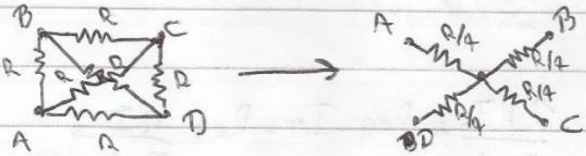
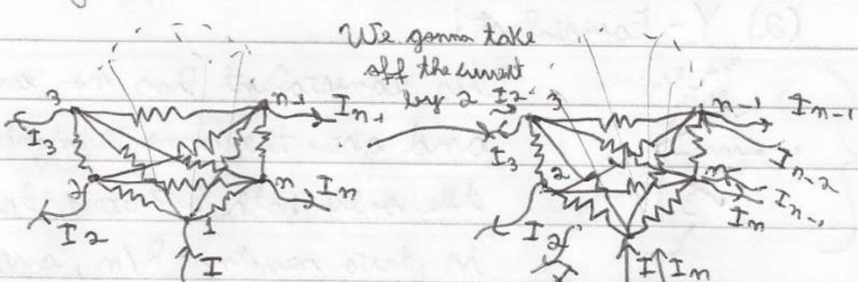
2. We gonna approach this problem by making a equivalent circuit; we can express a conex graph by a Y circuit (if all resistances are equal):

Examples:



$\left(Req_{AB} = \frac{R}{2} = \frac{2R}{4}\right)$

In general, the equivalent resistance between two points in this kind of graph is $2R/n$, being n the number of vertices; we can prove this by using that; if we put a current I in one vertex, we gonna have something like:



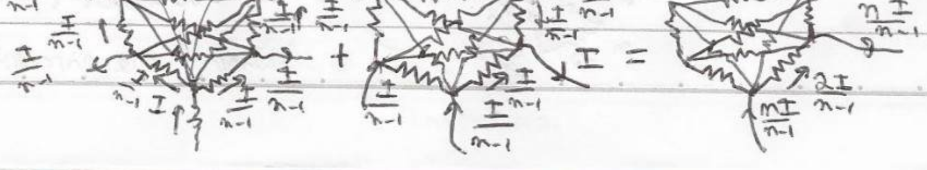
How in a system in we pull a current I from a node and take by the other, we don't have current entering the others, so:

$$I_2 = I_3 = I_4 = \dots = I_n \Rightarrow$$

$$\Rightarrow \text{if } I = I_2 + I_3 + I_4 + \dots + I_n = (n-1) I_2 \Rightarrow$$

$$\Rightarrow \boxed{I_2 = \frac{I}{n-1}}$$ $\longrightarrow$ the current flowing out of any node is $I/_{n-1}$, and

how as we have simmetry this works too for the current going from 1 to 2, 3, 4, ... or n, so we have:



So, using superposition we have a circuit with current onlyng entering A and just leaving B, and is easy to find the potencial difference AB in function of I, because we know the current AB (or 1 → 2):

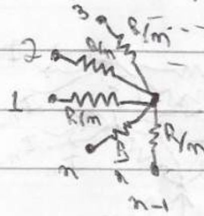
(1) Potential

$$V_{AB} = R I_{AB} = \frac{2RI}{n-1} = R_{eq} \cdot I_T = R_{eq} \cdot \frac{nI}{n-1} \rightarrow$$

$$\rightarrow \boxed{R_{eq} = \frac{2R}{n}}$$

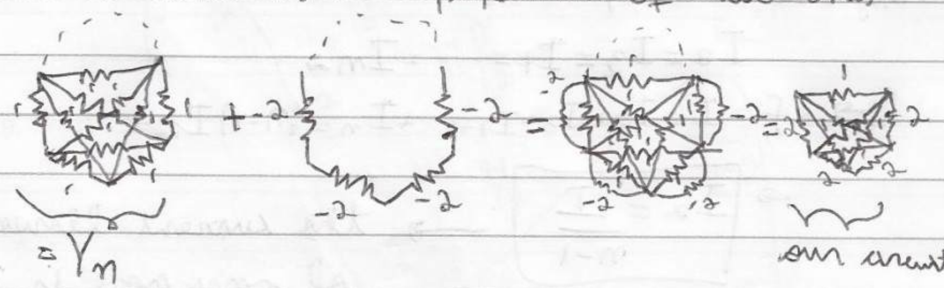
And, for simmetry, this is the same resistance for any two vertex, so we can represent this circuit by:

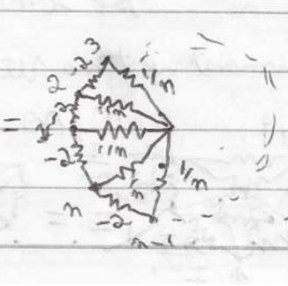
(2) Y - Equivalent:

Y_n 

This equivalent has no connections and gives the same resistance, because the only path between two vertex is two resistors R/n, giving the same response, there's no difference between the system's.

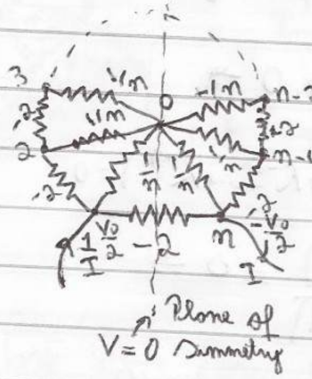
Now, the key idea is to transform our circuit that is a complex thing, into a "small" circuit, we can see our circuit as a superposition of two ones:



our circuit = 

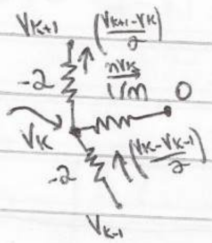
So, we gonna solve this circuit (that is way more simple).

We gonna set with a battery the ddp between the node 1 and n, being the $1 = V_0/2$ and $n = -\frac{V_0}{2}$, so:



In the node between all others (the center 0) the potential is zero, by the simmetry of the system, so we can easily find the current between a node and "0", so we gonna look to a node "k".

(3) Continuity of current:



$k \neq 1$

in $k=1$ we have a source of current, so the equation is not valid.

$$\underbrace{\frac{V_{k+1}-V_k}{2} + nV_k}_{\text{current getting out of } k} = \underbrace{\frac{V_k-V_{k-1}}{2}}_{\text{current entering } k.}$$

$$\boxed{V_{k+1} + 2(n-1)V_k + V_{k-1} = 0}$$

This is a second-order linear recurrence, with solutions of the form $V_k \sim \lambda^k \to$

$$\to \lambda^{k+1} + 2(n-1)\lambda^k + \lambda^{k-1} = 0 \to$$

$$\to \lambda^2 + 2(n-1)\lambda + 1 = 0 \to$$

$$\to \boxed{\lambda_\pm = -(n-1) \pm \sqrt{(n-1)^2-1}}$$

Let us define $\lambda_+ = \lambda = -(n+1) + \sqrt{(n+1)^2-1}$, and by $\lambda_+\lambda_- = 1 \to \lambda_- = 1/\lambda$; how $\lambda_\pm$ is solution, a linear combination of those also is, so:

$$\boxed{V_k = a\lambda^k + b\lambda^{-k}}$$

And, using that also in 0 we can't have charge varying in time, the net current is zero:

$$\sum_{k=1}^{k=n} I_k = \sum_{k=1}^{k=n}\left(a\lambda^k + b\lambda^{-k}\right) = 0 \rightarrow$$

$$\rightarrow a(\lambda+\lambda^2+\dots+\lambda^n) + b(\lambda^{-1}+\lambda^{-2}+\dots+\lambda^{-n}) = 0$$

$$\rightarrow \frac{a\lambda(\lambda^n-1)}{\lambda-1} + \frac{b}{\lambda}\frac{\left(\frac{1}{\lambda^n}-1\right)}{\left(\frac{1}{\lambda}-1\right)} = 0 \rightarrow$$

$$\rightarrow \frac{a\lambda(\lambda^n-1)}{\lambda-1} + \frac{b}{\lambda^n}\left(\frac{\lambda^n-1}{\lambda-1}\right) = 0 \xrightarrow{\text{being } \lambda\neq 1}$$

$$\rightarrow \boxed{b = -a\lambda^{n+1}}$$

Which implies that:

$$\boxed{V_k = a\left(\lambda^k - \lambda^{n+1-k}\right)} \rightarrow \boxed{V_k = V_{n-k} \text{ and if } k=\tfrac{n+1}{2} \Rightarrow V = 0 \text{ (plane of symmetry)}}$$

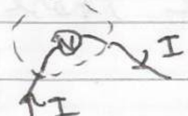
Using that $V_1 = \frac{V_0}{2} \rightarrow$

$$\rightarrow V_1 = a(\lambda-\lambda^n) = \frac{V_0}{2} \rightarrow \boxed{a = \frac{V_0}{2\lambda(1-\lambda^{n-1})}} \rightarrow$$

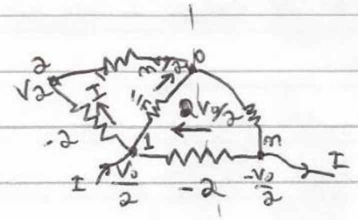
$$\rightarrow \boxed{V_k = \frac{V_0}{2\lambda(1-\lambda^{n-1})}\left(\lambda^k - \lambda^{n+1-k}\right)}$$

We can now find the relationship between V_0 and I, that is the net current entering and leaving the system, and with this we find the resistance:

$$\boxed{R_{eq} = \frac{\Delta V}{I_0} = \frac{\frac{V_0}{2}-\left(-\frac{V_0}{2}\right)}{I_0} = \frac{V_0}{I_0}}$$



Using OHM's LAW and Kirchoff:



$$I' = I_{12} = I - \left(\frac{n}{2} - \frac{1}{2}\right) V_0 \rightarrow$$

$$\rightarrow V_2 - V_1 = -(-2)\, I' =$$

$$= 2I - \left(\frac{2n}{2} - 1\right) V_0 =$$

$$\boxed{\Delta V = 2I - (n-1) V_0}$$

And, by our formula:

$$\Delta V = V_2 - V_1 = a\left(\lambda^2 - \lambda^{n-1}\right) - a\left(\lambda^1 - \lambda^n\right) =$$

$$= a\left[\lambda(\lambda - 1) + \lambda^{n-2}(\lambda - 1)\right]$$

$$= a\lambda(\lambda - 1)(1 + \lambda^{n-2})$$

$$\boxed{\Delta V = \frac{V_0}{2(1-\lambda^{n-1})}(\lambda - 1)(1 + \lambda^{n-2})} \rightarrow$$

$$\rightarrow 2I - (n-1)V_0 = \frac{V_0}{2(1-\lambda^{n-1})}(\lambda - 1)(1 + \lambda^{n-2}) \rightarrow$$

Ohm's

$$\rightarrow \boxed{Req_n = \frac{V_0}{I} = \frac{2}{n - 1 + \frac{(\lambda-1)(1+\lambda^{n-2})}{2(1-\lambda^{n-1})}}\ \Omega}$$

being $\lambda = -(n-1) + \sqrt{(n-1)^2 - 1}$, and n a integer represetating the number of nodes, and:

$$\boxed{\begin{aligned} R_3 &= Req_3 = \frac{4}{3}\ \Omega \\ R_4 &= Req_4 = \frac{5}{6}\ \Omega \\ R_5 &= Req_5 = \frac{32}{55}\ \Omega \end{aligned}}$$

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