Problem 03 - Diogo Correia Netto

- Let's analyse what happens in the respective time intervals.
- For $0 \leq t \leq \tau$: very slow growth of the magnetic field.

  According to the list of formulas to IPhO (Section IX, item 19), when a point charge has a circular orbit in a slowly varying magnetic field, the associated magnetic moment ($\sim v_\perp^2/B$) is an adiabatic invariant (and $v_\parallel$ doesn't change).
- Let's suppose a magnetic field along the z axis.
- If v_0 is the *rms* the speed at $t = 0$ and v_1 is the *rms* speed at $t = \tau$, the conservation law for the average magnetic moment allows us to write:

$$\frac{v_{0x}^2 + v_{0y}^2}{B_0} = \frac{v_{1x}^2 + v_{1y}^2}{2B_0} \quad \Rightarrow \quad v_{1x}^2 + v_{1y}^2 = 2\left(v_{0x}^2 + v_{0y}^2\right). \tag{1}$$

- We can calculate the temperature T_1 at $t = \tau$ using the equipartition theorem:

$$\frac{kT_0}{2} = \frac{mv_{0x}^2}{2} = \frac{mv_{0y}^2}{2} = \frac{mv_{0z}^2}{2}, \tag{2}$$

  and, as

$$\frac{m\left(v_{1x}^2 + v_{1y}^2 + v_{1z}^2\right)}{2} = \frac{3kT_1}{2}, \tag{3}$$

  we have

$$T_1 = \frac{5}{3}T_0 \tag{4}$$

- Between $\tau \leq t \leq \tau + T$: As $T \gg t_{\text{collisions}}$, the collisions are very likely to happen in this interval.

  Note that at $t = \tau$ the energy is not equally divided (kT_0 for the x and y axes, $kT_0/2$ for the z axis): the collisions have the effect of equally dividing the energy along the axes.

  Let v_2 be the *rms* speed at $t = \tau + T$. As the energy is equally divided:

$$\frac{mv_{2x}^2}{2} = \frac{mv_{2y}^2}{2} = \frac{mv_{2z}^2}{2} = \frac{kT_1}{2} = \frac{5}{6}kT_0 \tag{5}$$

Then:

$$v_{2x}^2 = v_{2y}^2 = v_{2z}^2 = \frac{kT_1}{m} = \frac{5}{6}\frac{kT_0}{m}. \tag{6}$$

- For $\tau + T \leq t \leq 2\tau + T$: very slow reduction of the magnetic field. Note that the magnetic moment is also conserved in this interval. Let v_3 be the *rms* speed at $t = 2\tau + T$, then:

$$\frac{v_{3x}^2 + v_{3y}^2}{B_0} = \frac{v_{2x}^2 + v_{2y}^2}{2B_0} \tag{7}$$

$$\Rightarrow v_{3x}^2 + v_{3y}^2 = \frac{1}{2}\left(2 \cdot \frac{5}{3}\frac{kT_0}{m}\right) = \frac{5}{3}\frac{kT_0}{m} \tag{8}$$

$$\Rightarrow \frac{m\left(v_{3x}^2 + v_{3y}^2\right)}{2} = \frac{5}{6}\frac{kT_0}{m} \tag{9}$$

As the velocity along z axis doesn't change we have:

$$\frac{3}{2}kT_{\text{final}} = \frac{m\left(v_{3x}^2 + v_{3y}^2 + v_{3z}^2\right)}{2} \tag{10}$$

- The final temperature is calculated using the equipartition theorem:

$$\frac{3kT_{\text{final}}}{2} = \frac{5}{3}kT_0 \quad \Rightarrow \quad T_{\text{final}} = \frac{10}{9}T_0 \tag{11}$$