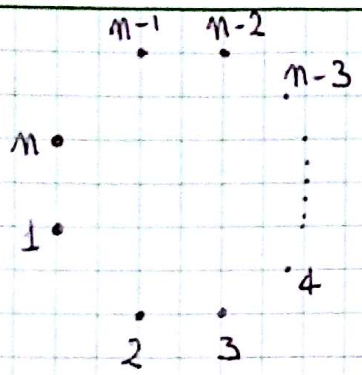

We have n vertices

Let V_i be the potential on the i-vertex

Let $V_1 = -1/2$ V and $V_n = 1/2$ V

Let $I_{i,j}$ be the direct current from i to j (it does not flow in any vertex different from either i or j).

As clear from the sketch, we have a vertical symmetry, so that $V_1 = -V_n$, $V_2 = -V_{n-1}, \dots, V_i = -V_{n-i+1}$

As a consequence:

$$\sum_{i=1}^{n} V_i = 0\,V \qquad (1)$$

Let's find all the $I_{i,j}$ values for a generic $i \neq 1, n$. Their sum must be equal to 0 A due to Kirchhoff's law.

$$I_{i,i-1} = (V_i - V_{i-1}) \cdot \frac{1}{2\Omega}$$

$$I_{i,i+1} = (V_i - V_{i+1}) \cdot \frac{1}{2\Omega}$$

In all the other cases we have that

$$I_{i,j} = (V_i - V_j) \cdot \frac{1}{1\Omega} \qquad \forall j \neq i-1, i+1$$

Note that there is no problem in including also $j = i$ (and, indeed, we are doing so)

We may now consider and evaluate the summation of the different $I_{i,j}$ for a fixed i, keeping in mind Kirchhoff's law:

$$\sum_{j=1}^{n} I_{i,j} = I_{i,1}+I_{i,2}+\dots+I_{i,n} = I_{i,1}+\dots+I_{i,i-1}+I_{i,i}+$$

$$+I_{i,i+1}+\dots+I_{i,n} = \frac{(V_i-V_1)}{1\Omega}+\dots+\frac{(V_i-V_{i-1})}{2\Omega}+$$

$$+\frac{(V_i-V_i)}{1\Omega}+\frac{(V_i-V_{i+1})}{2\Omega}+\dots+\frac{(V_i-V_n)}{1\Omega} = 0\,A \quad \Rightarrow$$

$$\Rightarrow \quad (n-1)\,V_i - \sum_{j=1}^{n} V_j + \frac{V_{i-1}}{2} + \frac{V_{i+1}}{2} = 0\,V$$

This ~~is~~ result is true for each generic vertex i which is not connected (directly) with an external circuit. So it is true if $2 \leq i \leq n-1$. Using equivalence (1):

$$2\,(n-1)\,V_i + V_{i+1} + V_{i-1} = 0\,V$$

$$\Rightarrow V_{i+1} + 2\,(n-1)\,V_i + V_{i-1} = 0\,V$$

Now we should turn this recursive property into an explicit form (it's "just" math):

$$x^2 + 2(n-1)\,x + 1 = 0 \Rightarrow x_{1,2} = 1-n \pm \sqrt{n^2-2n}$$

$$\lambda_+ = 1-n+\sqrt{n^2-2n} \qquad \lambda_- = 1-n-\sqrt{n^2-2n}$$

$$V_i = a\,\lambda_+^{i} + b\,\lambda_-^{i}$$

$$V_1 = a\lambda_+ + b\lambda_- = -\frac{1}{2}V \Rightarrow b = \frac{-\frac{1}{2}V - a\lambda_+}{\lambda_-}$$

$$V_n = a\lambda_+^n + b\lambda_-^n = \frac{1}{2}V \Rightarrow a = \frac{1}{2}V \cdot \frac{1+\lambda_-^{n-1}}{\lambda_+^n - \lambda_+ \cdot \lambda_-^{n-1}}$$

$$b = \frac{1}{2}V \cdot \frac{1+\lambda_+^{n-1}}{\lambda_-^n - \lambda_- \cdot \lambda_+^{n-1}}$$

$$\Rightarrow V_i = \frac{1}{2}V \cdot \left[\lambda_+^i \cdot \frac{1+\lambda_-^{n-1}}{\lambda_+^n - \lambda_+\lambda_-^{n-1}} + \lambda_-^i \cdot \frac{1+\lambda_+^{n-1}}{\lambda_-^n - \lambda_-\lambda_+^{n-1}}\right]$$

Let's now consider the case $i=1$. The ~~entering~~ negative exiting current $I_{1,2} + I_{1,3} + \ldots + I_{1,n}$ is simply the total entering current in 1, and due to Kirchhoff's law, it is the total current which flows in the circuit, which has a total "voltage" of $V_n - V_1 = 1\,V$. Using Ohm's 1st law and naming R_{TOT} the total resistance of the circuit:

$$R_{TOT} = \frac{1\,V}{-\sum_{j=1}^{n} I_{1,j}}$$

$$\sum_{j=1}^{n} I_{1,j} = \left[\frac{V_1 - V_2}{2} + (V_1 - V_3) + \ldots + (V_1 - V_{n-1}) + \frac{V_1 - V_n}{2}\right]\Omega^{-1} =$$

$$= \left[(n-1)V_1 - \sum_{j=1}^{n} V_j + \frac{V_2}{2} + \frac{V_n}{2}\right]\Omega^{-1}$$

So we need V_2:

$$V_2 = \frac{1}{2} V \cdot \left[\lambda_+^2 \cdot \frac{1+\lambda_-^{n-1}}{\lambda_+^n - \lambda_+ \lambda_-^{n-1}} + \lambda_-^2 \cdot \frac{1+\lambda_+^{n-1}}{\lambda_-^n - \lambda_- \lambda_+^{n-1}} \right] =$$

$$= \frac{1}{2\,(\lambda_+^{n-1} - \lambda_-^{n-1})} \left[\lambda_+ (\lambda_-^{n-1} + 1) - \lambda_- (\lambda_+^{n-1} + 1) \right] V$$

$$= t \quad V \qquad \text{where } t = \frac{\lambda_+ (\lambda_-^{n-1} + 1) - \lambda_- (\lambda_+^{n-1} + 1)}{2\,(\lambda_+^{n-1} - \lambda_-^{n-1})}$$ is a pure number

Then:

$$\sum_{j=1}^{n} I_{1,j} = \left[(n-1) V_1 + \frac{V_2}{2} + \frac{V_n}{2} \right] \Omega^{-1} =$$

$$= \left[(n-1)\left(-\frac{1}{2}\right) + \frac{t}{2} + \frac{1}{4} \right] A = \left[-\frac{1}{2} n + \frac{3}{4} + \frac{t}{2} \right] A \Rightarrow$$

$$\Rightarrow R_{TOT} = \frac{1\,V}{\left[\frac{1}{2} n - \frac{3}{4} - \frac{t}{2}\right] A} = \frac{4\,\Omega}{2n - 3 - 2t}$$

FOR $n = 3$ $\qquad R_{TOT} = \frac{4}{3}\,\Omega$

FOR $n = 4$ $\qquad \lambda_+ = -3 + 2\sqrt{2} \quad \lambda_- = -3 - 2\sqrt{2}$

$t = 3/35 \qquad R_{TOT} = \frac{140}{169}\,\Omega$

FOR $n = 5$ $\qquad \lambda_+ = -4 + \sqrt{15} \quad \lambda_- = -4 - \sqrt{15}$

$t = 1/16 \qquad R_{TOT} = \frac{32}{55}\,\Omega$