields $\Delta(1/\lambda) = 0.000\,72\,\mathrm{m}^{-1}$. **(0.2 pts)**

Now, $\mu = \frac{2\lambda^2}{\delta h}$. We measure h, the width of the gap, with a ruler to be $h = 7.7\,\mathrm{mm} \pm 0.3\,\mathrm{mm}$. The error is found by noting that the error of a ruler is half, or slightly less depending on how good your eye is, of the distance between two neighbouring ticks, 0.1 mm. Finally, we calculate $\mu = 57\,000$ **(0.4 pts)** with an error of

$$\Delta\mu = \mu\sqrt{\left(2\frac{\Delta(1/\lambda)}{(1/\lambda)}\right)^2 + \left(\frac{\Delta h}{h}\right)^2} = 3000. \qquad \textbf{(0.4 pts)}$$

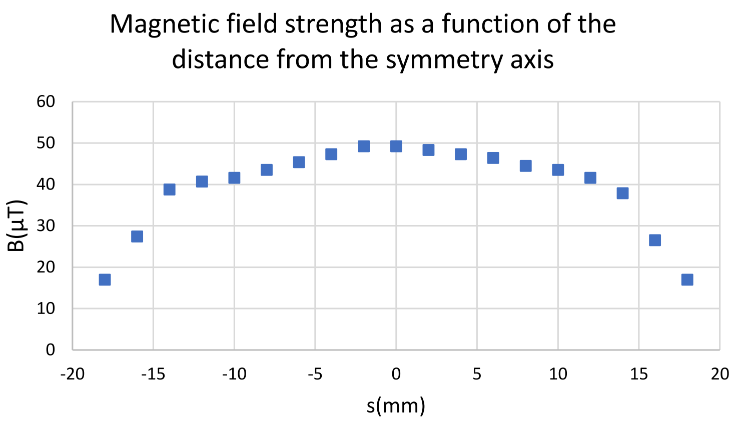
iii. (2 pts) The tabulated measurement data is given below. The voltages are translated to teslas using equation (3).

x(mm)	s(mm)	V(mV)	B(mT)
0	-18	18	0.170
2	-16	29	0.274
4	-14	41	0.388
6	-12	43	0.407
8	-10	44	0.416
10	-8	46	0.435
12	-6	48	0.454
14	-4	50	0.473
16	-2	52	0.492
18	0	52	0.492
20	2	51	0.483
22	4	50	0.473
24	6	49	0.464
26	8	47	0.445
28	10	46	0.435
30	12	44	0.416
32	14	40	0.379
34	16	28	0.265
36	18	18	0.170

less than 3 measurements **(0.1/0.6 pts)**
3 - 5 measurements **(0.3/0.6 pts)**
6 - 7 measurements **(0.4/0.6 pts)**
correct number of measurements **(0.6/0.6 pts)**

calculations **(0.4 pts)**

x is measured with respect to the first data point, s is with respect to the symmetry axis (found to be at $x = 18\,\mathrm{mm}$). The graph for B vs s is given below.



plotting **(0.6 pts)**

As can be seen, the magnetic field strength is uniform and with small deviations up to $\sim 10\,\%$ over the width of the strip. Outside the strip, the field strength starts dropping rapidly.

(0.4 pts)

iv. (2 pts) The main idea relies on the fact that magnetic field lines are conserved, or in other words the magnetic flux through a closed surface is 0. This is equivalent to Gauss' law. In the context of this problem, it implies that the flux entering the gap must come from the decrease of the flux flowing along the ferromagnetic strip. The flux flowing outside the strip will be negligible because of the high value of μ.

Let the total flux along the strip be Φ_{in}, z-axis be along the strip, and x-axis be horizontal, perpendicular to z. Also denote the flux through the $x-z$ plane intersecting the gap from $z=0$ to z as Φ_z. The Gauss' law can then be formulated as

$$\Phi_{\text{in}}(z) - \Phi_{\text{in}}(z=0) = \Phi_z. \tag{4}$$

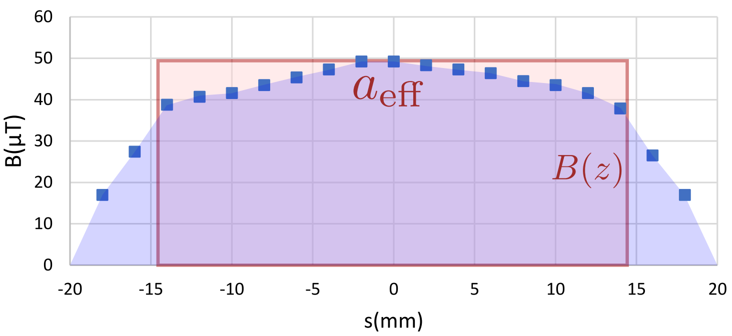
(0.5 pts)

Note that since the flux inside the ferromagnet drops exponentially, and judging from the tabulated data, it's reasonable to say that $\Phi_{\text{in}}(z) \gg \Phi_{\text{in}}(z=0)$. **(0.1 pts)**

We can approximate the magnetic field to be homogeneous throughout the cross-section of the ferromagnet (to very high accuracy, this can be verified using Ampère's law). Then $\Phi_{\text{in}}(z) = a\delta B_{\text{in}}(z)$, where a is the width of the ferromagnet, measured to be $a = 30\,\text{mm} \pm 0.3\,\text{mm}$, and $B_{\text{in}}(z)$ is the magnetic field inside the ferromagnet. **(0.1 pts)**

This means that if we calculate Φ_z, we can find B_{in} using equation (4). To find Φ_z, we need to sum the magnetic field over the z- and x-direction. In integral form, it looks like $\Phi_z = \int_0^z \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}x B(x,z')$. We can simplify this with the integral $\Phi_z = a_{\text{eff}} \int_0^z \mathrm{d}z' B(z')$, where a_{eff} is the effective width of the gap such that the area under the graph found in the previous part is equal to $B(z')a_{\text{eff}}$, where $B(z')$ is the maximal magnetic field in the gap found in part (ii).

From an approximate plot shown below, we find $a_{\text{eff}} = 0.955a = 28.7\,\text{mm}$.



Finding a_{eff} using a plot, or something equivalent **(0.2 pts)**

All that's left is to find $\int_0^z \mathrm{d}z' B(z')$. This can be found from the tabulated data found in part (ii) by summing over the data points using the trapezoid rule,

$$\begin{aligned}\Phi_i &= \Phi_{i-1} + \frac{B_i + B_{i-1}}{2}(z_i - z_{i-1})a_{\text{eff}} \\ &= \Phi_{i-1} + \frac{B_i + B_{i-1}}{2}\Delta A = \Phi_{i-1} + \Delta\Phi_i,\end{aligned}$$

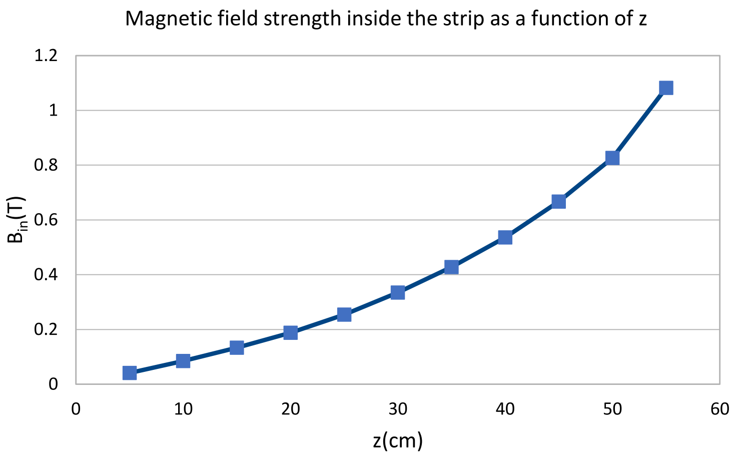
(0.3 pts)

where $\Delta A = (z_i - z_{i-1})a_{\text{eff}} = 5\,\text{cm} \cdot 28.7\,\text{mm} = 0.001\,44\,\text{m}^2$ is the effective area of the last segment and $\Delta\Phi_i = \frac{B_i + B_{i-1}}{2}\Delta A$ the flux through the corresponding surface. After that, the magnetic field inside the ferromagnetic is simply found using (4) as $B_{\text{in}} = \frac{\Phi_{\text{in}}}{a\delta}$.

The calculated data is given below alongside with the plot of B_{in} vs z.

z(cm)	B(mT)	$\Delta\Phi(\mu\text{T}\cdot\text{m}^2)$	$\Phi(\mu\text{T}\cdot\text{m}^2)$	B_{in}(T)
0	-	-	0	
5	0.232	0.334	0.334	0.041
10	0.246	0.354	0.688	0.085
15	0.27	0.389	1.077	0.133
20	0.308	0.444	1.521	0.188
25	0.374	0.539	2.06	0.254
30	0.454	0.654	2.714	0.335
35	0.521	0.75	3.464	0.428
40	0.61	0.878	4.342	0.536
45	0.738	1.063	5.405	0.667
50	0.894	1.287	6.692	0.826
55	1.438	2.071	8.763	1.082

calculations **(0.3 pts)**



plotting **(0.3 pts)**

Since in the graph found in part (ii), the saturation cut-off happened at the last data point, we can use the corresponding value for B_{in} as an estimate for B_s. Then $B_s \sim 1.1\,\text{T}$.

(0.2 pts)