CHANGE OF AIR TEMPERATURE WITH ALTITUDE, ATMOSPHERIC STABILITY AND AIR POLLUTION

Vertical motion of air governs many atmospheric processes, such as the formation of clouds and precipitation and the dispersal of air pollutants. If the atmosphere is *stable*, vertical motion is restricted and air pollutants tend to be accumulated around the emission site rather than dispersed and diluted. Meanwhile, in an *unstable* atmosphere, vertical motion of air encourages the vertical dispersal of air pollutants. Therefore, the pollutants' concentrations depend not only on the strength of emission sources but also on the *stability* of the atmosphere.

We shall determine the atmospheric stability by using the concept of *air parcel* in meteorology and compare the temperature of the air parcel rising or sinking adiabatically in the atmosphere to that of the surrounding air. We will see that in many cases an air parcel containing air pollutants and rising from the ground will come to rest at a certain altitude, called a *mixing height*. The greater the mixing height, the lower the air pollutant concentration. We will evaluate the mixing height and the concentration of carbon monoxide emitted by motorbikes in the Hanoi metropolitan area for a morning rush hour scenario, in which the vertical mixing is restricted due to a temperature inversion (air temperature increases with altitude) at elevations above 119 m.

Let us consider the air as an ideal diatomic gas, with molar mass μ = 29 g/mol.

Quasi equilibrium adiabatic transformation obey the equation $pV^{\gamma} = \text{const}$, where

$\gamma = \dfrac{c_p}{c_V}$ is the ratio between isobaric and isochoric heat capacities of the gas.

The student may use the following data if necessary:

The universal gas constant is $R = 8.31$ J/(mol.K).

The atmospheric pressure on ground is $p_0 = 101.3$ kPa

The acceleration due to gravity is constant, $g = 9.81$ m/s^2

The molar isobaric heat capacity is $c_p = \dfrac{7}{2}R$ for air.

The molar isochoric heat capacity is $c_V = \dfrac{5}{2}R$ for air.

Mathematical hints

a. $\int \frac{dx}{A+Bx} = \frac{1}{B}\int \frac{d(A+Bx)}{A+Bx} = \frac{1}{B}\ln(A+Bx)$

b. The solution of the differential equation $\frac{dx}{dt} + Ax = B$ (with A and B constant) is $x(t) = x_1(t) + \frac{B}{A}$ where $x_1(t)$ is the solution of the differential equation $\frac{dx}{dt} + Ax = 0$.

c. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

1. Change of pressure with altitude.

1.1. Assume that the temperature of the atmosphere is uniform and equal to T_0. Write down the expression giving the atmospheric pressure p as a function of the altitude z.

1.2. Assume that the temperature of the atmosphere varies with the altitude according to the relation

$$T(z) = T(0) - \Lambda z$$

where Λ is a constant, called the *temperature lapse rate* of the atmosphere (the vertical gradient of temperature is - Λ).

1.2.1. Write down the expression giving the atmospheric pressure p as a function of the altitude z.

1.2.2. A process called free convection occurs when the air density increases with altitude. At which values of Λ does the free convection occur?

2. Change of the temperature of an air parcel in vertical motion

Consider an air parcel moving upward and downward in the atmosphere. An air parcel is a body of air of sufficient dimension, several meters across, to be treated as an independent thermodynamical entity, yet small enough for its temperature to be considered uniform. The vertical motion of an air parcel can be treated as a quasi adiabatic process, i.e. the exchange of heat with the surrounding air is negligible. If the air parcel rises in the atmosphere, it expands and cools. Conversely, if it moves downward, the increasing outside pressure will compress the air inside the parcel and its temperature will increase.

As the size of the parcel is not large, the atmospheric pressure at different points on the parcel boundary can be considered to have the same value $p(z)$, with z - the altitude of the parcel center. The temperature in the parcel is uniform and equals to $T_{\text{parcel}}(z)$, which is generally different from the temperature of the surrounding air $T(z)$. In parts 2.1 and 2.2, we do not make any assumption about the form of $T(z)$.

2.1. The change of the parcel temperature T_{parcel} with altitude is defined by $\frac{dT_{\text{parcel}}}{dz} = -G$. Derive the expression of $G(T, T_{\text{parcel}})$.

2.2. Consider a special atmospheric condition in which at any altitude z the temperature T of the atmosphere equals to that of the parcel T_{parcel}, $T(z) = T_{\text{parcel}}(z)$. We use Γ to denote the value of G when $T = T_{\text{parcel}}$, that is $\Gamma = -\frac{dT_{\text{parcel}}}{dz}$ (with $T = T_{\text{parcel}}$). Γ is called *dry adiabatic lapse rate*.

2.2.1. Derive the expression of Γ

2.2.2. Calculate the numerical value of Γ.

2.2.3. Derive the expression of the atmospheric temperature $T(z)$ as a function of the altitude.

2.3. Assume that the atmospheric temperature depends on altitude according to the relation $T(z) = T(0) - \Lambda z$, where Λ is a constant. Find the dependence of the parcel temperature $T_{\text{parcel}}(z)$ on altitude z.

2.4. Write down the approximate expression of $T_{\text{parcel}}(z)$ when $|\Lambda z| << T(0)$ and $T(0) \approx T_{\text{parcel}}(0)$.

3. The atmospheric stability.

In this part, we assume that T changes linearly with altitude.

3.1. Consider an air parcel initially in equilibrium with its surrounding air at altitude

z_0, i.e. it has the same temperature $T(z_0)$ as that of the surrounding air. If the parcel is moved slightly up and down (e.g. by atmospheric turbulence), one of the three following cases may occur:

- The air parcel finds its way back to the original altitude z_0, the equilibrium of the parcel is stable. The atmosphere is said to be stable.
- The parcel keeps moving in the original direction, the equilibrium of the parcel is unstable. The atmosphere is unstable.
- The air parcel remains at its new position, the equilibrium of the parcel is indifferent. The atmosphere is said to be neutral.

What is the condition on Λ for the atmosphere to be stable, unstable or neutral?

3.2. A parcel has its temperature on ground $T_{\text{parcel}}(0)$ higher than the temperature $T(0)$ of the surrounding air. The buoyancy force will make the parcel rise. Derive the expression for the maximal altitude the parcel can reach in the case of a stable atmosphere in terms of Λ and Γ.

4. The mixing height

4.1. Table 1 shows air temperatures recorded by a radio sounding balloon at 7: 00 am on a November day in Hanoi. The change of temperature with altitude can be approximately described by the formula $T(z) = T(0) - \Lambda z$ with different lapse rates Λ in the three layers $0 < z < 96$ m, 96 m $< z <$ 119 m and 119 m$< z <$ 215 m.

Consider an air parcel with temperature $T_{\text{parcel}}(0) = 22^{\circ}$C ascending from ground. On the basis of the data given in Table 1 and using the above linear approximation, calculate the temperature of the parcel at the altitudes of 96 m and 119 m.

4.2. Determine the maximal elevation H the parcel can reach, and the temperature $T_{\text{parcel}}(H)$ of the parcel.

H is called the mixing height. Air pollutants emitted from ground can mix with the air in the atmosphere (e.g. by wind, turbulence and dispersion) and become diluted within this layer.

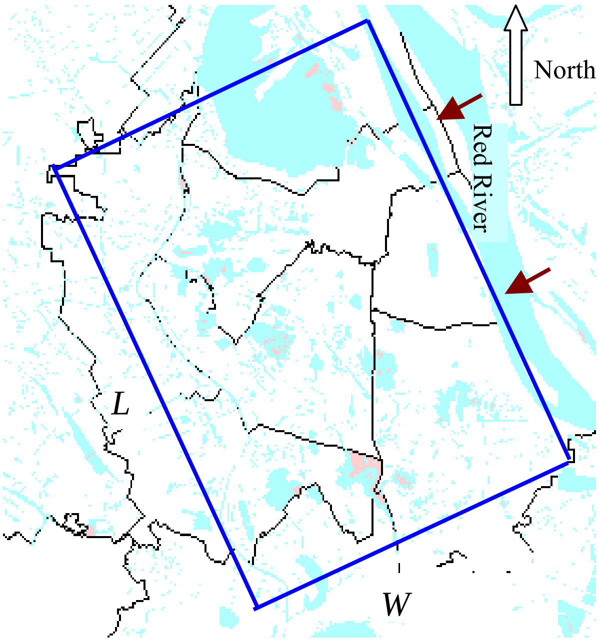
Table 1

Data recorded by a radio sounding balloon at 7:00 am on a November day in Hanoi.

Altitude, m	Temperature, °C
5	21.5
60	20.6
64	20.5
69	20.5
75	20.4
81	20.3
90	20.2
96	20.1
102	20.1
109	20.1
113	20.1
119	20.1
128	20.2
136	20.3
145	20.4
153	20.5
159	20.6
168	20.8
178	21.0
189	21.5
202	21.8
215	22.0
225	22.1
234	22.2
246	22.3
257	22.3

5. Estimation of carbon monoxide (CO) pollution during a morning motorbike rush hour in Hanoi.

Hanoi metropolitan area can be approximated by a rectangle with base dimensions L and W as shown in the figure, with one side taken along the south-west bank of the Red River.



It is estimated that during the morning rush hour, from 7:00 am to 8:00 am, there are 8×10^5 motorbikes on the road, each running on average 5 km and emitting 12 g of CO per kilometer. The amount of CO pollutant is approximately considered as emitted uniformly in time, at a constant rate M during the rush hour. At the same time, the clean north-east wind blows perpendicularly to the Red River (i.e. perpendicularly to the sides L of the rectangle) with velocity u, passes the city with the same velocity, and carries a part of the CO-polluted air out of the city atmosphere.

Also, we use the following rough approximate model:

- The CO spreads quickly throughout the entire volume of the mixing layer above the Hanoi metropolitan area, so that the concentration $C(t)$ of CO at time t can be assumed to be constant throughout that rectangular box of dimensions L, W and H.
- The upwind air entering the box is clean and no pollution is assumed to be lost from the box through the sides parallel to the wind.
- Before 7:00 am, the CO concentration in the atmosphere is negligible.

5.1. Derive the differential equation determining the CO pollutant concentration $C(t)$ as a function of time.

5.2. Write down the solution of that equation for $C(t)$.

5.3. Calculate the numerical value of the concentration $C(t)$ at 8:00 a.m.

Given $L = 15$ km, $W = 8$ km, $u = 1$ m/s.