

Let us define the z-axis as parallel to the magnetic field. Let us now look at a particle with velocity  $v_{xy}$  in the x-y-plane. Since  $\frac{dB}{dt} \ll \frac{B^2 e}{m}$ , ~~the~~ the radius of the electron's trajectory is kept nearly the same during one ~~revolution~~ revolution. This radius is  $r$ , then  $\frac{v_{xy}^2}{r} = \frac{m v_{xy} B}{e \hbar}$   $\Rightarrow r = \frac{m v_{xy}}{e \hbar B}$ . The potential difference induced by the changing magnetic field is  $\mathcal{E} = \frac{d\phi}{dt}$  per revolution. The energy gained per revolution is thus  $2\mathcal{E} = e \frac{d\phi}{dt} = e \pi r^2 \frac{dB}{dt} = \frac{2\pi m^2 v_{xy}^2}{e \hbar^2 B^2} = m v_{xy} \Delta v_{xy}$

$$m v_{xy} \Delta v_{xy} = 2\pi r^2 \frac{dB}{dt} = \frac{2\pi m^2 v_{xy}^2}{e^2 \hbar^2 B^2} \frac{dB}{dt} \Rightarrow \text{per revolution } \Delta v_{xy} = \frac{2\pi m v_{xy}}{e \hbar B^2} \frac{dB}{dt}$$

$$\text{From here } \frac{dv_{xy}}{dt} = \frac{2\pi m v_{xy}}{e \hbar B^2} \frac{dB}{dt} = \frac{2\pi m v_{xy}}{2 B} \frac{dB}{dt} \Rightarrow \frac{2 dv_{xy}}{v_{xy}} = \frac{dB}{B}$$

Integrating from the initial state to the state with  $B = 2B_0$

$$2 \int_{v_{xy0}}^{v_{xy1}} \frac{dv_{xy}}{v_{xy}} = \int_{B_0}^{2B_0} \frac{dB}{B} \Rightarrow 2 (\ln v_{xy1} - \ln v_{xy0}) = \ln 2B_0 - \ln B_0 = \ln \frac{2B_0}{B_0}$$

$$\ln \frac{v_{xy1}}{v_{xy0}} = \frac{\ln 2}{2} \Rightarrow v_{xy1} = \sqrt{2} \cdot v_{xy0}, \text{ where } v_{xy1} \text{ is the speed after the increase}$$

to  $B = 2B_0$  and  $v_{xy0}$  is the initial speed. This means that the ~~kinetic~~ kinetic energy in the x-y-plane is multiplied by  $(\sqrt{2})^2 = 2$ , meaning it is now  $2 \cdot 2 \cdot \frac{KT_0}{2} = 2KT_0$  per particle. ~~After~~ The

kinetic energy in the z-axis unaffected by this process, meaning the total kinetic energy is now

$$2KT_0 + \frac{KT_0}{2} = \frac{5KT_0}{2} \text{ or } \frac{5KT_0}{2 \cdot 3} = \frac{5}{6} KT_0 \text{ per degree of freedom. This means that after many collisions}$$

the energy in the x-y-plane is  $\frac{5}{6} \cdot 2KT_0 = \frac{5}{3} KT_0$ . Since the reduction from  $2B_0$  to  $B_0$  is the reverse

process of the initial increasing, the energy is divided by 2 during the ~~reduction~~ decreasing. Thus the energy

in the x-y-plane will be  $\frac{5}{6} KT_0 \cdot \frac{1}{2} = \frac{5}{12} KT_0$  and the total energy will be  $\frac{5}{6} KT_0 + \frac{5}{12} KT_0 = \frac{5}{4} KT_0 = \frac{3}{2} KT_1 =$

$$\Rightarrow T_1 = \frac{10}{9} T_0$$