Problem E1. Rolling cylinder *(points)*

Part A. Critical slopes *(1 points)*

We measure the board length $L = 680\,\mathrm{mm}$, **(0.1 pts)**
its uncertainty $\Delta L = 1\,\mathrm{mm}$.

Further we determine the steepest position of the board where the cylinder's rolling stops when it is pushed downwards, and measure the difference $H = h_u - h_l$ between the heights h_u and h_l of the upper and lower edges of the upper surface of the board. We find $h_l = 8\,\mathrm{mm}$ **(0.1 pts)**
and $h_u = 33\,\mathrm{mm}$ **(0.1 pts)**
so that $H = 25\,\mathrm{mm}$. Finally, we calculate angles according to the formula $\alpha_0 = \arcsin H/L \approx 2.1\,^\circ$; the final result needs to be within $2.1\,^\circ \pm 0.2\,^\circ$. **(0.1 pts)**
We repeat measurements: find the critical positions of the slope and measure h_u. Three or more measurements **(0.1 pts)**

We do the same for the second critical angle: if the result is within $h_u = 170\,\mathrm{mm}$, and $\alpha_2 \approx 14.5\,^\circ$; the final result needs to be within $14.5\,^\circ \pm 3\,^\circ$. **(0.1 pts)**
if three or more measurements were done. **(0.1 pts)**

Error estimates: h_l and L uncertainties are estimated as $1\,\mathrm{mm}$ or $0.5\,\mathrm{mm}$. **(0.1 pts)**
h_u uncertainty when determining α_0 is estimated in the range from $1\,\mathrm{mm}$ to $2\,\mathrm{mm}$, and when determining α_2 — in the range from $10\,\mathrm{mm}$ to $50\,\mathrm{mm}$. **(0.1 pts)**
This estimation can be done based on the standard deviation of the repeated measurements.

For small angles, in radians $\alpha \approx H/L$ so that

$$\Delta\alpha = \sqrt{\left(\frac{\Delta H}{H}\right)^2 + \left(\frac{\Delta L}{L}\right)^2},$$

and $\Delta H = \sqrt{\Delta h_u^2 + \Delta_l^2}$, numerically $\Delta\alpha_1 \approx 0.1\,^\circ - 0.3\,^\circ$ and $\Delta\alpha_2 \approx 1\,^\circ - 5\,^\circ$. If both calculations are made reasonably (it is also OK to apply addition of errors by modulus, instead of the Pythagorean rule, or upper-lower bound method) and without mistakes **(0.1 pts)**

Part B. Rolling speed *(3 points)*

We take first segment close to the upper edge of the board, and second segment close to the lower end of the board, so that $l_t = 60\,\mathrm{mm}$. Then the data will be as follows

h (mm)	t_1 (s)	v_1 (mm/s)	t_2 (s)	v_2 (mm/s)	t_l (s)	v_l (mm/s)
2.6	64.8	1.54	64.5	1.55	388.2	1.55
3	15.2	6.6	15.2	6.6	91.2	6.58
3.5	7.2	13.9	7.4	13.5	43.8	13.7
4	4.52	22.1	4.38	22.8	26.46	22.7
4.5	3.41	29.3	3.53	28.3	20.7	29.0
5	2.86	35.0	2.97	33.7	17.22	34.8
5.5	2.27	44.1	2.31	43.3	13.56	44.2
6	1.78	56.2	1.65	60.1	10.14	59.2
6.5	1.51	66.2	1.35	74.1	8.46	70.9
7	1.35	74.1	1.02	98.0	7.02	85.5
7.5	1.31	76.3				
8	1.21	82.6				
8.5	1.09	91.2				
9	0.98	102				
9.5	0.92	109				
10	0.84	119				
10.5	0.78	128				
11	0.72	139				

As we can see, the rolling time for the first and second segments start departing at $H = 65\,\mathrm{mm}$, which gives us the critical slope $\alpha_1 = 6.2\,^\circ$.

Grading: at least 10 plausible time values for the first segment; **(0.8 pts)**
If less than 10 time values are taken, no credit is given if the number of measurements is less than 3; each next plausible time value earns a partial credit. **(0.1 pts)**
At least 10 speed values are correctly calculated; **(0.5 pts)**
If less than 10 time values are taken, no credit is given if the number of measurements is less than 2; each next pair of correctly calculated speed value earns a partial credit. **(0.1 pts)**

At least 6 plausible time values for the second segm.; **(0.4 pts)**
If less than 6 time values are taken, no credit is given if the number of measurements is less than 3; each next plausible time value earns a partial credit. **(0.1 pts)**
At least 6 speed values are correctly calculated; **(0.2 pts)**
If less than 6 time values are taken, no credit is given if the number of measurements is less than 2; each next pair of correctly calculated speed value earns a partial credit. **(0.1 pts)**

At least 6 plausible time values for the long segment; **(0.4 pts)**
If less than 6 time values are taken, no credit is given if the number of measurements is less than 3; each next plausible time value earns a partial credit. **(0.1 pts)**

At least 6 speed values are correctly calculated; **(0.2 pts)**
If less than 6 time values are taken, no credit is given if the number of measurements is less than 2; each next pair of correctly calculated speed value earns a partial credit. **(0.1 pts)**

For the second segment, time values are taken for all those values of H for which there is no significant time difference between the first segment, and additionally, at time values for at least two next values of H are measured. **(0.2 pts)**

For the long segment, speed values are calculated for all those values of H for which there is no significant time difference between the first segment. **(0.2 pts)**

Value $\alpha_1 = 6.2\,^\circ \pm 0.4\,^\circ$ is reported. **(0.2 pts)**
If the reported value does not fall into this range, but falls into $\alpha_1 = 6.2\,^\circ \pm 0.7\,^\circ$, partial credit is given. **(0.1 pts)**

Part C. Force as a function of speed *(2.3 points)*

What keeps the cylinder in motion is that component of gravity force which is parallel to the board surface, $F_m = mg\sin\alpha$. In those cases when the force depends significantly on time, we need to take data of the first segment since at the beginning of rolling, the friction force is maximal.

Rolling height h (mm)	Rolling speed v (mm/s)	Force applied (F_m): (mN)
2.6	1.55	16.3
3	6.58	18.6
3.5	13.7	22.0
4	22.7	25.1
4.5	29.0	28.2
5	34.8	31.4
5.5	44.2	34.6
6	56.2	37.7
6.5	66.2	40.8
7	74.1	44.0
7.5	76.3	47.1
8	82.6	50.2
8.5	91.2	53.4
9	102	56.5
9.5	109	59.7
10	119	62.8
10.5	128	66.0
11	139	69.1

Based on these data, we build a graph, see next page. As one can see, the data lie on a line, which is described by the following law:

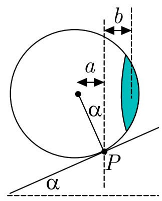
$$F = kv + F_0,$$

where $k = 390\,\mathrm{g/s}$ and $F_0 = 16.6\,\mathrm{mN}$. Grading: At least 5 correct data points in table for $\alpha \le \alpha_1$ **(0.6 pts)**
(force calculated correctly, speeds copied from the v_l column. Partial credit: less than 3 data points — no credit; each next correctly copied speed value earns a partial credit; **(0.1 pts)**
each next correctly calculated force value deserves a partial credit. **(0.1 pts)**
If speed values are copied from the column of v_1 or v_2, **0.1 pts** if four or more values are copied.
At least 5 correct data points in table for $\alpha > \alpha_1$ **(0.6 pts)**
(force calculated correctly, speeds copied from the v_1 column. Partial credit: less than 3 data points — no credit; each next correctly copied speed value earns a partial credit; **(0.1 pts)**
each next correctly calculated force value deserves a partial credit. **(0.1 pts)**
If speed values are copied from a wrong column — no credit.
Graph: axis marked and labelled correctly. **(0.1 pts)**
Scale on graph selected reasonably (the area of the smallest rectangle wich covers all the data is at least one third of the total graphical paper area. **(0.1 pts)**

At least 5 correctly marked data points on graph for $\alpha \le \alpha_1$ — full credit. **(0.3 pts)**
Partial credit: less than 3 data points — no credit; each next correctly marked data point earns a partial credit; **(0.1 pts)**
At least 5 correctly marked data points on graph for $\alpha > \alpha_1$ — full credit. **(0.3 pts)**
Partial credit: less than 3 data points — no credit; each next correctly marked data point earns a partial credit; **(0.1 pts)**

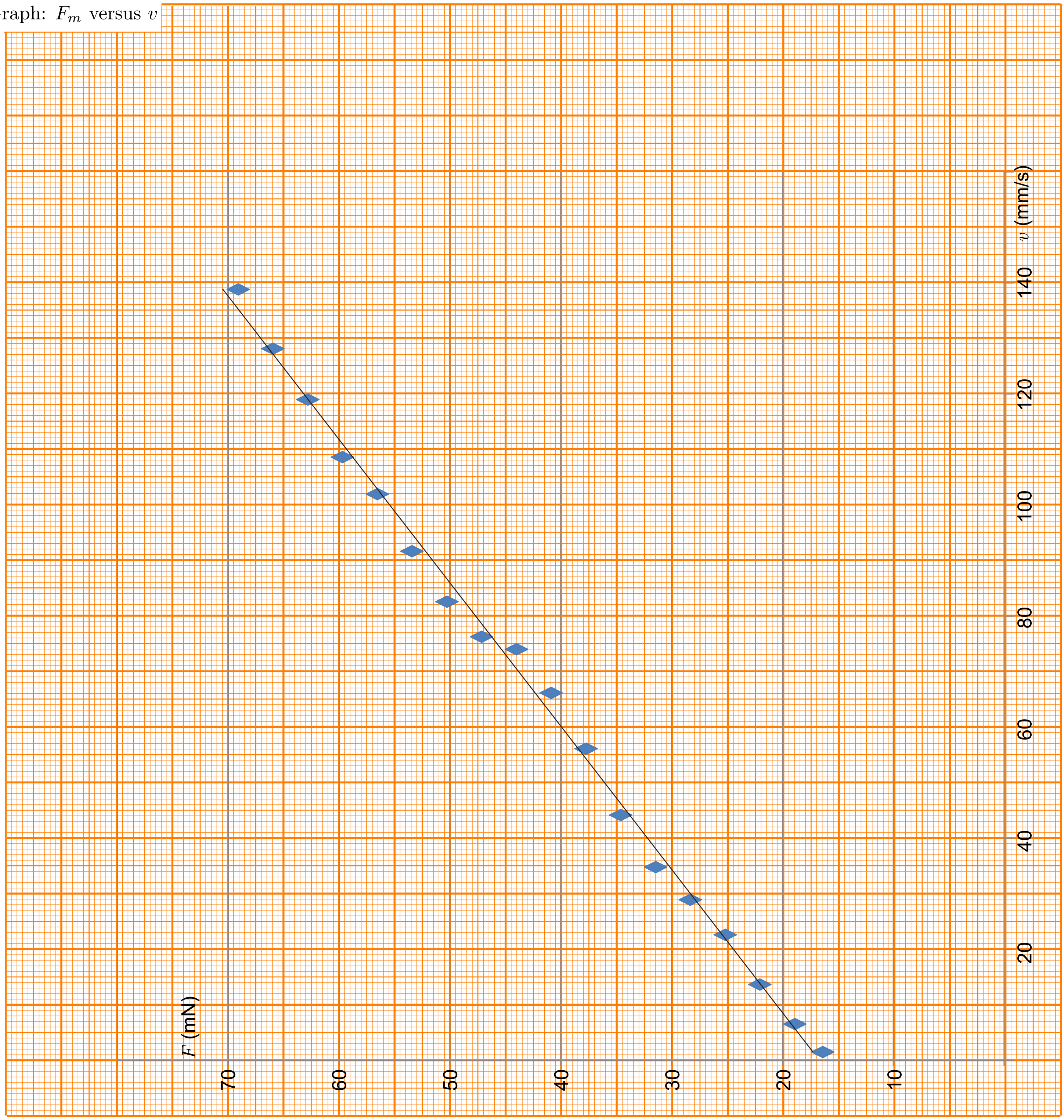
Correctly deduced functional dependence
$F = kv + F_0$; **(0.1 pts)**
$k = 390 \pm 40\,\mathrm{g/s}$; **(0.1 pts)**
$F_0 = 16.6 \pm 1\,\mathrm{mN}$. **(0.1 pts)**

Part D. Mass of liquid *(0.7 points)*



At the critical angle α_2, all the liquid will roll up and provide strongest resistance against rolling when the line conecting the middle point of the liquid and the centre of the cylinder form a vertical line, see figure. At larger angles, this position does not provide enough torque to stop downwards acceleration. Then, the centre of mass of the whole system must lay directly above the point P, therefore $m_l b = (M - m_l)a$, where m_l is the mass of liquid. Since $a = R\sin\alpha$ and $b = R - a$ (we neglect the thickness of the layer of liquid), we obtain $m_l(1 - \sin\alpha) = (M - m_l)\sin\alpha$, hence $m_l = M\sin\alpha \approx 12.5\,\mathrm{g}$. **Grading:** realizing that the ratio m_l/M is related to the critical angle α_2. **(0.3 pts)**
Obtaining quality $m_l = M\sin\alpha \approx 12.5\,\mathrm{g}$. **(0.2 pts)**
Correct numerical calculation. **(0.2 pts)**

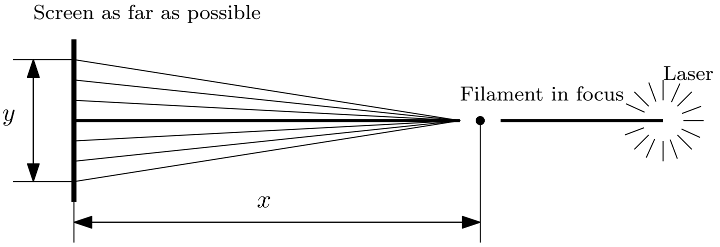
Graph: F_m versus v



Problem E2. Tungsten Filament *(13 points)*

Part A. Filament diameter *(1.5 points)*

The sketch the measurement setup:



(0.2 pts)

We focus the laser to the filament, holding the screen close to the filament during the adjustments helps focusing. For the measurements we place the screen perpendicular to the beam and reasonably far back ($x \geq 50\,\mathrm{cm}$) to get the maxima spaced out. **(0.2 pts)**
Partial credit if $30\,\mathrm{cm} \leq x < 50\,\mathrm{cm}$. **(0.1 pts)**

We measure the distance between two maxima or two minima. To get more accurate measurement we choose maxima that are far apart ($n \geq 5$). **(0.2 pts)**
Partial credit if $3 \leq n < 5$. **(0.1 pts)**

Formula for calculating diameter $d = \frac{n\lambda x}{y}$. **(0.2 pts)**

Most of the uncertainty in this case is due to the fact the diffraction pattern is fuzzy. To estimate the uncertainty we should perform repeated measurements (three or more).

n	x	y	$d = \frac{n\lambda x}{y}$
10	909 mm	131 mm	45.1 µm
7	905 mm	89 mm	46.3 µm
7	907 mm	87 mm	47.4 µm

(0.3 pts)

(Each line in table up to 3rd earns 0.1 pts.)
Filament diameter d and its uncertainty:

$$d = 46.3\,\mu\mathrm{m}$$

For $|d - 46.3\,\mu\mathrm{m}| \leq 2\,\mu\mathrm{m}$, **(0.2 pts)**
partial credit if $2\,\mu\mathrm{m} < |d - 46.3\,\mu\mathrm{m}| \leq 5\,\mu\mathrm{m}$, **(0.1 pts)**

Uncertainty is dominated by the uncertainty of y, $\Delta d \approx d\frac{\Delta x}{x} \approx 1.2\,\mu\mathrm{m}$. Reasonably estimated Δy, **(0.1 pts)**
correct calculation of Δd **(0.1 pts)**

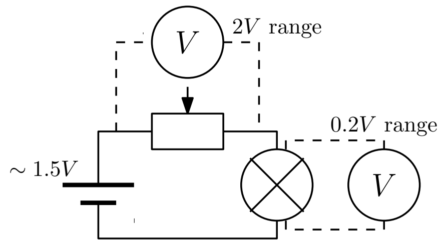
Part B. Filament's resistance *(2 points)*

The problem is that with this multimeter we cannot accurately measure the resistance of the filament directly when the knob is turned to the resistance measurement position, the resistance is too small for that. There are two issues: first, the multimeter is not accurate enough $\pm 0.5\% + 0.5\Omega$; second, the internal resistance can be in the same order of magnitude. If the filament's resistance is directly measured, no more than 0.5 points overall: 0.3 pts for the answer if it is within $0.8 \pm 0.4\,\Omega$, and 0.2 pts for the uncertainty if it is stated as either $0.5\,\Omega$ or $0.6\,\Omega$.
Thus, we need to pass a current through the bulb and measure the voltage. **(0.1 pts)**

The current needs to be small, otherwise we shall heat the filament. **(0.2 pts)**

To get the smallest possible current we use a single 1.5 V battery **(0.1 pts)**
in series with the rheostat. **(0.1 pts)**

We can measure accurately the voltage on the bulb, but the problem is the current, because the ammeter is not ideal. If we use it in the "mA"-range, we cannot take account its internal resistance, if we use it in 10A-range, the current measurement error will be large. So, we need to use the multimeter as a voltmeter. **(0.1 pts)**
Thus, we use the circuit as shown below. **(0.1 pts)**



The resistance of the rheostat can be measured directly, or using current/voltage measurements, $R_r = 25.3\,\Omega$ **(0.1 pts)**
$\Delta R_r = 0.7\,\Omega$ **(0.1 pts)**
Here and in what follows only reasonable results are accepted.
Measurement results: $U_r = 1.483\,\mathrm{V}$ **(0.1 pts)**
$U_b = 45.0\,\mathrm{mV}$ **(0.1 pts)**
$\Delta U_r = 0.012\,\mathrm{V}$, and $\Delta U_b = 0.5\,\mathrm{mV}$ **(0.1 pts)**

Formula for filament resistance $R = U_b R_r / U_r$ **(0.1 pts)**
Formula for filament resistance uncertainty
$\Delta R = R\sqrt{\left(\frac{\Delta U_r}{U_r}\right)^2 + \left(\frac{\Delta U_b}{U_b}\right)^2 + \left(\frac{\Delta R_r}{R_r}\right)^2}$ **(0.1 pts)**
Formula for filament length $l = \frac{Rd^2\pi}{4\rho_{25}}$ **(0.1 pts)**
Formula for filament length uncertainty
$\Delta l = l\sqrt{\left(\frac{\Delta R}{R}\right)^2 + 2\left(\frac{\Delta d}{d}\right)^2}$ **(0.1 pts)**
Filament resistance R and its uncertainty:
$R = 0.77\,\Omega$ **(0.1 pts)**

$\pm 0.03\,\Omega$ **(0.1 pts)**

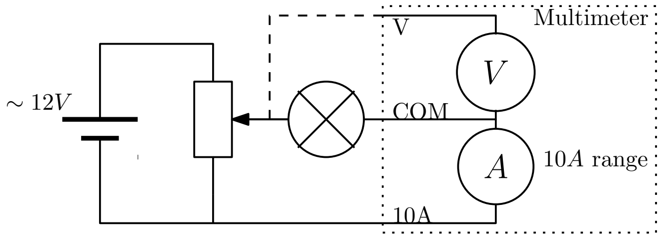
Filament length l and its uncertainty:
$l = 23\,\mathrm{mm}$ **(0.1 pts)**

$\pm 2\,\mathrm{mm}$ **(0.1 pts)**

Part C. Current–voltage curve *(2.5 points)*

Now we connect the bulb to the battery via rheostat as a potentiometer, i.e. according to the diagram below. Only that way will we be able to cover the whole range of voltages from 0 V to 12 V. **(0.2 pts)**

If we connect the rheostat in series, we'll miss low voltage values (unless we switch the power supply to a battery).



Usable correctly drawn circuit (even if the rheostat is connected in series) deserves credit. **(0.2 pts)**

If we leave ammeter connected during voltage measurements, the COM terminal must be connected to the bulb, because voltage drop on the ammeter is not negligible. Credit is given for any circuit which does not neglect the internal resistance of the ammeter. **(0.3 pts)**

U	I	U	I	U	I
100 mV	100 mA	1000 mV	230 mA	5500 mV	540 mA
200 mV	140 mA	1500 mV	280 mA	6000 mV	560 mA
300 mV	150 mA	2000 mV	320 mA	6500 mV	590 mA
400 mV	150 mA	2500 mV	360 mA	7000 mV	610 mA
500 mV	170 mA	3000 mV	390 mA	7500 mV	630 mA
600 mV	180 mA	3500 mV	430 mA	8000 mV	620 mA
700 mV	200 mA	4000 mV	460 mA	8300 mV	600 mA
800 mV	200 mA	4500 mV	490 mA		
900 mV	220 mA	5000 mV	510 mA		

At least 4 correct measurements below 1 V. **(0.2 pts)**
Partial credit if 3 measurements **(0.1 pts)**
Partial credit if 2 measurements **(0.05 pts)**
(Final score for this task is rounded up to a single decimal digit.)
At least 4 correct measurements for $1\,\text{V} \le U < 3\,\text{V}$ **(0.2 pts)**
Partial credit if 3 measurements **(0.1 pts)**
Partial credit if 2 measurements **(0.05 pts)**

At least 4 correct measurements for $3\,\text{V} \le U \le 5\,\text{V}$ **(0.2 pts)**
Partial credit if 3 measurements **(0.1 pts)**
Partial credit if 2 measurements **(0.05 pts)**

At least 4 correct measurements above 5 V **(0.2 pts)**
Partial credit if 3 measurements **(0.1 pts)**
Partial credit if 2 measurements **(0.05 pts)**

Formula for filament temperature expressed in terms of the current I_{last} and voltage U_{last} at which the tungsten filament broke:

$$T = T\left(\frac{U_{\text{last}}}{I_{\text{last}} R}\right)$$

(0.2 pts)

Correctly calculated temperature $T = 3190\,\text{K}$ **(0.1 pts)**
Credit is given if the result remains between 3000 K to 3700 K.

Graph is given at Pg. 7. Grading of the graph: axes marked with scales and units, and labelled correctly. **(0.1 pts)**
Scale is chosen appropriately (graph covers at least one third of the graphical paper area). **(0.1 pts)**
Data correctly carried over to the graph. **(0.3 pts)**
Partial credit: one clear mistake: 0.2 points, two clear mistakes: 0.1 points; if some points from the table are not copied, as long as there are 4 data points in each of the four ranges given above, no penalty. If this condition is not satisfied, subtract 0.1 points for each point which was not copied until no marks remains for the graph.
Curve connecting the points is drawn. **(0.1 pts)**
The drawn curve goes through origin. **(0.1 pts)**

Part D. Emissivity *(3.5 points)*

To verify the prediction we should build a plot of k versus T which should be constant. We could alternatively plot P versus T^4 which would be linear or we could also plot P versus T in logarithmic scale and measure the slope, these are the correct options (but second and third options make the follow-up questions somewhat harder to answer). **(0.5 pts)**

We can calculate temperature from $T = T(\frac{U}{IR})$ **(0.2 pts)**
We can calculate emissivity from $k = \frac{UI}{\pi d l \sigma T^4}$ **(0.3 pts)**

Calculated data (you don't have to fill the entire table):

T	k	T	k	T	k
380 K	2.52	1245 K	0.50	2531 K	0.38
496 K	2.43	1488 K	0.45	2633 K	0.37
648 K	1.34	1695 K	0.41	2691 K	0.39
823 K	0.69	1852 K	0.40	2777 K	0.38
893 K	0.70	2016 K	0.37	2855 K	0.37
993 K	0.59	2112 K	0.40	3033 K	0.31
1035 K	0.64	2230 K	0.39	3191 K	0.25
1160 K	0.47	2330 K	0.39		
1182 K	0.53	2455 K	0.37		

The full credit (0.6 pts) for the table breaks down as follows:
At least 4 correct data points below 1000 K. **(0.2 pts)**

Partial credit if 3 data points **(0.1 pts)**
Partial credit if 2 data points **(0.05 pts)**
(Final score for this task is rounded up to a single decimal digit.)
At least 4 correctly calculated data points between 1000 K and 2000 K **(0.2 pts)**
Partial credit if 3 data points **(0.1 pts)**
Partial credit if 2 data points **(0.05 pts)**

At least 4 correct data points above 2000 K **(0.2 pts)**
Partial credit if 3 data points **(0.1 pts)**
Partial credit if 2 data points **(0.05 pts)**

Graph is given at Pg. 8. Grading of the graph: axes marked with scales and units, and labelled correctly. **(0.1 pts)**
Scale is chosen appropriately (graph covers at least one third of the graphical paper area). **(0.1 pts)**
Data correctly carried over to the graph. **(0.3 pts)**
Partial credit: one clear mistake: 0.2 points, two clear mistakes: 0.1 points; if some points from the table are not copied, as long as there are 4 data points in each of the four ranges given above, no penalty. If this condition is not satisfied, subtract 0.1 points for each point which was not copied until no marks remains for the graph.
Curve connecting the points is drawn. **(0.1 pts)**
Range of constant k is shown. **(0.1 pts)**
At small temperatures, k is larger. **(0.1 pts)**

We can see that the emissivity in more or less constant in the middle of the graph $1350\,\mathrm{K} < T < 3000\,\mathrm{K}$ The lower limit of this range is $1350 \pm 250\,\mathrm{K}$ **(0.2 pts)**
Partial credit for results within the extended range of $1350 \pm 350\,\mathrm{K}$. **(0.1 pts)**
The upper limit is either the breaking temperature, or a value larger than 2900 K. **(0.1 pts)**

The emissivity k in that range $k = 0.4$ Answers in the range 0.3 to 0.5 give full credit. **(0.3 pts)**
Partial credit for results from 0.25 to 0.55 **(0.2 pts)**
and from 0.2 to 0.65. **(0.1 pts)**

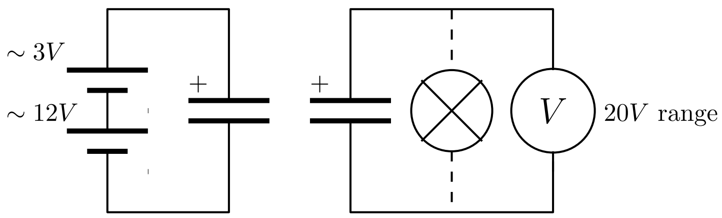
From the plot we can see that prediction fails when $T < 1350\,\mathrm{K}$ (the value stated above). Based on the graph on Pg. 8, one can say that it fails also at very high temperatures when $T > 3000\,\mathrm{K}$, but this is not always so and depends on how fast the measurements are taken. In this case the measurements were taken quite slowly and the resistance of the filament grew at the very end because tungsten deposited itself to the inside of the glass.

We can see that in the low temperatures it appears as if that $k > 1$. That is because in these lower temperatures our assumption that heat is transferred mainly by radiation fails and we can't neglect heat transfer by convection and conduction. **(0.5 pts)**

Part E. Specific heat capacity of tungsten *(3.5 points)*

We can measure how much energy it took to break the filament by connecting the bulb to the capacitor charged to a high voltage. By measuring the voltage on the capacitor before and after the process we can calculate the energy. **(0.5 pts)**

To get better accuracy we must make the radiated power as low as possible, that means we have to break the filament as fast as possible, that means we must charge the capacitor to highest possible voltage. For using the highest possible voltage of 15 V. **(0.3 pts)**



(0.2 pts)

Formula for quantity of heat $Q = \frac{(U_1^2 - U_2^2)C}{2}$ **(0.1 pts)**
$U_1 = 15.00\,\mathrm{V}$ **(0.1 pts)**
$U_2 = 14.27\,\mathrm{V}$ **(0.1 pts)**

Quantity of heat $Q = 0.5\,\mathrm{J}$ **(0.3 pts)**

We cannot perform (many) repeated measurements because we have only few bulbs.

For average specific heat c, measured quantities and calculations:

We use the breaking temperature T from previous part.

$c = \frac{4 \cdot Q}{\pi d^2 l D (T - 298.15\,\mathrm{K})}$ **(0.3 pts)**

$c = 187\,\frac{\mathrm{J}}{\mathrm{Kkg}}$ **(0.2 pts)**
Results from $100\,\frac{\mathrm{J}}{\mathrm{Kkg}}$ to $300\,\frac{\mathrm{J}}{\mathrm{Kkg}}$ are accepted, results within the extended range, from $60\,\frac{\mathrm{J}}{\mathrm{Kkg}}$ to $500\,\frac{\mathrm{J}}{\mathrm{Kkg}}$ give a partial credit. **(0.1 pts)**

We know the amount of energy that was taken from the capacitor fairly accurately. **(0.2 pts)**
Indeed, the magnitude of relative uncertainty that is caused by filament dimensions d is about 15% and the magnitude of relative uncertainty that is caused from the measurement of breaking temperature T is around 10%.

The largest source of error is from the amount of heat radiated Q_r away before the filament breaks. **(0.5 pts)**
We can estimate its value as follows. When we connect the bulb to the capacitor, the initial current is the largest, but it drops very fast as the filament heats up and its resistance grows.

That means most of the time is spent so that the filament is hot and has high resistance. Because the voltage drop on the capacitor was small we can estimate discharge time from

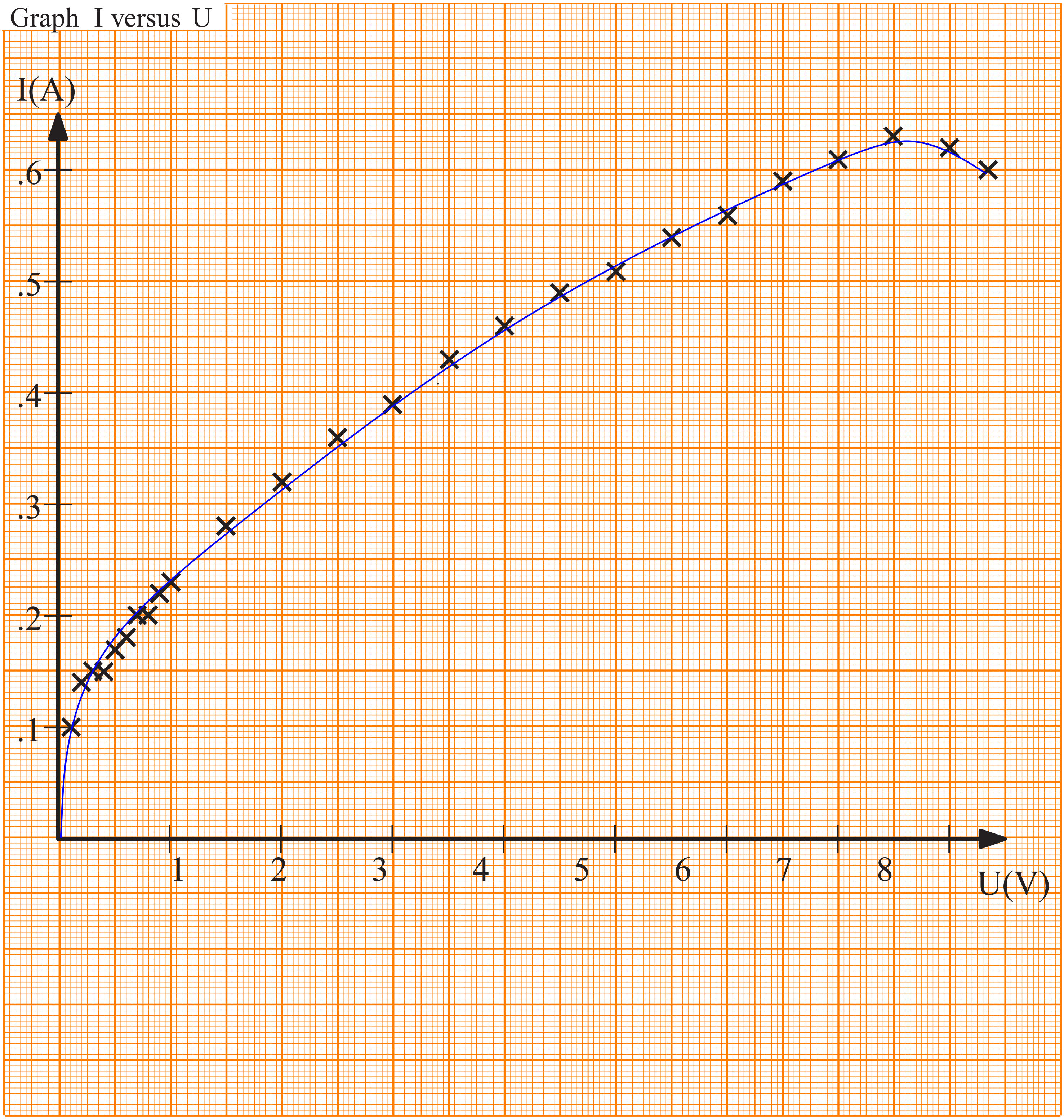
$$t \sim \frac{C\Delta U}{I_{\text{last}}} \sim \frac{C\Delta U R_{\text{last}}}{U_2} \sim \frac{C\Delta U U_{\text{last}}}{U_2 I_{\text{last}}} \approx 30\,\text{ms}$$

(0.3 pts)

Any reasonable estimation slightly departing from what is given above gives full credit. Power radiated away during that time is estimated as $Q_r \sim t U_{\text{last}} I_{\text{last}}$. **(0.3 pts)**

which numerically gives $Q_r \approx 0.15\,\text{J}$ **(0.1 pts)**

which is 30% of final result.



Graph k versus T