The 4th Gulf Physics Olympiad — Theoretical Competition

Dammam, Saudi Arabia — March 14th 2022

- The examination lasts for 5 hours. There are 3 problems worth in total 30 points.
- **You must not open the envelope with the problems before the signal of the beginning of competition.**
- **You are not allowed to leave your working place without permission.** If you need any assistance (broken calculator, need to visit a restroom, etc), please raise your hand until an organizer arrives.
- Use only the front side of the sheets of paper.
- For each problem, there are **dedicated Solution Sheets** (see header for the number and pictogram). Write your solutions onto the appropriate Solution Sheets. For each Problem, the Solution Sheets are numbered; use the sheets according to the enumeration. **Always mark which Problem Part and Question you are dealing with.** Copy the final answers into the appropriate boxes of the **Answer Sheets**. There are also **Draft** papers; use these for writing things which you don't want to be graded. If you have written something that you don't want to be graded onto the Solution Sheets (such as initial and incorrect solutions), cross these out.
- If you need more paper for a certain problem, please raise your hand and tell an organizer the problem number; you are given two Solution sheets (you can do this more than once).
- **You should use as little text as possible:** try to explain your solution mainly with equations, numbers, symbols and diagrams. Though in some places textual explanation may be unavoidable.
- **After the signal signifying the end of examination you must stop writing immediately.** Put all the papers into the envelope at your desk. **You are not allowed to take any sheet of paper out of the room.** If you have finished solving before the final sound signal, please raise your hand.

Problem T1. Dark matter *(10 points)*

One of the main unsolved problems in modern physics is concerning dark matter. It's a hypothetical form of matter which gets its name from its peculiar property not to interact with electromagnetic fields (i.e. light), making it very difficult to detect. It's thought to account for around 85 % of matter in the universe, including most of the mass in galaxies.

The existence of dark matter is motivated by various astrophysical observations which cannot be explained unless there's more matter that cannot be seen. The primary candidate for dark matter is a new elementary particle that's yet to be discovered. Experiments to directly detect and study dark matter particles are ongoing but have yet to succeed.

This task focuses on one indirect observation of dark matter and tests its agreement with a potential model for dark matter.

Part A. Rotation curves *(5.5 points)*

For this part, you may use the following physical constants and their numerical values:
Gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$.
$1\,\mathrm{pc} = 3.09 \times 10^{16}\,\mathrm{m}$.
Duration of one year is 365.24 days.

Rotation curves are a commonly used method to deduce the matter densities of galaxies. A rotation curve shows the orbital speeds of stars in a particular galaxy as a function of the radial distance from the galaxy's centre. The following figure shows the simplified rotation curve of the Milky Way. We approximate it into two regions A and B, one with a linearly increasing profile from 0 to $v_0 = 230\,\mathrm{km/s}$ for $r \le r_1 = 3\,\mathrm{kpc}$, and the other with a constant velocity profile $v = v_0$ for $r > r_1$.

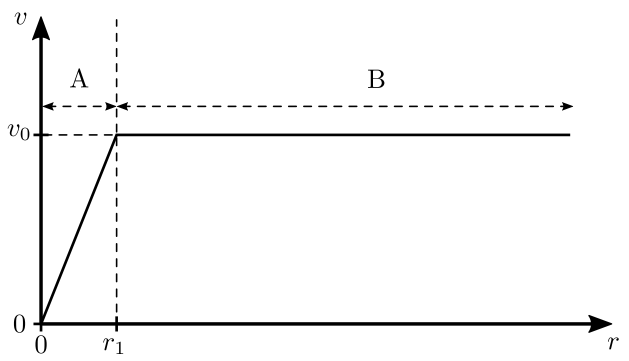


Figure 1: Milky Way's rotation curve

1. (1 pt) How far is the Sun from the centre of the galaxy if its orbital period is $T_\odot = 2.2 \times 10^8$ yrs?

2. (1 pt) Consider a test particle of negligible mass in a circular orbit of radius r around a point mass m. Find the orbital speed of the test particle.

3. (1.5 pts) Consider a cloud of gas of uniform density ρ_0 and radius r_0. Express the orbital speed of the aforementioned test particle for both when $r \le r_0$ and $r > r_0$ in terms of ρ_0, r_0, and the physical constants. Also find an expression for the potential energy per unit mass φ in the cloud.

4. (1.5 pts) We now focus on the Milky Way galaxy. Deduce the matter density as a function of r in both of the regions A and B shown on figure 1. You may assume spherical symmetry.

5. (0.5 pts) In the neighbourhood of the Sun, stars are typically separated by a distance of $d = 2\,\mathrm{pc}$ from each other. Estimate the density of visible mass (in the form of stars) in the solar neighbourhood. Compare this with the results from part **4.** and comment on potential discrepancies. Take the mass of a typical star to be $M = 1 \times 10^{30}\,\mathrm{kg}$.

Part B. Self-interacting dark matter *(4.5 points)*

Self-interacting dark matter, or SIDM for short, is one of the many proposed classes of dark matter particles. Like most DM models, it interacts weakly with ordinary matter via gravity. Its main characteristic is that the particles interact strongly with each-other, leading to energy and momentum exchange between dark matter particles. Specifically, the interactions happen with a rate large enough that the dark matter in a galaxy thermalizes and assumes a uniform temperature T. We additionally consider the dark matter gas to be sparse enough (i.e. classical) so that Boltzmann statistics can be used for describing how the particle densities and velocities are distributed.

1. (1.5 pts) According to Boltzmann statistics, the density of dark matter is proportional to

$$\rho \propto \mathrm{e}^{-\frac{m\varphi}{k_B T}},$$

where φ is the potential energy per unit mass at the respective position in space and m is the mass of a dark matter particle.

Assuming that most of matter in the Milky Way is made up of dark matter, which region(s) can agree with the implications of the SIDM model?

2. (1 pt) Find the thermal speed of the dark matter particles in the aforementioned region(s) that agree with the SIDM model.

3. (1 pt) One of the key assumption for the isothermal model to work is that the dark matter particles undergo enough collisions to thermalize, i.e. the mean free path length of dark matter particles is significantly smaller than the characteristic size of the Milky Way. Based on this, give an estimate for the lower bound of the ratio of the self-interaction cross section to the mass of a dark matter particle.

4. (1 pt) Another of our assumptions has been that the SIDM gas is classical in nature, i.e. the density of the gas is well below the limit where quantum effects become evident. This condition can generally be written as

$$n \ll \left(\frac{m k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}},$$

where n is the number density of dark matter, $k_B = 1.38 \times 10^{-23}\,\mathrm{J/K}$ the Boltzmann constant, and $\hbar = 1.05 \times 10^{-34}\,\mathrm{J\,s}$ the reduced Planck's constant. Based on this, give a lower bound for the mass of a dark matter particle m for the classical considerations to apply in the Milky Way.

Problem T2. Global warming *(10 points)*

The average temperature of planet Earth (15 °C) is the result of a combination of two factors: (a) Earth's surface absorbs a fraction $\alpha \approx 0.7$ of the incident solar radiation; (b) The surface of the Earth radiates according to Stefan-Boltzamann's law: the power radiated per unit area equals to $\sigma\varepsilon T^4$, where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$, and T denotes the surface temperature in Kelvin, and ε — the emissivity, the ratio of the energy radiated from a material's surface to that radiated from an ideal black body. Due to the second law of thermodynamics, the emission factor must be equal to the absorption coefficient α, but only if the radiation and absorption occur at the same wavelengths. The heat radiation from the Sun is mostly in the optical range for which the atmosphere is fully transparent (the above value $\alpha \approx 0.7$ corresponds to the optical wavelengths). The Earth emits predominantly infrared radiation, and in this wavelength range the Earth's surface (mostly water) has an average emissivity $\epsilon_0 \approx 0.9$. At the same time, the Earth's heat balance is affected by greenhouse gases, which partially reflect Earth's heat radiation back to Earth, and thus reducing the emissivity to a new effective value ϵ_{eff}.

Below, you can use the following data: The emissivity of the Sun is 1.00; Distance between the Sun and Earth $D = 1.47 \times 10^8\,\mathrm{km}$; Radius of the Sun $R_\odot = 6.96 \times 10^5\,\mathrm{km}$; the surface temperature of the Sun $T_\odot = 5780\,\mathrm{K}$; Average surface temperature of Earth $T_0 = 15\,°\mathrm{C}$; zero degrees celsius in Kelvin is 273.15 K; Earth's radius $R_0 = 6370\,\mathrm{km}$; total energy produced by mankind in one year: $A = 8.8 \times 10^{20}\,\mathrm{J}$ (assume that this energy ends up mostly being dissipated as heat into the environment).

1. (2 pts) What is the ratio of the total power consumed by mankind to the total solar radiation power absorbed by the Earth? A year has 365.24 days.

2. (1 pt) How large of a surface area on the Earth needs to be covered by solar panels to supply enough electrical energy to the humankind assuming a solar panel efficiency of $\eta = 20\%$? Compare this with the surface area of the Empty Quarter desert of about $650\,000\,\mathrm{km}^2$.

3. (2 pts) What is the average effective emissivity ϵ_{eff} of the Earth's surface at the wavelengths corresponding to the heat radiation of the Earth?

4. (2 pts) By how many degrees would the average temperature of the Earth's surface decrease if, at some point of time, humanity stopped producing any energy, and assuming that the value of ϵ_{eff} would remain unchanged?

5. (1.5 pts) The average surface temperature of the Earth has increased by $\Delta T = 0.9\,\mathrm{K}$ over the last 50 years and this is mainly explained by the increase in greenhouse gas concentrations in Earth's atmosphere. In a simplified manner, this effect can be modelled as the greenhouse gases absorbing a fraction k of the heat radiation emitted by Earth and then radiating the absorbed energy thermally so half of the radiated energy goes towards the surface of the Earth, and the other half goes into space. By how much has the factor k changed over the last 50 years?

6. (1.5 pts) Solar radiation is the main driving force of the atmospheric motion; on a global scale, the atmospheric circulation can be seen as a heat engine. To explain this, consider the so-called Hadley circulation: the Sun heats the ground, which in turn warms the air; the warm air initially travels along the ground, from the middle latitudes towards the equator, and at a certain point, when it has warmed up sufficiently, rises adiabatically and cools down. In the high atmospheric layers, at an altitude of $\sim 10\,\mathrm{km}$, the cooler air travels back to the middle latitudes and slowly radiates heat into space. During its travel, air becomes cooler and the density increasingly higher, until at a certain point, it becomes so dense that it descends down back to the ground level. After that, the process starts repeating. Based on the model of ideal heat engine and the data provided and calculated above, estimate an upper limit for the total power of producing the wind energy on Earth, assuming that the temperature in the atmosphere drops by about one degree celsius per every 100 meters increase in the altitude. How many times does it differ from the total energy produced by humanity?

Problem T3. Physics of sports *(10 points)*

Part A. Hammer throw *(4 points)*

A hammer is an iron ball of mass $m = 7.26\,\mathrm{kg}$ density and $\rho_v = 7900\,\mathrm{kg/m^3}$, fixed to steel wire with a grip on the other end. The length of the steel wire (grip included) is $L = 122\,\mathrm{cm}$; the mass and air drag of the wire and the grip can be neglected. During the flight, a drag force $F_D = 0.24 A\rho_a v^2$ acts on the hammer, where v is the speed of the hammer with respect to the air, A — the cross-sectional area of the iron ball, and $\rho_a = 1.23\,\mathrm{kg/m^3}$ — the density of air. Free fall acceleration is $g = 9.81\,\mathrm{m/s^2}$.

An athlete throws a hammer to a distance of $d = 80\,\mathrm{m}$. Assume that (a) the launching velocity of the hammer forms an angle of 45° with respect to the horizon; (b) immediately before the hammer is released, the hammer moves along a circle of radius $r = L + l$, where $l = 1\,\mathrm{m}$ denotes the length of the arms; (c) the height of the hammer above the ground immediately before it is released can be neglected (i.e. taken to be equal to zero).

1. (0.5 pts) What is the speed of the hammer v_0 just before being released if we neglect the effect of air drag on the hammer?

2. (1 pt) What is the force F_t with which the athlete pulls from the grip at the final stage of his throw?

3. (0.5 pts) From this part onwards, we consider the effects of air drag. Find the air drag F_{D0} acting on the hammer at the beginning of its flight.

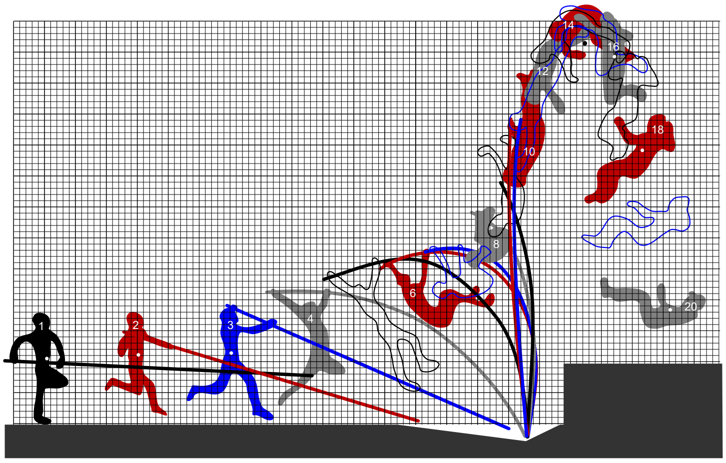
4. (1 pt) Estimate as precisely as you can by how much is the final speed of the hammer (just before hitting the ground) smaller than its initial speed.

5. (1 pt) Estimate as precisely as you can by how much would the distance of the throw be longer if there were no air drag.

Part B. Discus throw *(1 points)*

Usually tailwind helps in sports, but not in discus throw. Explain qualitatively, why headwind can increase the distance reached in discus throw. Use a force diagram to show which forces are acting on a discus during its flight and in which way they are influenced by headwind.

Part C. Pole vault *(5 points)*



The figure shows an athlete (Armand Duplantis) setting the new world record of 618 cm in pole vault (a bigger figure can be found in the next sheet). The figure is to scale and the inter-line distance of the grid is 10 cm. What you can see is an overlay of a series of 20 snapshots, the time interval between two subsequent snapshots τ is not known. For positions 1, 2, 3, 16, 18, and 20, the white dots show the position of the centre of mass of the athlete. The black dot shows the location of the bar. To avoid clutter, some position numbers are not shown.

1. (0.5 pts) For which position is the elastic energy of the pole the largest?

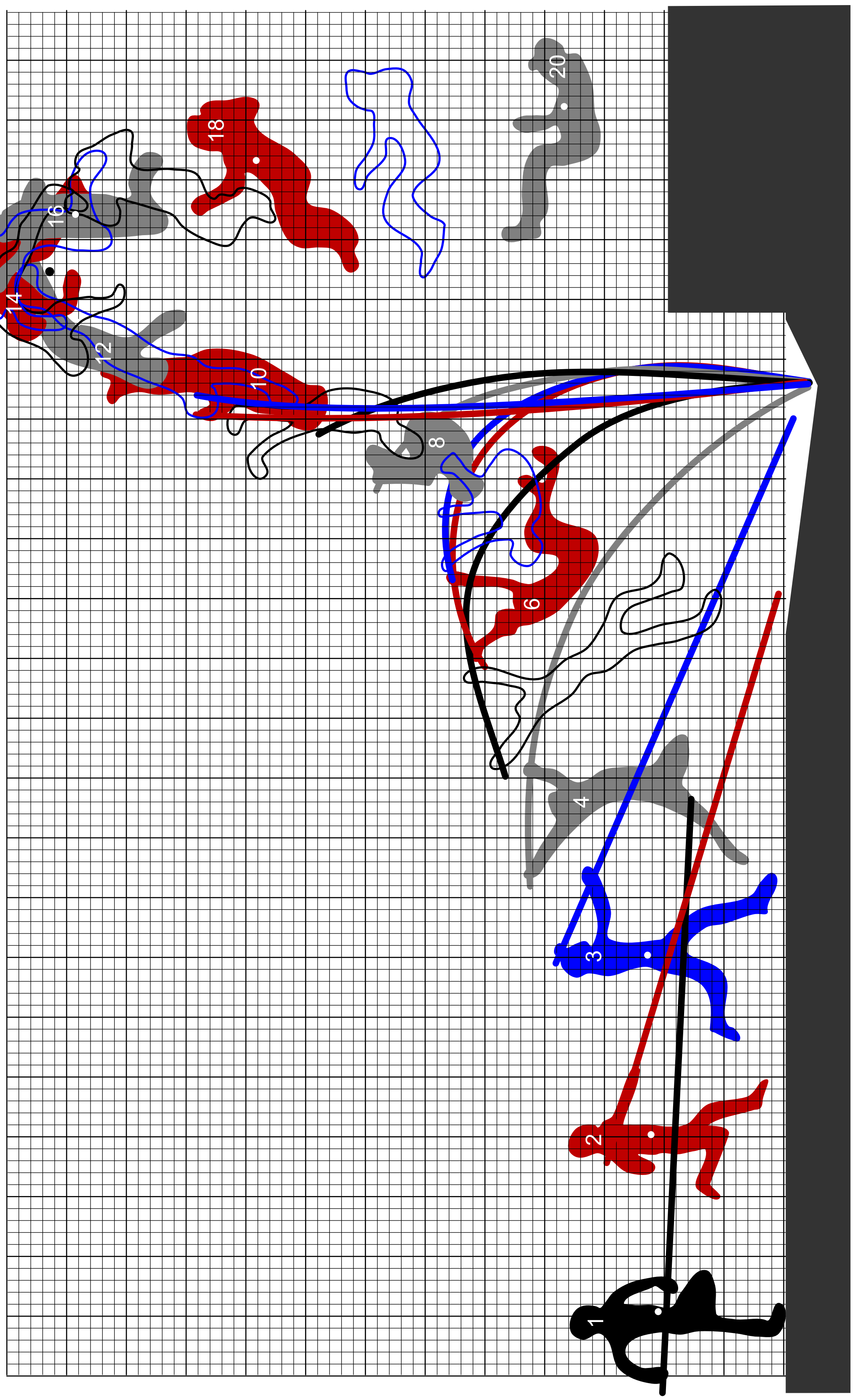
2. (2 pts) Determine the value of the time interval τ by taking appropriate readings from the figure. Free fall acceleration is $g = 9.81\,\mathrm{m/s^2}$.

Remark: if you are unable to do it, in what follows you may use a very approximate value of $\tau = 0.1\,\mathrm{s}$.

3. (0.5 pts) Find the speed of the athlete at the position No 2.

4. (1 pt) During the jump, from the position No 3 to 12, the athlete performs a certain amount of mechanical work W with his muscles. Find this work if the mass of the athlete is $m = 80\,\mathrm{kg}$. You may neglect the mass of the pole.

5. (1 pt) What was the maximum height of the centre of mass of the man during the jump? Give the height with respect to the ground (corresponding to the lower boundary of the grid).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